Integer conformal dimensions for type IIa flux vacua

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We give a concise argument that supersymmetric anti–de Sitter type IIA DeWolfe-Giryavets-Kachru-Taylor flux vacua on general Calabi-Yau's have, interpreted holographically, integer conformal dimensions for low-lying scalar primaries in the dual conformal field theory. These integers are independent of any compactification details, such as the background fluxes or triple intersection numbers of the compact manifold. For the Kähler moduli and dilaton, there is one operator with $\Delta = 10$ and $h_-^{1,1}$ operators with $\Delta = 6$, whereas the corresponding axions have $\Delta = 11$ and $\Delta = 5$. For the complex structure moduli, the $h^{2,1}$ saxions have $\Delta = 2$, and the axions $\Delta = 3$. We give a tentative discussion of the origin of these integers and effects that would modify these results.

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I. INTRODUCTION

Moduli stabilization is one of the necessary prerequisites for compactified string theories to describe a four-dimensional world with a hierarchical separation of scales between this and extra-dimensional physics. The traditional tools to construct such vacua, effective field theory and dimensional reduction, have come under close scrutiny with the advent of the swampland program [1–3]. For this reason, it was proposed in Refs. [4–7] that holography might provide an independent approach to test the consistency of anti–de Sitter (AdS) vacua, by analyzing the properties of the putative dual conformal field theories (CFTs).

One of the most studied scenarios of moduli stabilization featuring scale separation is the type IIA DeWolfe-Giryavets-Kachru-Taylor (DGKT) flux vacua [8], and scale separated IIA vacua have received considerable attention lately [7,9–14], as the scale separation property has been conjectured to lie in the swampland [15–18].[1] From a holographic point of view, scale separated vacua also admit a clear interpretation in terms of CFTs with a gapped spectrum, which could also point towards inconsistencies [19]. Therefore, DGKT provides a particularly interesting example to study in this context (see Ref. [20] for an early investigation).

In this paper we give a concise derivation of the mass matrix for general IIA DGKT flux vacua and show that, interpreted holographically, it has an extremely simple form. In particular, the conformal dimensions of scalar operators dual to the moduli are both integers and also highly degenerate. These results are suggestive of a hidden structure that is best understood from a dual CFT perspective.

Our argument establishes in full generality results hinted at in Refs. [6,7], where these results were found for simple specific examples of DGKT. Although the argument here is more concise, a derivation of the mass matrix for general DGKT vacua (although without a link to conformal dimensions) also appears spread across the two papers [21,22].

II. DGKT FLUX VACUA

In a type IIA setting, moduli stabilization with fluxes can be achieved at tree level due to the simultaneous presence of the Neveu-Schwarz-Neveu-Schwarz (NS-NS) B_2 form and the Ramond-Ramond (R-R) odd p forms. In particular, this is obtained by compactifying massive IIA string theory on Calabi-Yau orientifolds with O6 planes (for which the effective field theory was derived in Refs. [23,24]), which allows for stable AdS_4 vacua in the controlled limit of large volumes and weak string coupling [8]. This was demonstrated in Ref. [8] for a fully explicit example based on a $T^6/\mathbb{Z}_3 \times \mathbb{Z}_3$ orientifold with no complex structure (CS) moduli ($h^{2,1} = 0$). There, the axio-dilaton, Kähler moduli and axions were all stabilized by fluxes.

Similar results apply for a generic $\mathcal{N} = 1$ orientifold, where it is again possible to fix all moduli (except for the

[1] However, both the AdS moduli conjecture of Ref. [15] and a refined version of the strong AdS distance conjecture [17] presented in Ref. [18] are consistent with DGKT.

flat directions corresponding to the complex structure axions). Following the orientifold projection, one is left with $h^{1,1}_-$ Kähler moduli and $h^{2,1}$ CS moduli, which can be described with the formalism of $\mathcal{N}=1$ supergravity. The Kähler moduli can be expressed in terms of the complex scalar fields

$$t_a = b_a + i v_a, \quad a = 1, \ldots h^{1,1}_- \tag{1}$$

where the v_a are volumes of the 2-cycles ($\int J$) and the b_a axions arise from dimensional reduction of the B_2 form on this cycle. Their Kähler potential is given by

$$K^K = -\log\left(\frac{4}{3}\mathcal{V}\right), \tag{2}$$

where the volume is[2] $\mathcal{V} = \kappa_{abc} v_a v_b v_c$. This Kähler potential satisfies the well-known no-scale relations

$$K^{ab} K_a K_b = 3, \qquad K^{ab} K_b = -v_a. \tag{3}$$

The complex structure moduli, together with the axio-dilaton, can be packaged into $h^{2,1}+1$ complex fields

$$N_k = \frac{\xi_k}{2} + i\mathrm{Re}(C Z_k), \quad k = 0 \ldots \tilde{h},$$
$$T_\lambda = i\tilde{\xi}_\lambda - 2\mathrm{Re}(C g_\lambda), \qquad \lambda = \tilde{h}+1 \ldots h^{2,1} \tag{4}$$

where the Z_k, g_λ and the ξ_k, $\tilde{\xi}_\lambda$ are the coefficients of the holomorphic 3-form Ω and of C_3

$$\Omega = Z_{\hat{K}} \alpha_{\hat{K}} - g_{\hat{L}} \beta_{\hat{L}},$$
$$C_3 = \xi_{\hat{K}} \alpha_{\hat{K}} - \tilde{\xi}_{\hat{L}} \beta_{\hat{L}}, \tag{5}$$

expanded in a symplectic basis $\{\alpha_{\hat{K}}, \beta_{\hat{L}}\}$ of H^3. The basis can be split into an odd part $\{\alpha_\lambda, \beta_k\}$ and an even part $\{\alpha_k, \beta_\lambda\}$; it is only the components with respect to the latter which survive the orientifold projection and appear in Eq. (4). We have also introduced the compensator $C \equiv e^{-D + K_{\mathrm{cs}}/2}$, where the four-dimensional dilaton D is related to the ten-dimensional one by $e^D = e^\phi/\sqrt{\mathrm{Vol}}$ and

$$K_{\mathrm{cs}} = -\log\left(i \int \Omega \wedge \overline{\Omega}\right). \tag{6}$$

Their Kähler potential is given by

$$K^Q = -2\log\left(2\int \mathrm{Re}(C\Omega) \wedge *\mathrm{Re}(C\Omega)\right) = 4D \tag{7}$$

and also satisfies a no-scale relation (see Appendix C of Ref. [23]), namely

$$K^{N_k \bar{N}_k} K_{N_k} K_{\bar{N}_k} + K^{T_\lambda \bar{T}_\lambda} K_{T_\lambda} K_{\bar{T}_\lambda} = 4. \tag{8}$$

In a generic setting, both the NS-NS 3-form field strength H_3 and the R-R field strengths F_0, F_2, F_4, F_6 can thread fluxes through the internal manifold. Following the notation of Ref. [23], the background fluxes can be expressed in a basis of the appropriate cohomologies as

$$H_3 = q_\lambda \alpha_\lambda - p_k \beta_k, \qquad F_2 = -m_a w_a, \qquad F_4 = e_a \tilde{w}^a,$$
$$F_0 = m_0, \qquad F_6 = e_0. \tag{9}$$

The even 2-forms $\{w_a\}$ span a basis of $H^{1,1}_+$, while their duals $\{\tilde{w}^a\}$ are a basis of $H^{2,2}_+$. We briefly remark that the presence of the F_2 and F_6 fluxes is not needed to achieve moduli stabilization, but we nevertheless include them for full generality.[3]

The resulting superpotential is given by

$$W = e_0 + e_a t^a + \frac{1}{2}\kappa_{abc} m_a t_b t_c - \frac{m_0}{6}\kappa_{abc} t_a t_b t_c$$
$$- 2 p_k N_k - i q_\lambda T_\lambda. \tag{10}$$

A crucial simplification is that the superpotential (10) depends only on a linear combination of the complex structure moduli which, after a (holomorphic) rotation in field space, can be effectively taken to be a single modulus. When $h^{2,1} = 0$ this direction reduces entirely to the axio-dilaton,[4] and hence it will be denoted as

$$S \equiv \xi + is, \quad \text{with} \quad W \supset -2pS \tag{11}$$

as the form for the superpotential. Combined with the fact that the Kähler potential factorizes as a sum of two independent terms, this ensures a decoupling between the two sectors. Other than S, there will now be $h^{2,1}$ complex structure moduli

$$U_\alpha \equiv a_\alpha + i u_\alpha, \qquad \alpha = 1, \ldots h^{2,1} \tag{12}$$

which do not appear in the superpotential $W(t_a, S)$.

Assuming the tadpole conditions are satisfied (as they must be), the scalar potential then takes the standard $\mathcal{N}=1$ supergravity form [8,23]

$$V = e^K\left(\sum_{t_i, U_\alpha, S} K^{i\bar{j}} D_i W D_{\bar{j}} \overline{W} - 3|W|^2\right). \tag{13}$$

Supersymmetric vacua occur with vanishing F terms,

$$D_{t_i} W = 0, \qquad D_S W = 0, \qquad D_{U_\alpha} W = 0, \tag{14}$$

[2]Our definition differs from the proper volume by $\mathrm{Vol} = \mathcal{V}/6$.

[3]In fact, it usually leads to solutions which are either equivalent or qualitatively similar to the ones without F_2 or F_6 flux.

[4]In that case, the imaginary part of S will be related to the four-dimensional dilaton by $s = e^{-D}/\sqrt{2}$.

which also ensures extrema of the potential (13). For the superpotential (10) and Kähler potential (2), these imply the following relationships for the Kähler moduli at the minimum [8]:

$$3m_0^2\kappa_{abc}v^b v^c + 10m_0 e_a + 5\kappa_{abc}m^b m^c = 0,$$
$$s = -\frac{2m_0}{15p}\mathcal{V}, \qquad b_a = \frac{m_a}{m_0}, \qquad \xi = -\frac{e_0}{2p}. \tag{15}$$

This implies that at the minimum [8]

$$W = \frac{2i}{15}m_0\mathcal{V}. \tag{16}$$

Equations (15) and (16) imply the second derivatives of the superpotential take the simpler form

$$\begin{aligned}\partial_{v_a}\partial_{v_b}W &= -\partial_{t_a}\partial_{t_b}W \\ &= -\kappa_{cab}m_c + m_0\kappa_{abc}t_c = im_0\kappa_{abc}v_c \\ &= i\frac{m_0\mathcal{V}}{6}(K_aK_b - K_{ab}) = \frac{5}{4}(K_aK_b - K_{ab})W \end{aligned}\tag{17}$$

when Eq. (15) is satisfied. On the other hand, the F-term condition for the complex structure moduli simply implies

$$K_{U_\alpha} = \frac{1}{2i}K_{u_\alpha} = 0, \tag{18}$$

ensuring the absence of mixing with the Kähler moduli and S for fluctuations around the minimum. In addition, Eq. (18) turns the no-scale relation (8) into

$$K^{ss}K_sK_s = 4. \tag{19}$$

III. MASS MATRICES FOR THE KÄHLER MODULI AND DILATON

To obtain the mass matrix, we compute the second derivatives of the potential (13),

$$\begin{aligned}\partial_m\partial_n V = &-3e^K(K_{mn}|W|^2 + K_m\partial_n|W|^2 + \partial_m\partial_n|W|^2) \\ &+ 2e^K K^{i\bar{j}}(\partial_n D_i W)(\partial_m D_{\bar{j}}\overline{W}),\end{aligned}\tag{20}$$

in which terms proportional to a first derivative of the potential or D_iW are dropped, and m, n denote saxions. To evaluate the derivatives in the first line of Eq. (20), we note that the conditions (14) imply that

$$\partial_m|W|^2 = -K_m|W|^2, \tag{21}$$

and the second derivatives of $|W|^2$ equal

$$\begin{aligned}\partial_s^2|W|^2 &= \frac{1}{2}K_s^2|W|^2, \\ \partial_s\partial_b|W|^2 &= \frac{1}{2}K_sK_b|W|^2, \\ \partial_b\partial_a|W|^2 &= 2(\partial_{v_a}\partial_{v_b}W)\bar{W} + 2(\partial_{v_a}W)(\partial_{v_b}\bar{W}) \\ &= \left(3K_aK_b - \frac{5}{2}K_{ab}\right)|W|^2.\end{aligned}\tag{22}$$

For the second line in Eq. (20), we have[5]

$$\begin{aligned}K^{i\bar{j}}(\partial_n D_i W)(\partial_m D_{\bar{j}}\bar{W}) &= K^{i\bar{j}}\left(W_{in} + K_{in}W - \frac{K_iK_n}{2}W\right)\left(\bar{W}_{\bar{j}m} + K_{\bar{j}m}\bar{W} - \frac{K_{\bar{j}}K_m}{2}\bar{W}\right) \\ &= K^{i\bar{j}}W_{in}\bar{W}_{\bar{j}m} + 2K^{i\bar{j}}W_{in}K_{\bar{j}m}\bar{W} - K^{i\bar{j}}W_{in}K_{\bar{j}}K_m\bar{W} \\ &\quad + \left[K^{i\bar{j}}K_{in}K_{\bar{j}m} - K^{i\bar{j}}K_{in}K_{\bar{j}}K_m + \frac{1}{4}(K^{i\bar{j}}K_iK_{\bar{j}})K_nK_m\right]|W|^2 \\ &= K^{i\bar{j}}W_{in}\bar{W}_{\bar{j}m} + 4W_{nm}\bar{W} - K^{i\bar{j}}W_{in}K_{\bar{j}}K_m\bar{W} + \left[K_{nm} + \frac{3}{4}K_nK_m\right]|W|^2.\end{aligned}\tag{23}$$

If $n, m = a, b$ are both size moduli,

$$K^{i\bar{j}}W_{in}\bar{W}_{\bar{j}m} = \frac{25}{16}(K_aK_b + K_{ab})|W|^2, \tag{24}$$

and if $n = a$ is a size modulus

[5] Note that because $K = K(t_a + \bar{t}_a, S + \bar{S})$, the derivatives with respect to the complex moduli are $\partial_{t_a}K = \partial_{v_a}K/2i$, $\partial_S K = \partial K_s/2i$. The superpotential is holomorphic and so $\partial W_{t_a} = -i\partial_{v_a}W$, $\partial_S W = -i\partial_s W$.

$$K^{i\bar{j}}W_{in}K_{\bar{j}}K_m\bar{W} = 5K_aK_b|W|^2 \tag{25}$$

by substituting Eq. (17), and otherwise these terms vanish. Combining these leads to

$$\partial_a\partial_b V = e^K(9K_{ab} + 8K_aK_b)|W|^2, \tag{26}$$

$$\partial_a\partial_s V = e^K(-2K_aK_s)|W|^2, \tag{27}$$

$$\partial_s^2 V = e^K(-K_{ss} + 3K_s^2)|W|^2. \tag{28}$$

Hence, the mass matrix for the moduli in AdS units is given by

$$R_{\mathrm{AdS}}^2\partial_m\partial_n V = \begin{pmatrix} 9K_{ab} + 8K_aK_b & -2K_aK_s \\ -2K_aK_s & -K_{ss} + 3K_s^2 \end{pmatrix}, \tag{29}$$

where $R_{\mathrm{AdS}}^2 = -3/V_{\mathrm{min}}$. We can proceed similarly with Eq. (20) for the axions b_a, ξ, where now all derivatives of the Kähler potential will vanish, and for the superpotential derivatives we have

$$\partial_{b_a}W = -i\partial_{v_a}W, \qquad \partial_\xi W = -i\partial_s W. \tag{30}$$

This results in

$$\partial_{b_a}\partial_{b_b}V = e^K(5K_{ab} + 12K_aK_b)|W|^2, \tag{31}$$

$$\partial_{b_a}\partial_\xi V = -3e^KK_aK_s|W|^2, \tag{32}$$

$$\partial_\xi^2 V = 2e^KK_s^2|W|^2, \tag{33}$$

and so the mass matrix is

$$R_{\mathrm{AdS}}^2\partial_m\partial_n V = \begin{pmatrix} 5K_{ab} + 12K_aK_b & -3K_aK_s \\ -3K_aK_s & 2K_s^2 \end{pmatrix}. \tag{34}$$

These expressions match with the results obtained in Ref. [22] (see in particular Appendix B). However, this computation provides a more compact way to arrive at these results, using the ordinary $\mathcal{N} = 1$ structure of the potential and without having to rely on the formalism developed in Refs. [14,21,22]. Moreover, the derivation provided above makes it clear that the only properties feeding into the final result are the no-scale relations for the Kähler potential and the specific form of the superpotential leading to Eqs. (16) and (17).

IV. COMPLEX STRUCTURE MODULI

Let us briefly review the arguments that lead to both integer conformal dimensions for the complex structure moduli [which have already been discussed (implicitly or explicitly) in Refs. [6,25]] and the absence of any mixing with the Kähler sector. To establish the latter, one can follow the same steps as the ones leading to Eq. (27), finding that

$$\partial_{u_\alpha}\partial_{v_b}V = -2K_{u_\alpha}K_{v_b}e^K|W|^2 = 0 \tag{35}$$

because of Eq. (18). Similarly,

$$\partial_{u_\alpha}\partial_s V = -3e^K\left(-K_{u_\alpha}K_s + \frac{1}{2}K_{u_\alpha}K_s\right)|W|^2 + \frac{3}{2}K_{u_\alpha}K_se^K|W|^2 = 0. \tag{36}$$

Within the CS sector, nondiagonal terms are also absent

$$\partial_{u_\alpha}\partial_{u_\beta}V = 3e^KK_{u_\alpha}K_{u_\beta}|W|^2 = 0. \tag{37}$$

It was shown in Ref. [25] that the only nonvanishing contribution then comes from

$$\partial_{u_\alpha}\partial_{u_\alpha}V = -K_{u_\alpha u_\alpha}e^K|W|^2, \tag{38}$$

fixing all masses to the universal value of

$$m_u^2 = -2/3V_{\mathrm{min}} = -2/R_{\mathrm{AdS}}^2. \tag{39}$$

As the axions appear in neither the Kähler potential nor the superpotential, they are all massless.

V. SPECTRUM

Let us define the two block matrices

$$K_{mn} = \begin{pmatrix} K_{ab} & 0 \\ 0 & K_{ss} \end{pmatrix} \quad L_{mn} = \begin{pmatrix} 4K_aK_b & -K_aK_s \\ -K_aK_s & \frac{K_s^2}{4} \end{pmatrix}, \tag{40}$$

where K is simply the Kähler metric for the moduli (in real components) evaluated at the minimum of the potential. Then, from the results of the previous section the Hessians of the moduli and axion potential can be expressed as

$$H_{mn}^M \equiv R_{\mathrm{AdS}}^2\partial_m\partial_n V = 9K_{mn} + 2L_{mn} \tag{41}$$

and

$$H_{mn}^A \equiv R_{\mathrm{AdS}}^2\partial_m\partial_n V = 5K_{mn} + 3L_{mn}, \tag{42}$$

respectively. The physical masses can be obtained as the eigenvalues of $M = 2K^{-1}H$, which is related to the mass matrix

$$\tilde{M} = 2\sqrt{K^{-1}}^T H\sqrt{K^{-1}} \tag{43}$$

by a similarity transformation.

In principle, the stabilized values of the v_i's and the dilaton can be extracted by the system of $h_-^{1,1}+1$ equations in the first line of Eq. (15). Knowing the value of the axions at the minimum, it would then be possible to write the mass matrix as a function of the flux data only. Although the equations cannot be solved explicitly for arbitrary triple intersection numbers κ_{abc} and fluxes e_a, m_a, the "implicit" form of the matrix derived above is not only sufficient for our purposes, but it is more helpful to elucidate the surprising simplifications in the final expressions.

To compute the eigenvalues of $K^{-1}L$, one can begin by noticing that any vector of the form

$$x=(x_i,x_0) \quad \text{with} \quad x_0K_s=4K_ax^a \tag{44}$$

satisfies $K^{-1}Lx=0$. This essentially descends from the fact that M can be expressed as a tensor product $L_{mn}=A_mA_n$, with $A_m=(2K_a,-K_s/2)$. One therefore has a basis of $h_-^{1,1}$ eigenvectors with an eigenvalue of $\lambda_x^i=0$. Furthermore, the no-scale relations (3)–(8) imply that the vector

$$y=(v_i,K_s^{-1}), \tag{45}$$

is also an eigenvector of L, with eigenvalue $\lambda_y=13$.

Then, there is a basis in which the mass matrices in AdS units read

$$M_{mn}^M=18\delta_{mn}+52\delta_{1,m} \tag{46}$$

$$M_{mn}^A=10\delta_{mn}+78\delta_{1,m}, \tag{47}$$

also in agreement with the results of Ref. [22]. Through the relation $\Delta(\Delta-d)=m^2R_{\mathrm{AdS}}^2$, they correspond to conformal dimensions of

$$\Delta_1=10, \qquad \Delta_{2\ldots h_-^{1,1}+1}=6 \tag{48}$$

for the saxions, and

$$\Delta_1=11, \qquad \Delta_{2\ldots h_-^{1,1}+1}=5 \tag{49}$$

for the corresponding axions. Similar conclusions can be drawn for all the complex structure moduli as well. From Eq. (39), there are in principle two distinct possibilities for the saxions as both solutions to $\Delta(\Delta-d)=m^2R_{\mathrm{AdS}}^2$ satisfy the unitarity bound. However, $\mathcal{N}=1$ superconformal symmetry excludes the option $\Delta_{u_\alpha}=1$,[6] as the axion and saxion are in the same three-dimensional $\mathcal{N}=1$ supermultiplet [26]. Combined with the fact that the complex structure axions are all massless, one arrives at the conclusion that

$$\Delta_{u_\alpha}=2, \qquad \Delta_{a_\alpha}=3. \tag{50}$$

for all complex structure moduli U_α, where $\alpha=1\ldots h^{2,1}$.

VI. CONCLUSIONS

We have shown how—if we assume they actually exist—the spectrum of the CFT_3 duals to supersymmetric DGKT type IIA flux vacua is characterized by a spectrum of integer conformal dimensions for the low-lying primaries. This result generalizes an observation first made in Ref. [6] for the simplest case of a $T^6/\mathbb{Z}_3\times\mathbb{Z}_3$ orientifold, and later extended in Ref. [7] to a wide range of examples.

For a generic modulus, the conformal dimension is given by one of the very few integer values in Eqs. (48)–(50). It is surprising to see how these numbers do not bear any trace of the microscopic details of the compactification, i.e., the values of the fluxes and the geometry of the orientifold. Such a simple and universal behavior is suggestive of the fact that the holographic perspective offers a both novel and insightful viewpoint on these constructions.

Despite the simplicity of the result, the calculation itself does not seem to yield any clear insight into why such a striking property should hold. Its outcome relies on the no-scale relations for the Kähler potential, as well as some more specific conditions involving the superpotential and its derivatives, Eqs. (16) and (17).

One might wonder whether supersymmetry plays a role: already at the $\mathcal{N}=1$ level, the structure of the superconformal multiplets implies that the conformal dimensions of axions and saxions should differ by one. A further observation is that no-scale relations partially depend on the factorization of the Kähler potential into a sum of two terms (depending only on the Kähler and complex structure moduli respectively), which could be seen as a remnant of a more constraining $\mathcal{N}=2$ supersymmetry of the original supergravity theory, where the Kähler potential factorizes into hypermultiplet and vector multiplet terms.

Generally, one would expect that $\mathcal{N}=1$ corrections would break this factorization and introduce mixing between the Kähler and complex structure moduli. Examples of such corrections would be α' and g_s corrections which, generically, would not preserve the $\mathcal{N}=2$ structure. The DGKT vacua have a scale-separated limit of large volume and weak coupling; one would expect this limit to suppress such corrections, with an asymptotic limit in which corrections to the integer conformal dimensions vanish asymptotically.

More generally, it would be extremely fascinating to come up with a deeper explanation of either the integer dimensions or the large degeneracies amongst the operators. A related question one might hope to address in the future is whether any similar structure holds for the

[6] It would be interesting to understand how supersymmetry implements this from an AdS perspective, where the choices correspond to different boundary conditions for the scalar field.

nonsupersymmetric vacua as well, as suggested by the examples in Refs. [6,7].

In a long-term perspective, an interesting direction would involve trying to understand if one could either prove or disprove the consistency of these (or similar) four-dimensional vacua using ideas from AdS/CFT.

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