# Effect of Earth-Moon's gravity on TianQin's range acceleration noise. II. Impact of orbit selection

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The paper is a sequel to our previous work [Zhang *et al.* Phys. Rev. D 103[, 062001 \(2021\)](https://doi.org/10.1103/PhysRevD.103.062001)]. For proposed geocentric space-based gravitational wave detectors such as TianQin, gLISA, and GADFLI, the gravityfield disturbances, i.e., the so-called "orbital noise", from the Earth-Moon system on the sensitive intersatellite laser interferometric measurements should be carefully evaluated and taken into account in the concept studies. Based on TianQin, we investigate how the effect, in terms of frequency spectra, varies with different choices of orbital orientations and radii through single-variable studies, and present the corresponding roll-off frequencies that may set the lower bounds of the targeted detection bands. The results, including the special cases of geostationary orbits (gLISA/GADFLI) and repeat orbits, can provide a useful input to orbit and constellation design for future geocentric missions.

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## I. INTRODUCTION

In ground-based gravitational wave (GW) detectors such as LIGO [[1\]](#page-6-0), Virgo [[2\]](#page-6-1), and KAGRA [[3\]](#page-6-2), Newtonian (or gravity-gradient) noise due to nearby terrestrial gravity fluctuations constitutes a prominent obstacle to sensitivity improvement below ∼10 Hz [[4](#page-6-3)]. This difficulty on the ground, in addition to seismic noise and the need of long arms (typically  $\sim 10^6$  km), has motivated space-based detection in order to access sub-Hertz GW signals [\[5](#page-6-4)].

The space-based GW detection missions can be roughly categorized into heliocentric concepts, featured by LISA [\[6\]](#page-6-5), and geocentric concepts, featured by TianQin [\[7](#page-6-6)]. For geocentric missions, though the issue of gravity-field disturbances is expected to be less severe, it may still require careful assessment due to the relative proximity of the satellites to the Earth and the Moon. The goal is to ascertain that there can be a proper separation in the frequencies of gravity-field disturbances and the targeted sensitive frequency bands near mHz, which is prerequisite to a successful space-based GW detection. Unlike groundbased detectors, armlengths of space-based detectors are generally not fixed primarily due to gravitational perturbations to orbital motion. Varying armlengths also induces Doppler shifts (∼5 MHz) in the beat notes of the intersatellite heterodyne laser interferometric measurements. The frequency shifts may generate phase ramp-up, onto which GW-induced phase fluctuations are superimposed.

The former, containing gravity-field information, should be removed or separated with caution in the predata processing stage on the ground. Thereby, an accurate simulation of the orbital Doppler shifts would be needed to verify the phase measurement accuracy and time-delay interferometry [\[8\]](#page-6-7), at least for TianQin.

The nominal orbit design for TianQin and other geocentric missions should take into account and trade off between a variety of factors [\[9](#page-6-8)] such as science objectives, space environment, science payloads, etc. For example, in [\[10\]](#page-6-9), the impact of orbit selection on the constellation stability of TianQin has been studied, and the permitted ranges of the orbital elements that can meet the payload requirements were identified. In a similar manner, one should also investigate the impact of orbit selection on gravity-field disturbances so as to facilitate trade studies in the orbit design and help seek potential remedies if needed.

In the previous paper  $[11]$  $[11]$  $[11]$ , we have shown that, with the  $1 \times 10^5$  km orbital radius and a vertical orbital plane relative to the ecliptic for TianQin, the Earth-Moon's gravity-field disturbances are dominating in lower frequencies, and roll off rapidly at  $1 \times 10^{-4}$  Hz. The effect does not enter the detection frequency band of  $10^{-4}$  Hz–1 Hz where<br>the intersatellite residual acceleration noise requirement of the intersatellite residual acceleration noise requirement of  $\sqrt{2} \times 10^{-15}$  m/s<sup>2</sup>/Hz<sup>1/2</sup> is imposed, and hence presents no showstopper to the mission. For comparison, the gravity showstopper to the mission. For comparison, the gravity mapping mission GRACE-FO has a much higher roll-off frequency at  $4 \times 10^{-2}$  Hz [[12](#page-6-11)], owing to its low altitude of  $~\sim$ 500 km above the ground, at which the science instrument can be sensitive to the Earth's spherical harmonic [\\*](#page-0-1)

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gravity field to  $> 150$ th degree [\[13\]](#page-6-12). Thereby, a separation of  $1 \times 10^5$  km from the Earth is considered large enough for TianQin to push the gravity-field disturbances out of the detection band.

Nevertheless, these two isolated cases does not tell us much about other possible options in selecting orbital orientations and radii. It begs the questions on how the gravity-field effect changes with different orbit selections, and how one chooses a safe distance from gravity-field noise sources, particularly, the Earth. In this work, we intend to resolve these problems by evaluating the effect with respect to various inclinations, longitudes of ascending nodes, and orbital radii. The results may provide a useful reference to general orbit and constellation design for geocentric missions, such as GEOGRAWI/gLISA [[14](#page-6-13)], GADFLI  $[15]$ , B-DECIGO  $[16]$ , etc.

This is our sixth paper of the concept study series on TianQin's orbit and constellation [[10](#page-6-9),[11](#page-6-10),[17](#page-7-2)–[19\]](#page-7-3). For progress on other space environmental effects on TianQin, one may refer to [\[20](#page-7-4)–[24\]](#page-7-5). This article is structured as follows. Sec. [II](#page-1-0) describes the observable and force models used in the simulations. The results are shown in Sec. [III](#page-1-1), which also include geostationary orbits, and repeat orbits and resonant orbits are deferred to the appendixes. Finally, we reach the conclusions in Sec. [IV.](#page-5-0)

## II. SIMULATION SETUP

<span id="page-1-0"></span>Following [\[11\]](#page-6-10), we adopt the range acceleration  $\ddot{\rho}$  as the observable, which is simply the second time derivative of the range  $\rho$  between two satellites. The criteria is to compare the amplitude spectral densities (ASD) of  $\ddot{\rho}$  with the residual acceleration noise requirements between two free-floating test masses, which is generally in the order of  $10^{-15}$  m/s<sup>2</sup>/Hz<sup>1/2</sup> at mHz. To overcome the insufficiency of double precision, we again use the TQPOP (TianQin Quadruple Precision Orbit Propagator) program [[11](#page-6-10)] to propagate pure-gravity orbits in quadruple precision and with a constant step size of 50 seconds for 90 days. Here the step size is a balanced choice made among the factors of frequency band coverage, numerical accuracy, and computational run-time [[11](#page-6-10)]. Like TianQin, all constellations are assumed to be equilateral triangles barring small deviations, and the three satellites moving in the same nominal circular orbits with 120° phase separation (see, e.g., Table [I](#page-1-2)). The

<span id="page-1-2"></span>TABLE I. The nominal orbital elements of the TianQin constellation in the J2000-based Earth-centered ecliptic (Earth-MJ2000Ec) coordinate system. Here a denotes the semimajor axis, e the eccentricity, i the inclination,  $\Omega$  the longitude of ascending node,  $\omega$  the argument of periapsis, and  $\Delta \nu$  the difference in the true anomaly.

		$\omega$	
SC1,2,3 $1 \times 10^5$ km $0^{\circ}$ 94.7° $210.4^{\circ}$ 0° $120^{\circ}$			

<span id="page-1-3"></span>TABLE II. The list of force models implemented in TQPOP. If multiple options are given, the first one is used in the simulations.



spectral results are not sensitive to the specific year chosen. Without loss of generality, the initial epoch assumes 06 June 2004, 00∶00∶00 UTC as in [[11](#page-6-10)].

The detailed force models implemented in TQPOP are summarized in Table [II](#page-1-3). Further description of them can be found in [[11](#page-6-10)]. In particular, the program offers multiple options in modeling solar system ephemeris, Earth's precession and nutation, Earth's static gravity field, etc. The flexibility helps cross-checking model errors, which have been shown not to alter the spectral results for TianQin [\[11\]](#page-6-10). In the following simulations, the more recent options will be used (except for the ocean tides). A new addition to TQPOP lately includes two ocean tide models FES2014b [[25](#page-7-6)] and EOT11a [[26\]](#page-7-7). Based on the results of the test runs, all three models generate spectra consistent with one another in low frequencies. However, since FES2014b contains more high-frequency tidal components (e.g., M6, M8), we will use FES2014b in the following simulation.

### III. SIMULATION RESULTS

<span id="page-1-4"></span><span id="page-1-1"></span>We can assess the impact of orbit selection on the gravity-field effect from the changes of the amplitudes  $A_{kl}^{\pm}$  and frequencies  $f_{kl}^{\pm}$  of spectral peaks in the range<br>acceleration ASD curve. The frequencies  $f_{\pm}^{\pm}$  can be acceleration ASD curve. The frequencies  $f_{kl}^{\pm}$  can be generally expressed as generally expressed as

$$
f_{kl}^{\pm} = k \cdot f_{orb} \pm l \cdot f_{gra}, \qquad (1)
$$

where  $f_{orb}$  is the satellites' orbital frequency, and  $f_{gra}$  is the intrinsic frequencies of gravity-field disturbances, and  $k$ ,  $l$ are integers.

The goal is to find how the gravity-field effect and the roll-off frequency vary with orbital elements. Here the rolloff frequency refers to where the range acceleration ASD curve intersects the acceleration noise requirement.

The study cases involve three orbital elements, i.e., the inclination *i*, the longitude of ascending node  $\Omega$ , and the semimajor axis a. Starting from the nominal orbit of TianQin given in Table [I,](#page-1-2) we vary one single element at a time to make the trends more prominent. Besides, we discuss the particular case of geostationary orbits in Sec. [III C](#page-4-0).

# A. Impact of orbital orientations

<span id="page-2-5"></span>Orbital inclinations i and longitudes of ascending nodes  $Ω$  determine orbital orientations. For single-variable studies, we vary *i* and  $\Omega$  separately in the EarthMJ2000Ec coordinate system.

However, due to the existence of obliquity of the ecliptic, the influence of the Earth's static gravity field and its variation with orbital inclinations cannot be fully demonstrated in the EarthMJ2000Ec coordinate system by numerical methods. Fortunately, from the previous work [\[11\]](#page-6-10) (Sec. IVB), we have learned that the total gravity-field effect in terms of the range-acceleration ASD can be roughly divided into two primary parts. The high-frequency part (above  $\sim$ 5 × 10<sup>-5</sup> Hz) is dominated by the nonspherical gravity field of the Earth, and the low-frequency part (below  $~\sim$ 5 × 10<sup>-5</sup> Hz) is dominated by the Sun and the Moon. Therefore, based on symmetry considerations, we discuss the impact of the Earth's static gravity field with orbital orientations varying in the geocentric equatorial (EarthMJ2000Eq) coordinate system to supplement the effects of all the gravity-field disturbances in the EarthMJ2000Ec coordinate system. Here we mention that there is a 5° difference between the Moon's orbital plane

<span id="page-2-0"></span>TABLE III. The orbital elements with different inclinations given in the EarthMJ2000Ec coordinate system at the epoch 06 Jun. 2004, 00∶00∶00 UTC.

a		
$1 \times 10^5$ km	$0^{\circ}$ , 23.5°, 45°, 90°, 135°, 180°	$210.4^{\circ}$

<span id="page-2-2"></span>TABLE IV. The orbital elements with different longitudes of ascending node given in the EarthMJ2000Ec coordinate system at the epoch 06 Jun. 2004, 00∶00∶00 UTC.



<span id="page-2-3"></span>TABLE V. The orbital elements with different inclinations given in the EarthMJ2000Eq coordinate system at the epoch 06 Jun. 2004, 00∶00∶00 UTC, assuming circular orbits and  $\Delta \nu = 120^{\circ}$  for three satellites. Note that  $i = 94.7^{\circ}$ ,  $\Omega = 210.4^{\circ}$ (cf. Table [I](#page-1-2)) correspond to  $i_{eq} = 74.5^{\circ}$ ,  $\Omega_{eq} = 211.6^{\circ}$ .

a	$\iota_{\text{eq}}$	
$1 \times 10^5$ km	$0^\circ$ , 30 $^\circ$ , 60 $^\circ$ , 90 $^\circ$ , 120 $^\circ$ , 150 $^\circ$ , 180 $^\circ$	$211.6^{\circ}$

and the ecliptic plane, but it does not contribute significantly to the results.

In simulation, setting the orbital radius to a fixed value of  $1 \times 10^5$  km, we vary the inclination from  $i = 0^\circ$  to 180° by

<span id="page-2-1"></span>

FIG. 1. The range acceleration ASD between SC1 and SC2 calculated with the orbital parameters in Table [III](#page-2-0), for 90 days with a constant step size of 50 seconds.

<span id="page-2-4"></span>

FIG. 2. Examples of low-frequency peaks of Fig. [1](#page-2-1) and Eq. [\(1\),](#page-1-4) where  $f_{22}^{\pm} = 2f_{\text{orb}} \pm 2f_{\text{moon}}$ . The plot shows that the amplitude  $4^-$  falls and  $4^+$  rises as the inclination increases in geocentric  $A_{22}^-$  falls and  $A_{22}^+$  rises as the inclination increases in geocentric equinitie coordinate system ecliptic coordinate system.

<span id="page-3-0"></span>

FIG. 3. The range acceleration ASD of the Earth's static gravity field between SC1 and SC2 calculated with the orbital parameters in Table [V,](#page-2-3) for 90 days with a constant step size of 50 seconds.

 $\Delta i = 45^{\circ}$  intervals (see Table [III\)](#page-2-0), where  $i = 23.5^{\circ}$  corresponds to the orbit near the equatorial plane. Then we take four distinct values of longitudes of ascending nodes Ω from 30.4° to 300.4° by  $\Delta \Omega = 90^\circ$  intervals (Table [IV](#page-2-2)) in the EarthMJ2000Ec coordinate system. Besides, to supplement the above cases, we vary the inclination from  $i_{eq} = 0^{\circ}$ to 180° by  $\Delta i_{\text{eq}} = 30^{\circ}$  intervals (see Table [V\)](#page-2-3) in the EarthMJ2000Eq coordinate system. Due to rotational symmetry, the variation of the Earth's static gravity-field effect with different  $\Omega_{eq}$  is insignificant, which is also confirmed by our simulations. A few observations can be summarized as follows.

First, the low-frequency peaks of Fig. [1](#page-2-1) show that  $A_{kl}^ (l \neq 0)$  falls and  $A_{kl}^+(l \neq 0)$  rises as i increases, which, for example can be read from Fig. 2. Moreover, the peaks example, can be read from Fig. [2](#page-2-4). Moreover, the peaks  $k = 1$ ,  $l = 0$  corresponding to the orbital frequencies coincide with different i's in the plot. The difference in the amplitudes  $A_{10}$  has been discussed in [\[10\]](#page-6-9).

<span id="page-3-1"></span>

FIG. 4. The range acceleration ASD between SC1 and SC2 calculated with the orbital parameters in Table [IV,](#page-2-2) for 90 days with a constant step size of 50 seconds.

Second, for high-frequency components, Fig. [1](#page-2-1) shows the ASD curves extend to higher frequencies monotonically as  $i$  increases from  $45^\circ$ . Besides, Fig. [3](#page-3-0) demonstrates that the ASD curves of the Earth's static gravity-field effect extend to higher frequencies as  $i_{eq}$  increases. When  $0^{\circ} \leq$  $i \leq 45^{\circ}$  in the EarthMJ2000Ec coordinate system and  $i_{eq} \geq$ 150° in the EarthMJ2000Eq coordinate system, the roll-off frequency is insensitive to changes in inclinations, and hence it does not affect the range of the detection frequency band significantly.

Third, when the satellites are running on the equatorial plane ( $i_{eq} = 0^{\circ}$  and 180°), Fig. [3](#page-3-0) also shows the curves' trough values are obviously lower than other cases. This may be attributed to that the satellites experience less "raggedness" of the Earth's gravity field when orbiting in the equatorial plane (see also Fig. [7\)](#page-5-1).

Fourth, compared with the case of orbital inclinations, the gravity-field effect is less sensitive to changes of the longitude of ascending node  $Ω$ , as shown in Fig. [4.](#page-3-1) In high frequencies, the differences come from that the Earth's static gravity field varies with  $\Omega$  in the EarthMJ2000Ec coordinate system because of the obliquity of the ecliptic. In lower frequencies, the differences are partly due to the 5° mismatch between the Moon's orbital plane and the ecliptic plane. The situation is somewhat similar to that of

<span id="page-3-2"></span>TABLE VI. The orbital elements with different orbital radii given in the EarthMJ2000Ec coordinate system at the epoch 06 June 2004, 00∶00∶00 UTC.

$a/10^4$ km		$\Omega$ <sup>c</sup>
4, 6, 8, 10, 12	94.7	210.4

<span id="page-4-1"></span>

FIG. 5. The range acceleration ASD with different orbital radii from Table [VI](#page-3-2) and for 90 days with a constant step size of 50 seconds.

constellation stability [\[10\]](#page-6-9), which is related to an approximate rotational symmetry about the ecliptic pole.

#### B. Impact of orbital radii

Based on TianQin, we examine the orbital radius ranging from  $4 \times 10^4$  km to  $1.2 \times 10^5$  km by  $\Delta a = 2 \times 10^4$  km intervals. For special cases of repeat orbits and resonant orbits, see the appendixes. The orbital elements are given in Table [VI](#page-3-2).

It should be noted that simulations with smaller orbital radii require higher degrees and orders in the Earth's gravity-field model. With estimated gravitational acceleration from different degrees, it is sufficient to set  $n = 16$  for  $a = 4 \times 10^4$  km,  $n = 13$  for  $a = 6 \times 10^4$  km, and  $n = 12$ for  $a \geq 8 \times 10^4$  km [[11](#page-6-10)], so as to meet an accuracy below  $10^{-16}$  m/s<sup>2</sup>/Hz<sup>1/2</sup>. The corresponding results of the range acceleration ASD are presented in Fig. [5.](#page-4-1)

First, with the orbital radius increasing, the ASD curves appear to shift to low frequencies overall. Moreover, one can read out the roll-off frequencies from Fig. [5](#page-4-1), and the values are listed in Table [VII](#page-4-2) and shown in Fig. [6](#page-4-3).

<span id="page-4-2"></span>TABLE VII. The numerical values of the roll-off frequency  $f<sub>r</sub>$ and the index  $E_f$ .

$a$ /km	$f_r/10^{-4}$ Hz	$E_f$
40000	2.30	21.0
60000	1.46	22.2
80000	1.20	23.8
100000	1.00	26.0
120000	0.93	28.6

<span id="page-4-3"></span>

FIG. 6. The relation between roll-off frequency and orbital radius of Fig. [5.](#page-4-1)

Second, one can fit the ASD curves of the range acceleration near the roll-off frequencies  $f<sub>r</sub>$  by

$$
\widetilde{RA} = \sqrt{2} \times 10^{-15} \times \left(\frac{f_r}{f}\right)^{E_f} \text{ m/s}^2/\text{Hz}^{1/2},\qquad(2)
$$

where f is the Fourier frequency, and  $E_f$  is the power index of the ratio. The fitted values of  $E_f$  are given in Table [VII](#page-4-2). We mention that  $E_f$  roughly changes with  $a^2$  and  $f_r$ with  $1/a^2$ .

## <span id="page-4-0"></span>C. Geostationary orbits for gLISA/GADFLI

In this subsection we examine geostationary orbits proposed by gLISA/GADFLI. The initial orbital elements are given in Table [VIII,](#page-4-4) which have been optimized for 90 days to stabilize the constellation. Here to focus on the gravity-field effect, we ignore station-keeping maneuvers that may be needed roughly every two weeks [[14](#page-6-13)]. The corresponding range acceleration ASD is presented in Fig. [7](#page-5-1), which incorporates the cases with and without the Earth's nonspherical gravity field in the simulation.

Compared with Fig. [5,](#page-4-1) the plot shows a rather distinct feature in that the number of peaks is significantly reduced. This is due to the fact that by placing the satellites at the same location above the Earth's surface, the Earth's gravity

<span id="page-4-4"></span>TABLE VIII. The initial orbital elements of gLISA given in the EarthMJ2000Eq coordinate system at the epoch 06 June 2004, 00∶00∶00 UTC.

Satellite	a	$\boldsymbol{\rho}$		Ω	$\omega$	$\nu$ <sup>ini</sup>
SC <sub>1</sub> SC <sub>2</sub>	42158.65 km 42164.90 km	$\Omega^{\circ}$	$\Omega^{\circ}$	$\Omega^{\circ}$	$\Omega^{\circ}$	$210.0^{\circ}$ 330.8°
SC <sub>3</sub>	42171.00 km					$91.3^{\circ}$

<span id="page-5-1"></span>

FIG. 7. The range acceleration ASD between two satellites of gLISA/GADFLI, using the orbital parameters in Table [VIII](#page-4-4), and calculated for 90 days with a constant step size of 50 seconds.

experienced by the satellites becomes localized and nearly static. Therefore, the range acceleration ASD is instead dominated by the Moon and the Sun's gravity, with the high-frequency components from the Earth's nonspherical gravity field being suppressed. This is indicated by the overlapping of the blue and green curves in Fig. [7,](#page-5-1) where the latter is calculated without the Earth's nonspherical gravity field in orbit propagation. Moreover, the ASD curve intersects with the noise requirement  $3\sqrt{2} \times$  $10^{-15}$  m/s<sup>2</sup>/Hz<sup>1/2</sup> at  $1.69 \times 10^{-4}$  Hz, which is much lower than the proposed detection frequency band  $10^{-2}$ –1 Hz [[14](#page-6-13)].

# IV. CONCLUSION AND DISCUSSION

<span id="page-5-0"></span>In this paper we have simulated and analyzed the gravityfield effect of the Earth-Moon system on intersatellite range acceleration of geocentric detectors in circular orbits with various orbital orientations and radii. It can help fill the blank in the concept studies of geocentric missions (e.g., [\[39\]](#page-7-20)) that might have been overlooked. As the previous work has shown [\[11\]](#page-6-10), one common behavior is that the "orbital noise" dominates in the neighborhood of the orbital frequencies (e.g.,  $3 \times 10^{-6}$  Hz for TianQin), and then falls off rapidly below residual acceleration noise requirements  $({\sim}10^{-15} \text{ m/s}^2/\text{Hz}^{1/2})$  at specific higher frequencies. Three main conclusions can be drawn here.

First, for a fixed orbital radius  $1 \times 10^5$  km and the longitude of ascending node 210.4°, the roll-off frequency extends to higher frequencies with the inclination increasing when the orbital inclination is greater than 45° in the EarthMJ2000Ec coordinate system. When the inclination is between 0° and 45° in the EarthMJ2000Ec coordinate system or above 150° in the EarthMJ2000Eq coordinate system, the variation of roll-off frequency would not affect the choice of the detection frequency band significantly. For a given inclination, altering the longitude of ascending node has only a small impact.

Second, as the orbital radius increases, the gravity-field effect shifts to lower frequencies so that the detection frequency band can be extended accordingly (cf. Fig. [5](#page-4-1)).

Third, the gravity-field effect for gLISA in geostationary orbits intersects with the acceleration noise requirement  $3\sqrt{2} \times 10^{-15}$  m/s<sup>2</sup>/Hz<sup>1/2</sup> at  $1.69 \times 10^{-4}$  Hz. It allows for sufficient clearance from the targeted detection band of sufficient clearance from the targeted detection band of 10<sup>−</sup><sup>2</sup> Hz–1 Hz [[14](#page-6-13)], and hence presents no showstopper to the mission.

It should be pointed out that the results appear to favor small inclinations (see Sec. [III A](#page-2-5)). However, for TianQin, the inclination selection is largely driven by other more pressing technical demands. These include sciencecraft thermal control [\[20,](#page-7-4)[40](#page-7-21)] and the need of lowering eclipse occurrence [\[18\]](#page-7-22), etc., which instead favor a 90° inclination. With a vertical orbital plane relative to the ecliptic, setting the radius equal to or greater than  $1 \times 10^5$  km is sufficient for TianQin to meet the requirement of extending the detection band to as low as  $1 \times 10^{-4}$  Hz. Meanwhile, the upper limit of the orbital radius is likely to be set by the constellation stability requirement [\[10\]](#page-6-9). For future work, it would be desirable to develop an analytic model of the orbital dynamics to account for the range acceleration ASD at greater depths.

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## APPENDIX A: REPEAT ORBITS

A repeat orbit has the property that the satellite covers the same ground track periodically. Such orbits may have certain engineering benefits. With the orbital plane facing J0806, we set the orbital periods from one day to four days, and give the initial orbital radii in Table [IX.](#page-5-2) The corresponding results of the range acceleration ASD are presented in Fig. [8.](#page-6-14) Notably, the ASD of the geosynchronous orbit ( $T = 1$  day) bears great resemblance to Fig. [7](#page-5-1).

<span id="page-5-2"></span>TABLE IX. The initial optimized orbital elements of different repeat orbits for the detectors in the EarthMJ2000Ec coordinate system at the epoch 06 June 2004, 00∶00∶00 UTC.

$T$ /day	SC1/km	SC2/km	SC3/km
	42164.14	42162.22	42163.60
	66934.50	66936.30	66932.00
	87714.00	87719.70	87710.00
	106265.00	106278.50	106261.00

<span id="page-6-14"></span>

FIG. 8. The range acceleration ASD of different repeat orbital radii between SC1 and SC2, using orbital parameters in Table [IX,](#page-5-2) for 90 days with a constant step size of 50 seconds.

# APPENDIX B: RESONANT ORBITS

Although for TianQin, the work of [[10](#page-6-9)] has ruled out the orbits resonant with the Moon due to their inferior constellation stability performance. However, the results are still included for the sake of completeness.

We consider orbital period ratios  $T_{\text{moon}}/T_{orb} = 6, 7, 8$ . Table [X](#page-6-15) shows the corresponding orbital radii, and the other orbital elements are the same as Table [VI](#page-3-2). The results are presented in Fig. [9.](#page-6-16) Compared with nonresonant orbits, the peaks corresponding to the Earth-Moon coupling are enhanced. On thewhole, the plot show a similar trend as Fig. [5](#page-4-1).

<span id="page-6-15"></span>TABLE X. The initial orbital elements of different orbital resonance for the detectors in the EarthMJ2000Ec coordinate system at the epoch 06 June 2004, 00∶00∶00 UTC.

$T_{\rm moon}/T_{\rm orb}$	SC1/km	SC2/km	SC3/km
8	95796.617	95805.000	95791.000
	104715.621	104728.000	104710.500
6	116049.338	116070.000	116040.000

<span id="page-6-16"></span>

FIG. 9. The range acceleration ASD of different orbital resonance altitude between SC1 and SC2, using the orbital parameters in Table [X](#page-6-15), for 90 days with a constant step size of 50 seconds.

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