Mechanism for the quantum natured gravitons to entangle masses

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This paper points out the importance of the quantum nature of the gravitational interaction with matter in a linearized theory of quantum gravity induced entanglement of masses. We will show how the quantum interaction entangles the steady states of a closed system (eigenstates) of two test masses placed in the harmonic traps, and how such a quantum matter-matter interaction emerges from an underlying quantum gravitational field. We will rely upon quantum perturbation theory highlighting the critical assumptions for generating a quantum matter-matter interaction and showing that a classical gravitational field does not render such an entanglement. We will consider two distinct examples: one where the two harmonic oscillators are static, and the other where the harmonic oscillators are nonstatic. In both cases it is the quantum nature of the gravitons interacting with the harmonic oscillators that are responsible for creating an entangled state with the ground and the excited states of harmonic oscillators as the Schmidt basis. We will compute the concurrence as a criterion for the above entanglement and compare the two ways of entangling the two harmonic oscillators.

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I. INTRODUCTION

The classical theory of general relativity (GR) is outstanding in matching the observations on large scales, especially from the solar system tests to the observations from the detection of the gravitational waves [1]. Despite these successes, the classical theory fails at very short distances and early times. The classical GR predicts black hole and cosmological singularity where the notion of spacetime breaks down [2].

Although it is believed that the quantum theory of gravity will alleviate some of these challenges, however, we still do not know whether gravity is indeed quantum or not. Moreover, there are also many candidates for a quantum theory of gravity [3]. From an effective field theory perspective and at low energies, it is believed that the gravitational interaction is being mediated by a massless spin-2 graviton, which can be canonically quantized [4–7]. Although the perturbative quantum theory of gravity also possesses many challenges, such as the issues of renormalizability at very high energies and the issue of finiteness at low energies where the day to day experiments are performed, it is still a very good effective field theory description of nature [8].

Given the feeble interaction strength of gravity, it is extremely hard to detect a graviton in a detector by the momentum transfer [9]. Indirect detection of the quantum properties of the graviton remains elusive in the primordial nature of the gravitational waves (GWs) [10,11]. Astrophysical and cosmological uncertainties shroud any validation of the quantum nature of spacetime by modifying the photon dispersion relationship [12]. Moreover, the strict constraint on the graviton mass indirectly arising from the propagation of the GWs detected by the LIGO observatory hints no departure from GR in the infrared [13].

Given all these challenges, it is worth asking how to test the quantum nature of a graviton in a laboratory at low energies. Recently, there has been a proposal to test the quantum nature of gravity by witnessing the spin entanglement between the two quantum superposed test masses, known as the quantum gravity induced entanglement of masses (QGEM) [14,15]. The idea is to create a spatial quantum superposition of two test masses and bring them adjacent to each other in a controlled environment such that their only dominant interaction that remains is the exchange of a massless graviton. It is possible to realize such a daunting experiment but there are many challenges that need to be overcome.[1]

[1]The detailed analysis of the demanding nature of the QGEM experiment (such as creating Schrödinger cat states with massive test masses along with achieving the required coherence lifetime required to detect the entanglement) has been discussed already in [14]. A related idea was also proposed in [16]. These initial works [14–16] garnered extensive interest in the research community [17–44].

In this paper we will review the conceptual underpinnings of the QGEM mechanism. The entanglement of the two masses emerges from "local operation and quantum communication (LOQC)," whereas no entanglement would occur by "local operations and classical communication (LOCC)" [45]. The LOCC principle states that the two quantum states cannot be entangled via a classical channel if they were not entangled to begin with, or entanglement cannot be increased by local operations and classical communication. The classical communication is the critical ingredient which can be put to the test when it comes to graviton mediated interaction between the two masses. If the graviton is quantum, it would mediate the gravitational attraction between the two masses, and it would also entangle them, hence confirming the QGEM proposal [14,15].

One of the aims of the current paper is to sharpen the argument of LOCC for the purpose of QGEM, and highlight the role of the quantum nature of the interactions for entangling the two quantum systems. We will use basic quantum mechanics and perturbation theory to show how the perturbed wave functions of the matter systems become entangled *solely* by the virtue of the quantum natured graviton. We will furthermore highlight the relevant degrees of freedom of the graviton which interacts with the quantized matter, and they are responsible for the entanglement in both the static and nonstatic cases.

We will study this problem in the number state basis of two harmonic oscillator states, and we will show that the perturbed state is an entangled state even at the *first order* in a quantum perturbation theory [46]. The *quantum* interaction between the two matter systems emerges from the change in the graviton vacuum energy due to the presence of the two quantum harmonic oscillators. In the QFT community this is a well-known way to understand how contact interactions emerge (see [47]). We will show that in a static limit this change in the vacuum energy is the same as that of Newton's potential at the lowest order in Newton's constant, which appears at the *second order* in the perturbation theory. Furthermore, the Newtonian potential is the energy shift of the gravitational vacuum. In this case the relevant gravitational degrees of freedom required to be quantized is composed of both the *spin-2* and *spin-0* components [4,15]. A similar interpretation applies to the nonstatic case, except there are some details in the components of the graviton which will get modified.

In particular, if the matter is quantized, then the energy shift in the gravitational field becomes an *operator valued* interaction. Since we have the quantum superpositions for the matter systems, the energy shift in the gravitational field will not be a real number, resulting in the gravitational field itself being a nonclassical entity.

We will calculate the *concurrence* [48] as a way to measure the entanglement between the two harmonic oscillators and show that the concurrence is always positive for the quantum interaction between the graviton and the matter states.[2,3]

This paper is organized in the following way. We will first briefly recap the known results, i.e., the two quantum harmonic oscillators (Sec. II), and show how the quantum interaction is responsible for generating the entanglement (Sec. III). We will then quantify the degree of entanglement using concurrence which we will compute using perturbation theory. We then discuss the special case where the interaction potential is generated by the gravitational field in the regime of weak gravity (Sec. IV). In particular, we will first show how the $\hat{T}_{00}$ component of the stress-energy tensor generates entanglement—$|00\rangle$ and $|11\rangle$ are the Schmidt basis of the entangled state, where $|nN\rangle \equiv |n\rangle|N\rangle$ and $|n\rangle$ ($|N\rangle$) denote the number state of the first (second) harmonic oscillator (Sec. V). We will then consider entanglement via graviton in the nonstatic case (Sec. VI). We will find that the $\hat{T}_{0i}$ components of the stress-energy tensor generate a two-mode squeezed state of the two harmonic oscillators (Sec. VII). In addition, we will show that the $\hat{T}_{ij}$ components of the stress-energy tensor (which give rise to the GWs) generate entanglement—$|00\rangle$ and $|22\rangle$ are the Schmidt basis of the entangled state, in line with the quadrupole nature of the gravitational radiation (Sec. VIII). We will finally conclude with the consequences for the classical/quantum communication (Sec. IX).

II. TWO QUANTUM HARMONIC OSCILLATORS

Let us consider the two matter systems, denoted by A and B, which are placed in the harmonic traps located at $\pm d/2$. We suppose that the harmonic oscillators are well-localized, such that

$$\hat{x}_A = -\frac{d}{2} + \delta\hat{x}_A, \qquad \hat{x}_B = \frac{d}{2} + \delta\hat{x}_B, \tag{1}$$

where $\hat{x}_A$ and $\hat{x}_B$ are the positions, and $\delta\hat{x}_A$ and $\delta\hat{x}_B$ denote small displacements from the equilibrium. The usual Hamiltonian for the two harmonic oscillators is given by

[2]In this paper we will consider only pure states to highlight the conceptual points, but the analysis could be readily extended to more realistic situations with mixed states to account for the internal/external noise sources and environmental decoherence.

[3]The entanglement features of harmonic oscillators in the presence of the interaction are quite well-known in the quantum optics literature (see for example [49]). Typically, the quantum nature of the photon plays the role of the quantum interaction. However, our aim here is to concentrate on the quantum nature of the graviton, especially highlighting the graviton's dynamical degrees of freedom which are responsible for the quantum interaction in enabling the entanglement feature of the quantum harmonic oscillators. These dynamical degrees of freedom of the graviton are very different in nature compared to the photon.

$$\hat{H}_{\rm matter} = \frac{\hat{p}_A^2}{2m} + \frac{\hat{p}_B^2}{2m} + \frac{m\omega_{\rm m}^2}{2}\delta\hat{x}_A^2 + \frac{m\omega_{\rm m}^2}{2}\delta\hat{x}_B^2, \qquad (2)$$

where $\hat{p}_A$ and $\hat{p}_B$ are the conjugate momenta and $\omega_{\rm m}$ is the harmonic frequency of the two traps (assumed equal for the two particles for simplicity). We now introduce the adimensional mode operators for the matter by writing

$$\delta\hat{x}_A = \sqrt{\frac{\hbar}{2m\omega_{\rm m}}}(\hat{a}+\hat{a}^\dagger), \quad \delta\hat{x}_B = \sqrt{\frac{\hbar}{2m\omega_{\rm m}}}(\hat{b}+\hat{b}^\dagger), \qquad (3)$$

$$\hat{p}_A = i\sqrt{\frac{\hbar m\omega_{\rm m}}{2}}(\hat{a}^\dagger-\hat{a}), \quad \hat{p}_B = i\sqrt{\frac{\hbar m\omega_{\rm m}}{2}}(\hat{b}^\dagger-\hat{b}), \qquad (4)$$

which satisfy the usual canonical commutation relationships (the only nonzero commutators are given by $[a,a^\dagger]=1$ and $[b,b^\dagger]=1$). Using this notation the Hamiltonian can be written succinctly as

$$\hat{H}_{\rm matter} = \hat{H}_A + \hat{H}_B, \qquad (5)$$

where $\hat{H}_A = \hbar\omega_{\rm m}\hat{a}^\dagger\hat{a}$ and $\hat{H}_B = \hbar\omega_{\rm m}\hat{b}^\dagger\hat{b}$. We will now want to investigate the steady state when the system is perturbed by an interaction Hamiltonian H_{AB}. In particular, we will show that in general any *quantum* interaction will entangle the two harmonic oscillators.

III. QUANTUM INTERACTION INDUCES ENTANGLEMENT

Let us assume that the initial state of the matter system is given by

$$|\psi_{\rm i}\rangle = |0\rangle_A|0\rangle_B, \qquad (6)$$

where $|0\rangle_A$ ($|0\rangle_B$) denote the ground state of the first (second) harmonic oscillator (in the following we will omit the subscripts A and B for the states to ease the notation). Suppose we now introduce an interaction potential λH_{AB} between the two matter systems, where λ is a small bookkeeping parameter. The perturbed state is given by

$$|\psi_{\rm f}\rangle \equiv \frac{1}{\sqrt{\mathcal{N}}}\sum_{n,N} C_{nN}|n\rangle|N\rangle, \qquad (7)$$

where $|n\rangle$ and $|N\rangle$ denote the number states and the overall normalization is given by $\mathcal{N} = \sum_{n,N}|C_{nN}|^2$. We have that $C_{00}\equiv 1$ (coefficient of the unperturbed state), while the other coefficients are given by

$$C_{nN} = \lambda\frac{\langle n|\langle N|\hat{H}_{AB}|0\rangle|0\rangle}{2E_0 - E_n - E_N}, \qquad (8)$$

where E_0 is the ground-state energy for the harmonic oscillators (equal for the two harmonic oscillators as we have assumed the same trap frequency) and E_n and E_N denote the energies of the excited states.

Here we note the role of $\hat{H}_{AB}$ being a quantum operator. If H_{AB} were *classical*, it would have an associated c-number (complex number), which would yield $\langle n|\langle N|H_{AB}|0\rangle|0\rangle = 0$, by virtue of the orthogonality of the ground and the excited states ($|0\rangle|0\rangle$ and $|n\rangle|N\rangle$) of the two quantum harmonic oscillators, as $n, N > 0$ in Eq. (8).[4] By the same argument, interactions acting as operators on only one of the two quantum systems (i.e., without products of operators acting on the two matter systems) cannot entangle the two systems. It is thus instructive to rewrite the state in Eq. (7) in the following way [46]:

$$|\psi_{\rm f}\rangle \sim \left(|0\rangle + \sum_{n>0}A_n|n\rangle\right)\left(|0\rangle + \sum_{N>0}B_N|N\rangle\right) + \sum_{n,N>0}(C_{nN} - A_nB_N)|n\rangle|N\rangle, \qquad (9)$$

where $A_n \equiv C_{n0}$ and $B_N \equiv C_{0N}$. The first line in Eq. (9) would yield a separable state, while the second line is responsible for *entanglement* of the two matter systems (the A_n and B_N terms will not contribute to the entanglement at first order in perturbation theory). We can already see the stark difference between the LOQC and the LOCC.[5] The nontrivial part of a LOQC mechanism is now encoded in the terms of the interaction Hamiltonian $\hat{H}_{AB}$ producing the second line in Eq. (9). On the other hand, a LOCC mechanism could produce the first line of Eq. (9), but not the second line, as a classical interaction cannot entangle the two quantum states if they were not entangled to begin with.[6]

To quantify the degree of entanglement we can compute the *concurrence* [48,50]

[4]Let us clarify what we mean by the Hamiltonian acting on a quantum state in a Hilbert space, which is by definition an operator, to be associated with a number. Essentially, we mean that it could (a) be proportional to the identity operator multiplied by a number, or (b) be something nontrivial, but acts on an eigenbasis. Our statement above holds for both the definitions.

[5]The above discussion, of course, relies on initially pure states evolving unitarily under a fixed Hamiltonian so that they remain pure. The general notion of LOCC [45], as used in quantum information is broader, distinguishing entangled states from classically mutually correlated states. The above discussion of Eq. (9) can, of course, be easily generalized to mixed states and probabilistic operations (simply several repeats of our argument for different initial states and different Hamiltonians with their corresponding probabilities).

[6]A similar discussion was first adopted in the momentum space entanglement in a perturbative quantum field theory, Ref. [46], where they argued that the entanglement entropy of and mutual information between subsets of field theoretic degrees of freedom at different momentum scales are natural observables in quantum field theory. Here we will compare the degree of entanglement by computing the *concurrence* (see the discussion below).

$$C \equiv \sqrt{2(1 - \mathrm{tr}[\hat{\rho}_A^2])}, \quad (10)$$

where $\hat{\rho}_A$ can be computed by tracing away the B state

$$\hat{\rho}_A = \sum_N \langle N|\psi_\mathrm{f}\rangle\langle\psi_\mathrm{f}|N\rangle. \quad (11)$$

We will recall that the larger the concurrence C is, the larger is the degree of entanglement—$C = 0$ corresponds to a separable state, while $C = \sqrt{2}$ is obtained for a maximally entangled state. Inserting Eq. (7) into Eq. (11) we find

$$\hat{\rho}_A = \frac{1}{\mathcal{N}} \sum_{n,n',N} C_{nN} C^*_{n'N} |n\rangle\langle n'|. \quad (12)$$

We will then insert Eq. (11) back into Eq. (10) to eventually find

$$C \equiv \sqrt{2\left(1 - \sum_{n,n',N,N'} C_{nN} C^*_{n'N} C_{n'N'} C^*_{nN'}/\mathcal{N}^2\right)}. \quad (13)$$

In the next sections, we will consider the entanglement of two harmonic oscillators induced by the quantum nature of gravitons. For this case, the entanglement will be induced by the terms C_{11} and C_{22} at the lowest order in the perturbation theory when the potential $\hat{H}_{AB}$ is generated by the quantized gravitational field in the regime of weak gravity.

IV. QUANTUM GRAVITATIONAL INTERACTION

We will consider the setup of two quantum harmonic oscillators (introduced in the previous sections) in the presence of the gravitational field. In particular, we will work in the regime of small perturbations $|h_{\mu\nu}| \ll 1$ about the Minkowski background $\eta_{\mu\nu}$. The metric is given by $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ [where μ, $\nu = 0$, 1, 2, 3 and we are using the $(-,+,+,+)$ signature throughout]. We will promote the fluctuations into the quantum operators,

$$\hat{h}_{\mu\nu} = \mathcal{A} \int d\boldsymbol{k} \sqrt{\frac{\hbar}{2\omega_{\boldsymbol{k}}(2\pi)^3}} (\hat{P}^\dagger_{\mu\nu}(\boldsymbol{k}) e^{-i\boldsymbol{k}\cdot\boldsymbol{r}} + \mathrm{H.c.}), \quad (14)$$

where $\boldsymbol{k}$ is the three-vector and $d\boldsymbol{k} \equiv d^3k$. The prefactor is denoted by $\mathcal{A} = \sqrt{16\pi G/c^2}$, where G is Newton's constant, and $\hat{P}_{\mu\nu}$ and $\hat{P}^\dagger_{\mu\nu}$ denote the graviton annihilation and the creation operator. We will discuss in detail the properties of the graviton and the relevant degrees of freedom below.

Around the Minkowski background, the graviton coupling to the stress-energy tensor $\hat{T}_{\mu\nu}$ is given by the following *operator valued* interaction term:

$$\hat{H}_\mathrm{int} = -\frac{1}{2} \int d\boldsymbol{r} \hat{h}^{\mu\nu}(\boldsymbol{r}) \hat{T}_{\mu\nu}(\boldsymbol{r}), \quad (15)$$

where $\boldsymbol{r}$ denotes the three-vector.

We will now consider separately the coupling induced by the component $\hat{T}_{00}$ in the static limit and by the full stress-energy tensor $\hat{T}_{\mu\nu}$ in the nonstatic case.

V. ENTANGLEMENT VIA GRAVITON IN THE STATIC LIMIT

Let us consider two particles of mass m (which will form the two oscillating systems). The two particles are generating the following current in the static limit:

$$\hat{T}_{00}(\boldsymbol{r}) \equiv mc^2(\delta(\boldsymbol{r} - \hat{\boldsymbol{r}}_A) + \delta(\boldsymbol{r} - \hat{\boldsymbol{r}}_B)), \quad (16)$$

where $\hat{\boldsymbol{r}}_A = (\hat{x}_A, 0, 0)$ and $\hat{\boldsymbol{r}}_B = (\hat{x}_B, 0, 0)$ denote the positions of the two matter systems. The Fourier transform of the current is given by

$$\hat{T}_{00}(\boldsymbol{k}) = \frac{mc^2}{\sqrt{(2\pi)^3}} (e^{i\boldsymbol{k}\cdot\hat{\boldsymbol{r}}_A} + e^{i\boldsymbol{k}\cdot\hat{\boldsymbol{r}}_B}), \quad (17)$$

where $\boldsymbol{k}$ denotes 3-momentum.

Following the canonical quantization of graviton in a weak field regime [4], we decompose $\hat{h}_{\mu\nu} = \hat{\gamma}_{\mu\nu} - (1/2)\eta_{\mu\nu}\hat{\gamma}$ around a Minkowski background (where we use the convention $\gamma \equiv \eta_{\mu\nu}\gamma^{\mu\nu}$). The two distinct modes, i.e., the spin-2, $\gamma_{\mu\nu}$, and the spin-0, γ, can be treated as independent variables. They are promoted as self-adjoint operators and decomposed into

$$\hat{\gamma}_{\mu\nu} = \mathcal{A} \int d\boldsymbol{k} \sqrt{\frac{\hbar}{2\omega_{\boldsymbol{k}}(2\pi)^3}} (\hat{P}^\dagger_{\mu\nu}(\boldsymbol{k}) e^{-i\boldsymbol{k}\cdot\boldsymbol{r}} + \mathrm{H.c.}), \quad (18)$$

$$\hat{\gamma} = 2\mathcal{A} \int d\boldsymbol{k} \sqrt{\frac{\hbar}{2\omega_{\boldsymbol{k}}(2\pi)^3}} (\hat{P}^\dagger(\boldsymbol{k}) e^{-i\boldsymbol{k}\cdot\boldsymbol{r}} + \mathrm{H.c.}), \quad (19)$$

where

$$[\hat{P}_{\mu\nu}(\boldsymbol{k}), \hat{P}^\dagger_{\lambda\rho}(\boldsymbol{k}')] = [\eta_{\mu\lambda}\eta_{\nu\rho} + \eta_{\mu\rho}\eta_{\nu\lambda}]\delta(\boldsymbol{k} - \boldsymbol{k}'), \quad (20)$$

$$[\hat{P}(\boldsymbol{k}), \hat{P}^\dagger(\boldsymbol{k}')] = -\delta(\boldsymbol{k} - \boldsymbol{k}'). \quad (21)$$

The graviton Hamiltonian is now given by [4]

$$\hat{H}_g = \int d\boldsymbol{k} \hbar\omega_{\boldsymbol{k}} \left(\frac{1}{2} \hat{P}^\dagger_{\mu\nu}(\boldsymbol{k}) \hat{P}^{\mu\nu}(\boldsymbol{k}) - \hat{P}^\dagger(\boldsymbol{k}) \hat{P}(\boldsymbol{k})\right). \quad (22)$$

We are interested in computing the change in the energy $\Delta\hat{H}_g$—*the shift of the energy of the graviton vacuum arising from the interaction with the matter*. In the static limit (where we neglect the motion of the two harmonic oscillators), the interaction Hamiltonian can be written in a simple form:

$$\hat{H}_{\text{int}} = \frac{1}{2}\int d\boldsymbol{r}[\hat{\gamma}_{00}(\boldsymbol{r}) + (1/2)\hat{\gamma}(\boldsymbol{r})]\hat{T}_{00}(\boldsymbol{r}). \tag{23}$$

We can now compute the shift to the energy of the graviton vacuum using the perturbation theory. The first order term vanishes,[7] while the second order term in the perturbation theory yields

$$\Delta\hat{H}_g \equiv \int d\boldsymbol{k}\frac{\langle 0|\hat{H}_{\text{int}}|\boldsymbol{k}\rangle\langle\boldsymbol{k}|\hat{H}_{\text{int}}|0\rangle}{E_0 - E_{\boldsymbol{k}}}, \tag{24}$$

where $|\boldsymbol{k}\rangle = (\hat{P}^\dagger_{00}(\boldsymbol{k}) + \hat{P}^\dagger(\boldsymbol{k}))|0\rangle$ is the one-particle state constructed in the unperturbed vacuum, $E_{\boldsymbol{k}} = E_0 + \hbar\omega_{\boldsymbol{k}}$ is the energy of the one-particle state, and E_0 is the energy of the vacuum state. The mediated graviton is now off-shell/virtual by virtue of the integration of all possible momentum $\boldsymbol{k}$—and hence does not obey classical equations of motion. Using Eqs. (14), (18), (19), and (23) we readily find[8]

$$\langle\boldsymbol{k}|\hat{H}_{\text{int}}|0\rangle = \frac{\mathcal{A}}{2}\sqrt{\frac{\hbar}{2\omega_{\boldsymbol{k}}}}\hat{T}_{00}(\boldsymbol{k}), \tag{25}$$

where we have used the definition of the Fourier transform

$$\hat{T}_{00}(\boldsymbol{k}) = \sqrt{\frac{1}{(2\pi)^3}}\int d\boldsymbol{r}e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}\hat{T}_{00}(\boldsymbol{r}). \tag{26}$$

From Eq. (25) we then obtain a simple expression,

$$\langle 0|\hat{H}_{\text{int}}|\boldsymbol{k}\rangle\langle\boldsymbol{k}|\hat{H}_{\text{int}}|0\rangle = \frac{\hbar\mathcal{A}^2\hat{T}^\dagger_{00}(\boldsymbol{k})\hat{T}_{00}(\boldsymbol{k})}{8\omega_{\boldsymbol{k}}}. \tag{27}$$

From Eq. (24) we then readily find

$$\Delta\hat{H}_g = -\mathcal{A}^2\int d\boldsymbol{k}\frac{\hat{T}^\dagger_{00}(\boldsymbol{k})\hat{T}_{00}(\boldsymbol{k})}{8c^2\boldsymbol{k}^2}. \tag{28}$$

Performing the momentum integration using spherical coordinates, we then find the result

$$\Delta\hat{H}_g = -\frac{\mathcal{A}^2m^2c^2}{16\pi|\hat{\boldsymbol{r}}_A - \hat{\boldsymbol{r}}_B|}, \tag{29}$$

where we have omitted the self-energy terms of the individual particles.[9] We will finally insert $\mathcal{A} = \sqrt{16\pi G/c^2}$ into Eq. (29) to find Newton's potential[10]:

[7]The first order contribution to the energy is given by $\langle 0|\hat{H}_{\text{int}}|0\rangle = 0$, where $|0\rangle$ denotes the unperturbed graviton vacuum. This is due to the fact that $\hat{H}_{\text{int}}$ depends linearly on $\hat{\gamma}_{\mu\nu}, \hat{\gamma}$ which are themselves linear combinations of creation and the annihilation operators, $\hat{P}^\dagger_{\mu\nu}, \hat{P}_{\mu\nu}, \hat{P}^\dagger, \hat{P}$. Hence $\langle 0|\hat{H}_{\text{int}}|0\rangle$ depends only linearly on $\hat{P}^\dagger_{\mu\nu}, \hat{P}_{\mu\nu}, \hat{P}^\dagger, \hat{P}$ and thus vanishes (as $\hat{P}|0\rangle = 0$ and $\langle 0|\hat{P}^\dagger = 0$ and similarly for the other operators). The nonvanishing contribution will come from the second order term in the perturbation theory [15,51,52].

[8]Inserting the definition of $\hat{H}_{\text{int}}$ from Eq. (23) [and the definitions of $\hat{\gamma}_{\mu\nu}$ and $\hat{\gamma}$ from Eqs. (18) and (19), respectively], we encounter the following expression: $\langle\boldsymbol{k}|(\hat{P}^\dagger_{00}(\boldsymbol{k}') + \hat{P}^\dagger(\boldsymbol{k}'))|0\rangle$. Using $|\boldsymbol{k}\rangle = (\hat{P}^\dagger_{00}(\boldsymbol{k}) + \hat{P}^\dagger(\boldsymbol{k}))|0\rangle$ we then find

$$\langle 0|(\hat{P}_{00}(\boldsymbol{k})\hat{P}^\dagger_{00}(\boldsymbol{k}') + \hat{P}(\boldsymbol{k})\hat{P}^\dagger(\boldsymbol{k}'))|0\rangle,$$

while the other terms vanish as the vacuum state satisfies $\hat{P}_{00}(\boldsymbol{k})|0\rangle = \hat{P}(\boldsymbol{k})|0\rangle = 0$. The two terms on the right-hand side can then be rewritten as $\langle 0|[\hat{P}_{00}(\boldsymbol{k}), \hat{P}^\dagger_{00}(\boldsymbol{k}')]|0\rangle$ and $\langle 0|[\hat{P}(\boldsymbol{k}), \hat{P}^\dagger(\boldsymbol{k}')]|0\rangle$, where we have used the definition of the commutator $[\hat{O}_1, \hat{O}_2] = \hat{O}_1\hat{O}_2 - \hat{O}_2\hat{O}_1$ (as well as again the definition of the vacuum state). Using now the commutation relations defined in Eqs. (20) and (21) and summing the two terms, we then finally obtain

$$\langle\boldsymbol{k}|(\hat{P}^\dagger_{00}(\boldsymbol{k}') + \hat{P}^\dagger(\boldsymbol{k}'))|0\rangle = \delta(\boldsymbol{k} - \boldsymbol{k}').$$

[9]There are self-energy contributions which provide the ultraviolet (UV) corrections and tend to generate infinities in the limit when the graviton momentum goes to infinity, i.e., $k \to \infty$. This is an example of a UV divergence appearing in a perturbative quantum gravity. We are interested in the infrared (IR) limit where we are neglecting the UV aspects of the quantum gravity.

[10]There is a covariant formulation also to obtain the same answer by using the time-ordered graviton propagator discussed in [15]. The Einstein-Hilbert action can be written as in terms of the fluctuations $h_{\mu\nu}$ up to quadratic in order:

$$S = (1/4)\int d^4x h_{\mu\nu}\mathcal{O}^{\mu\nu\lambda\sigma}h_{\lambda\sigma} + \mathcal{O}(h^3),$$

where $\mathcal{O}^{\mu\nu\lambda\sigma} = (1/4)(\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho})\Box - (1/2)\eta^{\mu\nu}\eta^{\rho\sigma}\Box + (1/2)(\eta^{\mu\nu}\partial^\rho\partial^\sigma + \eta^{\rho\sigma}\partial^\mu\partial^\nu - \eta^{\mu\rho}\partial^\nu\partial^\sigma - \eta^{\mu\sigma}\partial^\nu\partial^\rho)$, where the d'Alembertain operator is $\Box = g_{\mu\nu}\nabla^\mu\nabla^\nu$. The propagator for the graviton [15,52] $h_{\mu\nu}$ can be recast in terms of

$$\Pi\mu\nu\rho\sigma(\boldsymbol{k}) = (1/2\boldsymbol{k}^2)(\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\nu\rho}\eta_{\mu\sigma} - \eta_{\mu\nu}\eta_{\rho\sigma}).$$

With the help of this propagator, one can find the gravitational potential, i.e., the nonrelativistic scattering due to an exchange of an off-shell graviton. The gravitational potential is given by $\Phi(\boldsymbol{r}) = -(8\pi G/(2\pi)^3)\int d^3\boldsymbol{k}T_1^{00}\Pi_{0000}(k)T_2^{00}(-k)e^{i\boldsymbol{k}\cdot\boldsymbol{r}} = -4\pi Gm^2\int d^3\boldsymbol{k}e^{i\boldsymbol{k}\cdot\boldsymbol{r}}/\boldsymbol{k}^2 = -Gm^2/\boldsymbol{r}$. This result is the same as what we have obtained in Eq. (30). The only difference here is that we have computed the potential by using the full graviton propagator and the scattering amplitude between the two masses via the exchange of spin-2 and spin-0 components of the graviton (see the Appendix of Ref. [15]). In the text we have computed the change in the graviton vacuum. However, in the nonrelativistic limit both the results give rise to the same conclusion.

$$\Delta\hat{H}_g = -\frac{Gm^2}{|\hat{x}_A - \hat{x}_B|}. \tag{30}$$

We thus find that the change in the graviton energy, $\Delta\hat{H}_g$, due to the interaction between the graviton and the matter is an operator valued function of the two matter systems, i.e.,

$$\Delta\hat{H}_g \equiv f(\hat{x}_A, \hat{x}_B). \tag{31}$$

If the two matter systems do not have sharply defined positions (such as when placed in a spatial superposition or some other nonclassical state), then the change in the graviton energy $\Delta\hat{H}_g$ will not be a real number, as required in a classical theory of gravity, but rather an operator-valued quantity, a bona fide quantum entity.

We now wish to calculate the excited wave function $|\psi_f\rangle$ of the two harmonic oscillators to establish the link between entanglement and LOQC discussed in Sec. III. We first use Eq. (1) and expand Eq. (30) to find

$$\Delta\hat{H}_g \approx -\frac{Gm^2}{d} + \frac{Gm^2}{d^2}(\delta\hat{x}_B - \delta\hat{x}_A) - \frac{Gm^2}{d^3}(\delta\hat{x}_B - \delta\hat{x}_A)^2. \tag{32}$$

The last term gives the lowest order matter-matter interaction[11]

$$\hat{H}_{\mathrm{AB}} \equiv \frac{2Gm^2}{d^3}\delta\hat{x}_A\delta\hat{x}_B. \tag{33}$$

Note that the interaction Hamiltonian $\hat{H}_{\mathrm{AB}}$ contains only the operators of the two harmonic oscillators $\delta\hat{x}_A, \delta\hat{x}_B$. Yet it is critical to realize that the product $\delta\hat{x}_A\delta\hat{x}_B$ would not have arisen if we had assumed a real-valued shift of the energy of the gravitational field. Indeed, a classical gravitational field is unable to produce the operator-valued shift in Eq. (30) [and hence the quantum interaction potential in Eq. (33)]. We must thus conclude that gravitationally induced entanglement is indeed a quantum signature of the gravitational field.[12]

We will now use the modes in Eq. (3) to find

$$\hat{H}_{\mathrm{AB}} \approx \hbar\mathfrak{g}(\hat{a}\,\hat{b} + \hat{a}^\dagger\hat{b} + \hat{a}\hat{b}^\dagger + \hat{a}^\dagger\hat{b}^\dagger), \tag{34}$$

where we have defined the coupling

$$\mathfrak{g} \equiv \frac{Gm}{d^3\omega_{\mathrm{m}}}. \tag{35}$$

Using $\hat{H}_{\mathrm{AB}}$ as the interaction Hamiltonian in Eq. (8), we find that the only nonzero coefficient emerges from the term $\sim\hat{a}^\dagger\hat{b}^\dagger$ and is given by

$$C_{11} = -\frac{\mathfrak{g}}{2\omega_{\mathrm{m}}}. \tag{36}$$

We note that the $a^\dagger b^\dagger$ term generates the first excited states in the harmonic oscillators (with energy $E_1 = E_0 + \hbar\omega_{\mathrm{m}}$). In addition, we also have the term $C_{00} = 1$ corresponding to the unperturbed state.

The final state in Eq. (7) thus simplifies to (up to first order in the perturbation theory, and by setting $\lambda = 1$)

$$|\psi_{\mathrm{f}}\rangle \equiv \frac{1}{\sqrt{1 + (\mathfrak{g}/(2\omega_{\mathrm{m}}))^2}}\left[|0\rangle|0\rangle - \frac{\mathfrak{g}}{2\omega_{\mathrm{m}}}|1\rangle|1\rangle\right], \tag{37}$$

which is *an entangled state involving the ground and the first excited states of the two harmonic oscillators*. We compute the reduced density matrix by tracing system B (we recall that our notation is $|n\rangle|N\rangle = |n\rangle_A|N\rangle_B$). The concurrence in Eq. (13) reduces to

$$C \equiv \sqrt{2\left(1 - \frac{1 + (\mathfrak{g}/(2\omega_{\mathrm{m}}))^4}{1 + (\mathfrak{g}/(2\omega_{\mathrm{m}}))^2}\right)} \approx \sqrt{2}\frac{\mathfrak{g}}{\omega_{\mathrm{m}}}, \tag{38}$$

which is valid when the parameter $\mathfrak{g}/\omega_{\mathrm{m}} \ll 1$ is small. Inserting the coupling from Eq. (35), we find the concurrence is given by

$$C = \frac{\sqrt{2}Gm}{d^3\omega_{\mathrm{m}}^2}. \tag{39}$$

We thus see that the degree of entanglement grows linearly with the mass of the oscillator and inversely with the distance between the two oscillators (inverse cubic) as well as with the frequency of the harmonic trap (inverse square).

Let us reiterate the key finding. If the underlying gravitational field were classical (specifically, obeying LOCC), then the final state of the matter components, i.e., the two harmonic oscillator states, would have never evolved to the entangled state $|\psi_f\rangle$, but would have rather remained in an unentangled/separable state. Conversely, if the gravitational field is quantized (and hence obeys LOQC), then we have shown that it can give rise to the entangled state $|\psi_f\rangle$.

[11]It is instructive to compare the obtained results for two harmonic oscillators to the results obtained previously for two interferometers. In both cases, the action is proportional to $S = E\tau/\hbar$, where the interaction energy of the system is given by $E \sim H_{AB}$ and τ is the coherence timescale. Considering the setup in [14,16], and setting $\Delta\phi \sim S$, we then recover the entanglement phase $\Delta\phi \sim (2Gm^2/\hbar d)(\delta x/d)^2\tau$, where we have assumed $\delta x_A \sim \delta x_B \sim \delta x$ for the localized spatial superpositions of the two test masses.

[12]The above expression, Eq. (33), has been the starting point for the entanglement of the two harmonic oscillators with $1/r$-potential in many analyses (see [36,39–42,53]), but here we have shown how this interaction arises by noting how the vacuum of the spin-2 and spin-0 components of the graviton has shifted due to the quantum nature of the harmonic oscillators.

VI. ENTANGLEMENT VIA GRAVITON IN THE NONSTATIC CASE

In this section, we are interested in the coupling of the gravitational field to the $\hat{T}_{ij}$ components of the stress-energy tensor. In our specific case we consider two particles (in harmonic traps) moving along the x-axis such that the only nonzero components are given by T_{00}, T_{01}, and T_{11} (with $T_{10} = T_{01}$). Hence the relevant components of the graviton are given by $\hat{h}_{00} = \hat{\gamma}_{00} + (1/2)\hat{\gamma}$ (already present in the static case), by $\hat{h}_{01} = \hat{h}_{10} = \hat{\gamma}_{01}$, and by $\hat{h}_{11} = \hat{\gamma}_{11} - (1/2)\hat{\gamma}$ (which can be identified with the degrees of freedom of the GWs as discussed below). We will find that the energy shift in the graviton vacuum induces a coupling between the two harmonic oscillator states, which leads to the entanglement only when we assume that the $\hat{h}_{00}, \hat{h}_{01}, \hat{h}_{11}$ components are quantum.

The computation follows the analogous steps as the ones discussed in the previous section. The basic assumption is that these graviton modes are quantized and act as a quantum communicator, or serve as a quantum interaction between the two harmonic oscillators. The interaction Hamiltonian now has two contributions:

$$\begin{aligned}\hat{H}_{\rm int} = &\frac{1}{2}\int d\boldsymbol{r}[\hat{\gamma}_{00}(\boldsymbol{r}) + (1/2)\hat{\gamma}(\boldsymbol{r})]\hat{T}_{00}(\boldsymbol{r}) \\ &+ \int d\boldsymbol{r}\hat{\gamma}_{01}(\boldsymbol{r})\hat{T}_{01}(\boldsymbol{r}) \\ &+ \frac{1}{2}\int d\boldsymbol{r}[\hat{\gamma}_{11}(\boldsymbol{r}) - (1/2)\hat{\gamma}(\boldsymbol{r})]\hat{T}_{11}(\boldsymbol{r}), \end{aligned} \tag{40}$$

where the first line coincides with the interaction considered in Eq. (23), while the second and third lines arise from the degrees of freedom of the GWs corresponding to the +polarization.[13]

[13]We recall that in the transverse-traceless (TT) gauge we have the interaction Hamiltonian given by [54]

$$\hat{H}_{\rm int} = -\frac{1}{2}\int d\boldsymbol{r}\hat{h}_{ij}(\boldsymbol{r})\hat{T}_{ij}(\boldsymbol{r}), \tag{41}$$

where we implicitly assume the summation over the indices i, $j = 1, 2, 3$. The propagating, on-shell, graviton is described by the two helicity states $(+, \times)$:

$$\hat{h}_{ij} = \mathcal{A}\int d\boldsymbol{k}\sqrt{\frac{\hbar}{2\omega_{\boldsymbol{k}}(2\pi)^3}}P^{\dagger}_{\lambda}(\boldsymbol{k})e^{\lambda}_{ij}(\boldsymbol{k})e^{-i\boldsymbol{k}\cdot\boldsymbol{r}} + \text{H.c.}, \tag{42}$$

where we have assumed the summation over the two polarizations $(+, \times)$ (e^{λ}_{jk} denote the basis for the two polarization states), and the annihilation and the creation operator satisfy

$$[\hat{P}_{\lambda}(\boldsymbol{k}), \hat{P}^{\dagger}_{\lambda}(\boldsymbol{k}')] = \delta(\boldsymbol{k} - \boldsymbol{k}'). \tag{43}$$

The trace-reversed perturbation $\hat{h}_{ij}(\boldsymbol{r})$ in Eq. (42) can be identified with $\hat{\gamma}_{ij}(\boldsymbol{r}) - (1/2)\eta_{ij}\hat{\gamma}(\boldsymbol{r})$. In particular, in our specific case the $+$ polarization GW $\hat{h}_{11}(\boldsymbol{r})$ can be identified with $\hat{\gamma}_{11}(\boldsymbol{r}) - (1/2)\hat{\gamma}(\boldsymbol{r})$.

Let us first rewrite the interaction term in Eq. (40) using the definitions in Eqs. (18) and (19):

$$\begin{aligned}\hat{H}_{\rm int} = &\frac{\mathcal{A}}{2}\int d\boldsymbol{k}\sqrt{\frac{\hbar}{2\omega_{\boldsymbol{k}}}}([\hat{P}_{00}(\boldsymbol{k}) + \hat{P}(\boldsymbol{k})]\hat{T}_{00}(\boldsymbol{k}) + \text{H.c.}) \\ &+ \mathcal{A}\int d\boldsymbol{k}\sqrt{\frac{\hbar}{2\omega_{\boldsymbol{k}}}}(\hat{P}_{01}(\boldsymbol{k})\hat{T}_{01}(\boldsymbol{k}) + \text{H.c.}) \\ &+ \frac{\mathcal{A}}{2}\int d\boldsymbol{k}\sqrt{\frac{\hbar}{2\omega_{\boldsymbol{k}}}}([\hat{P}_{11}(\boldsymbol{k}) - \hat{P}(\boldsymbol{k})]\hat{T}_{11}(\boldsymbol{k}) + \text{H.c.}),\end{aligned} \tag{44}$$

where we have introduced the Fourier transform of the stress-energy tensor

$$\hat{T}_{\mu\nu}(\boldsymbol{k}) = \frac{1}{\sqrt{(2\pi)^3}}\int d\boldsymbol{r}e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}\hat{T}_{\mu\nu}(\boldsymbol{r}). \tag{45}$$

Since we are considering the two harmonic oscillators to be moving along the x-axis such that the only nonzero components are given by

$$\hat{T}_{\mu\nu}(\boldsymbol{r}) \equiv \frac{\hat{p}_{\mu}\hat{p}_{\nu}}{E/c^2}(\delta(\boldsymbol{r} - \hat{\boldsymbol{r}}_A) + \delta(\boldsymbol{r} - \hat{\boldsymbol{r}}_B)), \tag{46}$$

where $p_{\mu} = (-E/c, \boldsymbol{p})$, $E = \sqrt{\boldsymbol{p}^2c^2 + m^2c^4}$, $\mu, \nu = 0, 1$, and $\hat{\boldsymbol{r}}_A = (\hat{x}_A, 0, 0)$, $\hat{\boldsymbol{r}}_B = (\hat{x}_B, 0, 0)$ denote the positions of the two matter systems. Here we have promoted the classical expression of the stress-energy tensor to a quantum operator following the Weyl quantization prescription to ensure that the quantum stress-energy tensor is a Hermitian operator. In order to simplify the notation we will, however, write the unsymmetrized expressions (e.g., $\hat{x}\hat{p}$), implicitly assuming that all expressions need to be interpreted in the symmetrized ordering [e.g., $(\hat{x}\hat{p} + \hat{p}\hat{x})/2$]. Using Eqs. (45) and (46), we find the following Fourier space components:

$$\hat{T}_{00}(\boldsymbol{k}) = \frac{1}{\sqrt{(2\pi)^3}}(\hat{E}_A e^{i\boldsymbol{k}\cdot\hat{\boldsymbol{r}}_A} + \hat{E}_B e^{i\boldsymbol{k}\cdot\hat{\boldsymbol{r}}_B}), \tag{47}$$

$$\hat{T}_{01}(\boldsymbol{k}) = -\frac{1}{\sqrt{(2\pi)^3}}(\hat{p}_A c e^{i\boldsymbol{k}\cdot\hat{\boldsymbol{r}}_A} + \hat{p}_B c e^{i\boldsymbol{k}\cdot\hat{\boldsymbol{r}}_B}), \tag{48}$$

$$\hat{T}_{11}(\boldsymbol{k}) = \frac{1}{\sqrt{(2\pi)^3}}\left(\frac{\hat{p}_A^2c^2}{E_A}e^{i\boldsymbol{k}\cdot\hat{\boldsymbol{r}}_A} + \frac{\hat{p}_B^2c^2}{E_B}e^{i\boldsymbol{k}\cdot\hat{\boldsymbol{r}}_B}\right). \tag{49}$$

We can readily extend the computation from Sec. V to Eq. (44) by including in the computation the intermediate graviton states: $|\boldsymbol{k}\rangle = \frac{1}{\sqrt{2}}\hat{P}^{\dagger}_{00}(\boldsymbol{k})|0\rangle$, $\frac{1}{\sqrt{2}}\hat{P}^{\dagger}_{11}(\boldsymbol{k})|0\rangle$, $\hat{P}^{\dagger}_{01}(\boldsymbol{k})|0\rangle$, and $\hat{P}^{\dagger}(\boldsymbol{k}))|0\rangle$ (where the prefactor $\frac{1}{\sqrt{2}}$ in the first two states ensured the correct normalization[14]). The energy shift of the graviton vacuum $|0\rangle$ is thus given by the second order perturbation theory (while the first order perturbation will vanish[7]):

$$\Delta\hat{H}_g \equiv \sum \int d\boldsymbol{k} \frac{\langle 0|\hat{H}_{\rm int}|\boldsymbol{k}\rangle\langle\boldsymbol{k}|\hat{H}_{\rm int}|\boldsymbol{0}\rangle}{E_0 - E_{\boldsymbol{k}}}, \tag{50}$$

where the sum indicates summation over the one-particle projectors[15] $|\boldsymbol{k}\rangle\langle\boldsymbol{k}|$ constructed on the unperturbed vacuum, E_0 is the energy of the vacuum state, and $E_{\boldsymbol{k}} = E_0 + \hbar\omega_{\boldsymbol{k}}$ is the energy of the one-particle state. We can readily evaluate

$$\langle 0|\hat{P}(\boldsymbol{k})\hat{H}_{\rm int}|\boldsymbol{0}\rangle = \frac{\mathcal{A}}{2}\sqrt{\frac{\hbar}{2\omega_{\boldsymbol{k}}}}(\hat{T}_{00}(\boldsymbol{k}) - \hat{T}_{11}(\boldsymbol{k})), \tag{51}$$

$$\langle 0|\hat{P}_{01}(\boldsymbol{k})\hat{H}_{\rm int}|\boldsymbol{0}\rangle = \mathcal{A}\sqrt{\frac{\hbar}{2\omega_{\boldsymbol{k}}}}\hat{T}_{01}(\boldsymbol{k}), \tag{52}$$

$$\langle 0|\hat{P}_{00}(\boldsymbol{k})\hat{H}_{\rm int}|\boldsymbol{0}\rangle = \mathcal{A}\sqrt{\frac{\hbar}{2\omega_{\boldsymbol{k}}}}\hat{T}_{00}(\boldsymbol{k}), \tag{53}$$

$$\langle 0|\hat{P}_{11}(\boldsymbol{k})\hat{H}_{\rm int}|\boldsymbol{0}\rangle = \mathcal{A}\sqrt{\frac{\hbar}{2\omega_{\boldsymbol{k}}}}\hat{T}_{11}(\boldsymbol{k}). \tag{54}$$

By using Eqs. (51)–(54), we then find from Eq. (50)

$$\begin{aligned}\Delta\hat{H}_g = &-\mathcal{A}^2\int d\boldsymbol{k}\frac{\hat{T}^\dagger_{00}(\boldsymbol{k})\hat{T}_{00}(\boldsymbol{k}) + \hat{T}^\dagger_{11}(\boldsymbol{k})\hat{T}_{11}(\boldsymbol{k})}{8c^2\boldsymbol{k}^2}\\ &-\mathcal{A}^2\int d\boldsymbol{k}\frac{(\hat{T}^\dagger_{00}(\boldsymbol{k})\hat{T}_{11}(\boldsymbol{k}) + \text{H.c.})}{8c^2\boldsymbol{k}^2}\\ &+4\mathcal{A}^2\int d\boldsymbol{k}\frac{\hat{T}^\dagger_{01}(\boldsymbol{k})\hat{T}_{01}(\boldsymbol{k})}{8c^2\boldsymbol{k}^2}.\end{aligned} \tag{55}$$

We now use the fact that the two particles are confined along the x-axis, where we set $\hat{p}_{Ay} = \hat{p}_{Az} = \hat{p}_{By} = \hat{p}_{Bz} = 0$ and write $\hat{p}_A \equiv \hat{p}_{Ax}$, $\hat{p}_B \equiv \hat{p}_{Bx}$, $\hat{\boldsymbol{r}}_A = (x_A, 0, 0)$, $\hat{\boldsymbol{r}}_B = (x_B, 0, 0)$, and $\boldsymbol{k} = (k_x, k_y, k_z)$. We then insert Eqs. (47)–(49) to find[16]

$$\begin{aligned}\Delta\hat{H}_g = -\frac{\mathcal{A}^2}{(2\pi)^3}\int d\boldsymbol{k}\Bigg(&\frac{\hat{E}_A\hat{E}_B + \frac{\hat{p}_A^2c^2}{E_A}\frac{\hat{p}_B^2c^2}{E_B}}{8c^2\boldsymbol{k}^2}\\ &+\frac{\hat{E}_A\frac{\hat{p}_B^2c^2}{E_B} + \hat{E}_B\frac{\hat{p}_A^2c^2}{E_A}}{8c^2\boldsymbol{k}^2} - 4\frac{\hat{p}_Ac\hat{p}_Bc}{8c^2\boldsymbol{k}^2}\Bigg)\\ &\times(e^{ik_x(\hat{x}_A-\hat{x}_B)} + e^{-ik_x(\hat{x}_A-\hat{x}_B)}).\end{aligned} \tag{56}$$

Performing the integration and expanding in powers of $1/c^2$, we find that Eq. (56) simplifies to[17]

$$\begin{aligned}\Delta\hat{H}_g = &-\frac{Gm^2}{|\hat{x}_A - \hat{x}_B|}\\ &-\frac{G(3\hat{p}_A^2 - 8\hat{p}_A\hat{p}_B + 3\hat{p}_B^2)}{2c^2|\hat{x}_A - \hat{x}_B|}\\ &-\frac{G(5\hat{p}_A^4 - 18\hat{p}_A^2\hat{p}_B^2 + 5\hat{p}_B^4)}{8c^4m^2|\hat{x}_A - \hat{x}_B|}.\end{aligned} \tag{58}$$

Equation (56) contains the exact couplings between the two masses up to order $\mathcal{O}(1/c^4)$ and to the leading order IR contributions in Newton's constant, G. Note that if we set $\hat{p}_A = \hat{p}_B = 0$, the last two terms vanish. However, a quantum system retains its zero point fluctuations, and hence we

[14]The normalization of the states can be computed using the commutation relations in Eqs. (20) and (21). Let us consider first $\hat{P}^\dagger_{00}(\boldsymbol{k})|0\rangle$. We note that $\langle 0|\hat{P}_{00}(\boldsymbol{k})\hat{P}^\dagger_{00}(\boldsymbol{k}')|0\rangle = \langle 0|[\hat{P}_{00}(\boldsymbol{k}), \hat{P}^\dagger_{00}(\boldsymbol{k}')]|0\rangle$, where we have used the definition of the commutator $[\hat{O}_1, \hat{O}_2] = \hat{O}_1\hat{O}_2 - \hat{O}_2\hat{O}_1$ (as well as the definition of the vacuum state). Using (20) we then readily find $\langle 0|\hat{P}_{00}(\boldsymbol{k})\hat{P}^\dagger_{00}(\boldsymbol{k}')|0\rangle = 2\delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}')$. Using analogous steps we find $\langle 0|\hat{P}_{11}(\boldsymbol{k})\hat{P}^\dagger_{11}(\boldsymbol{k}')|0\rangle = 2\delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}')$, $\langle 0|\hat{P}_{01}(\boldsymbol{k})\hat{P}^\dagger_{01}(\boldsymbol{k}')|0\rangle = -\delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}')$, and $\langle 0|\hat{P}(\boldsymbol{k})\hat{P}^\dagger(\boldsymbol{k}')|0\rangle = -\delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}')$.

[15]The projectors $\frac{|\boldsymbol{k}\rangle\langle\boldsymbol{k}|}{\langle\boldsymbol{k}|\boldsymbol{k}\rangle}$ are given by $\frac{1}{2}\hat{P}^\dagger_{00}(\boldsymbol{k})|0\rangle\langle 0|\hat{P}_{00}(\boldsymbol{k})|$, $\frac{1}{2}\hat{P}^\dagger_{11}(\boldsymbol{k})|0\rangle\langle 0|\hat{P}_{11}(\boldsymbol{k})|$, $-\hat{P}^\dagger_{01}(\boldsymbol{k})|0\rangle\langle 0|\hat{P}_{01}(\boldsymbol{k})|$, and $-\hat{P}^\dagger(\boldsymbol{k})|0\rangle\times\langle 0|\hat{P}(\boldsymbol{k})|$. The normalization prefactors $\frac{1}{\langle\boldsymbol{k}|\boldsymbol{k}\rangle} = \frac{1}{2}, \frac{1}{2}, -1, -1$ are a direct consequence of the commutation relations in Eqs. (20) and (21) which fix the normalization of the states.[14] With these definitions of the projectors we find that $\frac{|\boldsymbol{k}\rangle\langle\boldsymbol{k}|}{\langle\boldsymbol{k}|\boldsymbol{k}\rangle}|\boldsymbol{k}\rangle = +1|\boldsymbol{k}\rangle$; i.e., the projectors give a positive eigenvalue $+1$ as expected. In Eq. (50) we are thus implicitly using the normalized projectors $\frac{|\boldsymbol{k}\rangle\langle\boldsymbol{k}|}{\langle\boldsymbol{k}|\boldsymbol{k}\rangle}$ when we write $|\boldsymbol{k}\rangle\langle\boldsymbol{k}|$.

[16]In Eq. (56) we have omitted cross terms between each particle with itself, i.e., terms involving only one of the two particles such as $\sim\hat{E}_A\hat{E}_A, \hat{E}_B\hat{E}_B, \ldots$. Such terms are known as the self-energy terms and do not contribute to the interaction between the two particles. Analogous self-energy terms appear also in electromagnetism when we try to compute the interaction between the two charges (see for example Ref. [51]).

[17]It is instructive to compare the gravitational potential obtained in Eq. (58) to the results for classical point particles in the literature. We first transform from the reference frame of the two traps to the center-of-mass reference frame where we have $p \equiv p_A = -p_B$ and denote $r \equiv |x_A - x_B|$. From Eq. (58) we then find the potential

$$\Delta H_g = -\frac{Gm^2}{r} - 7\frac{Gp^2}{c^2r} - \frac{Gp^4}{c^4m^2r}, \tag{57}$$

which matches the results previously obtained using different methods [55–58].

find $\langle \hat{p}_A^2 \rangle = \langle \hat{p}_B^2 \rangle \sim \hbar m \omega_m$ even for ground states of the two harmonic oscillators [using Eq. (4) and the canonical commutation relations].

Let us make a brief comment on Eq. (58). By quantizing the graviton we have obtained

$$\Delta \hat{H}_g \equiv f(\hat{p}_A, \hat{p}_B, \hat{x}_A, \hat{x}_B), \tag{59}$$

which is an operator-valued shift in the vacuum energy depending on the matter operators. On the other hand, if we would have assumed a classical gravitational field, we could have only generated a real-valued shift ΔH_g in a complete analogy to what we have discussed in Eq. (30).

We will be interested in computing the lowest order corrections for the final matter state $|\psi_f\rangle$ due to the second and third terms on the right-hand side of Eq. (58) (the first term has already been discussed in Sec. V).

VII. COMPUTING THE CONCURRENCE FOR CASE-1

We first discuss the second term on the right-hand side of Eq. (58). We can extract the lowest order nontrivial quantum interaction term[18]:

$$\hat{H}_{AB} \sim 4 \frac{G \hat{p}_A \hat{p}_B}{c^2 d} + \cdots. \tag{60}$$

Note that at the lowest order in the expansion of the denominator, $\hat{x}_A$ and $\hat{x}_B$ do not occur, and the interaction Hamiltonian is dominated by the momentum operators $\hat{p}_A$ and $\hat{p}_B$. We will now use the modes in Eq. (4) to find

$$\hat{H}_{AB} \approx \hbar \mathfrak{g} (\hat{a}^\dagger - \hat{a})(\hat{b}^\dagger - \hat{b}), \tag{61}$$

where the coupling is given by

$$\mathfrak{g} = \frac{2 G m \omega_{\mathrm{m}}}{c^2 d}. \tag{62}$$

As we will see, the only term that is relevant in our case is $\hat{a}^\dagger \hat{b}^\dagger$, which signifies that the final matter state is a linear combination of $|0\rangle|0\rangle$ and $|1\rangle|1\rangle$. In particular, using $\hat{H}_{\mathrm{AB}}$ as the interaction Hamiltonian in Eq. (8) we find that the only nonzero perturbation coefficient emerges from the term $\sim \hat{a}^\dagger \hat{b}^\dagger$, and it is given by

$$C_{11} = -\frac{\mathfrak{g}}{2\omega_{\mathrm{m}}}. \tag{63}$$

Here we have used the fact that energy momentum conservation constraints

$$E_1 = E_0 + \hbar \omega_{\mathrm{m}}. \tag{64}$$

Note that it is twice the frequency of the harmonic oscillators. In addition, we also have the term $C_{00} = 1$, corresponding to the unperturbed state.

We find that the final state in Eq. (7) thus simplifies to (setting $\lambda = 1$)

$$|\psi_{\mathrm{f}}\rangle \equiv \frac{1}{\sqrt{1 + (\mathfrak{g}/(2\omega_{\mathrm{m}}))^2}} \left[|0\rangle|0\rangle - \frac{\mathfrak{g}}{2\omega_{\mathrm{m}}} |1\rangle|1\rangle \right], \tag{65}$$

which is *an entangled state involving the ground and the first excited states of the harmonic oscillators* (up to first order in the perturbation theory). We compute the reduced density matrix by tracing away system B (we recall that our notation is $|n\rangle|N\rangle = |n\rangle_A |N\rangle_B$). The concurrence in Eq. (13) reduces to

$$C \equiv \sqrt{2\left(1 - \frac{1 + (\mathfrak{g}/(2\omega_{\mathrm{m}}))^4}{1 + (\mathfrak{g}/(2\omega_{\mathrm{m}}))^2}\right)} \approx \sqrt{2} \frac{\mathfrak{g}}{\omega_{\mathrm{m}}}, \tag{66}$$

which is valid when the parameter $\mathfrak{g}/\omega_{\mathrm{m}} \ll 1$. After inserting the coupling from Eq. (62), we find the concurrence to be

$$C = \frac{2\sqrt{2} G m}{c^2 d}. \tag{67}$$

Note that the degree of entanglement grows linearly with the mass of the harmonic oscillators, does not depend on the frequency, and scales inversely with the distance between the two oscillators.

We find that the concurrence in the case of a static limit given in Eq. (39) dominates over the nonstatic case, provided

$$\frac{\omega_{\mathrm{m}} d}{c} < \frac{1}{\sqrt{2}}. \tag{68}$$

For example, with $\omega_{\mathrm{m}} \sim 10^8$ Hz we find that the threshold value is obtained already at $d \sim 1$ m. Hence, such effects could in principle be tested already with a small tabletop experiment, but the feasibility of the experiment has to be studied separately.

VIII. COMPUTING THE CONCURRENCE FOR CASE-2

From the last term in Eq. (58) we can extract the lowest order nontrivial quantum interaction term[19]:

[18]Intuitively, it is again interesting to estimate the entanglement phase. We find $\Delta\phi \sim 4 G p_a p_B \tau/(c^2 \hbar d)$, where τ is the coherence timescale. As expected, such effects are thus typically suppressed in comparison to the phase accumulated from the exchange of graviton in the static case[11].

[19]It is again interesting to estimate the approximate entanglement phase. We find $\Delta\phi \sim 9 G p_a^2 p_B^2 \tau/(4 c^4 m^2 \hbar d)$, where τ is the coherence timescale. As expected such effects are thus typically suppressed in comparison to the phase accumulated from the exchange of the graviton in the static case[11].

$$\hat{H}_{AB} \sim -\frac{9G\hat{p}_A^2\hat{p}_B^2}{4c^4m^2d} + \cdots. \tag{69}$$

Note that at the lowest order in the expansion of the denominator, $\hat{x}_A$ and $\hat{x}_B$ do not occur, and the interaction Hamiltonian is dominated by the momentum operators $\hat{p}_A$ and $\hat{p}_B$. We will now use the modes in Eq. (4) to find

$$\hat{H}_{AB} \approx -\hbar\mathfrak{g}(\hat{a}^\dagger - \hat{a})^2(\hat{b}^\dagger - \hat{b})^2, \tag{70}$$

where the coupling is given by

$$\mathfrak{g} = \frac{9G\hbar\omega_{\mathrm{m}}^2}{16c^4d}. \tag{71}$$

The only term that is relevant in our case is $(\hat{a}^\dagger\hat{b}^\dagger)^2$, which signifies that the final matter state is a linear combination of $|0\rangle|0\rangle$ and $|2\rangle|2\rangle$. Hence, at the lowest order the gravitons carry twice the energy of the harmonic oscillators, i.e., $\omega_{\boldsymbol{k}} = 2\omega_{\boldsymbol{m}}$.

In particular, using $\hat{H}_{\mathrm{AB}}$ as the interaction Hamiltonian in Eq. (8), we find that the only nonzero perturbation coefficient emerges from the term $\sim\hat{a}^{\dagger 2}\hat{b}^{\dagger 2}$ and is given by

$$C_{22} = \frac{\mathfrak{g}}{2\omega_{\mathrm{m}}}. \tag{72}$$

Here we have used the fact that energy momentum conservation constraints

$$E_2 = E_0 + 2\hbar\omega_{\mathrm{m}}. \tag{73}$$

Note that it is twice the frequency of the harmonic oscillators. In addition, we also have the term $C_{00} = 1$, corresponding to the unperturbed state.

We find that the final state in Eq. (7) thus simplifies to (setting $\lambda = 1$)

$$|\psi_{\mathrm{f}}\rangle \equiv \frac{1}{\sqrt{1+(\mathfrak{g}/(2\omega_{\mathrm{m}}))^2}}\left[|0\rangle|0\rangle + \frac{\mathfrak{g}}{2\omega_{\mathrm{m}}}|2\rangle|2\rangle\right], \tag{74}$$

which is *an entangled state involving the ground and the second excited states of the harmonic oscillators* (up to first order in the perturbation theory).

Note that the occurrence of the second excited states from the initial ground states requires the transition $n \to n+2$, where n is the number eigenvalue of the harmonic oscillator. This distinct $n \to n+2$ transition can be traced back to the coupling to the gravitational field [see Eqs. (58), (69), and (70)]. In particular, it emerges from the coupling $\propto \hat{h}_{11}\hat{T}_{11}$, where $\hat{h}_{11}$ can be identified with the degrees of freedom associated with the "+" gravitational waves.[13] In our case we have $\hat{T}_{11} \sim (\hat{a}^\dagger)^2, (\hat{b}^\dagger)^2$, and thus we find the couplings $(\hat{a}^\dagger)^2\hat{h}_{11}$ and $(\hat{b}^\dagger)^2\hat{h}_{11}$, which lead to the transition $n \to n+2$ for the two harmonic oscillators. In general, one can expect the transitions $n \to n \pm 2$ whenever we have a coupling of the gravitational field to a harmonic oscillator.[20] For example, it occurs also in the case of absorption/emission of GWs of a specific polarization "+" [23].

We now compute the reduced density matrix by tracing away system B (we recall that our notation is $|n\rangle|N\rangle = |n\rangle_A|N\rangle_B$). The concurrence in Eq. (13) reduces to

$$C \equiv \sqrt{2\left(1 - \frac{1+(\mathfrak{g}/(2\omega_{\mathrm{m}}))^4}{1+(\mathfrak{g}/(2\omega_{\mathrm{m}}))^2}\right)} \approx \sqrt{2}\frac{\mathfrak{g}}{\omega_{\mathrm{m}}}, \tag{77}$$

which is valid when the parameter $\mathfrak{g}/\omega_{\mathrm{m}} \ll 1$. After inserting the coupling from Eq. (71), we find the concurrence to be

$$C = \frac{9\sqrt{2}G\hbar\omega_{\mathrm{m}}}{16c^4d}. \tag{78}$$

Note that the degree of entanglement grows linearly with the frequency of the harmonic oscillators, does not depend on the mass, and scales inversely with the distance between the two oscillators. The concurrence in the case of the exchange of a graviton in the static limit dominates over the nonstatic case, provided

$$\omega_{\mathrm{m}}^2 < \frac{mc^4}{\hbar\omega_{\mathrm{m}}d^2}. \tag{79}$$

[20]We will bring an intuitive understanding on the origin of the transition $n \to n+2$. We can decompose the gravitational field into the plane waves $\sim e^{-i(\omega_k t - kx)}$, and Taylor expand in small displacements up to order $\mathcal{O}(x^2)$:

$$\hat{h}_{11}(t,x) \sim \hat{h}_{11}(t,0) + \left.\frac{\partial\hat{h}_{11}(t,x)}{\partial x}\right|_{x=0} ikx - \frac{1}{2}\left.\frac{\partial^2\hat{h}_{11}(t,x)}{\partial x^2}\right|_{x=0} k^2x^2 + \cdots, \tag{75}$$

where $k = \omega_k/c$ and ω_k is the angular frequency of the gravitational field mode. The first term on the right-hand side of Eq. (75) is a constant and can be omitted, while the second linear term $\sim kx$ can be shown to vanish by considering the Fermi normal coordinates [59], as a consequence of the equivalence principle. From the remaining last term, we thus find:

$$\hat{h}_{11}(t,x) \sim -\frac{1}{2c^2}\left.\frac{\partial^2\hat{h}_{11}(t,x)}{\partial t^2}\right|_{x=0} x^2, \tag{76}$$

where we have used $\omega_k = kc$. From the gravitational coupling to the matter component, $\hat{T}_{11}(x) \propto \delta(\hat{x} - x)$, where $\hat{x}$ is the position of the harmonic oscillator, we find the required quadratic coupling, i.e., $\hat{h}_{11}\hat{T}_{11} \propto \hat{x}^2$. It is this matter-gravity coupling $\propto\hat{x}^2$ that leads to the transition $n \to n+2$. Since $\hat{x} \propto (\hat{a} + \hat{a}^\dagger)$ and $\hat{x} \propto (\hat{b} + \hat{b}^\dagger)$, we find the terms $(\hat{a}^\dagger)^2$ and $(\hat{b}^\dagger)^2$, respectively. Combining these two terms, we then get precisely the term $(\hat{a}^\dagger\hat{b}^\dagger)^2$ that we found using the perturbation theory [see derivation below Eq. (69)].

In the original QGEM proposal [14], the proposed inter-separation distance between the two quantum superposition of particles with mass $m \sim 10^{-14}$ kg is kept roughly at $d \sim 100 \times 10^{-6}$ m in order to avoid Casimir induced entanglement [14]. If we wish to witness the entanglement in the nonstatic case, we would require extremely high frequency oscillators [i.e., from Eqs. (79) we find $\omega_{\rm m} \gtrsim 10^{21}$ Hz], beyond the reach of the current state of the art in a laboratory.

Let us highlight the link between LOCC/LOQC and the quantized graviton. If the graviton were treated classically, then the final state of the two harmonic oscillator states would have never evolved to an entangled state such as Eqs. (65) and (74)—in this case this amounts to $\hat{\gamma}_{11}$ and $\hat{\gamma}$ components. Indeed, a classical field is unable to give the operator-valued shift of the vacuum energy in Eq. (59) which led to the quantum coupling in Eq. (61) (i.e., a cross product of matter operators).

IX. DISCUSSION

In this paper we have considered a specific example to reinforce the importance of the quantum gravitational interaction in the QGEM protocol. The crucial observation here is that the quantum nature of the gravitational interaction yields an operator valued shift in the gravitational Hamiltonians, $\Delta\hat{H}_g$ [see Eqs. (30) and (58)]. Classical gravity will only yield a real-valued shift in ΔH_g.

In particular, we considered the two quantum harmonic oscillators separated by a distance d interacting via the exchange of a graviton comprising the spin-2 and spin-0 components. We have shown that the quantum nature of the graviton [for both spin-2 and spin-0, $\hat{h}_{00} \equiv \hat{\gamma}_{00} + (1/2)\hat{\gamma}$] is essential to create an entangled state with the ground and excited states of the harmonic oscillators forming the Schmidt basis.

Similar physics arises in the nonstatic case as well. The quantum nature of the graviton (i.e., $\hat{h}_{01} \equiv \hat{\gamma}_{01}$ component) will generate a two-mode squeezed state of the two harmonic oscillators, Eq. (65). On the other hand, the $\hat{h}_{11} \equiv \hat{\gamma}_{11} - (1/2)\hat{\gamma}$ component is crucial to entangle with the ground and the second excited states of the harmonic oscillators. It is also interesting to note that these latter states, Eq. (74), have never been presented previously, to our knowledge, in any context in the vast literature on entangled harmonic oscillators (for example in the quantum optics or in the allied literature). They are particular to the nature of spin-2 graviton.

We have obtained all the results relying only on the elementary perturbation theory; the wave function was evaluated up to the first order, and the correction to the graviton vacuum was computed up to the second order (to obtain the nonvanishing contribution to the vacuum energy). Both the wave function calculations and the correction to the energy of the vacuum suggest that the *quantum interaction between the graviton and the matter* is crucial to obtain entanglement, reinforcing that the LOCC cannot yield or lead to the increment in the entanglement.[21]

We computed the entanglement concurrence and showed that the concurrence is always positive for a quantum gravitational field (indicating entanglement), but would remain zero for a classical gravitational field (no entanglement). Moreover, the entanglement can be regarded as due to the operator valued shifts of the vacuum energy.

So far we have kept our investigation limited to the local quantum interaction between matter and the gravitational field—our $\hat{H}_{\rm int}$ was strictly local. It would be interesting to study what would happen if the locality in the gravitational interaction is abandoned [15,60–63]. Giving up local gravitational interaction will help us to further investigate the entanglement in theories beyond GR, and in quantum theories of gravity where nonlocal interactions enter in various manifestations (see [3,64–66]). We can also attempt to compute the entanglement by modifying the graviton propagator in nonperturbative formulations of quantum gravity [67,68]. Similar computations to the entanglement can be computed within perturbative quantum gravity but with higher post-Newtonian Hamiltonians in $3+1$ dimensions (see [69,70]).

In summary, our results corroborate the importance of the QGEM experiment, which relies on the fact that the two quantum superposed masses kept at a distance can entangle via the quantum nature of the graviton. This would be crucial in unveiling the quantum properties of the spin-2 graviton which is hitherto a hypothetical particle responsible for the fluctuations of the spacetime in the context of a perturbative quantum gravity.

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[21]If one limits the discussion to the nonrelativistic models of gravity and simply postulates the interaction term as $\sim 1/|\hat{\boldsymbol{r}}_a - \hat{\boldsymbol{r}}_b|$, one cannot say much about the underlying dynamical degrees of freedom of the gravitation field. Here, we have shown that in the perturbative canonical quantum theory of gravity we can account for the dynamical degrees of freedom. These are crucial to obtain the correct shift in the operator valued gravitational energy which gives rise to the quantum matter-matter interaction. Other theories beyond GR would require a similar analysis of the dynamical degrees of freedom of the gravitational field.

[1] C. M. Will, The confrontation between general relativity and experiment, Living Rev. Relativity **17**, 4 (2014).
[2] S. W. Hawking and G. F. R. Ellis, *The Large Scale Structure of Space-Time* (Cambridge University Press, Cambridge, England, 1973).
[3] C. Kiefer, *Quantum Gravity*, Springer Handbook of Spacetime (Springer, Berlin, Heidelberg, 2014), pp. 709–722.
[4] S. N. Gupta, Quantization of Einstein's gravitational field: Linear approximation, Proc. Phys. Soc. London Sect. A **65**, 161 (1952).
[5] S. N. Gupta, Gravitation and electromagnetism, Phys. Rev. **96**, 1683 (1954).
[6] R. P. Feynman, Quantum theory of gravitation, Acta Phys. Pol. **24**, 697 (1963).
[7] B. S. DeWitt, Quantum theory of gravity. 1. The canonical theory, Phys. Rev. **160**, 1113 (1967); Quantum theory of gravity. 2. The manifestly covariant theory?, Phys. Rev. **162**, 1195 (1967).
[8] J. F. Donoghue, General relativity as an effective field theory: The leading quantum corrections, Phys. Rev. D **50**, 3874 (1994).
[9] F. Dyson, Is a graviton detectable, in *XVIIth International Congress on Mathematical Physics* (World Scientific, Singapore, 2014), pp. 670–682.
[10] A. Ashoorioon, P. S. Bhupal Dev, and A. Mazumdar, Implications of purely classical gravity for inflationary tensor modes, Mod. Phys. Lett. A **29**, 1450163 (2014).
[11] J. Martin and V. Vennin, Obstructions to Bell CMB experiments, Phys. Rev. D **96**, 063501 (2017).
[12] A. Addazi *et al.*, Quantum gravity phenomenology at the dawn of the multi-messenger era—A review, Prog. Part. Nucl. Phys. **125**, 103948 (2022).
[13] B. P. Abbott *et al.* (LIGO Scientific and Virgo Collaborations), Tests of General Relativity with GW150914, Phys. Rev. Lett. **116**, 221101 (2016); Erratum, Phys. Rev. Lett. **121**, 129902 (2018).
[14] S. Bose, A. Mazumdar, G. W. Morley, H. Ulbricht, M. Toroš, M. Paternostro, A. Geraci, P. Barker, M. S. Kim, and G. Milburn, Spin Entanglement Witness for Quantum Gravity, Phys. Rev. Lett. **119**, 240401 (2017).
[15] R. J. Marshman, A. Mazumdar, and S. Bose, Locality and entanglement in table-top testing of the quantum nature of linearized gravity, Phys. Rev. A **101**, 052110 (2020).
[16] C. Marletto and V. Vedral, Gravitationally Induced Entanglement between Two Massive Particles is Sufficient Evidence of Quantum Effects in Gravity, Phys. Rev. Lett. **119**, 240402 (2017).
[17] A. Belenchia, R. M. Wald, F. Giacomini, E. Castro-Ruiz, C. Brukner, and M. Aspelmeyer, Quantum superposition of massive objects and the quantization of gravity, Phys. Rev. D **98**, 126009 (2018).
[18] D. L. Danielson, G. Satishchandran, and R. M. Wald, Gravitationally mediated entanglement: Newtonian field vs gravitons, Phys. Rev. D **105**, 086001 (2022).
[19] H. C. Nguyen and F. Bernards, Entanglement dynamics of two mesoscopic objects with gravitational interaction, arXiv:1906.11184.
[20] J. S. Pedernales, G. W. Morley, and M. B. Plenio, Motional Dynamical Decoupling for Matter-Wave Interferometry, Phys. Rev. Lett. **125**, 023602 (2020).
[21] R. J. Marshman, A. Mazumdar, R. Folman, and S. Bose, Large splitting massive Schrödinger kittens, Phys. Rev. Research **4**, 023087 (2022).
[22] T. W. van de Kamp, R. J. Marshman, S. Bose, and A. Mazumdar, Quantum gravity witness via entanglement of masses: Casimir screening, Phys. Rev. A **102**, 062807 (2020).
[23] M. Toroš, A. Mazumdar, and S. Bose, Loss of coherence of matter-wave interferometer from fluctuating graviton bath, arXiv:2008.08609.
[24] H. Chevalier, A. Paige, and M. Kim, Witnessing the nonclassical nature of gravity in the presence of unknown interactions, Phys. Rev. A **102**, 022428 (2020).
[25] M. Toroš, T. W. Van De Kamp, R. J. Marshman, M. S. Kim, A. Mazumdar, and S. Bose, Relative acceleration noise mitigation for nanocrystal matter-wave interferometry: Applications to entangling masses via quantum gravity, Phys. Rev. Research **3**, 023178 (2021).
[26] M. Christodoulou and C. Rovelli, On the possibility of laboratory evidence for quantum superposition of geometries, Phys. Lett. B **792**, 64 (2019).
[27] M. Christodoulou and C. Rovelli, On the possibility of experimental detection of the discreteness of time, Front. Phys. **8**, 207 (2020).
[28] Y. Margalit, O. Dobkowski, Z. Zhou, O. Amit, Y. Japha, S. Moukouri, D. Rohrlich, A. Mazumdar, S. Bose, C. Henkel *et al.*, Realization of a complete stern-gerlach interferometer: Toward a test of quantum gravity, Sci. Adv. **7**, eabg2879 (2021).
[29] J. Tilly, R. J. Marshman, A. Mazumdar, and S. Bose, Qudits for witnessing quantum-gravity-induced entanglement of masses under decoherence, Phys. Rev. A **104**, 052416 (2021).
[30] D. Carney, H. Müller, and J. M. Taylor, Using an atom interferometer to infer gravitational entanglement generation, PRX Quantum **2**, 030330 (2021).
[31] D. Carney, H. Müller, and J. M. Taylor, Comment on "Using an atom interferometer to infer gravitational entanglement generation", arXiv:2111.04667.
[32] K. Streltsov, J. S. Pedernales, and M. B. Plenio, On the significance of interferometric revivals for the fundamental description of gravity, Universe **8**, 58 (2022).
[33] R. Howl, V. Vedral, D. Naik, M. Christodoulou, C. Rovelli, and A. Iyer, Non-Gaussianity as a signature of a quantum theory of gravity, PRX Quantum **2**, 010325 (2021).
[34] D. Miki, A. Matsumura, and K. Yamamoto, Entanglement and decoherence of massive particles due to gravity, Phys. Rev. D **103**, 026017 (2021).
[35] A. Matsumura and K. Yamamoto, Gravity-induced entanglement in optomechanical systems, Phys. Rev. D **102**, 106021 (2020).
[36] S. Qvarfort, S. Bose, and A. Serafini, Mesoscopic entanglement through central potential interactions, J. Phys. B **53**, 235501 (2020).
[37] H. Miao, D. Martynov, H. Yang, and A. Datta, Quantum correlations of light mediated by gravity, Phys. Rev. A **101**, 063804 (2020).
[38] M. Schut, J. Tilly, R. J. Marshman, S. Bose, and A. Mazumdar, Improving resilience of the Quantum Gravity Induced Entanglement of Masses (QGEM) to decoherence using 3 superpositions, Phys. Rev. A **105**, 032411 (2022).

[39] A. Datta and H. Miao, Signatures of the quantum nature of gravity in the differential motion of two masses, Quantum Sci. Technol. **6**, 045014 (2021).

[40] T. Krisnanda, G. Y. Tham, M. Paternostro, and T. Paterek, Observable quantum entanglement due to gravity, npj Quantum Inf. **6**, 12 (2020).

[41] F. Cosco, J. S. Pedernales, and M. B. Plenio, Enhanced force sensitivity and entanglement in periodically driven optomechanics, Phys. Rev. A **103**, L061501 (2021).

[42] T. Weiss, M. Roda-Llordes, E. Torrontegui, M. Aspelmeyer, and O. Romero-Isart, Large Quantum Delocalization of a Levitated Nanoparticle Using Optimal Control: Applications for Force Sensing and Entangling via Weak Forces, Phys. Rev. Lett. **127**, 023601 (2021).

[43] S. Rijavec, M. Carlesso, A. Bassi, V. Vedral, and C. Marletto, Decoherence effects in non-classicality tests of gravity, New J. Phys. **23**, 043040 (2021).

[44] B. Yi, U. Sinha, D. Home, A. Mazumdar, and S. Bose, Massive spatial qubits for testing macroscopic non-classicality and casimir induced entanglement, arXiv:2106.11906v2.

[45] C. H. Bennett, D. P. DiVincenzo, J. A. Smolin, and W. K. Wootters, Mixed-state entanglement and quantum error correction, Phys. Rev. A **54**, 3824 (1996).

[46] V. Balasubramanian, M. B. McDermott, and M. Van Raamsdonk, Momentum-space entanglement and renormalization in quantum field theory, Phys. Rev. D **86**, 045014 (2012).

[47] A. Zee, *Quantum Field Theory in a Nutshell* (Princeton University Press, Princeton, NJ, 2010), Vol. 7.

[48] S. Hill and W. K. Wootters, Entanglement of a Pair of Quantum Bits, Phys. Rev. Lett. **78**, 5022 (1997).

[49] W. P. Bowen and G. J. Milburn, *Quantum Optomechanics* (CRC Press, Boca Raton, 2016).

[50] P. Rungta, V. Bužek, C. M. Caves, M. Hillery, and G. J. Milburn, Universal state inversion and concurrence in arbitrary dimensions, Phys. Rev. A **64**, 042315 (2001).

[51] C. Cohen-Tannoudji, J. Dupont-Roc, and G. Grynberg, *Photons and Atoms: Introduction to Quantum Electrodynamics* (J. Wiley, New York, 1997).

[52] M. D. Scadron, *Advanced Quantum Theory and Its Applications Through Feynman Diagrams* (Springer-Verlag, Berlin, Heidelberg, 1991).

[53] D. Kafri, J. M. Taylor, and G. J. Milburn, A classical channel model for gravitational decoherence, New J. Phys. **16**, 065020 (2014).

[54] S. Weinberg, *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity* (John Wiley & Sons, New York, 1972).

[55] G. Grignani, T. Harmark, M. Orselli, and A. Placidi, Fixing the non-relativistic expansion of the 1PM potential, J. High Energy Phys. 12 (2020) 001.

[56] A. Cristofoli, N. E. Bjerrum-Bohr, P. H. Damgaard, and P. Vanhove, Post-Minkowskian Hamiltonians in general relativity, Phys. Rev. D **100**, 084040 (2019).

[57] Y. Iwasaki, Quantum theory of gravitation vs classical theory: Fourth-order potential, Prog. Theor. Phys. **46**, 1587 (1971).

[58] K. Hiida and H. Okamura, Gauge transformation and gravitational potentials, Prog. Theor. Phys. **47**, 1743 (1972).

[59] M. Rakhmanov, Fermi-normal, optical, and wave-synchronous coordinates for spacetime with a plane gravitational wave, Classical Quantum Gravity **31**, 085006 (2014).

[60] E. T. Tomboulis, Superrenormalizable gauge and gravitational theories, arXiv:hep-th/9702146.

[61] T. Biswas, A. Mazumdar, and W. Siegel, Bouncing universes in string-inspired gravity, J. Cosmol. Astropart. Phys. 03 (2006) 009.

[62] T. Biswas, E. Gerwick, T. Koivisto, and A. Mazumdar, Towards Singularity and Ghost Free Theories of Gravity, Phys. Rev. Lett. **108**, 031101 (2012).

[63] L. Modesto, Super-renormalizable quantum gravity, Phys. Rev. D **86**, 044005 (2012).

[64] C. de Lacroix, H. Erbin, S. P. Kashyap, A. Sen, and M. Verma, Closed superstring field theory and its applications, Int. J. Mod. Phys. A **32**, 1730021 (2017).

[65] R. Loll, Quantum gravity from causal dynamical triangulations: A review, Classical Quantum Gravity **37**, 013002 (2020).

[66] R. D. Sorkin, Does locality fail at intermediate length-scales, arXiv:gr-qc/0703099.

[67] C. Rovelli, Graviton Propagator from Background-Independent Quantum Gravity, Phys. Rev. Lett. **97**, 151301 (2006).

[68] E. Bianchi, L. Modesto, C. Rovelli, and S. Speziale, Graviton propagator in loop quantum gravity, Classical Quantum Gravity **23**, 6989 (2006).

[69] Z. Bern, C. Cheung, R. Roiban, C. H. Shen, M. P. Solon, and M. Zeng, Scattering Amplitudes and the Conservative Hamiltonian for Binary Systems at Third Post-Minkowskian Order, Phys. Rev. Lett. **122**, 201603 (2019).

[70] D. Bini, T. Damour, and A. Geralico, Binary dynamics at the fifth and fifth-and-a-half post-Newtonian orders, Phys. Rev. D **102**, 024062 (2020).