Rotating structure of the Euler-Heisenberg black hole

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In this work, we deal with the QED interpretation of the Euler-Heisenberg nonlinear electrodynamics as the effective theory after a one-loop of nonperturbative quantization. It endows the vacuum with an effective dielectric constant, the polarizability and magnetizability of which are determined by clouds of virtual charges surrounding the real currents and charges. Therefore, we study the Euler-Heisenberg nonlinear electrodynamics as a screened Maxwell theory. We generate a rotating electrically charged Einstein-Euler-Heisenberg black hole solution and interpret it as a Kerr-Newman-like one with screened electric charge, as it happens for the static electrically charged black hole Einstein-Euler-Heisenberg solution, which is considered as a Reissner-Nordström-like solution with screened electric charge.

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I. INTRODUCTION

Quantum electrodynamical vacuum corrections to the Maxwell-Lorentz theory can be accounted for by an effective nonlinear theory derived by Euler and Heisenberg [1,2], using the Dirac electron-positron theory. Schwinger reformulated this one–loop effective Lagrangian in the quantum electrodynamics theory [3]. The vacuum is treated as a specific type of medium, the polarizability and magnetizability properties of which are determined by the clouds of virtual charges surrounding the real currents and charges. This fact can be interpreted as an effective dielectric constant of the vacuum. When the electric fields are stronger than the critical value, $D_c \equiv \frac{m^2 c^3}{e\hbar}$, spontaneous electron-positron pair production takes place, lowering the vacuum energy [1,3,4]. Recently, Bordin *et al.* [5] proposed a possible direct measurement of the Euler–Heisenberg effect. This theory is a valid physical theory [6,7].

It is worthwhile to stress the fact that the QED one-loop effective Lagrangian obtained by Euler and Heisenberg [1] predicts rates of nonlinear field interaction processes since it takes into account vacuum polarization to one loop and is valid for electromagnetic fields that change slowly compared to the inverse electron mass [8,9].

On the other hand, the existence of black holes is widely accepted since the observations reported in 2019 by the Event Horizon Telescope team [10]. Also gravitational waves and orbits of the S stars around Sgr. A^* have been reported. The interest in Petrov type-D metrics is based on their physical relevance, as Schwarzschild, Reissner-Nordström, Kerr, and Kerr-Newman black hole solutions belong to the type-D metrics.

Furthermore, the static charged black holes solutions in gravitating nonlinear electrodynamics have been studied since the 1930's by Hoffmann *et al.* [11,12]. Recently, Ruffini *et al.* [13] considered the contributions of Euler-Heisenberg effective Lagrangian in order to formulate the Einstein-Euler-Heisenberg theory and to study the spherically symmetric black hole solutions endowed with electric and magnetic monopole charges. They reduced the problem to screened Reissner-Nordström solutions and collected the Euler-Heisenberg corrections in the screening terms of the electromagnetic charges; i.e., the Euler-Heisenberg theory is looked at as a screened Maxwell one. They calculate the QED corrections to the black hole horizon, entropy, total energy, and maximally extractable energy.

A similar approach was studied by Yajima *et al.* [14] in which the effective Lagrangian is considered as low-energy limit of the Born-Infeld theory and the nonlinearity parameters are treated as free parameters and analyze either numerically or analytically the properties of

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spherically symmetric black holes solutions of the Einstein-Euler-Heisenberg theory. Other approaches exist like the one proposed by Bandos *et al.* [15,16], the so–called modified Maxwell approach (ModMax), where the nonlinear corrections to the Maxwell theory are contained in a constant screening of the electric charges.

In this paper, in the framework of the Einstein-Euler-Heisenberg theory, we will follow the approach of Ruffini *et al.* for studying the conventional electrically charged rotating black hole, endowed with electric and magnetic fields, the rotating Kerr-Newman-like black hole. We assume that the nonlinear effects act only in the screening of the electric charge generating virtual charges around the real charges and currents and affects the geometry only through the screened values of the real charges; i.e., we study the screened Kerr-Newman black hole.

It is important to mention that when considering the QED field of the electron, the gravitational and electromagnetic background fields of the Kerr-Newman black hole are stationary [17]. Hence, according to the equivalence principle, phenomena like the Sauter-Euler-Heisenberg-Schwinger process or vacuum polarization effects over a flat space-time can be locally applied to the case of the curved Kerr-Newman-like geometry.

The outline of the paper is as follows: in Sec. II, we revisit the Einstein-Euler-Heisenberg theory and its formulation in terms of the dual Plebański variables. In Sec. III, the Einstein-Euler-Heisenberg static electrically charged black hole solution is presented. In Sec. IV, the Kerr-Newman spacetime is reviewed. In Sec. V, we deduce the Einstein-Euler-Heisenberg rotating black hole as a screened Kerr-Newman one. In Sec. VI, a summary, some conclusions, and an outlook are presented.

II. THE EINSTEIN-EULER-HEISENBERG THEORY AND DUAL VARIABLES

We revisit in this section the basic features of the weak field one loop QED approximation of the full nonlinear electrodynamics proposed by Euler and Heisenberg [1] in the formalism introduced by Plebański [18] for stationary solutions of Petrov type D.

The action for Einstein gravity minimally coupled to the Euler-Heisenberg theory reads [1,19,20]

$$S = \int_{M_4} d^4x \mathcal{L}_{\rm GR} + \int_{M_4} d^4x \mathcal{L}_{\rm EH} \tag{1}$$

$$= \frac{1}{16\pi G}\int_{M_4} d^4x\sqrt{-g}R + \frac{1}{4\pi}\int_{M_4} d^4x\sqrt{-g}\left(-X+\frac{2\alpha^2}{45m^4}\{4X^2+7Y^2\}\right), \tag{2}$$

where R is the Ricci scalar curvature, $\sqrt{-g}$ is the square root of the determinant of the metric $g_{\mu\nu}$, G is the Newton's constant, which we will take $G=1$, m the electron mass, and α the fine structure constant, and the variables X and Y are the only two independent relativistic invariants constructed from the Maxwell field in four dimensions, which are defined as

$$X = \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \qquad Y = \frac{1}{4}F_{\mu\nu}{}^{*}F^{\mu\nu}, \tag{3}$$

${}^{*}F^{\mu\nu}$ is the dual of the Faraday tensor $F_{\mu\nu} = A_{\mu,\nu} - A_{\nu,\mu}$, and it is defined as usual ${}^{*}F^{\mu\nu} = \frac{1}{2\sqrt{-g}}\epsilon^{\mu\nu\sigma\rho}F_{\sigma\rho}$, and $\epsilon_{\mu\nu\sigma\rho}$ is the completely antisymmetric tensor that satisfies $\epsilon_{\mu\nu\sigma\rho}\epsilon^{\mu\nu\sigma\rho} = -4!$ [21]. The invariant $X = \frac{1}{2}(\mathbf{E}^2 - \mathbf{B}^2)$ and the pseudoinvariant $Y = -\mathbf{E}\cdot\mathbf{B}$. Notice that $F_{0i} = E_i$, is the electric field, and ${}^{*}F_{0i} = -\frac{1}{2}\epsilon_{0ijk}F^{jk} = B_i$, is the magnetic field strength.

The equations of motion for the coupled Einstein-Euler-Heisenberg system: the Faraday, the Maxwell, and the Einstein equations read [22]

$$dF = 0, \qquad d^{*}P = 0, \qquad R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi T_{\mu\nu}, \tag{4}$$

where d is the standard exterior derivative, $F = F_{\mu\nu}dx^\mu \wedge dx^\nu$ the electromagnetic two-form, $P = P_{\mu\nu}dx^\mu \wedge dx^\nu$ the Plebański two-form that corresponds to the electric field strength $\mathbf{D}$ and to the magnetic field $\mathbf{H}$, and ${}^{*}P^{\mu\nu} = \frac{1}{2\sqrt{-g}}\epsilon^{\mu\nu\sigma\rho}P_{\sigma\rho}$. The energy-momentum tensor reads

$$4\pi T_{\mu\nu} = L_X(-F_\mu{}^\rho F_{\nu\rho} + g_{\mu\nu}X) + g_{\mu\nu}(L - XL_X - YL_Y), \tag{5}$$

where L is the Euler-Heisenberg Lagrange density and the subscripts on L denote differentiation.

Explicitly, the energy-momentum tensor for the Euler-Heisenberg nonlinear electromagnetic field is given by

$$4\pi T_{\mu\nu} = (F_\mu{}^\beta F_{\nu\beta} - g_{\mu\nu}X)\left(1 - \frac{16\alpha^2}{45m^4}X\right) - \frac{16\alpha^2}{45m^4}g_{\mu\nu}\frac{1}{2}\left(X^2 + \frac{7}{4}Y^2\right). \tag{6}$$

The equations of motion derived from this action are more easily written in terms of the Legendre dual description of nonlinear electrodynamics [18], which involves the introduction of the Plebański tensor $P_{\mu\nu}$ defined by

$$dL(X,Y) = -\frac{1}{2}P^{\mu\nu}dF_{\mu\nu}, \tag{7}$$

where $L(X,Y)$ is the Lagrangian density for the Euler-Heisenberg nonlinear electrodyna-mics. Note that $P_{\mu\nu}$ coincides with $F_{\mu\nu}$ for the linear Maxwell theory. In general, it reads

$$P^{\mu\nu} = -\frac{1}{4\pi}(L_X F^{\mu\nu} + L_Y {}^*F^{\mu\nu}). \tag{8}$$

In our case, we have

$$P^{\mu\nu} = \frac{1}{4\pi}\left[F^{\mu\nu} - \frac{4\alpha^2}{45m^4}\{4XF^{\mu\nu} + 7Y^*F^{\mu\nu}\}\right]. \tag{9}$$

Notice that $P_{0i} = D_i$, is the electric field strength, and ${}^*P_{0i} = -\frac{1}{2}\epsilon_{0ijk}P^{jk} = H_i$, is the magnetic field; therefore, (9) are the constitutive relations of the Euler-Heisenberg nonlinear electrodynamics. It is interesting to note that ${}^*P_{\mu\nu} = B_{\mu,\nu} - B_{\nu,\mu}$.

We denote by s and t the two independent invariants in terms of the dual Plebański variables $P_{\mu\nu}$ defined in the following way:

$$s = -\frac{1}{4}P_{\mu\nu}P^{\mu\nu}, \qquad t = -\frac{1}{4}P_{\mu\nu}{}^*P^{\mu\nu}, \tag{10}$$

where ${}^*P^{\mu\nu} = \frac{1}{2\sqrt{-g}}\epsilon^{\mu\nu\sigma\rho}P_{\sigma\rho}$. The invariant $s = \frac{1}{2}(\mathbf{D}^2 - \mathbf{H}^2)$ and the pseudoinvariant $t = -\mathbf{D}\cdot\mathbf{H}$.

The structural function $H(s,t)$ is written as

$$H(s,t) = -\frac{1}{2}P^{\mu\nu}F_{\mu\nu} - L. \tag{11}$$

For the Euler-Heisenberg theory, the structural function (up to terms of higher order in α) reads

$$H(s,t) = s - \frac{2\alpha^2}{45m^4}\{4s^2 + 7t^2\}. \tag{12}$$

The equations of the motion for the coupled system read [22]

$$d^*P = 0, \qquad dF = 0, \qquad R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi T_{\mu\nu}, \tag{13}$$

with the energy-momentum tensor,

$$T_{\mu\nu} = \frac{1}{4\pi}[H_s P_\mu{}^\beta P_{\nu\beta} + g_{\mu\nu}(2sH_s + tH_t - H)], \tag{14}$$

where subscripts on H denote differentiation. The energy-momentum tensor for the Euler-Heisenberg nonlinear electromagnetic field is given by

$$T_{\mu\nu} = \frac{1}{4\pi}\left[(P_\mu{}^\beta P_{\nu\beta} + g_{\mu\nu}s)\left(1 - \frac{16\alpha^2}{45m^4}s\right) - \frac{16\alpha^2}{45m^4}g_{\mu\nu}\left(\frac{1}{2}s^2 + \frac{7}{8}t^2\right)\right]. \tag{15}$$

To obtain the original variables, we use the constitutive or material equations that relate $F_{\mu\nu}$ with $P_{\mu\nu}$. These are

$$F_{\mu\nu} = H_s P_{\mu\nu} + H_t{}^*P_{\mu\nu} = P_{\mu\nu} - \frac{16\alpha^2}{45m^4}\left[sP_{\mu\nu} + \frac{7}{4}t^*P_{\mu\nu}\right]. \tag{16}$$

III. ELECTRICALLY CHARGED STATIC EINSTEIN-EULER-HEISENBERG BLACK HOLE

In order to obtain the Einstein-Euler-Heisenberg generalization of the Reissner-Nordström solution, we consider the following static and spherically symmetric black hole metric:

$$ds^2 = -\left(1 - \frac{2m(r)}{r}\right)dt^2 + \frac{dr^2}{1 - \frac{2m(r)}{r}} + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{17}$$

and look for electrically charged black hole solutions.

For the electrically charged case, we assume the following ansatz for the electromagnetic field [13,23]:

$$P_{\mu\nu} = \frac{Q}{r^2}(\delta^0_\mu\delta^1_\nu - \delta^1_\mu\delta^0_\nu). \tag{18}$$

This ansatz satisfies the electromagnetic Eq. (13). The invariant s and the pseudoinvariant t, Eq. (10), read

$$s = \frac{Q^2}{2r^4}, \qquad t = 0. \tag{19}$$

Therefore, integrating the Einstein equations, the electrically charged static black hole solution reads

$$m(r) = M - \frac{\tilde{Q}^2}{2r}, \tag{20}$$

where the black hole charge is screened due to the Euler-Heisenberg vacuum polarization effect [13,23],

$$Q^2 \to \tilde{Q}^2 = Q^2\left(1 - \frac{\alpha}{225\pi}E_Q^2\right). \tag{21}$$

This solution is interpreted as a screened Reissner-Nordström one. When the electric field $E_Q(r) \equiv \frac{Q}{r^2 E_c}$ of the charged black hole is overcritical, electron-positron pair productions take place, and E_Q is screened down to its critical value $E_c \equiv \frac{m^2c^3}{e\hbar}$. This solution behaves asymptotically as the Reissner-Nordstrom one.

IV. KERR-NEWMAN SPACE–TIME

In this section, we will revisit the procedure for obtaining the Kerr-Newman black hole solution without using the

Newman-Janis algorithm or some other complexification methods. We write explicitly the ansatz for a Kerr-like metric, the ansatz for the Maxwell electromagnetic field, and solve the Einstein equations including the existing hidden symmetry, which reduces the problem to solve only two of the ten Einstein equations.

A. Kerr-Newman geometry

We begin with a Kerr-like metric, whose coefficients in Boyer-Lindquist coordinates are independent of the coordinates t and ϕ. Therefore, the spacetime geometry is stationary and axially symmetric. The Killing vectors associated with these two symmetries are $\frac{\partial}{\partial t}$ and $\frac{\partial}{\partial \phi}$. From [24,25], we write the Kerr–like line element as follows:

$$ds^2 = -\left(1 - \frac{2m(r)r}{\Sigma}\right)dt^2 + \frac{\Sigma}{\Delta}dr^2 - \frac{4m(r)ra\sin^2\theta}{\Sigma}dtd\phi + \Sigma d\theta^2 + \left(r^2 + a^2 + \frac{2m(r)ra^2\sin^2\theta}{\Sigma}\right)\sin^2\theta d\phi^2, \tag{22}$$

where

$$\Sigma = r^2 + a^2\cos^2\theta, \quad \Delta = r^2 + a^2 - 2m(r)r, \quad \sqrt{-g} = \Sigma\sin\theta, \tag{23}$$

a is the angular momentum per unit mass (Kerr parameter), and the mass–energy function $m(r)$ is a function to be determined from the Einstein equations for the electrovacuum Kerr-Newman case.

B. Kerr-Newman electromagnetic field

The Maxwell's electromagnetic field of the Kerr-Newman solution is assumed to read [26]

$$A = A_\alpha dx^\alpha = -\frac{Qr}{\Sigma}(dt - a\sin^2\theta d\phi). \tag{24}$$

The Faraday tensor $F = dA$ is thus the following;

$$F = \frac{Q}{\Sigma^2}(r^2 - a^2\cos^2\theta)dr \wedge (dt - a\sin^2\theta d\phi) + \frac{Q}{\Sigma^2}ar\sin 2\theta d\theta \wedge [(r^2 + a^2)d\phi - adt], \tag{25}$$

or in component form and introducing the rotationally induced magnetic dipole moment $\mathcal{M} = Qa$, they acquire the form,

$$F_{10} = \frac{Q}{\Sigma^2}(r^2 - a^2\cos^2\theta) = \frac{Q}{\Sigma^2}r^2 - \frac{\mathcal{M}}{\Sigma^2}a\cos^2\theta, \tag{26}$$

$$F_{20} = -\frac{Q}{\Sigma^2}a^2r\sin 2\theta = -\frac{\mathcal{M}}{\Sigma^2}ar\sin 2\theta, \tag{27}$$

$$F_{13} = -\frac{Q}{\Sigma^2}(r^2 - a^2\cos^2\theta)a\sin^2\theta = -\left(\frac{Q}{\Sigma^2}r^2 - \frac{\mathcal{M}}{\Sigma^2}a\cos^2\theta\right)a\sin^2\theta, \tag{28}$$

$$F_{23} = \frac{Q}{\Sigma^2}ar\sin 2\theta(r^2 + a^2) = \frac{\mathcal{M}}{\Sigma^2}r\sin 2\theta(r^2 + a^2). \tag{29}$$

The components are not independent; they are related by $F_{13} = a\sin^2\theta F_{01}$ and $aF_{23} = (r^2 + a^2)F_{02}$. As mentioned above, $F_{01} = \mathbf{E}$ is the electric field, and ${}^*F_{01} = -F_{02}/(a\sin\theta) = \mathbf{B}$ is the magnetic field strength.

It is straightforward to check that the electromagnetic field Eq. (25) satisfies the field equations,

$$dF = 0, \qquad d^*F = 0. \tag{30}$$

Asymptotically, the electric and magnetic fields have dominant components,

$$E_r = \frac{Q}{r^2}, \tag{31}$$

$$B_r = \frac{2\mathcal{M}\cos\theta}{r^3} \tag{32}$$

$$B_\theta = \frac{\mathcal{M}\sin\theta}{r^3}. \tag{33}$$

This fact implies that Q is the charge of the black hole, and $\mathcal{M} = Qa$ is the induced magnetic moment [26].

C. The underlying symmetry

The Kerr-Newman spacetime belongs to the Plebański-Carter [A] family [27,28], characterized by the metric form,

$$ds^2 = \frac{W}{P}dp^2 + \frac{P}{W}(d\tau + q^2d\sigma)^2 + \frac{W}{\hat{R}}dq^2 + \frac{\hat{R}}{W}(d\tau - p^2d\sigma)^2, \tag{34}$$

that can be brought to the Kerr-like form Eq. (22) by transforming coordinates $p = -a\cos\theta$, $q = r$, $\tau = t + a\phi$, $\sigma = \phi/a$, that gives $P = a^2\sin^2\theta$, $\hat{R} = r^2 + a^2 - 2rm(r) = \Delta$, $W = p^2 + q^2 = r^2 + a^2\cos^2\theta = \Sigma$; this means that the relevant coordinates are only r and θ, and the Kerr-Newman spacetime is a Petrov type D metric with the only nonvanishing Weyl scalar being

$$\Psi_2 = -\frac{m(r)(r + ia\cos\theta) - Q^2}{(r - ia\cos\theta)^3(r + ia\cos\theta)}, \tag{35}$$

with an algebraically general electromagnetic field aligned along the Debever-Penrose directions of the Weyl tensor. In [29], it is shown that the Hamilton-Jacobi equation is completely separable for the Kerr-family metrics, due to the presence of the so-called Carter's constant. This is connected with the existence of the Stäckel-Killing tensor K_{ij} of rank 2 associated with a Killing-Yano tensor Y_{ij} [30] given by

$$Y = r\sin\theta d\theta\wedge[-adt + (r^2+a^2)d\phi] + a\cos\theta dr\wedge[dt - a\sin^2\theta d\phi]. \tag{36}$$

Following Visinescu [30], it is straightforward to show that the condition the electromagnetic field tensor $F_{\mu\nu}$ in Eq. ((25) fulfils in order to preserve the hidden symmetry of the system is

$$F_{k[i}Y^k_{j]} = 0. \tag{37}$$

The symmetry is manifested in the relations between the Einstein tensor components $G_{\mu\nu}$ of Eq. (22) and the fulfilment of the analogous for the energy-momentum tensor $T_{\mu\nu}$ in Eq. (38), then being this a proof that the $T_{\mu\nu}$ components from the Kerr-Newman theory are compatible with Einstein equations for a Kerr-like metric. Therefore, solving the equations $G_{rr} = 8\pi T_{rr}$ and $G_{\theta\theta} = 8\pi T_{\theta\theta}$, guarantees the fulfilment of the whole set of the Einstein equations.

D. Kerr-Newman energy-momentum tensor

The energy-momentum tensor of the linear Maxwell theory is defined as follows:

$$8\pi T_{\mu\nu} = 2(F_{\mu\alpha}F^{\alpha}_{\nu.} + Xg_{\mu\nu}). \tag{38}$$

Hence,

$$8\pi T_{rr} = -\frac{Q^2}{\Delta\Sigma}, \tag{39}$$

$$8\pi T_{\theta\theta} = \frac{Q^2}{\Sigma}, \tag{40}$$

$$8\pi T_{t\phi} = -\frac{Q^2 a\sin^2\theta}{\Sigma^3}(r^2+\Delta) - \frac{\mathcal{M}^2 a\sin^2\theta}{\Sigma^3} \tag{41}$$

$$8\pi T_{tt} = \frac{Q^2}{\Sigma^3}\Delta + \frac{\mathcal{M}^2}{\Sigma^3}\sin^2\theta, \tag{42}$$

$$8\pi T_{\phi\phi} = \frac{Q^2\sin^2\theta}{\Sigma^3}(r^2+a^2)^2 + \frac{\mathcal{M}^2\Delta\sin^4\theta}{\Sigma^3}. \tag{43}$$

It is straightforward to see that the hidden symmetry implies the following relations for the components of the Kerr-Newman energy-momentum tensor in terms of the two independent components T_{rr} and $T_{\theta\theta}$,

$$T_{t\phi} = \frac{\Delta^2 a\sin^2\theta}{\Sigma^2}T_{rr} - \frac{(a^2+r^2)}{\Sigma^2}a\sin^2\theta T_{\theta\theta}, \tag{44}$$

$$T_{tt} = -\frac{\Delta^2}{\Sigma^2}T_{rr} + \frac{a^2\sin^2\theta}{\Sigma^2}T_{\theta\theta}, \tag{45}$$

$$T_{\phi\phi} = -\frac{a^2\sin^4\theta\Delta^2}{\Sigma^2}T_{rr} + \frac{(a^2+r^2)^2\sin^2\theta}{\Sigma^2}T_{\theta\theta}, \tag{46}$$

and $T_{rr} = -T_{\theta\theta}/\Delta$.

E. Solving the Einstein field equations

It is easy to check that as a consequence of the hidden symmetry, the components $G_{t\phi}$, G_{tt}, and $G_{\phi\phi}$ of the Einstein tensor can be written in terms of G_{rr} and $G_{\theta\theta}$ as follows:

$$\begin{aligned} G_{t\phi} &= \frac{\Delta^2 a\sin^2\theta}{\Sigma^2}G_{rr} - \frac{(a^2+r^2)}{\Sigma^2}a\sin^2\theta G_{\theta\theta}, \\ G_{tt} &= -\frac{\Delta^2}{\Sigma^2}G_{rr} + \frac{a^2\sin^2\theta}{\Sigma^2}G_{\theta\theta}, \\ G_{\phi\phi} &= -\frac{a^2\sin^4\theta\Delta^2}{\Sigma^2}G_{rr} + \frac{(a^2+r^2)^2\sin^2\theta}{\Sigma^2}G_{\theta\theta}, \end{aligned} \tag{47}$$

with

$$G_{rr} = -\frac{2r^2 m'(r)}{\Sigma\Delta}, \quad G_{\theta\theta} = -\frac{1}{\Sigma}[r\Sigma m''(r) + 2a^2\cos^2\theta m'(r)]. \tag{48}$$

The Einstein field equations read

$$G_{\mu\nu} = 8\pi T_{\mu\nu}. \tag{49}$$

Therefore, the Einstein equation for the (rr) component reads

$$G_{rr} = -\frac{2r^2 m'(r)}{\Sigma\Delta} = 8\pi T_{rr} = -\frac{Q^2}{\Sigma\Delta}, \tag{50}$$

from which we get

$$m'(r) = \frac{Q^2}{2r^2}, \quad \text{hence } m''(r) = -\frac{Q^2}{r^3}. \tag{51}$$

Replacing Eq. (51) in the Einstein equation for the $(\theta\theta)$ component,

$$G_{\theta\theta} = -\frac{1}{\Sigma}[r\Sigma m''(r) + 2a^2\cos^2\theta m'(r)] = 8\pi T_{\theta\theta} = \frac{Q^2}{\Sigma}, \tag{52}$$

it is straightforward to see that one obtains an identity. Thus, the mass-energy function $m(r)$ is given by

$$m(r) = M - \frac{Q^2}{2r}. \tag{53}$$

Furthermore, the other Einstein equations are automatically satisfied,

$$G_{t\phi} = 8\pi T_{t\phi} = -\frac{Q^2 a \sin^2\theta}{\Sigma^3}(r^2+\Delta) - \frac{\mathcal{M}^2 a \sin^2\theta}{\Sigma^3} \tag{54}$$

$$G_{tt} = 8\pi T_{tt} = \frac{Q^2}{\Sigma^3}\Delta + \frac{\mathcal{M}^2}{\Sigma^3}\sin^2\theta, \tag{55}$$

$$G_{\phi\phi} = 8\pi T_{\phi\phi} = \frac{Q^2 \sin^2\theta}{\Sigma^3}(r^2+a^2)^2 + \frac{\mathcal{M}^2 \Delta \sin^4\theta}{\Sigma^3}. \tag{56}$$

Finally, replacing $m(r)$, Eq. (53), in Eq. (22), one obtains the Kerr–Newman line element,

$$\begin{aligned} ds^2 = &-\left(1 - \frac{2Mr - Q^2}{\Sigma}\right)dt^2 - \frac{(2Mr - Q^2)2a\sin^2\theta}{\Sigma}dt d\phi \\ &+ \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 \\ &+ \left(r^2 + a^2 + \frac{(2Mr - Q^2)a^2\sin^2\theta}{\Sigma}\right)\sin^2\theta d\phi^2, \end{aligned} \tag{57}$$

where

$$\Sigma = r^2 + a^2\cos^2\theta, \qquad \Delta = r^2 + a^2 - 2Mr + Q^2. \tag{58}$$

It is important to note that by setting $a = 0$ the static Reissner-Nordström solution is recovered.

F. Event horizon of the Kerr-Newman black hole

In order to calculate the event horizon, one needs to solve the equation $\Delta = 0$, this gives the two horizons:

Outer horizon,

$$r_+ = M + \sqrt{M^2 - a^2 - Q^2}. \tag{59}$$

Inner horizon,

$$r_- = M - \sqrt{M^2 - a^2 - Q^2}. \tag{60}$$

The surface area of the charged rotating Kerr-Newman black hole is

$$\mathcal{A} = 4\pi(r_+^2 + a^2). \tag{61}$$

A static limit surface ($g_{tt} = 0$) is defined as $r = r_{st}$, with

$$r_{st} = M + \sqrt{M^2 - Q^2 - a^2\cos^2\theta}. \tag{62}$$

The static limit surface is located outside the event horizon and crosses it at two polar points $\theta = 0$ and $\theta = 2\pi$. The region $r_+ < r < r_{st}$ between the static limit surface and the horizon is called the ergosphere.

V. ROTATING EINSTEIN-EULER-HEISENBERG BLACK HOLE

In this section, we will derive the rotating electrically charged Einstein-Euler-Heisenberg black hole solution. According to the QED interpretation of the Euler-Heisenberg nonlinear electrodynamics, we assume that the nonlinear effects act only in the screening of the electric charge generating virtual charges around the real charges and currents and affects the geometry only through the screened values of the real charges; i.e., we obtain the screened Kerr-Newman black hole solution [17].

A. Rotating Einstein-Euler-Heisenberg ansatz

We assume that the Einstein-Euler-Heisenberg metric coefficients in Boyer-Lindquist coordinates are independent of t and ϕ. Hence, the spacetime geometry is stationary and axially symmetric. The Killing vectors associated with these two symmetries are $\frac{\partial}{\partial t}$ and $\frac{\partial}{\partial \phi}$. Therefore, we begin with the same ansatz Eqs. (22) and (23) as for the Kerr-Newman case. This is once again a Kerr-like type of line element.

B. Euler-Heisenberg electromagnetic field ansatz

The ansatz for the Euler-Heisenberg nonlinear electromagnetic potential of the dual Plebański variables reads

$$B = B_\alpha dx^\alpha = -\frac{Qa\cos\theta}{\Sigma}\left(dt - \frac{(r^2+a^2)}{a}d\phi\right). \tag{63}$$

Hence, the dual Plebański two-form ${}^*P = dB$ is given by

$$\begin{aligned} {}^*P = &\frac{2Q}{\Sigma^2}ar\cos\theta dr \wedge (dt - a\sin^2\theta d\phi) \\ &+ \frac{Q}{\Sigma^2}(r^2 - a^2\cos^2\theta)a\sin\theta d\theta \wedge [dt - (r^2+a^2)d\phi], \end{aligned} \tag{64}$$

or explicitly in components form and introducing the rotationally induced magnetic dipole moment $\mathcal{M} = Qa$,

$${}^*P_{01} = -\frac{2Q}{\Sigma^2}ar\cos\theta = -\frac{2\mathcal{M}}{\Sigma^2}r\cos\theta, \tag{65}$$

$$\begin{aligned} {}^*P_{02} &= -\frac{Q}{\Sigma^2}(r^2 - a^2\cos^2\theta)a\sin\theta \\ &= -\frac{Q}{\Sigma^2}r^2 a\sin\theta + \frac{\mathcal{M}}{\Sigma^2}a^2\cos^2\theta\sin\theta, \end{aligned} \tag{66}$$

$$ {}^{*}P_{13} = -\frac{Q}{\Sigma^2} a^2 r \sin 2\theta \sin\theta = -\frac{2\mathcal{M}}{\Sigma^2} ar\cos\theta \sin^2\theta, \quad (67) $$

$$ {}^{*}P_{23} = -\frac{Q}{\Sigma^2}(r^2 - a^2\cos^2\theta)(r^2+a^2)\sin\theta \quad (68) $$

$$ = -\frac{Q}{\Sigma^2} r^2 (r^2+a^2)\sin\theta + \frac{\mathcal{M}}{\Sigma^2} a\cos^2\theta (r^2+a^2)\sin\theta. \quad (69) $$

The components are not independent; they are related by ${}^{*}P_{13} = a\sin^2\theta\, {}^{*}P_{01}$ and $a\, {}^{*}P_{23} = (r^2+a^2)\, {}^{*}P_{02}$.

The Plebański two-form P reads

$$ P = \frac{Q}{\Sigma^2}(r^2 - a^2\cos^2\theta) dr \wedge (dt - a\sin^2\theta d\phi) + \frac{Q}{\Sigma^2} ar \sin 2\theta d\theta \wedge [(r^2+a^2)d\phi - a dt], \quad (70) $$

or explicitly, in components form,

$$ P_{01} = -\frac{Q}{\Sigma^2}(r^2 - a^2\cos^2\theta) = -\frac{Q}{\Sigma^2} r^2 + \frac{\mathcal{M}}{\Sigma^2} a\cos^2\theta, \quad (71) $$

$$ P_{02} = \frac{Q}{\Sigma^2} ra^2 \sin 2\theta = \frac{\mathcal{M}}{\Sigma^2} ar\sin 2\theta, \quad (72) $$

$$ P_{13} = -\frac{Q}{\Sigma^2}(r^2 - a^2\cos^2\theta) a\sin^2\theta = -\left(\frac{Q}{\Sigma^2} r^2 - \frac{\mathcal{M}}{\Sigma^2} a\cos^2\theta\right) a\sin^2\theta, \quad (73) $$

$$ P_{23} = \frac{Q}{\Sigma^2} ra\sin 2\theta (r^2+a^2) = \frac{\mathcal{M}}{\Sigma^2} r\sin 2\theta (r^2+a^2). \quad (74) $$

The components are not independent; they are related by $P_{13} = a\sin^2\theta P_{01}$ and $aP_{23} = (r^2+a^2)P_{02}$. Clearly when $a = 0$ the static electromagnetic field is recovered, $P_{10} = Q/r^2$. $P_{01} = \mathbf{D}$ is the electric field strength, and ${}^{*}P_{01} = -P_{02}/(a\sin\theta) = \mathbf{H}$ is the magnetic field.

It is straightforward to check that the electromagnetic field Eq. (70), its dual Eq. (64), and using Eq. (16) the field equations,

$$ d^{*}P = 0, \qquad dF = 0, \quad (75) $$

are satisfied.

Asymptotically, the electromagnetic field is given by

$$ D_r = \frac{Q}{r^2}, \quad (76) $$

$$ H_r = \frac{2\mathcal{M}\cos\theta}{r^3} \quad (77) $$

$$ H_\theta = \frac{\mathcal{M}\sin\theta}{r^3}. \quad (78) $$

The invariants s and t read

$$ s = \frac{1}{2}(D^2 - H^2) = \frac{1}{2}\left(P_{01}^2 - \frac{P_{02}^2}{a^2\sin^2\theta}\right) = \frac{Q^2}{2\Sigma^2} - \frac{4\mathcal{M}^2 r^2\cos^2\theta}{\Sigma^4}, \quad (79) $$

$$ t = -\mathbf{D}\cdot\mathbf{H} = -\frac{P_{01}P_{02}}{a\sin\theta} = \frac{2Qr\cos\theta}{\Sigma^4}\mathcal{M}(r^2 - a^2\cos^2\theta). \quad (80) $$

It is important to mention that according to Ruffini *et al.* [17], when considering the QED field of the electron, gravitational and electromagnetic background fields of the Kerr-Newman-like black holes are stationary. As far as only QED phenomena, such as pair production and vacuum polarization effects, are concerned, it is possible to consider the electric and magnetic fields defined by Eq. (70) as constants in the neighborhood of few wavelengths around any event. Hence, according to the equivalence principle, phenomena like the Sauter-Euler-Heisenberg-Schwinger process over a flat spacetime can be locally applied to the case of the curved Kerr-Newman-like geometry. Therefore, we would restrict the effects of the vacuum polarization only to the screening of the electric charge of the Euler-Heisenberg nonlinear electrodynamics as in flat spacetime.

C. The underlying symmetry

The rotating Einstein-Euler-Heisenberg spacetime Eq. (22) is assumed to belong to the Plebañski-Carter [A] family [27], characterized by the metric form Eq. (34), which can be brought to the Kerr-like form with the same transformation as for the Kerr-Newman one; this means that the relevant coordinates are only r and θ, and the rotating Einstein-Euler-Heisenberg spacetime is a Petrov type D metric with the only nonvanishing Weyl scalar given by Eq. (35) with an algebraically general electromagnetic field aligned along the Debever-Penrose directions of the Weyl tensor. Moreover, this geometry admits a hidden symmetry encapsulated in a Stäckel-Killing tensor K_{ij} of rank 2 associated with a Killing-Yano tensor Y_{ij} [30], which reads as in Eq. (36), and it can be checked in a straightforward way that the Plebański dual electromagnetic field $P_{\mu\nu}$ given in Eq. (70) fulfils the sufficient condition to preserve the hidden symmetry, i.e., considering the Killing-Yano tensor Y_{ij} given by (36). Note that by using the constitutive or material relation Eq. (16) the fulfilment of the condition $F_{k[i}Y^{k}_{j]} = 0$ reads

$$ F_{k[i}Y^{k}_{j]} = H_s P_{k[i}Y^{k}_{j]} + H_t {}^{*}P_{k[i}Y^{k}_{j]} = P_{k[i}Y^{k}_{j]} - \frac{16\alpha^2}{45m^4}\left[sP_{k[i}Y^{k}_{j]} + \frac{7}{4}t^{*}P_{k[i}Y^{k}_{j]}\right] = 0. \quad (81) $$

It can be shown by straightforward calculation that both conditions,

$$P_{k[i}Y^k_{j]} = 0, \quad \text{and} \quad {}^*P_{k[i}Y^k_{j]} = 0, \tag{82}$$

are satisfied. Therefore, the Plebański tensor $P_{\mu\nu}$ and its dual ${}^*P_{\mu\nu}$, both maintain the hidden symmetry of the Kerr-like family of metrics.

Conversely, using the constitutive or material relation Eq. (9), the fulfilment of the conditions $P_{k[i}Y^k_{j]} = 0$ and ${}^*P_{k[i}Y^k_{j]} = 0$ implies that

$$P_{k[i}Y^k_{j]} = F_{k[i}Y^k_{j]} - \frac{4\alpha^2}{45m^4}(4XF_{k[i}Y^k_{j]} + 7Y{}^*F_{k[i}Y^k_{j]}) = 0. \tag{83}$$

Hence, it reduces to the conditions,

$$F_{k[i}Y^k_{j]} = 0, \qquad {}^*F_{k[i}Y^k_{j]} = 0. \tag{84}$$

The symmetry is manifested in the relations between the $G_{\mu\nu}$ components Eq. (47) and the fulfilment of the analogous for the energy-momentum tensor $T_{\mu\nu}$ in Eqs. (44)–(46), then being this a proof that the $T_{\mu\nu}$ components from the Euler-Heisenberg theory given by Eq. (15) are compatible with Einstein equations for a Kerr-like metric. Therefore, the fulfilment of equations $G_{rr} = 8\pi T_{rr}$ and $G_{\theta\theta} = 8\pi T_{\theta\theta}$ guarantees the fulfilment of the whole set of the Einstein equations by the Euler-Heisenberg matter.

D. Einstein-Euler-Heisenberg energy-momentum tensor

The energy-momentum tensor for the Euler-Heisenberg nonlinear electromagnetic field is given by

$$T_{\mu\nu} = \frac{1}{4\pi}\left[(P_\mu{}^\beta P_{\nu\beta} + g_{\mu\nu}s)\left(1 - \frac{16\alpha^2}{45m^4}s\right) - \frac{16\alpha^2}{45m^4}g_{\mu\nu}\left(\frac{1}{2}s^2 + \frac{7}{8}t^2\right)\right]. \tag{85}$$

First, we calculate the component T_{rr} of the energy-momentum tensor,

$$8\pi T_{rr} = -\frac{1}{\Delta}\frac{Q^2}{\Sigma}\left(1 - \frac{16\alpha^2}{45m_e^4}s\right) - \frac{16\alpha^2}{45m_e^4}\frac{\Sigma}{\Delta}\left(s^2 + \frac{7}{4}t^2\right). \tag{86}$$

Let us calculate the $T_{\theta\theta}$ component of the energy-momentum tensor,

$$8\pi T_{\theta\theta} = \Sigma\left[\frac{Q^2}{\Sigma^2}\left(1 - \frac{16\alpha^2}{45m_e^4}s\right) - \frac{16\alpha^2}{45m_e^4}\left(s^2 + \frac{7}{4}t^2\right)\right]. \tag{87}$$

One can easily notice that $T_{tt}, T_{\phi\phi}$, and $T_{t\phi}$ can be written in terms of T_{rr} and $T_{\theta\theta}$ as it happens for the case of Kerr-Newman solution, Eqs. (44)–(46), due to the underlying symmetry.

E. Solving the Einstein-Euler-Heisenberg field equations

According to the QED interpretation of the Euler-Heisenberg nonlinear electrodynamics, all nonlinear effects are associated with the dielectric constant of the vacuum, i.e., clouds of virtual charges surrounding the real charges, which account for such vacuum dielectric constant.

We calculate the components G_{rr} and $G_{\theta\theta}$, the other three components can be written down from the relationship with these ones as consequence of the hidden symmetry, namely,

$$\begin{aligned}
G_{t\phi} &= \frac{\Delta^2 a \sin^2\theta}{\Sigma^2}G_{rr} - \frac{(a^2+r^2)}{\Sigma^2}a\sin^2\theta G_{\theta\theta},\\
G_{tt} &= -\frac{\Delta^2}{\Sigma^2}G_{rr} + \frac{a^2\sin^2\theta}{\Sigma^2}G_{\theta\theta},\\
G_{\phi\phi} &= -\frac{a^2\sin^4\theta\Delta^2}{\Sigma^2}G_{rr} + \frac{(a^2+r^2)^2\sin^2\theta}{\Sigma^2}G_{\theta\theta},
\end{aligned} \tag{88}$$

with

$$G_{rr} = -\frac{2r^2m'(r)}{\Sigma\Delta}, \quad G_{\theta\theta} = -\frac{1}{\Sigma}[r\Sigma m''(r) + 2a^2\cos^2\theta m'(r)]. \tag{89}$$

The Einstein field equations now read

$$G_{\mu\nu} = 8\pi T_{\mu\nu}. \tag{90}$$

The Einstein field equation for the (rr) component reads

$$\begin{aligned}
G_{rr} &= -\frac{2r^2m'(r)}{\Sigma\Delta} = 8\pi T_{rr}\\
&= -\frac{1}{\Delta}\frac{Q^2}{\Sigma}\left(1 - \frac{16\alpha^2}{45m_e^4}s\right) - \frac{16\alpha^2}{45m_e^4}\frac{\Sigma}{\Delta}\left(s^2 + \frac{7}{4}t^2\right),
\end{aligned} \tag{91}$$

then

$$m'(r) = \frac{Q^2}{2r^2}\left(1 - \frac{16\alpha^2}{45m_e^4}s\right) + \frac{16\alpha^2}{45m_e^4}\left(\frac{\Sigma^2}{2r^2}\right)\left(s^2 + \frac{7}{4}t^2\right). \tag{92}$$

Now we prove the consistency between $G_{rr} = 8\pi T_{rr}$ and $G_{\theta\theta} = 8\pi T_{\theta\theta}$ by deriving the previous expression and showing that it is consistent with m'' obtained from

$G_{\theta\theta} = 8\pi T_{\theta\theta}$, remember, as mentioned in the Introduction, we are considering the nonlinear effects, arising from the Euler-Heisenberg electrodynamics, i.e., the terms with the nonlinear parameter of the theory, being quasiconstant affecting only the electric charge. Therefore, from (92), one obtains

$$m'' = -\frac{Q^2}{r^3}\left(1 - \frac{16\alpha^2}{45m_e^4}s\right) + \frac{16\alpha^2}{45m_e^4}\left[\frac{\Sigma}{r^3}(r^2 - a^2\cos^2\theta)\right]\left(s^2 + \frac{7}{4}t^2\right). \tag{93}$$

While from $G_{\theta\theta} = 8\pi T_{\theta\theta}$,

$$G_{\theta\theta} = -rm'' - \frac{2a^2\cos^2\theta m'}{\Sigma} = 8\pi T_{\theta\theta} = \Sigma\left[\frac{Q^2}{\Sigma^2}\left(1 - \frac{16\alpha^2}{45m_e^4}s\right) - \frac{16\alpha^2}{45m_e^4}\left(s^2 + \frac{7}{4}t^2\right)\right], \tag{94}$$

clearing out m'' and then replacing m' from Eq. (92), one gets

$$m'' = -\frac{Q^2}{r^3}\left(1 - \frac{16\alpha^2}{45m_e^4}s\right) + \frac{16\alpha^2}{45m_e^4}\left[\frac{\Sigma}{r^3}(r^2 - a^2\cos^2\theta)\right]\left(s^2 + \frac{7}{4}t^2\right), \tag{95}$$

which is exactly the same as Eq. (93), proving the consistency between $G_{rr} = 8\pi T_{rr}$ and $G_{\theta\theta} = 8\pi T_{\theta\theta}$ and the fulfillment of the $(\theta\theta)$ component of the Einstein field equations.

Then we obtain the screening of the charge from m', Eq. (92), replacing the expressions for s and $(s^2 + 7t^2/4)$ as follows:

$$\left(s^2 + \frac{7}{4}t^2\right) = \frac{Q^4}{4\Sigma^4}\left[1 + 12\frac{r^2a^2\cos^2\theta}{\Sigma^4}(r^2 - a^2\cos^2\theta)^2\right], \tag{96}$$

then the expression for m' can be written as

$$m' = \frac{Q^2}{2r^2}\left\{1 - \frac{4\alpha^2}{45m_e^4}\frac{Q^2}{\Sigma^2}\left[1 - 4\left(7\frac{a^2\cos^2\theta}{\Sigma} - 12\frac{a^4\cos^4\theta}{\Sigma^2} + 12\frac{a^6\cos^6\theta}{\Sigma^3}\right)\left(1 - \frac{a^2\cos^2\theta}{\Sigma}\right)\right]\right\}, \tag{97}$$

in such a way that we can identify the charge screening as

$$\begin{aligned}\tilde{Q}^2 &= Q^2\left\{1 - \frac{4\alpha^2}{45m^4}\frac{1}{\Sigma^2}\left[Q^2 - 4\frac{\mathcal{M}^2\cos^2\theta}{\Sigma}\left(7 - 12\frac{a^2\cos^2\theta}{\Sigma} + 12\frac{a^4\cos^4\theta}{\Sigma^2}\right)\left(1 - \frac{a^2\cos^2\theta}{\Sigma}\right)\right]\right\},\\ &= Q^2\left\{1 - \frac{4\alpha^2}{45m^4}\frac{Q^2}{\Sigma^2} + \frac{16\alpha^2}{45m^4}H^2\right\} + \frac{16\alpha^2}{15m^4}\Sigma^2(\mathbf{D}\cdot\mathbf{H})^2,\\ &= Q^2\left\{1 - \frac{\alpha}{225\pi}D_Q^2 + \frac{4\alpha}{45\pi}H_Q^2\right\} + \frac{4\alpha}{45\pi}\Sigma^2(D_QH_Q)^2,\end{aligned} \tag{98}$$

where the square of the radial components of the electromagnetic fields read

$$D_Q^2 = \frac{Q^2}{\Sigma^2 D_c^2}, \qquad H_Q^2 = \frac{\mathcal{M}^2\cos^2\theta}{\Sigma^3 D_c^2}, \tag{99}$$

and we have expressed the nonlinear parameter of the theory in terms of the critical field defined in [13] as $D_c = m^2c^3/(e\hbar)$.

In the Euler-Heisenberg system, it is not proper to make the integration in Eq. (92), since the integrand comes from the Euler-Heisenberg effective Lagrangian, which endows the vacuum with a dielectric constant, and therefore, it is valid only for quasiconstant fields, we consider it as a kind of adiabatic correction to the Maxwell charge. Therefore,

$$m' = \frac{\tilde{Q}^2}{2r^2}. \tag{100}$$

Following Ruffini *et al.* [13] according with the QED interpretation, the screening of the Maxwell charge is provided by the contribution of the Euler-Heisenerg nonlinear electrodynamics. Therefore, we restrict ourselves only to the geometric contribution to $m'(r)$ and integrate only the explicit dependence on the r-coordinate, in order to remain in the framework of the Plebanski-Carter [A] class of metrics and to have a type D solution. Thus, the mass-energy function reads

$$m(r) = M - \frac{\tilde{Q}^2}{2r}, \tag{101}$$

with the screening given by Eq. (98).

The other Einstein equations are automatically satisfied,

$$G_{t\phi} = 8\pi T_{t\phi} = -\frac{\tilde{Q}^2 a \sin^2\theta}{2\Sigma^3}(r^2 + \Delta) - \frac{\tilde{\mathcal{M}}^2 a \sin^2\theta}{2\Sigma^3}, \quad (102)$$

$$G_{tt} = 8\pi T_{tt} = \frac{\tilde{Q}^2}{2\Sigma^3}\Delta + \frac{\tilde{\mathcal{M}}^2}{2\Sigma^3}\sin^2\theta, \quad (103)$$

$$G_{\phi\phi} = 8\pi T_{\phi\phi} = \frac{\tilde{Q}^2 \sin^2\theta}{2\Sigma^3}(r^2 + a^2)^2 + \frac{\tilde{\mathcal{M}}^2 \Delta \sin^4\theta}{2\Sigma^3}, \quad (104)$$

with $\tilde{\mathcal{M}}^2 = \tilde{Q}^2 a^2$.

Hence, the rotating Einstein-Euler-Heisenberg black hole space–time reads

$$\begin{aligned} ds^2 &= -\left(1 - \frac{2Mr - \tilde{Q}^2}{\Sigma}\right)dt^2 + \frac{\Sigma}{\Delta}dr^2 \\ &\quad - \frac{(2Mr - \tilde{Q}^2)2a\sin^2\theta}{\Sigma}dtd\phi + \Sigma d\theta^2 \\ &\quad + \left(r^2 + a^2 + \frac{(2Mr - \tilde{Q}^2)a^2\sin^2\theta}{\Sigma}\right)\sin^2\theta d\phi^2, \\ \Sigma &= r^2 + a^2\cos^2\theta, \\ \Delta &= r^2 + a^2 - (2Mr - \tilde{Q}^2), \\ m(r) &= M - \frac{\tilde{Q}^2}{2r}. \end{aligned} \quad (105)$$

We have reduced the Einstein-Euler-Heisenberg rotating black hole solution to a Kerr-Newman-like black hole one. By setting $a = 0$, the static screened Reissner-Nordstrom solution is recovered. In order to gain some physical insight into the energy-mass function, we could allow Eq. (101) to vary from point to point in the spacetime. In this framework, the solution behaves asymptotically as the Kerr-Newman one.

F. Event horizon of the Einstein-Euler-Heisenberg black hole

In order to calculate the event horizon, one needs to solve de equation $\Delta = 0$; this gives the two horizons.

Outer horizon,

$$r_+ = M + \sqrt{M^2 - a^2 - \tilde{Q}^2}. \quad (106)$$

Inner horizon,

$$r_- = M - \sqrt{M^2 - a^2 - \tilde{Q}^2}. \quad (107)$$

The surface area of a charged rotating black hole is

$$\mathcal{A} = 4\pi(r_+^2 + a^2). \quad (108)$$

A static limit surface ($g_{tt} = 0$) is defined as $r = r_{\rm st}$, with

$$r_{\rm st} = M + \sqrt{M^2 - \tilde{Q}^2 - a^2\cos^2\theta}. \quad (109)$$

The static limit surface is located outside the event horizon and crosses it at two polar points $\theta = 0$ and $\theta = 2\pi$. The region $r_+ < r < r_{\rm st}$ between the static limit surface and the horizon is called the ergosphere.

VI. SUMMARY, CONCLUSIONS, AND OUTLOOK

Quantum electrodynamical vacuum corrections to the Maxwell-Lorentz theory can be accounted for by the Euler-Heisenberg effective nonlinear theory. The vacuum is treated as a specific type of medium, the polarizability and magnetizability properties of which are determined by the clouds of virtual charges surrounding the real currents and charges; this fact can be interpreted as a kind of dielectric constant of the vacuum.

In this work, in the framework of the QED interpretation of the Euler-Heisenberg electrodynamics, we have studied an electrically charged rotating black hole, i.e., the rotating Kerr-Newman-like black hole. Since when considering the QED field of the electron, the gravitational and electromagnetic background fields of the Kerr-Newman black hole are stationary, and according to the equivalence principle, phenomena like the Sauter-Euler-Heisenberg-Schwinger process or vacuum dielectric constant effects over a flat space-time can be locally applied to the case of the curved Kerr-Newman-like geometry. We assume that the nonlinear effects of the Euler-Heisenberg electrodynamics influence only the electric charge generating virtual charges around the real charges and currents by means of a screening of it and affects the geometry only through the screened values of the real charges; i.e., we obtained the screened Kerr-Newman black hole solution.

First, we obtain the Kerr-Newman black hole solution without using the Newman-Janis algorithm or some other complexification methods. We write explicitly the ansatz for a Kerr-like metric, the ansatz for the Maxwell electromagnetic field, and solve the Einstein equations, including the existing hidden symmetry which reduces the problem to solve only two of the ten Einstein equations. Then, we assume the same ansatz for a Kerr-like space-time and also the ansatz for the Euler-Heisenberg electromagnetic field for the dual Plebański variables. We assume the same symmetries to the proposed space-time structure in order to still have the Petrov type D characteristics and proceed to calculate the screening of the electric charge and solve the Einstein equations.

The next step would be to analyze the effects of the screening in the trajectories of the different kinds of massive particles as well as of the photons and the modifications of the shape of the shadow of the screened Kerr-Newman black hole. We are presently working on that issue.

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