

Expanding edges of quantum Hall systems in a cosmology language: Hawking radiation from de Sitter horizon in edge modes

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Expanding edge experiments are promising to open new physics windows of quantum Hall systems. In a static edge, the edge excitation, which is described by free fields decoupled with the bulk dynamics, is gapless, and the dynamics preserve conformal symmetry. When the edge expands, such properties need not be preserved. We formulate a quantum field theory in $1 + 1$ -dimensional curved spacetimes to analyze the edge dynamics. We propose methods to address the following questions using edge waveforms from the expanding region: Does the conformal symmetry survive? Is the nonlinear interaction of the edge excitations induced by edge expansion? Do the edge excitations interact with the bulk excitations? We additionally show that the expanding edges can be regarded as expanding universe simulators of two-dimensional dilaton-gravity models, including the Jackiw-Teitelboim gravity model. As an application, we point out that our theoretical setup might simulate emission of analog Hawking radiation with the Gibbons-Hawking temperature from the future de Sitter horizon formed in the expanding-edge region.

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I. INTRODUCTION

How did our Universe develop from its earliest moments? To answer this major question of quantum cosmology, great efforts have been made both in theory and in the attempts of verifying the theory by observations of the cosmic background radiation and black holes. An ideal scenario would be the reproduction of the origin and evolution of the Universe or producing black holes in a laboratory and thereby experimentally verifying the theory through controlling key parameters. Regarding this, analog experiments have been performed in various systems [1–4].

Quantum Hall (QH) systems are unique and promising. A QH system [5,6] emerges when a strong perpendicular magnetic field (B) is applied to two-dimensional (2D) electrons when the Landau level filling factor $\nu = 2\pi\hbar n_e/(eB)$ becomes an integer or a rational fraction, where \hbar and e are the reduced Planck constant and the elementary charge, respectively.

The theoretical study of QH systems has contributed to modern understanding of condensed matter physics.

Notably, the QH systems are regarded as typical topological materials consisting of the bulk and edge. The dynamics in the bulk yield a large energy gap in its dispersion relations. In the bulk, there exist various gapped excitations such as magnetorotons [7–10] and anyonic quasiparticles [11–13]. The dispersion relation of the edge currents are firmly protected owing to the topological structure of the systems, and the edge excitations are always in gapless modes. The effective theory of the bulk is given by a topological field theory referred to as the Chern-Simons gauge theory [14–18]. In the topological field theory, no local dynamics appears, and properties of the system are stable under local perturbations in the bulk. The bulk action is not invariant under the gauge transformation at the edge. This unsatisfactory gauge-variant behavior of the bulk is compensated by adding a gauge-variant edge action with quantum anomaly. Hence, this predicts the edge dynamics [19]. It is possible to express the same edge degrees of freedom by both a fermionic field and bosonic field via the statistics transmutation in one-dimensional space. It is known that the bosonic field corresponds to the charge density of the edge current and can be directly observed by measuring the voltage deviation in the edge experiments [10,20–25]. The edge effective theories are given by free field theories with a chiral condition. Here, the chiral condition implies that the excitations move in one direction

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along the edge and do not return to the upstream region. The theories belong to a class of conformal field theory (CFT) in $1 + 1$ spacetime dimensions, which preserves an infinite-dimensional conformal symmetry, referred to as the Virasoro symmetry [26]. The total gauge symmetry of the bulk-edge composite system ensures the gapless property of the edge excitations.

Until now, all experiments of QH systems have been performed in a static situation, except for local edge excitations [10,20–24]. The electrons are confined in the bulk region by the static electric field created by the surface potential of host semiconductors of the 2D electrons. Thus, the edge attached to the bulk remains unchanged in time. In Ref. [27], expanding edges were proposed. The edge expands by gradually relaxing the external electric fields through continuous electron supply into the bulk. The excitations moving along the edge are affected by the expansion. In such situations, it is quite nontrivial that the conformal symmetry still holds. Recall that the static conformal symmetry is protected by topological properties of QH systems which are insensitive to microscopic details. It is known that such topologically robust phenomena are not limited to static systems but may also emerge in some periodically driven quantum systems [28]. From this viewpoint, it can be expected that the conformal symmetry still survives in the expanding QH edges. On the other hand, there exists a risk of terminating the conformal symmetry in the realistic experiments. It is considered that the conformal symmetry does not survive in some $1 + 1$ -dimensional acoustic systems described by a minimally coupled massless scalar field [29]. Also the conformal symmetry can be broken by inhomogeneity induced by impurities and/or time dependence of the realistic QH systems. About the effective field theory for expanding-edge systems, it should be stressed that answers to the following fundamental questions are unknown at present:

- (1) Does the edge current remain gapless in the expansion? Specifically, is the conformal symmetry preserved at the boundary?
- (2) Beyond the free field theory, does nontrivial interaction among edge excitations emerge in the expansion?
- (3) During the expansion, do excitations in the edge interact with gapped excitations in the bulk?

The expanding-edge experiments will provide crucial results for the answers to the above questions. Note that the experiments are able to achieve this aim even in classical regimes of the systems. Thus, the experiments are quite promising to reveal new physics of the systems under such dynamical backgrounds.

In quantum regimes with extremely low noise, QH edges may be applied to future development in quantum information science. The edge excitations transport quantum states, i.e., quantum information along an edge connected between distant points inside a quantum device. This means

that the QH edges play a role of quantum channels. In such situations, the expanding region provides a useful quantum gate referred to as quantum squeezing for the quantum states [27]. In addition, measurement of zero-point fluctuation at the downstream of the edge and its feedback to the upstream are capable of extracting local zero-point energy at the upstream accompanied by generation of a negative energy density region in the edge [25]. The protocols are referred to as quantum energy teleportation (QET) [30–32]. The inevitable energy cost E_{in} of the measurement at the downstream is regarded as the input energy of QET, and the extracted positive energy E_{out} at the upstream is regarded as the output energy of QET satisfying $0 < E_{\text{out}} < E_{\text{in}}$. It is expected that long-distance QET may be attained utilizing quantum entanglement of the expanding-edge systems [27].

The proposed experiment of expanding edges in [27] will also have large implications to cosmology. The expanding edge can be regarded as a simulator for the expanding universes in $1 + 1$ spacetime dimensions. In particular, the charge density of the edge current is capable of playing the analog role of a quantum field in curved spacetimes. In curved spacetime field theories, many intriguing phenomena are predicted [33]. One of these phenomena is the Hawking radiation emission out of black hole horizons [34]. The black hole evaporation induced by Hawking radiation has not yet been confirmed in astrophysical observations, but many believe that black holes are evaporating. If a black hole completely evaporated and its Hawking radiation remained in space, the thermal radiation would be in a mixed state with a single parameter, that is, its temperature. Thus, at a glance, the radiation does not appear to carry the huge amount of information originally stored inside black holes. Where has the information gone? Suppose that the initial state of the black hole is a pure state and that the evolution to the mixed state is realized resulting in a loss of quantum coherence without any environmental interaction. Then unitarity would be broken, which is one of fundamental laws of quantum mechanics. This unresolved problem is referred to as the information loss problem.

In the exploration of information loss, crucial missing links of Hawking's original analysis are known [35]. For example, his calculation is only semiclassical. The matter is quantized, but the spacetime remains classical in his analysis. Thus, his theory does not cover the final burst of black holes. In the theory of general relativity for classical spacetime, a curvature singularity appears and loses predictability by the theory after the burst. To avoid this flaw, quantum theory is required, which can treat quantum spacetimes with no singularities. Although several candidates such as string theory exist, quantum gravity theory remains elusive owing to the lack of strong guiding principles, actual measurements, and convincing observations to determine the theory uniquely. To resolve the

information loss, any findings and inspiration, which are delivered by analog black hole experiments in condensed matter physics including QH systems, are considered useful [36].

Such analog experiments also shed light on the trans-Planckian problem [37]. The typical wavelength of a thermal quantum particle emitted from a black hole with mass M is computed as $\lambda = O(GM/c^2)$, using the Hawking temperature $T = \hbar c^3/(8\pi k_B GM)$ [34]. Here, c is the velocity of light, G is the gravitational constant, and k_B is the Boltzmann constant. The particle experiences severe redshift by gravitational potential when it propagates to spatial infinity. At the previous point near the horizon where the particle is created, the wavelength of the particle mode is much shorter than the Planck length $l_P = \sqrt{G\hbar/c^3}$. Thus, a precise description of the mode in the past regime is required for fundamental microscopic theories such as string theory. It is possible that Hawking's approximation is incorrect and has serious discrepancy. However, his results appear to be correct. In reality, the formula for Hawking temperature T and derived entropy $S = k_B c^3 \mathcal{A}/(4G\hbar)$ for black holes with horizon area \mathcal{A} are consistent with the other theoretical results of generalized thermodynamics [38,39] and statistical mechanics with state counting in string theory [40]. Thus, why Hawking's analysis works so well despite being semiclassical is a mystery. This is called the trans-Planckian problem.

We can reconsider the trans-Planckian problem in condensed matter physics. If Hawking's prediction is correct, analog black holes also emit Hawking radiation, no matter what the analog black holes are made of [41]. Analog systems corresponding to black holes possess natural cutoff length, just like the magnetic length $l_B = \sqrt{\hbar/(eB)}$ of electron in QH systems. This phenomenon poses an analog version of the trans-Planckian problem by regarding l_B as l_P . Thus, when the original problem is considered, it is significant to analyze the analog problem. Notably, the trans-Planckian problem also appears in an inflationary expanding universe owing to the existence of the cosmological horizon in the universe. Hawking radiation is emitted out of the horizon and provides nondecaying thermal fluctuation even in the extreme expansion of the universe. Therefore, expanding-edge experiments using QH systems are also appropriate for exploration of the corresponding trans-Planckian problems associated with an expanding universe.

In the inflationary universe, there exists another conceptual issue related to quantum field theory in the expanding universe. The accelerated expansion in the inflationary universe provides large-scale quantum fluctuations of the inflaton field over the Hubble horizon scale. This primordial fluctuation leads to gravitational instabilities that ultimately form large-scale structures in our Universe [42]. However, it is not known how the quantum-classical transition of field fluctuation occurs in a long

wavelength region beyond the Hubble horizon scale. Because the details and mechanisms of the transition process are not known, it remains far from profound understanding. In the analysis, quantum entanglement is known to be capable of capturing the quantum-classical transition [43–46]. The bipartite entanglement between two spatial regions will be lost in the course of evolution [43,46], and entanglement harvesting from spatially separated regions becomes impossible [44,45]. Additionally a study of entanglement structure in quantum states in de Sitter space [47] may be useful to resolve the issue. It is possible to simulate the generation of analog primordial fluctuations experimentally, using expanding analog universes of QH systems. We are able to detect and study the entanglement of long wavelength quantum fluctuations in the expanding universe. This direction of analog experiments in QH systems facilitates us to capture a detailed understanding of the quantum-classical transition process in the early Universe.

In this paper, we formulate a general theory of the expanding edge of QH systems in a cosmology language, i.e., quantum field theory in a curved spacetime. Taking account of the great success of effective field theory of static edge systems, we adopt the similar description of effective field theory for the time-dependent edge systems. It is assumed that such an effective theory description is valid as long as we consider a long wavelength regime. In fact, the QH edge experiments usually observe edge excitation wavelengths about 100 times larger than the magnetic length l_B [10,24]. Hence the effective theory description may be suitable for the realistic experiments. The derivation of the effective equations obeyed by edge modes from a microscopic Hamiltonian of the dynamical QH system is out of our aim in this paper and will be reported in future. The theory will be directly applied to analyses of future experiments of the edge expansion and stimulate an interdisciplinary interest connecting condensed matter physics and cosmology. First, we divide the entire edge into three regions, as depicted in Fig. 1: input flat region I, expanding region II, and output flat region III. Region I is defined as $x > +L/2$. Region II is defined as $-L/2 \leq x \leq +L/2$. Region III is defined as $x < -L/2$. Here, L denotes the width of region II when the edge is static. Edge excitation starts from region I and runs through the expanding region II, and finally enters region III.

The structure of this paper is as follows. In Sec. II, conformally flat coordinates x^\pm of general relativity are introduced for the expanding region II. Additionally, we define another conformally flat set of coordinates x_{out}^\pm of the expanding region II, which are smoothly connected with the conformally flat coordinates of the output region III. Similarly, we define different conformally flat coordinates x_{in}^\pm of the expanding region II, which are smoothly connected with the conformally flat coordinates of the input

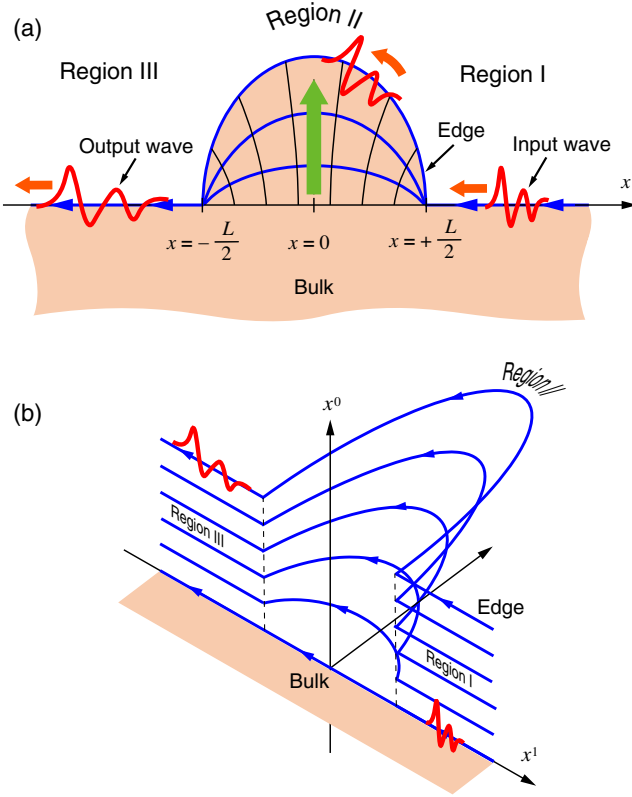


FIG. 1. Schematic of setup. (a) Top view of the QH system consisting of the bulk and the edge. Regions I and III are flat and region II is the expanding region. (b) The QH edge in 2D spacetime using the coordinates $x^0 = vt$ and $x^1 = x$.

region II. The coordinate transformation is derived between x_{out}^{\pm} and x_{in}^{\pm} . We introduce Bogoliubov coefficients between in-mode functions for region I and out-mode functions for region III. The implication of the Bogoliubov coefficients measurement in realistic experiments is discussed for future experiments. In Sec. III, we show that the expanding edges can be regarded as simulators of homogeneous and isotropic universes referred to as Friedmann-Lemaître-Robertson-Walker (FLRW) universes in $1+1$ dimensions. The dynamics are described by 2D gravity models with a scalar field Φ , which is called the dilaton field. The models include the famous Jackiw-Teitelboim (JT) model with a negative constant curvature $R = -4\lambda^2$ [48,49]. The JT model has recently attracted considerable attention because the gravity model is closely connected with a many-body model called the Sachdev-Ye-Kitaev model [50–52], and it suggests a nontrivial example of an AdS/CFT correspondence [53]. To connect the edge system with the AdS/CFT correspondence, the three-dimensional anti-de Sitter (AdS) chiral gravity may be interesting, which is dual to 2D CFT [54]. Kerr/CFT correspondence may be also related to the edge dynamics [55–57]. The JT model with positive cosmological constant is referred to as the de Sitter JT (dSJT) model in this paper. The dSJT model achieves an exponentially fast expansion of the analog universe. Thus,

the rapidly expanding-edge experiments are quite fascinating for the study of the trans-Planckian problem and the quantum-classical transition problem in the dSJT model. In Sec. IV, as an application of our experimental setup, we present the analog Hawking radiation from region II. With the de Sitter-type expanding-edge region, a future horizon is formed and the power of signals measured in region III shows the Planckian distribution with Hawking temperature $\lambda/(\sqrt{2}\pi)$. We summarize our results in Sec. V. In the Appendix, useful formulas are attached for 2D gravity.

II. CONFORMALLY FLAT COORDINATES FOR EXPANDING REGION

Let us first consider static edge systems. First, we define the edge excitation as the left mover with respect to the spatial coordinate x at the edge. Specifically, excitation runs from the right region with $x > 0$ to the left region with $x < 0$. Subsequently, v denotes the edge current velocity of the system. Introducing relativity notations, as in cosmology, is useful. The time t is replaced by $x^0 = vt$. The spatial coordinate x is denoted by x^1 , and light cone coordinates are then defined as $x^{\pm} = x^0 \pm x^1$, even though v is not the actual velocity of the light. The edge excitation is represented by a free real field $\varphi(x^+)$. The field $\varphi(x^+)$ is proportional to the voltage fluctuation on the edge and can be directly measured in experiments [58]. In this situation, the spacetime for the edge is just a flat spacetime called the Minkowski spacetime. The square of the spacetime-invariant distance, which is called metric form, is given by

$$ds^2 = -v^2 dt^2 + dx^2 = -(dx^0)^2 + (dx^1)^2 = -dx^+ dx^-. \quad (1)$$

The aforementioned equation fixes the metric matrix for each coordinate system as

$$\begin{bmatrix} g_{00} & g_{01} \\ g_{10} & g_{11} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} g_{++} & g_{+-} \\ g_{-+} & g_{--} \end{bmatrix} = \begin{bmatrix} 0 & -1/2 \\ -1/2 & 0 \end{bmatrix},$$

where $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$. Note that $\varphi(x^+)$ satisfies the massless Klein-Gordon equation,

$$[-(\partial_0)^2 + (\partial_1)^2]\varphi = -4\partial_+ \partial_- \varphi = 0, \quad (2)$$

and the chirality condition $\partial_- \varphi = 0$ implying that the edge current moves only in the left direction.

Let region II start expanding uniformly at $t = 0$, with the remaining regions I and III being unchanged. The space-time metric in region II is described by the following equation:

$$ds^2 = -v^2 d\tau^2 + a^2(\tau) dx^2, \quad (3)$$

where τ is the proper time for an observer at $x = \text{const}$, and it is equal to t when $t < 0$. The positive function $a(\tau)$ of τ

satisfies $a(\tau) = 1$ when $\tau < 0$. At a fixed time τ , the physical distance l between two points at $x = x_1$ and $x = x_2 (> x_1)$ is computed as $l = a(\tau)(x_2 - x_1)$. Thus, $a(\tau)$ determines the real size of the spatial region using the coordinate values and is called the scale factor of the expanding universe. Let us rewrite Eq. (3) as

$$ds^2 = a^2(\tau) \left[-v^2 \left(\frac{d\tau}{a(\tau)} \right)^2 + dx^2 \right]. \quad (4)$$

By defining the relativistic coordinates as

$$x^0 = vt = v \int_0^\tau \frac{d\tau'}{a(\tau')}, \quad x^1 = x, \quad (5)$$

and introducing a real function Θ of x^0 , which obeys $\Theta(x^0) = \ln a(\tau)$, we obtain the following metric form in the light cone coordinates:

$$ds^2 = -\exp \left[2\Theta \left(\frac{x^+ + x^-}{2} \right) \right] dx^+ dx^-, \quad (6)$$

where $x^0 = (x^+ + x^-)/2$. The form of Eq. (6) is defined as conformally flat.

Note that, at least in a local spacetime region, any metric form in 1 + 1 dimensions can be rewritten using a general coordinate transformation to the conformally flat form $ds^2 = -\exp(2\Theta(x^+, x^-)) dx^+ dx^-$, where $\Theta(x^+, x^-)$ is a real function of the light cone coordinates x^\pm . This is because we have 2 degrees of freedom as a general coordinate transformation: $x'^+ = f_+(x^+, x^-)$ and $x'^- = f_-(x^+, x^-)$. By considering the two functions f_\pm in the appropriate forms, the two conditions $g'_{++} = 0$ and $g'_{--} = 0$ are satisfied in the new coordinates x'^\pm . Hence, we always obtain conformally flat metric forms. The factor $\exp(2\Theta(x^+, x^-))$ is called the conformal factor.

A. Conformally flat coordinates connecting to the expanding and output regions

The conformally flat coordinates x^\pm in Eq. (6) as defined in region II cannot extend to region III as the conformal factor jumps at the boundary $x = x^1 = -L/2$. However, another set of conformally flat coordinates $x_{\text{out}}^\pm = x_{\text{out}}^0 \pm x_{\text{out}}^1$ can be introduced, which smoothly connect regions II and III at the boundary $x = x_1 = -L/2$. To ensure that x_{out}^\pm are conformally flat coordinate systems, the original coordinates x^\pm must be x_{out}^\pm -independent functions, i.e., $x^\pm = x^\pm(x_{\text{out}}^\pm)$. Then, the metric form in Eq. (6) is given by the new coordinates x_{out}^\pm as

$$ds^2 = -\exp \left[2\Theta \left(\frac{x^+(x_{\text{out}}^+) + x^-(x_{\text{out}}^-)}{2} \right) - \ln \left(\frac{dx_{\text{out}}^+}{dx^+} \frac{dx_{\text{out}}^-}{dx^-} \right) \right] dx_{\text{out}}^+ dx_{\text{out}}^-. \quad (7)$$

The above metric form remains the flat metric form in region III as $ds^2 = -dx_{\text{out}}^+ dx_{\text{out}}^-$. The coordinate transformation $x^\pm = x^\pm(x_{\text{out}}^\pm)$ is uniquely determined by the following two conditions:

- (i) The spatial position of the boundary in the new coordinates is given by $x^1 = x_{\text{out}}^1 = -L/2$.
- (ii) The time coordinates at the boundary coincide with each other: $x^0 = x_{\text{out}}^0$.

Here, conditions (i) and (ii) can be replaced by (i) and a conformal factor continuity condition, i.e., the conformal factor in x_{out}^\pm is continuous at the boundary between regions II and III such that

(ii)'

$$1 = \exp \left(2\Theta \left(\frac{x^+(x_{\text{out}}^+) + x^-(x_{\text{out}}^-)}{2} \right) - \ln \left(\frac{dx_{\text{out}}^+}{dx^+} \frac{dx_{\text{out}}^-}{dx^-} \right) \right) \quad \text{at } x^1 = -L/2. \quad (8)$$

Condition (i) yields the following condition:

$$x_{\text{out}}^+ \left(x^0 - \frac{L}{2} \right) - x_{\text{out}}^- \left(x^0 + \frac{L}{2} \right) = -L. \quad (9)$$

By taking the derivative with respect to x^0 , we get the following relation:

$$\frac{dx_{\text{out}}^+}{dx^+} \left(x^0 - \frac{L}{2} \right) = \frac{dx_{\text{out}}^-}{dx^-} \left(x^0 + \frac{L}{2} \right). \quad (10)$$

Using condition (ii)' the following relation is derived:

$$\frac{dx_{\text{out}}^+}{dx^+} \left(x^0 - \frac{L}{2} \right) \frac{dx_{\text{out}}^-}{dx^-} \left(x^0 + \frac{L}{2} \right) = \exp(2\Theta(x^0)). \quad (11)$$

From the above equation and Eq. (10), x_{out}^+ obeys the following relation:

$$\frac{dx_{\text{out}}^+}{dx^+} \left(x^0 - \frac{L}{2} \right) = \exp(\Theta(x^0)). \quad (12)$$

By changing the free parameter x^0 to $x^+ = x^0 - L/2$, this equation can be replaced by

$$\frac{dx_{\text{out}}^+}{dx^+}(x^+) = \exp \left(\Theta \left(x^+ + \frac{L}{2} \right) \right). \quad (13)$$

Integration of this equation yields

$$x_{\text{out}}^+(x^+) = \int_0^{x^+} dy \exp\left(\Theta\left(y + \frac{L}{2}\right)\right), \quad (14)$$

where the integration constant is fixed, such that $x_{\text{out}}^+ = 0$ when $x^+ = 0$. As the coordinate function $x_{\text{out}}^+(x^+)$ is a monotonically increasing function of x^+ , it has the inverse function $x^+ = F_{\text{out}}(x_{\text{out}}^+)$, which satisfies $x = F_{\text{out}}(x_{\text{out}}^+(x))$, i.e.,

$$x^+ = F_{\text{out}}(x_{\text{out}}^+) \Leftrightarrow x_{\text{out}}^+(x^+) = \int_0^{x^+} dy \exp\left(\Theta\left(y + \frac{L}{2}\right)\right). \quad (15)$$

Similarly, the coordinate function $x_{\text{out}}^-(x^-)$ can be derived from Eq. (9) to be

$$\begin{aligned} x_{\text{out}}^-(x^-) &= x_{\text{out}}^+(x^- - L) + L \\ &= \int_0^{x^- - L} dy \exp\left(\Theta\left(y + \frac{L}{2}\right)\right) + L, \end{aligned} \quad (16)$$

where Eq. (14) is used within the right-hand side. In the same way as the above discussion, it is possible to introduce continuous conformally flat coordinates x_{in}^\pm , which connect regions I and II. The coordinate functions $x_{\text{in}}^+(x^+)$ and $x_{\text{in}}^-(x^-)$ obey

$$x_{\text{in}}^+\left(x^0 + \frac{L}{2}\right) - x_{\text{in}}^-\left(x^0 - \frac{L}{2}\right) = L, \quad (17)$$

at the boundary $x^1 = x_{\text{in}}^1 = L/2$. As region I remains flat, the metric form is given by $ds^2 = -dx_{\text{in}}^+ dx_{\text{in}}^-$ for $x_{\text{in}}^1 > L/2$. By changing $L \rightarrow -L$ in Eq. (14), we obtain the following relation:

$$x_{\text{in}}^+(x^+) = \int_0^{x^+} dy \exp\left(\Theta\left(y - \frac{L}{2}\right)\right). \quad (18)$$

Here, we introduce the inverse function $x^+ = F_{\text{in}}(x_{\text{in}}^+)$, which satisfies $x = F_{\text{in}}(x_{\text{in}}^+(x))$ such that

$$x^+ = F_{\text{in}}(x_{\text{in}}^+) \Leftrightarrow x_{\text{in}}^+(x^+) = \int_0^{x^+} dy \exp\left(\Theta\left(y - \frac{L}{2}\right)\right). \quad (19)$$

In Eq. (16), changing $L \rightarrow -L$ yields

$$\begin{aligned} x_{\text{in}}^-(x^-) &= x_{\text{in}}^+(x^- + L) - L \\ &= \int_0^{x^- + L} dy \exp\left(\Theta\left(y - \frac{L}{2}\right)\right) - L. \end{aligned} \quad (20)$$

B. Conformal symmetry of expanding-edge excitations

In this section, we introduce the coordinate transformation between x_{out}^+ and x_{in}^+ . Let $F_{\text{in}}^{-1}(x)$ denote the inverse function of $F_{\text{in}}(x)$, i.e., $F_{\text{in}}^{-1}(F_{\text{in}}(x)) = x$. It should be noted

that $F_{\text{out}}^{-1}(F_{\text{out}}(x)) = x$ holds true. We then define the composite function $F(x) = F_{\text{in}}^{-1}(F_{\text{out}}(x))$, which satisfies $x_{\text{in}}^+ = F(x_{\text{out}}^+)$. From Eqs. (14) and (18), the following relation holds:

$$\begin{aligned} X &= \int_{L/2}^{F_{\text{out}}(X)+L/2} dy \exp(\Theta(y)), \\ F(X) &= \int_{-L/2}^{F_{\text{out}}(X)-L/2} dy \exp(\Theta(y)). \end{aligned} \quad (21)$$

By combining two equations in Eq. (21), the following relation is derived:

$$\begin{aligned} F(X) &= X - \int_{F_{\text{out}}(X)-L/2}^{F_{\text{out}}(X)+L/2} dy \exp(\Theta(y)) \\ &\quad + \int_{-L/2}^{L/2} dy \exp(\Theta(y)). \end{aligned} \quad (22)$$

If the function $\Theta(vt)$ is a constant function independent of t , the above equations reduce to $F(X) = X$. However, if $\Theta(vt)$ depends on t , then $F(X)$ is a nontrivial function, which describes the dynamics of the expanding-edge region.

If the conformal symmetry (Virasoro symmetry) survives even in the expansion, the time evolution of the edge excitation is given in regions I and II simply by $\varphi_{\text{in}}(x_{\text{in}} + vt_{\text{in}})$, where $\varphi_{\text{in}}(x_{\text{in}}^+)$ is the initial configuration in region I. In region II, the configuration can be expressed with respect to x_{out}^+ as $\varphi_{\text{in}}(F(x_{\text{out}}^+))$. Assuming the conformal symmetry, the configuration in region III takes the same form as that within region II,

$$\varphi_{\text{out}}(x_{\text{out}} + vt_{\text{out}}) = \varphi_{\text{out}}(x_{\text{out}}^+) = \varphi_{\text{in}}(F(x_{\text{out}}^+)). \quad (23)$$

In future experiments of the expanding-edge systems, confirmation of the above equation directly implies verification of the conformal symmetry preservation.

C. Measurement of Bogoliubov coefficients and its implications

In this section, we consider the dynamical analysis of excitations in the expanding edge using the Bogoliubov coefficients in the curved spacetime field theory. Suppose plane wave mode functions are defined by

$$u_k(x) = \frac{e^{-ikx}}{\sqrt{4\pi k}}, \quad (24)$$

where the wave number k has continuous positive values. Then the input wave $\varphi_{\text{in}}(x_{\text{in}}^+)$ in region I can be expanded as

$$\varphi_{\text{in}}(x_{\text{in}}^+) = \int_0^\infty dk (a_k^{(\text{in})} u_k(x_{\text{in}}^+) + a_k^{(\text{in})*} u_k^*(x_{\text{in}}^+)), \quad (25)$$

where $a_k^{(\text{in})}$ are complex coefficients in this expansion. From Eq. (23), the output wave is then calculated as

$$\varphi_{\text{out}}(x_{\text{out}}^+) = \int_0^\infty dk' (a_{k'}^{(\text{in})} u_{k'}(F(x_{\text{out}}^+)) + a_{k'}^{(\text{in})*} u_{k'}^*(F(x_{\text{out}}^+))). \quad (26)$$

Note that the output waveform is given by $\varphi_{\text{out},k'}(x_{\text{out}}^+) = u_{k'}(F(x_{\text{out}}^+))$ if the input wave is the plane wave as in Eq. (24) with $k = k'$. To extract information about the dynamics in region II induced by the edge expansion, let us expand $u_{k'}(F(x_{\text{out}}^+))$ with respect to the plane wave mode functions as

$$u_{k'}(F(x_{\text{out}}^+)) = \int_0^\infty dk (\alpha(k, k') u_k(x_{\text{out}}^+) + \beta(k, k') u_k^*(x_{\text{out}}^+)). \quad (27)$$

The coefficients $\alpha(k, k')$ and $\beta(k, k')$ are referred to as Bogoliubov coefficients and satisfy the unitarity conditions [33] as follows:

$$\int_0^\infty dq (\alpha(k, q) \alpha^*(k', q) - \beta(k, q) \beta^*(k', q)) = \delta(k - k'), \quad (28a)$$

$$\int_0^\infty dq (\alpha(k, q) \beta(k', q) - \beta(k, q) \alpha(k', q)) = 0. \quad (28b)$$

The output wave $\varphi_{\text{out}}(x_{\text{out}}^+)$ can be expanded using the plane wave mode functions in Eq. (24) as

$$\varphi_{\text{out}}(x_{\text{out}}^+) = \int_0^\infty dk (b_k^{(\text{out})} u_k(x_{\text{out}}^+) + b_k^{(\text{out})*} u_k^*(x_{\text{out}}^+)), \quad (29)$$

where $b_k^{(\text{out})}$ are the expansion coefficients and are computed as follows:

$$b_k^{(\text{out})} = \int_0^\infty dk' (a_{k'}^{(\text{in})} \alpha(k, k') + a_{k'}^{(\text{in})*} \beta^*(k, k')). \quad (30)$$

The coefficients $\alpha(k, k')$ and $\beta(k, k')$ play a crucial role in the quantum regime [33]. By introducing creation operators $\hat{a}_k^{(\text{in})\dagger}$ and $\hat{b}_k^{(\text{out})\dagger}$ and annihilation operators $\hat{a}_k^{(\text{in})}$ and $\hat{b}_k^{(\text{out})}$, satisfying

$$[\hat{a}_k^{(\text{in})}, \hat{a}_{k'}^{(\text{in})\dagger}] = \hbar \delta(k - k'), \quad [\hat{b}_k^{(\text{out})}, \hat{b}_{k'}^{(\text{out})\dagger}] = \hbar \delta(k - k'), \quad (31)$$

the field operators $\hat{\varphi}_{\text{in}}(x_{\text{in}}^+)$ and $\hat{\varphi}_{\text{out}}(x_{\text{out}}^+)$ are given by

$$\hat{\varphi}_{\text{in}}(x_{\text{in}}^+) = \int_0^\infty dk (\hat{a}_k^{(\text{in})} u_k(x_{\text{in}}^+) + \hat{a}_k^{(\text{in})\dagger} u_k^*(x_{\text{in}}^+)), \quad (32a)$$

$$\hat{\varphi}_{\text{out}}(x_{\text{out}}^+) = \int_0^\infty dk (\hat{b}_k^{(\text{out})} u_k(x_{\text{out}}^+) + \hat{b}_k^{(\text{out})\dagger} u_k^*(x_{\text{out}}^+)). \quad (32b)$$

Using $\alpha(k, k')$ and $\beta(k, k')$, we obtain proof of the following relations:

$$\hat{b}_k^{(\text{out})} = \int_0^\infty dk' (\alpha(k, k') \hat{a}_{k'}^{(\text{in})} + \beta^*(k, k') \hat{a}_{k'}^{(\text{in})\dagger}), \quad (33a)$$

$$\hat{b}_k^{(\text{out})\dagger} = \int_0^\infty dk' (\alpha^*(k, k') \hat{a}_{k'}^{(\text{in})\dagger} + \beta(k, k') \hat{a}_{k'}^{(\text{in})}). \quad (33b)$$

Note that the input vacuum state $|0_{\text{in}}\rangle$ is defined by $\hat{a}_k^{(\text{in})}|0_{\text{in}}\rangle = 0$. Thus, $\beta(k, k')$ describes particle creation in region III from the input vacuum in region I. The expectation values of particle number density $\hat{n}_k^{(\text{out})}$, with wave number k , are computed as

$$\langle \hat{n}_k^{(\text{out})} \rangle = \frac{1}{\hbar} \langle 0_{\text{in}} | \hat{b}_k^{(\text{out})\dagger} \hat{b}_k^{(\text{out})} | 0_{\text{in}} \rangle = \int_0^\infty dk' |\beta(k, k')|^2. \quad (34)$$

The total number of particles is evaluated as

$$\langle \hat{N}^{(\text{out})} \rangle = \int_0^\infty dk \langle \hat{n}_k^{(\text{out})} \rangle.$$

This implies that quantum particle creation in the expanding analog universes can be predicted only from $\beta(k, k')$, which can be measured in the classical regime of the system. Thus, experimentally determining $\beta(k, k')$ is crucial.

Here it may be useful to comment about a relation between the conformal symmetry and the coefficient $\beta(k, k')$. If a conformally flat coordinate system covers the entire space-time region (region I + region II + region III) in the experiments and remains the flat metric in regions I and III, x_{out}^+ is equal to x_{in}^+ up to a factor and a constant. Thus, the conformal symmetry always results in $\beta(k, k') = 0$. It should be stressed that this is not the case in our situation since we do not have such a global conformally flat coordinate system. Due to this fact, the function $u_{k'}(F(x_{\text{out}}^+))$ becomes a nontrivial function with a transient behavior in a region of x_{out}^+ and includes negative frequency out modes. This implies that $\beta(k, k') \neq 0$ in general even if the conformal symmetry is preserved.

As regions I and III remain flat during the expansion of region II, the excitations in the regions are always described by the free fields in Eqs. (32a) and (32b). The coefficients $\alpha(k, k')$ and $\beta(k, k')$ can be estimated using the experimental data in region III, even if the conformal symmetry is broken in region II. Thus, the question of whether the conformal symmetry survives can be answered by the experimental data analysis of the waveform $\varphi_{\text{out}}(x)$ in Eq. (23), $\alpha(k, k')$, and $\beta(k, k')$ in Eq. (27). Let us consider a deviation $\delta\varphi_{\text{out},k'}(x)$ from the predicted function form $\varphi_{\text{out},k'}(x)$. Then the following relation holds for the deviations of $\alpha(k, k')$ and $\beta(k, k')$:

$$\delta\varphi_{\text{out},k'}(x_{\text{out}}^+) = \int_0^\infty dk (\delta\alpha(k,k')u_k(x_{\text{out}}^+) + \delta\beta(k,k')u_k^*(x_{\text{out}}^+)). \quad (35)$$

From the above equation, the following relations are directly computed via the Fourier transformation:

$$\delta\alpha(k,k') = \sqrt{\frac{k}{\pi}} \int_{-\infty}^\infty dx^+ \delta\varphi_{\text{out},k'}(x^+) \exp(ikx^+), \quad (36)$$

$$\delta\beta(k,k') = \sqrt{\frac{k}{\pi}} \int_{-\infty}^\infty dx^+ \delta\varphi_{\text{out},k'}(x^+) \exp(-ikx^+). \quad (37)$$

By substituting observed deviation $\delta\varphi_{\text{out},k'}$ into Eqs. (36) and (37), we are able to quantify the conformal symmetry breaking for every wavelength k' of the input wave and wavelength k of the output plane wave. It is possible that the conformal symmetry breaking may occur only for some regions of (k, k') . In such a case, Eqs. (36) and (37) provide the information of the detailed structure of the symmetry breaking.

If the future experiments will confirm breakdown of the conformal symmetry, then we will be able to consider various kinds of extended effective field theories in region II to compute $\alpha(k, k')$ and $\beta(k, k')$, and compare the predictions with the experimental results. To construct simple effective theories describing the possible breakdown of the conformal symmetry, we propose to assume the general covariance, i.e., symmetry under general coordinates transformations, as a working hypothesis for the dynamics. Then the equation of motion takes a general form as follows:

$$\left(\nabla^2 - \left(\frac{m(t)v}{\hbar} \right)^2 - \xi R(t) \right) \varphi(t, x) - U'(\varphi, t) = J(t, x). \quad (38)$$

Here ∇^2 is the covariant Laplacian operator in region II, $m(t)$ is the time-dependent effective mass induced by the expansion. The scalar curvature of the FLRW spacetime in region II is denoted by $R(t)$, and ξ is a real parameter controlling the curvature interaction. The terms with $m(t)$ and $\xi R(t)$ induce different velocity of the edge excitation in region II for each wave number k and break the conformal symmetry. The term $U'(\varphi, t)$ represents a time-dependent nonlinear interaction of the field $\varphi(t, x)$, which may also break the conformal symmetry. The term $J(t, x)$ is a possible source. When the source term $J(t, x)$ exists, an additional wave $\varphi_J(t, x)$ is classical-mechanically generated by the source, even in the zero input wave case. Note that the source $J(t, x)$ also breaks the conformal symmetry, but it remains harmless when the nonlinear interaction does not exist or when the amplitude of the input wave is so small that the nonlinear interaction can be neglected. In such cases, it turns out that the source term can be

eliminated by subtracting $\varphi_J(t, x)$ from $\varphi(t, x)$. The dynamics information about the edge excitations in region II is imprinted onto $\alpha(k, k')$ and $\beta(k, k')$, which enables us to check the validity of Eq. (38).

Increasing the amplitude of the initial wave $\varphi_{\text{in}}(x_{\text{in}}^+)$ in region I, the experiments are able to determine whether nonlinear interactions like $U'(\varphi, t)$ are generated in region II. If the amplitude of $\varphi_{\text{out}}(x_{\text{out}}^+)$ does not increase linearly as the amplitude of $\varphi_{\text{in}}(x_{\text{in}}^+)$, the nonlinear interaction exists. Furthermore, it should be emphasized that an experimental check of the edge unitarity conditions of $\alpha(k, k')$ and $\beta(k, k')$ in Eqs. (28a) and (28b) may detect the existence of bulk-edge interactions in region II. If the edge excitations interact with bulk excitations like magnetorotons, the edge dynamics are coupled with the bulk dynamics and cannot be determined only by the edge information. Specifically, the relations in Eqs. (28a) and (28b) may be extended to the following relations:

$$\begin{aligned} & \int_0^\infty dq (\alpha(k, q)\alpha^*(k', q) - \beta(k, q)\beta^*(k', q)) \\ & + \int d^2q (\tilde{\alpha}(k, \vec{q})\tilde{\alpha}^*(k', \vec{q}) - \tilde{\beta}(k, \vec{q})\tilde{\beta}^*(k', \vec{q})) = \delta(k - k'), \\ & \int_0^\infty dq (\alpha(k, q)\beta(k', q) - \beta(k, q)\alpha(k', q)) \\ & + \int d^2q (\tilde{\alpha}(k, \vec{q})\tilde{\beta}(k', \vec{q}) - \tilde{\beta}(k, \vec{q})\tilde{\alpha}(k', \vec{q})) = 0, \end{aligned}$$

where $\tilde{\alpha}(k, \vec{q})$ and $\tilde{\beta}(k', \vec{q})$ are contributions to the Bogoliubov coefficients induced by the bulk mode functions. This implies that Eqs. (28a) and (28b) are not satisfied if the bulk-edge interactions appear in region II. Consequently, the experiments are able to provide upper bounds of coupling constants of bulk-edge interactions in various situations. More detailed analyses in this section will be reported in forthcoming papers.

III. EXPANDING EDGES AS UNIVERSE SIMULATORS IN 2D DILATON GRAVITY

In this section, we show that the expanding edges of QH systems are regarded as universe simulators of 2D dilaton-gravity models. Some useful formulas for 2D gravity are given in the Appendix. Let us suppose the following action for a real scalar field Φ , which is called the dilaton field, and the metric field $g_{\alpha\beta}$:

$$S = \int d^2x \sqrt{-g} (\Phi R - 4\lambda^2 V(\Phi)), \quad (39)$$

where λ is a positive constant and $V(\Phi)$ is the potential term of Φ . As shown later, the determination of the time schedule of edge expansion corresponds to the determination of a form of $V(\Phi)$. Thus, the field φ in Eq. (38) can be interpreted as a matter field in a curved spacetime

background of 2D dilation gravity with the potential term $V(\Phi)$. The metric equation $\delta S/\delta g^{\alpha\beta}(x) = 0$ derived from Eq. (39) is written as

$$(g_{\alpha\beta}\nabla^2 - \nabla_\alpha\nabla_\beta)\Phi + 2\lambda^2 V(\Phi)g_{\alpha\beta} = 0. \quad (40)$$

Here we have used a property of the 2D gravity, such that $R_{\alpha\beta} - Rg_{\alpha\beta}/2 = 0$. The field equation $\delta S/\delta\Phi(x) = 0$ reads

$$R = 4\lambda^2 V'(\Phi), \quad (41)$$

where $V'(\Phi)$ is the derivative function of $V(\Phi)$ with respect to Φ . As already mentioned in Sec. II, any 2D spacetime is able to be described, at least locally, by the conformally flat metric as $ds^2 = -\exp(2\Theta(x^+, x^-))dx^+dx^-$. In this coordinate system, Eq. (40) is simplified to the following three equations:

$$\partial_+\partial_-\Phi - \lambda^2 V(\Phi)e^{2\Theta} = 0, \quad (42a)$$

$$\partial_+^2\Phi - 2\partial_+\Theta\partial_+\Phi = 0, \quad (42b)$$

$$\partial_-^2\Phi - 2\partial_-\Theta\partial_-\Phi = 0. \quad (42c)$$

Similarly, Eq. (41) is reduced to

$$2\partial_+\partial_-\Theta - \lambda^2 V'(\Phi)e^{2\Theta} = 0. \quad (43)$$

To describe FLRW universes, let us consider metric forms such that Θ does not have any x dependence,

$$ds^2 = -e^{2\Theta((x^++x^-)/2)}dx^+dx^- = e^{2\Theta(vt)}(-v^2dt^2 + dx^2).$$

The conformal factor $e^{2\Theta(vt)}$ directly corresponds to the conformal factor in Eq. (6) of the expanding-edge realistic experiments. In this case, Eqs. (42a) and (43) become

$$\frac{d^2\Phi}{dt^2} - 4v^2\lambda^2 V(\Phi(t))e^{2\Theta(vt)} = 0, \quad (44a)$$

$$\frac{d^2\Theta}{dt^2} - 2v^2\lambda^2 V'(\Phi(t))e^{2\Theta(vt)} = 0. \quad (44b)$$

Equations (42b) and (42c) result in the same equation as

$$\frac{d^2\Phi}{dt^2} - 2\frac{d\Theta}{dt}\frac{d\Phi}{dt} = 0. \quad (45)$$

Let us divide both sides by $(d\Phi/dt)$ in Eq. (45), yielding $d/dt(\ln(d\Phi/dt) - 2\Theta(vt)) = 0$. By integrating the aforementioned equation with respect to t , we obtain $d\Phi/dt = Ae^{2\Theta(vt)}$, where A is a positive integration constant. The constant A can be replaced by λv using a transformation such that $\Theta(vt) \rightarrow \Theta(vt) - 1/2 \ln A + 1/2 \ln(\lambda v)$. Thereby, the equation can be written as

$$\frac{d\Phi}{dt} = \lambda v e^{2\Theta(vt)}. \quad (46)$$

By integrating Eq. (46), we obtain the following equation:

$$\Phi(t) = \lambda v \int_0^t dt' e^{2\Theta(vt')} + \Phi_0, \quad (47)$$

where Φ_0 is a constant. In the expanding-edge experiment, the factor $e^{2\Theta(vt)}$ is fixed as a function of t . This determines the monotonically increasing function $\Phi(t)$ in Eq. (47). Subtracting Eq. (44a) from Eq. (45) yields

$$2\frac{d\Theta}{dt}\frac{d\Phi}{dt} = 4v^2\lambda^2 V(\Phi(t))e^{2\Theta(vt)}.$$

By substituting Eq. (46) into the above equation, the potential as a function of time t can be determined by

$$V(\Phi(t)) = \frac{1}{2v\lambda} \frac{d\Theta}{dt}(vt). \quad (48)$$

Here we define the inverse function of $\Phi(t)$ as $t(\Phi)$. Since $\Phi(t(\Phi)) = \Phi$ holds, the potential term $V(\Phi)$ is fixed by the function $\Theta(vt)$ as

$$V(\Phi) = \frac{1}{2v\lambda} \frac{d\Theta}{dt}(vt(\Phi)). \quad (49)$$

It is worth highlighting that various function forms of $\Theta(vt)$ can be realized by changing the time schedule of the edge expansion in the experiments. This implies that the expanding-edge experiments provide universe simulators of the dilaton-gravity models.

When we consider $V(\Phi) = -\Phi$, the model becomes the Jackiw-Teitelboim model [48,49]. The equation of motion possesses a solution of AdS spacetime with negative constant curvature $R = -4\lambda^2$. The edge expansion experiment is capable of simulating the JT model by taking the conformal factor as

$$e^{\Theta(vt)} = \frac{1}{\cosh(Hvt)}, \quad (50)$$

where we introduced $H = \sqrt{2}\lambda$; H^{-1} corresponds to the AdS curvature radius. The conformal factor yields the following FLRW metric of AdS spacetime with proper time τ :

$$ds^2 = -v^2d\tau^2 + \cos^2(Hv\tau)dx^2. \quad (51)$$

From Eq. (47), the time evolution of the dilaton field is given by

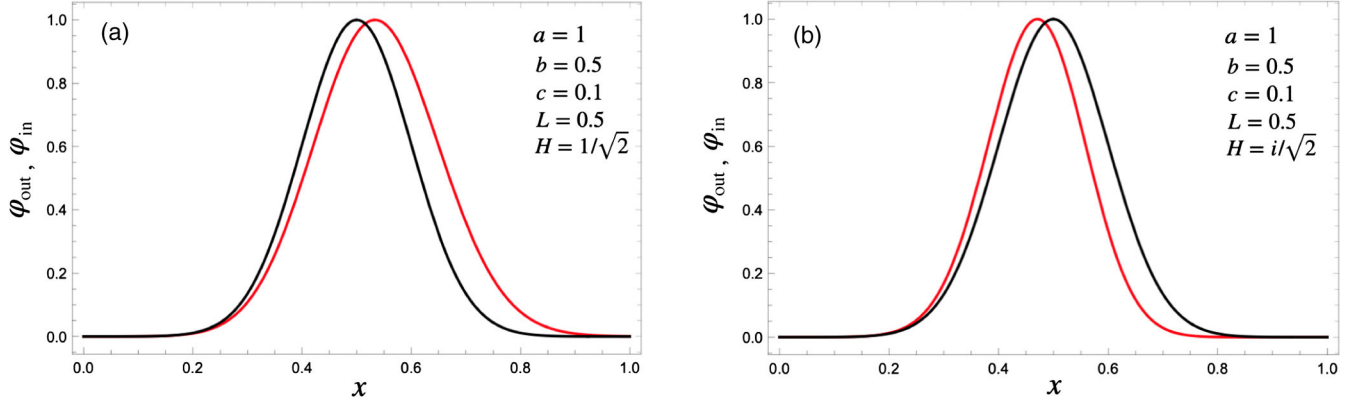


FIG. 2. Behavior of φ_{in} (black curves) and φ_{out} (red curves). (a) de Sitter ($H = 1/\sqrt{2}$), (b) AdS spacetime ($H = i/\sqrt{2}$) cases. All parameters, i.e., a , b , c , L , are identical except for H . Here x denotes x_{in}^+ for φ_{in} and x_{out}^+ for φ_{out} .

$$\begin{aligned}\Phi(t) &= \frac{Hv}{\sqrt{2}} \int_0^t \frac{dt'}{\cosh^2(Hvt')} + \Phi_0 \\ &= \frac{1}{\sqrt{2}} \tanh(Hvt) + \Phi_0.\end{aligned}\quad (52)$$

Next, let us consider the dSJT model, which is constructed by $H \rightarrow iH$ in the JT model with the negative cosmological constant $-\lambda^2$. The corresponding potential term in Eq. (39) is given by $V(\Phi) = \Phi$. This model reproduces the de Sitter spacetime with a positive constant curvature as $R = 4\lambda^2$. The conformal factor is fixed as

$$e^{\Theta(vt)} = \frac{1}{\cos(Hvt)}. \quad (53)$$

In this case, H^{-1} corresponds to the Hubble radius of de Sitter spacetime. Since the FLRW metric is computed using proper time τ as

$$ds^2 = -v^2 d\tau^2 + \cosh^2(Hv\tau) dx^2, \quad (54)$$

the analog universe experiences an inflationary expansion when $\tau > 0$. From Eq. (47), the time evolution of the dilaton field is derived as follows:

$$\Phi(t) = \frac{1}{\sqrt{2}} \tan(Hvt) + \Phi_0.$$

We demonstrate the effect of the expanding (shrinking) region on the edge mode's waveform. The mode function in region III is related to the mode function in region I by Eq. (23), which is explicitly obtained by finding the function $F(X)$. The function $F(X)$ is explicitly given by Eq. (56) when the boundary of region II is the de Sitter spacetime (54), while $F(X)$ for the anti-de Sitter spacetime is given by the analytic continuation. The derivation of $F(X)$ for the de Sitter case is presented in the next section.

From Eq. (23), the form of edge excitation $\varphi_{\text{out}}(x_{\text{out}}^+)$ in region III is predicted, assuming the conformal symmetry in region II.

Figures 2(a) and 2(b) show the behavior of the input wave φ_{in} (black curves) and output wave φ_{out} (red curves) in de Sitter and AdS spacetime. Here, the input wave is assumed to be the following Gaussian form:

$$\varphi_{\text{in}}(x_{\text{in}}^+) = a \exp\left(-\frac{(x_{\text{in}}^+ - b)^2}{2c^2}\right),$$

where a , b , and c are arbitrary parameters. Note that the parameter H in de Sitter spacetime (54) is replaced by iH in AdS spacetime (51). In our case, de Sitter spacetime is expanding, so the output wave is also spreading, but, in AdS spacetime, it is shrinking, so the output wave is narrowing. In both cases, the longer the input wave stays in region II, the more it will be affected by the expansion or shrinkage, and the output wave will behave as if it is expanding or shrinking.

When region II is the expanding de Sitter spacetime (54), i.e., inflationary universe, there appear the wave modes whose wavelength stretched infinitely. Namely, the cosmological horizon appears. This predicts the Hawking radiation in the edge modes on the quantum Hall systems, which we will discuss in the next section.

IV. EDGE MODES' HAWKING RADIATION

In this section, we will discuss the Hawking radiation when region II is an inflationary universe. We first evaluate $F_{\text{out}}(X)$ in (21) for the inflationary universe with the conformal factor (53), which covers a whole $(1+1)$ -dimensional de Sitter spacetime. From Eq. (21), $F_{\text{out}}(X)$ obeys the following equation:

$$X = \frac{1}{2H} \ln \left[\frac{1 + \sin(H(F_{\text{out}}(X) + L/2))}{1 - \sin(H(F_{\text{out}}(X) + L/2))} \times \frac{1 - \sin(HL/2)}{1 + \sin(HL/2)} \right].$$

Thus, the functions $F_{\text{out}}(X)$ and $F(X)$ are computed as

$$F_{\text{out}}(X) = -\frac{L}{2} + \frac{1}{H} \arcsin \left[\frac{(1 + \sin(HL/2))e^{2HX} - (1 - \sin(HL/2))}{(1 + \sin(HL/2))e^{2HX} + (1 - \sin(HL/2))} \right], \quad (55)$$

$$F(X) = \frac{1}{2H} \ln \left[\frac{1 + \sin(H(F_{\text{out}}(X) - L/2))}{1 - \sin(H(F_{\text{out}}(X) - L/2))} \times \frac{1 + \sin(HL/2)}{1 - \sin(HL/2)} \right]. \quad (56)$$

The function $F(X)$ behaves as is shown in Fig. 3. For a negative value of $X = X_* < 0$, $F(X_*) = -\infty$ and for $X \rightarrow +\infty$, $F \rightarrow \text{const}$. The domain and the range of $F(X)$ are semi-infinite, and this behavior indicates existence of horizons in the present spacetime. About $X = 0$, $F(X) \approx X$ which corresponds to behavior of flat spacetime. Indeed, in the limit of $L \rightarrow 0$, a whole spacetime region becomes flat and $F(X) = X$ is recovered.

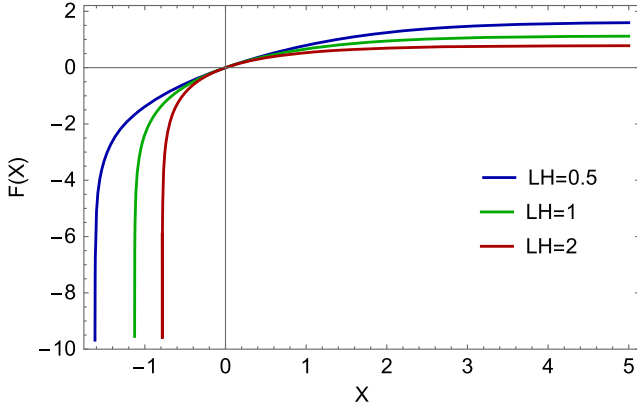


FIG. 3. Behavior of the function $F(X)$ which relates x_{in}^+ and x_{out}^+ as $x_{\text{in}}^+ = F(x_{\text{out}}^+)$. The domain and the range of $F(X)$ become semi-indefinite, which indicates existence of horizons in the present spacetime.

Figure 4 shows Penrose diagrams of the analog spacetime for the QH system with an expanding and contracting edge. Regions I and III are flat spacetimes, and region II is a part of the de Sitter spacetime representing the expanding edge, whose metric is described by (54) with the global chart. This form of de Sitter metric covers a whole region of the de Sitter spacetime and represents a contracting universe for $t < 0$ and an expanding universe for $t > 0$. Three spacetime regions I–III are joined along spatial points $x_{\text{in}} = L/2$ and $x_{\text{out}} = -L/2$. Owing to its global structure, only a part of the data prepared on \mathcal{I}^- of region I can reach \mathcal{I}^+ of region III, which implies existence of the event horizon in this spacetime (\mathcal{H}^+ in Fig. 4).

Reachability of signals from the in-region I to the out-region III depends on the size L of the expanding-edge region II (Fig. 5); for $\pi/2 < LH$, signals from region I cannot reach region III. For $LH < \pi/4$, signals prepared at $t = 0$ in region I reach region III at $t > 0$.

Figure 6 shows the waveform $\varphi_{\text{out}}(X) = \varphi_{\text{in}}(F(X))$ with $\varphi_{\text{in}}(X) = e^{-ikX}$. Owing to the existence of the future horizon \mathcal{H}^+ , the wave is stretched and freezes out as $X \rightarrow \infty$. This behavior of the wave is the same as that for black hole formation via gravitational collapse. Thus, we expect the emission of Hawking radiation from the vicinity of the future horizon \mathcal{H}^+ if the scalar field is quantized and the vacuum condition for the in-vacuum state is imposed at \mathcal{I}^- in region I, which corresponds to the Unruh vacuum state in the standard scenario of Hawking radiation via

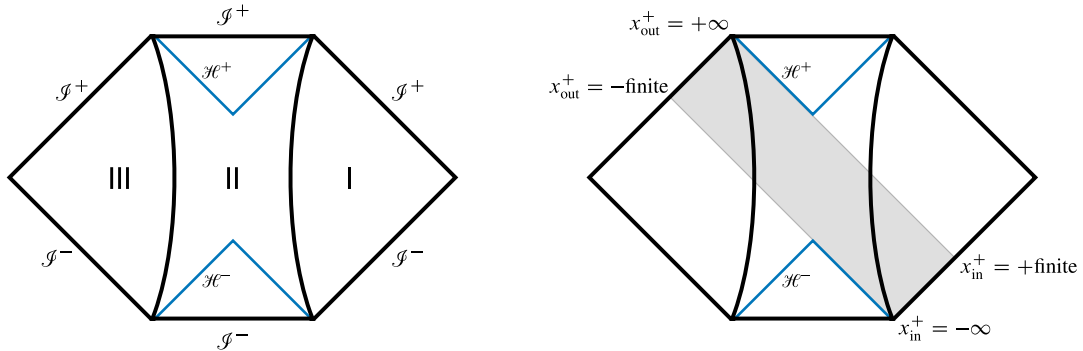


FIG. 4. Penrose diagram for the QH system with an expanding ($t > 0$) and a contracting ($t < 0$) edge region II, which is assumed to be a part of the de Sitter spacetime with the global chart. Owing to its global structure, this spacetime possesses a future horizon \mathcal{H}^+ and a past horizon \mathcal{H}^- . The shaded region of the right panel shows the region that the edge modes in \mathcal{I}^- in region I move to reach \mathcal{I}^+ in region III.

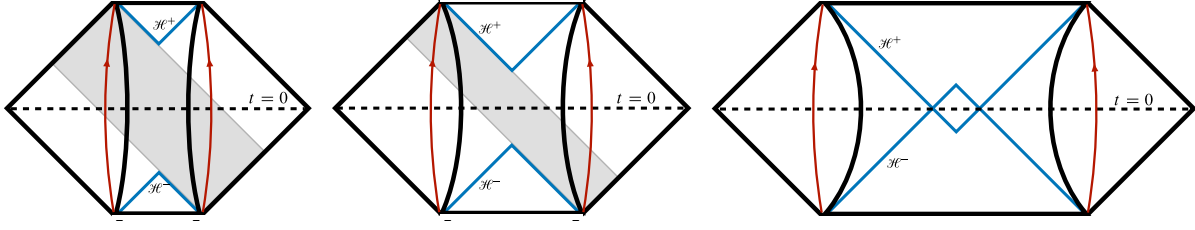


FIG. 5. L dependence of the global structure of spacetime. Red lines represent world lines for fixed spatial points in regions I and III. From left to right: $0 < LH < \pi/4$, $\pi/4 < LH < \pi/2$, $\pi/2 < LH$. For $\pi/2 < LH$, signals emitted from the in-region I cannot reach the out-region III.

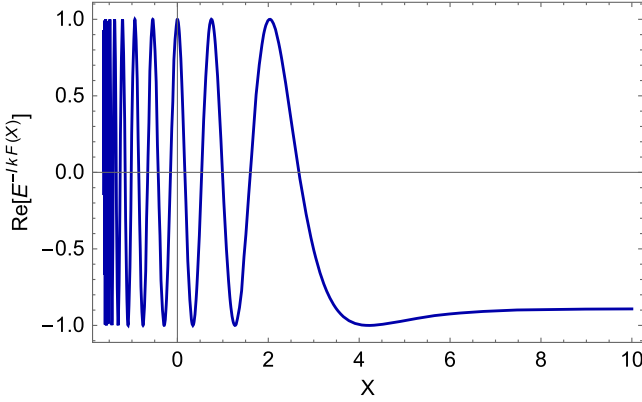


FIG. 6. The waveform received at a detection point in region III ($LH = 0.5$). X corresponds to x_{out}^+ . The input signal is a plane wave $\varphi_{\text{in}} = e^{-ikX}$ with $k = 10$. For $X \rightarrow X_* \approx -1.6$, the input wave is infinitely blueshifted owing to the past de Sitter horizon \mathcal{H}^- in region II, and for $X \rightarrow +\infty$, the input wave is infinitely redshifted owing to the future de Sitter horizon \mathcal{H}^+ in region II.

gravitational collapse [33,34,59]. We do not discuss details of the quantum effect in this paper (we will discuss this aspect in our forthcoming paper), but just investigate the Bogoliubov coefficient which can be obtained from the relation between classical wave modes φ_{in} and φ_{out} .

For the input plane wave $\varphi_{\text{in}}(X) = e^{-ikX}$, from Eq. (27), the Bogoliubov coefficients are obtained by the following Fourier transformation:

$$\begin{aligned}\alpha(k, k') &= \sqrt{\frac{k}{k'}} \int_{-\infty}^{\infty} dX e^{-ik'F(X)} e^{ikX}, \\ \beta(k, k') &= \sqrt{\frac{k}{k'}} \int_{-\infty}^{\infty} dX e^{-ik'F(X)} e^{-ikX}.\end{aligned}\quad (57)$$

To extract information of late time particle creations in region II, we consider the asymptotic behavior of the function $F(X)$ for $HX \gg 1$,

$$F(X) \approx -c_0 - c_1 e^{-HX}, \quad (58)$$

with

$$c_0 = \frac{2}{H} \ln(\tan(HL/2)), \quad c_1 = \frac{4}{H} \sqrt{\frac{1 - \sin(HL/2)}{1 + \sin(HL/2)}}. \quad (59)$$

Using this asymptotic form of $F(X)$, we obtain the Bogoliubov coefficient as

$$\begin{aligned}\beta(k, k') &\approx \sqrt{\frac{k}{k'}} \frac{e^{ik'c_0}}{H} \int_0^{\infty} dy y^{-ik/H-1} e^{ik'c_1 y} \\ &= \sqrt{\frac{k}{k'}} \frac{e^{ik'c_0}}{H} (-ik'c_1)^{-ik/H} \Gamma\left(\frac{ik}{H}\right).\end{aligned}\quad (60)$$

Therefore,

$$|\beta(k, k')|^2 \approx \frac{2\pi}{Hk'} \frac{1}{\exp(2\pi k/H) - 1}, \quad (61)$$

and it shows the Planckian distribution with a temperature

$$T_H = \frac{H}{2\pi}. \quad (62)$$

This temperature coincides with the Gibbons-Hawking temperature in the de Sitter spacetime. Equation (61) is a signal of the classical counterpart of Hawking radiation from the future horizon in region II. Although region II corresponds to an expanding de Sitter universe, owing to our setup of experiment, the global structure of spacetime resembles the situation of black hole formations by gravitational collapse, and it is possible to detect Hawking radiation from the analog black hole. The temperature of the emitted Hawking radiation is the same as that of the de Sitter horizon. More detailed analysis on Hawking radiation from the expanding-edge region will be presented in our forthcoming paper.

V. SUMMARY

In this paper, we formulated a general theory for the analysis of future experiments of the expanding edges of QH systems. The dynamics of edge expansion is described by the waveform $\varphi_{\text{out}}(x_{\text{out}}^+)$ of edge excitation measured in region III and its Bogoliubov coefficients $\alpha(k, k')$ and $\beta(k, k')$. Based on Eq. (23), the experiments determine

whether the conformal symmetry survives in region II. By increasing the amplitude of the initial wave $\varphi_{\text{in}}(x_{\text{in}}^+)$ in region I, the experiments determine whether nonlinear interactions of the edge waves are generated in region II. If the amplitude of the output wave $\varphi_{\text{out}}(x_{\text{out}}^+)$ does not increase linearly with the increase in the amplitude of $\varphi_{\text{in}}(x_{\text{in}}^+)$, the nonlinear interactions exist. If the experiments show a breakdown of the edge unitarity conditions of $\alpha(k, k')$ and $\beta(k, k')$ in Eqs. (28a) and (28b), the edge excitations interact with bulk excitations. More detailed analyses will be reported in forthcoming papers.

We have also shown that the expanding edges can be regarded as analogs of expanding universes in 2D dilaton-gravity models, including the JT model and dSJT model. By controlling the time schedule of the edge expansion, the experiments are capable of simulating the gravity models with the potential term $V(\Phi)$ in Eq. (49). Furthermore, we demonstrated the analog Hawking radiation from the de Sitter horizon formed in the expanding-edge region, which might be detected in our experimental setup. Since the field φ in Eq. (38) for a QH system in a low-noise situation is regarded as a quantum field in an expanding analog universe, the time-dependent QH systems enable us to explore the trans-Planckian problem and the quantum-classical transition problem in 2D dilaton gravity.

In conclusion, future studies of the expanding-edge experiments are expected to reveal new physics of QH systems, yielding numerous milestones that endeavour to resolve the fundamental problems in the early Universe.

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APPENDIX: FORMULA FOR 2D GRAVITY

- (i) Flat spacetime metric,

$$ds^2 = -c^2 dt^2 + dx^2.$$

- (ii) Definition of Christoffel symbols for general metric form $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$,

$$\Gamma_{\beta\gamma}^\alpha = \frac{1}{2} g^{\alpha\mu} (\partial_\beta g_{\mu\gamma} + \partial_\gamma g_{\mu\beta} - \partial_\mu g_{\beta\gamma}).$$

- (iii) Definition of Riemann curvature tensor,

$$R_{\beta\mu\nu}^\alpha = \partial_\mu \Gamma_{\nu\beta}^\alpha - \partial_\nu \Gamma_{\mu\beta}^\alpha + \Gamma_{\mu\gamma}^\alpha \Gamma_{\nu\beta}^\gamma - \Gamma_{\nu\gamma}^\alpha \Gamma_{\mu\beta}^\gamma, \\ R_{\alpha\beta\mu\nu} = g_{\alpha\gamma} R_{\beta\mu\nu}^\gamma.$$

- (iv) Definition of Ricci curvature tensor,

$$R_{\alpha\beta} = R_{\alpha\mu\beta}^\mu = g^{\mu\nu} R_{\mu\alpha\nu\beta}.$$

- (v) Definition of scalar curvature,

$$R = g^{\alpha\beta} R_{\alpha\mu\beta}^\mu = g^{\alpha\beta} g^{\mu\nu} R_{\mu\alpha\nu\beta}.$$

- (vi) In 2D gravity theory, $R_{\alpha\beta\mu\nu}$ and $R_{\alpha\beta}$ are uniquely determined by R and $g_{\mu\nu}$ as

$$R_{\alpha\beta\mu\nu} = \frac{1}{2} R (g_{\alpha\mu} g_{\beta\nu} - g_{\alpha\nu} g_{\beta\mu}), \\ R_{\alpha\beta} = \frac{1}{2} g_{\alpha\beta} R.$$

- (vii) Light cone coordinate systems,

$$x^\pm = ct \pm x = x^0 \pm x^1.$$

- (viii) Partial derivatives with respect to x^\pm ,

$$\partial_+ := \frac{\partial}{\partial x^+} = \frac{\partial x^0}{\partial x^+} \partial_0 + \frac{\partial x^1}{\partial x^+} \partial_1 = \frac{1}{2} (\partial_0 + \partial_1), \\ \partial_- := \frac{\partial}{\partial x^-} = \frac{\partial x^0}{\partial x^-} \partial_0 + \frac{\partial x^1}{\partial x^-} \partial_1 = \frac{1}{2} (\partial_0 - \partial_1).$$

- (ix) Any metric form can be rearranged into a conformally flat metric form as

$$ds^2 = -\exp(2\Theta(x^+, x^-)) dx^+ dx^-,$$

at least, in any local region of whole spacetime.

- (x) Nonvanishing $\Gamma_{\beta\gamma}^\alpha$ in conformally flat coordinate system,

$$\Gamma_{++}^+ = 2\partial_+ \Theta, \\ \Gamma_{--}^- = 2\partial_- \Theta.$$

- (xi) Scalar curvature in a conformally flat coordinate system,

$$R = 8e^{-2\Theta} \partial_+ \partial_- \Theta.$$

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