Investigation of the possible $D\bar{D}^*/B\bar{B}^*$ and $DD^*/\bar{B}\bar{B}^*$ bound states

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In this work, we systematically study the $D\bar{D}^*/B\bar{B}^*$ and $DD^*/\bar{B}\bar{B}^*$ systems with the Bethe-Salpeter equation in the ladder and instantaneous approximations for the kernel. By solving the Bethe-Salpeter equation numerically with the kernel containing the direct and crossed one-particle exchange diagrams and introducing three different form factors (monopole, dipole, and exponential form factors) at the vertices, we find only the isoscalar $D\bar{D}^*/B\bar{B}^*$ and $DD^*/\bar{B}\bar{B}^*$ systems can exist as bound states. This indicate that the $X(3872)$ and T_{cc}^+ could be accommodated as $I^G(J^{PC}) = 0^+(1^{++})$ $D\bar{D}^*$ and $(I)J^P = (0)1^+$ DD^* bound states while the bound-state explanations for $Z_b(10610)$ and $Z_c(3900)$ are excluded.

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I. INTRODUCTION

Since the discovery of $X(3872)$ in 2003 [1], more than twenty exotic-state candidates containing $c\bar{c}$ and $b\bar{b}$ quarks have been found and studied by the LHCb, ATLAS, CMS, BESIII, Belle, *BABAR*, CDF, and D0 experiments [2,3]. The structures of these exotic states are more complex than the standard $q\bar{q}$ mesons. Recently, the LHCb Collaboration reported the first doubly open charmed tetraquark state T_{cc}^+ in proton-proton collisions with a signal significance over 10σ [4,5], and with its mass and width being

$$\delta_m = m_{T_{cc}^+} - (m_{D^{*+}} + m_{D^0}) = -273 \pm 61 \pm 5^{+11}_{-14}\ \text{keV},$$
$$\Gamma = 410 \pm 165 \pm 43^{+18}_{-38}\ \text{keV}, \tag{1}$$

respectively. This exotic state with a mass of about 3875 MeV manifests itself as a narrow peak in the mass spectrum of $D^0D^0\pi^+$ mesons just below the $D^{*+}D^0$ mass threshold, and is consistent with the ground isoscalar $cc\bar{u}\bar{d}$ state with the spin-parity quantum numbers $J^P = 1^+$. Although there have been many studies on the exotic states, the puzzle about the nature of these states remains unsolved so far.

Up to now, three exotic states [$X(3872)$, $Z_c(3900)$, and T_{cc}^+] with their masses close to the threshold of DD^* have been found experimentally. To describe these exotic states a variety of phenomenological models have been proposed, including the chiral effective field theory [6–17], the Bethe-Salpeter approach [18–22], the constituent quark model [23–28], QCD sum rules [29–33], and the relativized quark model [34–38], etc. In these models some physical pictures have also been developed to understand the known exotic states, such as hadronic molecule and tetraquark state. Since the masses of these exotic states are close to the threshold of the two S-wave lowest-lying standard mesons (D and D^*), one would naturally identify them as molecular states of standard mesons. Therefore, it is very interesting to investigate whether this is plausible for these exotic states, which will be very helpful to reveal the structures of these exotic states.

In this paper we will focus on the $D\bar{D}^*$ and DD^* bound states and their b partners. Our purpose is to investigate whether the bound states of the $D\bar{D}^*$ and DD^* systems and their b partners via the interaction through exchanging scalar mesons (σ), vector mesons (ρ and ω), and pseudoscalar mesons (π and η) can exist. As the relativistic equation describing the bound state of two particles, the Bethe-Salpeter (BS) equation is an effective method to deal with nonperturbative QCD effects and has been applied to

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many theoretical studies concerning standard heavy hadrons [39–41] and exotic states [42–46]. The kernel of the BS equation can be derived from the relevant effective Lagrangian, based on which numerical solutions for the BS wave functions can be obtained in the covariant instantaneous approximation and the ladder approximation, and these solutions can be used to judge whether these bound states exist.

The remainder of this paper is organized as follows. In Sec. II we discuss the $D\bar{D}^*$ and DD^* system hadronic wave function and derive the BS equation for the vector and pseudoscalar mesons system. We also discuss the kernel derived from the relevant effective Lagrangian. In Sec. III, we present numerical solutions for $D\bar{D}^*$ and DD^* bound states and their b partners with three different form factors. Finally, Sec. IV contains a summary.

II. THE BETHE-SALPETER FORMALISM FOR $D\bar{D}^*$ AND DD^* SYSTEMS

The $D\bar{D}^*$ + c.c. and DD^* systems have isospin 0 or 1. The flavor wave functions of the $D\bar{D}^*$ + c.c. systems are [15,21,47]

$$\begin{aligned}
|X^0_{D\bar{D}^*}\rangle_{I=0} &= \frac{1}{2}[(|D^{*+}D^-\rangle + |D^{*0}\bar{D}^0\rangle) \\
&\quad + c(|D^+D^{*-}\rangle + |D^0\bar{D}^{*0}\rangle)], \\
|X^0_{D\bar{D}^*}\rangle_{I=1} &= \frac{1}{2}[(|D^*D^-\rangle - |D^*\bar{D}^0\rangle) \\
&\quad + c(|D^+D^{*-}\rangle - |D^0\bar{D}^{*0}\rangle)], \\
|X^+_{D\bar{D}^*}\rangle_{I=1} &= \frac{1}{\sqrt{2}}(|D^{*+}\bar{D}^0\rangle + c|D^+\bar{D}^{*0}\rangle), \\
|X^-_{D\bar{D}^*}\rangle_{I=1} &= \frac{1}{\sqrt{2}}(|D^{*-}\bar{D}^0\rangle + c|D^-\bar{D}^{*0}\rangle),
\end{aligned} \tag{2}$$

where $c = \pm 1$ correspond to C parity $C = \mp$ respectively. The wave functions for the DD^* systems are

$$\begin{aligned}
|T^+_{cc}\rangle_{I=0} &= \frac{1}{\sqrt{2}}(|D^+D^{*0}\rangle - |D^0D^{*+}\rangle), \\
|T^+_{cc}\rangle_{I=1} &= \frac{1}{\sqrt{2}}(|D^+D^{*0}\rangle + |D^0D^{*+}\rangle), \\
|T^{++}_{cc}\rangle_{I=1} &= |D^+D^{*+}\rangle, \\
|T^0_{cc}\rangle_{I=1} &= |D^0D^{*0}\rangle.
\end{aligned} \tag{3}$$

The wave functions of the hidden-bottom and doubly bottom states can be obtained analogously.

Since the bound states are composed of a vector meson (D^*) and a pseudoscalar meson (D), we can define the BS wave function of the bound state by

$$\chi^\alpha_P(x_1, x_2, P) = \langle 0|TD^{*\alpha}(x_1)D(x_2)|P\rangle = e^{-iPX}\chi^\alpha_P(x), \tag{4}$$

where $P = Mv$ is the total momentum of the bound state, and v is its velocity. $D^{*\alpha}(x_1)$ and $D(x_2)$ are the field operators of the vector meson D^* and the pseudoscalar meson D at space coordinates x_1 and x_2, respectively. $X \equiv \lambda_1 x_1 + \lambda_2 x_2$ is the coordinate of the center of mass and $x \equiv x_1 - x_2$ is the relative coordinate with $\lambda_1 \equiv m_1/(m_1 + m_2)$ and $\lambda_2 \equiv m_2/(m_1 + m_2)$, with m_1 and m_2 being the masses of D^* and D, respectively. The BS wave function in momentum space is defined as

$$\chi^\alpha_P(x_1, x_2, P) = e^{-iP\cdot X}\int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot x}\chi^\alpha_P(p), \tag{5}$$

where the relative momentum $p = \lambda_2 p_1 - \lambda_1 p_2$, the $p_1 = \lambda_1 P + p$ and $p_2 = \lambda_2 P - p$ are the D^* meson and D meson momenta, respectively.

The BS equation for the bound state consisting of a vector and a pseudoscalar mesons can be written in the following form

$$\chi^\alpha_P(p) = S^{\alpha\lambda}(p_1)\int \frac{d^4q}{(2\pi)^4}\bar{K}_{\lambda\tau}(P, p, q)\chi^\tau_P(q)S(p_2), \tag{6}$$

where $S^{\alpha\lambda}(p_1)$ and $S(p_2)$ are the propagators of D^* and D, respectively, and $K_{\lambda\tau}(P, p, q)$ is the four-point truncated irreducible kernel. The $K_{\lambda\tau}(P, p, q)$ can be derived from four-point Green's function as follows:

$$\begin{aligned}
&G^{\alpha\beta}(x_1, x_2; y_2, y_1) \\
&= G^{\alpha\beta}_{(0)}(x_1, x_2; y_2, y_1) + \int d^4u_1 d^4u_2 d^4v_1 d^4v_2 \\
&\quad \times G^{\alpha\lambda}_{(0)}(x_1, x_2; u_2, u_1)\bar{K}_{\lambda\tau}(u_1, u_2; v_2, v_1) \\
&\quad \times G^{\tau\beta}(v_1, v_2; y_2, y_1),
\end{aligned} \tag{7}$$

where $G^{\alpha\beta}_{(0)}$ is related to the forward scattering disconnected four-point amplitude,

$$G^{\alpha\beta}_{(0)}(x_1, x_2; y_2, y_1) = S^{\alpha\beta}(x_1, y_1)S(x_2, y_2), \tag{8}$$

$S^{\alpha\beta}(x_1, y_1)$ and $S(x_2, y_2)$ are the propagators of constituent particles D^* and D in coordinate space, respectively.

For convenience, we define $p_l (= p\cdot v)$ and $p^\mu_t (= p^\mu - p_l v^\mu)$ to be the longitudinal and transverse projections of the relative momentum (p) along the bound state momentum (P). Then the D^* propagator has the form

$$S^{\alpha\beta}(p_1) = \frac{-i[g^{\alpha\beta} - (\lambda_1 Mv + p_l v + p_t)^\alpha(\lambda_1 Mv + p_l v + p_t)^\beta/m_1^2]}{(\lambda_1 M + p_l)^2 - \omega_1^2 + i\epsilon}, \tag{9}$$

and the propagator of D meson has the form

$$S(p_2) = \frac{i}{(\lambda_2 M - p_l)^2 - \omega_2^2 + i\epsilon}, \tag{10}$$

where $\omega_{1(2)} = \sqrt{m_{1(2)}^2 - p_t^2}$.

In general, for $D\bar{D}^*$ and DD^* systems, $\chi_P^\alpha(p)$ can be written as

$$\chi_P^\alpha(p) = f_1(p)\varepsilon^{\alpha\beta\mu\nu} g_{\mu\nu}\epsilon_\beta(P) + f_2(p)\varepsilon^{\alpha\beta\mu\nu} P_\mu P_\nu \epsilon_\beta(P) + f_3(p)\varepsilon^{\alpha\beta\mu\nu} p_\mu P_\nu \epsilon_\beta(P) + f_4(p)\varepsilon^{\alpha\beta\mu\nu} p_\mu p_\nu \epsilon_\beta(P), \tag{11}$$

where $\epsilon_\beta(P)$ represents the polarization vector of the bound state and f_i ($i = 1, 2, 3, 4$) are Lorentz-scalar functions. With the constraints imposed by parity and Lorentz transformations, it is easily to prove that $\chi_P^\alpha(p)$ can be simplified as

$$\chi_P^\alpha(p) = f(p)\varepsilon^{\alpha\beta\mu\nu} p_\mu P_\nu \epsilon_\beta, \tag{12}$$

where the function $f(p)$ contains all the dynamics and is a Lorentz scalar function of p.

In order to obtain the interaction kernel $K_{\lambda\tau}(P, p, q)$ of $D\bar{D}^*$ and DD^* systems, the Lagrangian for heavy mesons interacting with light mesons are needed, which can be described from the heavy-meson chiral perturbation theory as the following [48,49]:

$$\begin{aligned}
\mathcal{L}_{DD\sigma} &= -2g_\sigma m_D D_a D_a^\dagger \sigma - 2g_\sigma m_D \bar{D}_a \bar{D}_a^\dagger \sigma, \\
\mathcal{L}_{D^*D^*\sigma} &= 2g_\sigma m_{D^*} \mathcal{D}_a^{*\alpha} D_{\alpha a}^{*\dagger}\sigma + 2g_\sigma m_{D^*} \bar{D}_a^{*\alpha} \bar{D}_{\alpha a}^{*\dagger}\sigma, \\
\mathcal{L}_{DD^*\mathbb{P}} &= g_{DD^*\mathbb{P}}(D_b D_{\alpha a}^{*\dagger} + D_{\alpha b}^* D_a^\dagger)\partial^\alpha \mathbb{P}_{ba} + g_{\bar{D}\bar{D}^*\mathbb{P}}(\bar{D}_{\alpha a}^{*\dagger}\bar{D}_b + \bar{D}_a^\dagger \bar{D}_{\alpha b}^*)\partial^\alpha \mathbb{P}_{ab}, \\
\mathcal{L}_{DD^*\mathbb{V}} &= -2ig_{DD^*\mathbb{V}}\varepsilon_{\mu\nu\alpha\beta}(\partial^\mu \mathbb{V}^\nu)_{ba}[(D_a^\dagger \partial^\alpha D_b^{*\beta} - D_b^{*\beta}\partial^\alpha D_a^\dagger) + (D_a^{*\beta\dagger}\partial^\alpha \mathcal{D}_b - \mathcal{D}_b \partial^\alpha D_a^{*\beta\dagger})] \\
&\quad - 2ig_{\bar{D}\bar{D}^*\mathbb{V}}\varepsilon_{\mu\nu\alpha\beta}(\partial^\mu \mathbb{V}^\nu)_{ab}[(\bar{D}_a^\dagger \partial^\alpha \bar{D}_b^{*\beta} - \bar{D}_b^{*\beta}\partial^\alpha \bar{D}_a^\dagger) + (\bar{D}_a^{*\beta\dagger}\partial^\alpha \bar{D}_b - \bar{D}_b \partial^\alpha \bar{D}_a^{*\beta\dagger})], \\
\mathcal{L}_{DD\mathbb{V}} &= ig_{DD\mathbb{V}}(D_b \partial_\alpha D_a^\dagger - D_a^\dagger \partial_\alpha D_b)(\mathbb{V}^\alpha)_{ba} + ig_{\bar{D}\bar{D}\mathbb{V}}(\bar{D}_b \partial_\alpha \bar{D}_a^\dagger - \bar{D}_a^\dagger \partial_\alpha \bar{D}_b)(\mathbb{V}^\alpha)_{ab}, \\
\mathcal{L}_{D^*D^*\mathbb{V}} &= ig_{D^*D^*\mathbb{V}}(D_{\beta,b}^* \partial^\alpha D_a^{*\beta\dagger} - D_a^{*\beta\dagger}\partial^\alpha D_{\beta,b}^*)(\mathbb{V}_\alpha)_{ba} - ig'_{D^*D^*\mathbb{V}} D_{\alpha,a}^{*\dagger} D_{\beta,b}^*(\partial^\alpha \mathbb{V}^\beta - \partial^\beta \mathbb{V}^\alpha)_{ba} \\
&\quad + ig_{\bar{D}^*\bar{D}^*\mathbb{V}}(\bar{D}_{\beta,b}^* \partial^\alpha \bar{D}_a^{*\beta\dagger} - \bar{D}_a^{*\beta\dagger}\partial^\alpha \bar{D}_{\beta,b}^*)(\mathbb{V}_\alpha)_{ab} - ig'_{\bar{D}^*\bar{D}^*\mathbb{V}} \bar{D}_{\alpha,a}^{*\dagger} \bar{D}_{\beta,b}^*(\partial^\alpha \mathbb{V}^\beta - \partial^\beta \mathbb{V}^\alpha)_{ab},
\end{aligned} \tag{13}$$

where D, $\bar{D}$, D^*, and $\bar{D}^*$ are heavy flavored meson fields, with $D = (D^0, D^+, D_s^+)$, $\bar{D} = (\bar{D}^0, D^-, D_s^-)$, $D^* = (D^{*0}, D^{*+}, D_s^{*+})$, and $\bar{D}^* = (\bar{D}^{*0}, \bar{D}^{*-}, \bar{D}_s^{*-})$, a and b denote the light quark flavor indices, the octet pseudoscalar $\mathbb{P}$ and the nonet vector $\mathbb{V}$ meson matrices are defined as

$$\mathbb{P} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}, \tag{14}$$

and

$$\mathbb{V} = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}, \tag{15}$$

respectively, and the coupling constants are given as

$$\begin{aligned}
&g_{DD^*\mathbb{P}} = -g_{\bar{D}\bar{D}^*\mathbb{P}} = -\frac{2g}{f_\pi}\sqrt{m_D m_{D^*}}, \\
&g_{DD^*\mathbb{V}} = g_{\bar{D}\bar{D}^*\mathbb{V}} = \frac{\lambda g_{\mathbb{V}}}{\sqrt{2}}, \\
&g_{DD\mathbb{V}} = -g_{\bar{D}\bar{D}\mathbb{V}} = -g_{D^*D^*\mathbb{V}} = g_{\bar{D}^*\bar{D}^*\mathbb{V}} = \frac{\beta g_{\mathbb{V}}}{\sqrt{2}}, \\
&g'_{D^*D^*\mathbb{V}} = -g'_{\bar{D}^*\bar{D}^*\mathbb{V}} = -\sqrt{2}\lambda g_V m_{D^*} \\
&g_{\mathbb{V}} = \frac{m_\rho}{f_\pi}, \quad g_\sigma = \frac{g_\pi}{2\sqrt{6}}, \quad g = 0.59, \quad \beta = 0.9, \\
&\lambda = 0.56\,\mathrm{GeV}^{-1}, \quad f_\pi = 132\,\mathrm{MeV}, \quad g_\pi = 3.73.
\end{aligned} \tag{16}$$

In the so-called ladder approximation, the kernel $K_{\lambda\tau}(P, p, q)$ is replaced by its lowest-order form. Then with the Lagrangian for heavy mesons interacting with light mesons, for the kernel of the bound states induced by ρ, ω, σ, π, and η exchanges, we have

$$\bar{K}^{\lambda\tau,\sigma}_{DD^*-\text{direct}}(p_1,p_2;q_2,q_1) = (2\pi)^4\delta^4(q_1+q_2-p_1-p_2)4c_I^d g_\sigma^2 m_{D^*}m_D\Delta(k,m_\sigma)g^{\lambda\tau},$$
$$\bar{K}^{\lambda\tau,P}_{DD^*-\text{crossed}}(p_1,p_2;q_2,q_1) = -(2\pi)^4\delta^4(q_1+q_2-p_1-p_2)c_I^c g^2_{DD^*\mathbb{P}}k^\lambda k^\tau\Delta(k,m_P),$$
$$\bar{K}^{\lambda\tau,V}_{DD^*-\text{direct}}(p_1,p_2;q_2,q_1) = -(2\pi)^4\delta^4(q_1+q_2-p_1-p_2)c_I^d g_{DDV}(p_2+q_2)_\nu\Delta^{\mu\nu}(k,m_V)$$
$$\times[g_{D^*D^*\mathbb{V}}(p_1+q_1)_\mu g_{\lambda\tau}+g'_{D^*D^*\mathbb{V}}(k_\lambda g_{\tau\mu}-k_\tau g_{\lambda\mu})],$$
$$\bar{K}^{\lambda\tau,V}_{DD^*-\text{crossed}}(p_1,p_2;q_2,q_1) = -(2\pi)^4\delta^4(q_1+q_2-p_1-p_2)4c_I^c f^2_{DD^*\mathbb{V}}\epsilon^{\mu\beta\sigma\lambda}\epsilon^{\nu\rho\gamma\tau}$$
$$\times k_\mu k_\nu(p_1+q_2)_\sigma(q_1+p_2)_\gamma\Delta_{\beta\rho}(k,m_V), \tag{17}$$

for the DD^* bound state, and

$$\bar{K}^{\lambda\tau,\sigma}_{D\bar{D}^*-\text{direct}}(p_1,p_2;q_2,q_1) = \bar{K}^{\lambda\tau,\sigma}_{DD^*-\text{direct}}(p_1,p_2;q_2,q_1),$$
$$\bar{K}^{\lambda\tau,P}_{D\bar{D}^*-\text{crossed}}(p_1,p_2;q_2,q_1) = -\bar{K}^{\lambda\tau,P}_{DD^*-\text{crossed}}(p_1,p_2;q_2,q_1),$$
$$\bar{K}^{\lambda\tau,V}_{D\bar{D}^*-\text{direct}}(p_1,p_2;q_2,q_1) = \bar{K}^{\lambda\tau,V}_{DD^*-\text{direct}}(p_1,p_2;q_2,q_1),$$
$$\bar{K}^{\lambda\tau,V}_{D\bar{D}^*-\text{crossed}}(p_1,p_2;q_2,q_1) = \bar{K}^{\lambda\tau,V}_{DD^*-\text{crossed}}(p_1,p_2;q_2,q_1), \tag{18}$$

for the $D\bar{D}^*$ bound state, where m_σ, m_P, and m_V represent the masses of the exchanged σ, the pseudoscalar light meson and the vector light meson, respectively. c_I^d and c_I^c are the isospin coefficients for the direct and crossed diagrams, and the values for different exchange mesons are listed in Table I, and the derivation process shown in Appendix A. $\Delta^{\mu\nu}$ represents the propagator for a vector meson and Δ represents the scalar or pseudoscalar meson propagator, and they have the following forms,

$$\Delta^{\mu\nu}(k,m_V) = \frac{-i}{k^2-m_V^2}\left(g_{\mu\nu}-\frac{k_\mu k_\nu}{m_V^2}\right),$$
$$\Delta(k,m_{\sigma(P)}) = \frac{i}{k^2-m^2_{\sigma(P)}}. \tag{19}$$

Considering the size effects of constituent particles, we introduce the monopole, exponential and dipole form factors at each vertex. The form factors are respectively defined as

$$F_M(k^2) = \frac{\Lambda^2-m^2}{\Lambda^2-k^2},$$
$$F_E(k^2) = e^{(k^2-m^2)/\Lambda^2},$$
$$F_D(k^2) = \frac{(\Lambda^2-m^2)^2}{(\Lambda^2-k^2)^2}. \tag{20}$$

From Refs. [50,51], we can find the wave function curves of π and ρ mesons do not differ much. Furthermore, since σ, π, η, ρ, and ω are light mesons, their radii should not be very different. Since the form factors are determined by the overlap integrals of the wave functions of the hadrons at the vertices, we assumed the same value of Λ for all the exchanged particles.

In the following we will drive explicitly the integral equation for the BS scalar wave function. Substituting Eqs. (9), (10), (12), and (17) or (18) into the BS equation (6), we can obtain the simplified BS equation and we listed it in Appendix B.

In Eq. (B1) there are poles in p_l at $-\lambda_1 M - w_1 + i\epsilon$, $-\lambda_1 M + w_1 - i\epsilon$, $\lambda_2 M + w_2 - i\epsilon$, and $\lambda_2 M - w_2 + i\epsilon$. By choosing the appropriate contour, we integrate over p_l on both sides of Eq. (B1) and obtain the three-dimensional integral equation. Then the scalar BS wave function is rotationally invariant, and $f(\mathbf{p}_\mathrm{t})$ $[f(\mathbf{p}_\mathrm{t}) = \int dp_l f(p)]$ only depends on the norm of the three-momentum, $\mathbf{p}_\mathrm{t}$. Therefore, after completing the azimuthal integration, the above BS equation becomes a one-dimensional integral equation, which is

$$f(|\mathbf{p}_\mathrm{t}|) = \int d|\mathbf{p}_\mathrm{t}|A(|\mathbf{p}_\mathrm{t}|,|\mathbf{q}_\mathrm{t}|)f(|\mathbf{q}_\mathrm{t}|), \tag{21}$$

TABLE I. The isospin factors c_I^d and c_I^c for direct and crossed Feynman diagrams with $I=0$ and $I=1$.

		c_I^d					c_I^c				
		ρ	ω	π	η	σ	ρ	ω	π	η	σ
$D\bar{D}^*$	$I=0$	3/2	1/2	⋯	⋯	1	3c/2	c/2	3c/2	c/6	⋯
	$I=1$	−1/2	1/2	⋯	⋯	1	−c/2	c/2	−c/2	c/6	⋯
DD^*	$I=0$	−3/2	1/2	⋯	⋯	1	3/2	−1/2	3/2	−1/6	⋯
	$I=1$	1/2	1/2	⋯	⋯	1	1/2	1/2	1/2	1/6	⋯

where $f(|\mathbf{p}_t|)$ is the one-dimensional Lorentz-scalar BS function. The propagators and kernels after one-dimensional simplification are included in $A(|\mathbf{p}_t|, |\mathbf{q}_t|)$.

III. NUMERICAL SOLUTIONS FOR THE BS WAVE FUNCTION

In this section we solve the integral equation—Eq. (21). The method discretizes the integration region into sufficiently large n pieces by the Gaussian quadrature method. In this way, the BS scalar function $f(|\mathbf{p}_t|)$ becomes n dimensional vector and the integral equation becomes an eigenvalue equation. The matrix equation obtained in this way can be written in the form $f^{(n)}(|\mathbf{p}_t|) = A^{n\times n}(|\mathbf{p}_t|, |\mathbf{q}_t|)f^{(n)}(|\mathbf{q}_t|)$.

There are two parameters in our model, the cutoff Λ and the binding energy E_b. The cutoff Λ contains the information about the nonpoint interaction of the hadrons which is nonperturbative. Therefore, the cutoff Λ cannot be determined exactly. Fortunately, the cutoff was found in the study of the deuteron to be around 1 GeV. In this work, we vary the cutoff Λ over a much wider range (0.8 GeV–5 GeV) to find all possible solutions of the $D\bar{D}^*/B\bar{B}^*$ and $DD^*/\bar{B}\bar{B}^*$ bound states. The binding energy E_b is defined as $E_b = E - m_1 - m_2$, and we will vary E_b from −30 MeV–0 MeV. Fixing a value of the cutoff Λ and varying the binding energy E_b we will obtain a series of the trial eigenvalues. For some (not all) values of the cutoff, we could find that the binding energy in the range −30 MeV–0 MeV corresponds to the eigenvalue closest to 1.0. Our task is to find out all these cutoff values.

In the present paper, we will systematically study the S-wave $D\bar{D}^*/B\bar{B}^*$ and $DD^*/\bar{B}\bar{B}^*$ bound states. As studied in Ref. [47], the $D\bar{D}^*/B\bar{B}^*$ carry different C parity for the isoscalar and isovector states, respectively. From the effective Lagrangian listed in Eq. (13), we can find the kernels in the bottom sector which have the same form as those in the charm sector. We work in the rest frame of the bound state in which $P = (M, 0)$ and take the averaged masses of the mesons from the PDG [2], $m_D = 1867.24$ MeV, $m_{D^*} = 2008.56$ MeV, $m_B = 5279.48$ MeV, $m_{B^*} = 5324.70$ MeV, $m_\pi = 137.28$ MeV, $m_\eta = 547.86$ MeV, $m_\rho = 775.26$ MeV, $m_\omega = 782.65$ MeV, and $m_\sigma = 550$ MeV. The contribution of σ exchange is permitted by the chiral perturbation theory and thus is included in our work, despite the large uncertainties in its mass and structure. In our previous work [45] and Ref. [48], it found that the contribution from σ exchange is very small and not sufficient to form bound states. The numerical results from our calculation for the charm and bottom systems are listed in Tables II and III, respectively.

From the results in Tables II and III, we can draw three main conclusions. First, only the bound states with isospin $I = 0$ exist. Second, for the same binding energy E_b, the values of the cutoff Λ is smaller in the bottom sector than those in the charm sector, which means the interactions in the bottom bound states are stronger than the interactions in the charm sector as expected. Finally, the values of the cutoff Λ for the dipole form factor are larger than the corresponding values for the monopole and exponential form factors for both the charm states or bottom states.

For the $D\bar{D}^*$ bound state, there are two relevant states $X(3872)$ and $Z_c(3900)$ observed by the experiments. The quantum numbers of $X(3872)$ and $Z_c(3900)$ are $I^G(J^{PC}) = 0^+(1^{++})$ and $I^G(J^{PC}) = 1^+(1^{+-})$, respectively [2]. From our calculations, there are no $I = 1$ $D\bar{D}^*$ bound states existing with the cutoff Λ in the range 0.8 GeV–5 GeV. Therefore, $Z_c(3900)$ cannot exist as an $I = 0$ $D\bar{D}^*$ bound state in our model. Our result consistent with many other studies. In Ref. [52], using the local hidden gauge approach, the authors found a state with 3900 MeV could not be easily interpreted as a $D\bar{D}^*$ ($\bar{D}D^*$) molecular state. Albaladejo *et al.* [53] found the $Z_c(3900)$ cannot be a $D\bar{D}^*$ bound state considering that the Z_c enhancement was originated from a resonance with a mass around the

TABLE II. The numerical results for the $D\bar{D}^*$ and DD^* systems. $E_b = -4.15$ and -0.71 correspond to the binding energy of T_{cc} and $X(3872)$, respectively. The units of E_b and Λ are MeV.

	$D\bar{D}^*$												DD^*					
	$C=1$						$C=-1$											
	$I=0$			$I=1$			$I=0$			$I=1$			$I=0$			$I=1$		
E_b	Λ_M	Λ_E	Λ_D	Λ_M	Λ_E	Λ_D	Λ_M	Λ_E	Λ_D	Λ_M	Λ_E	Λ_D	Λ_M	Λ_E	Λ_D	Λ_M	Λ_E	Λ_D
−5	1056	1013	1456	…	…	…	1340	1250	1823	…	…	…	1093	1097	1540	…	…	…
−10	1117	1100	1562	…	…	…	1402	1337	1928	…	…	…	1146	1177	1635	…	…	…
−15	1165	1166	1643	…	…	…	1448	1401	2006	…	…	…	1184	1234	1701	…	…	…
−20	1208	1224	1714	…	…	…	1488	1455	2072	…	…	…	1214	1279	1755	…	…	…
−25	1247	1276	1779	…	…	…	1523	1502	2130	…	…	…	1240	1318	1800	…	…	…
−30	1284	1324	1839	…	…	…	1555	1544	2182	…	…	…	1264	1352	1840	…	…	…
−4.15	…	…	…	…	…	…	…	…	…	…	…	…	957	885	1292	…	…	…
−0.71	1043	993	1432	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…

TABLE III. The numerical results for the $B\bar{B}^*$ and $\bar{B}\bar{B}^*$ systems. The units of E_b and Λ are MeV.

	$B\bar{B}^*$												$\bar{B}\bar{B}^*$					
	$C=1$						$C=-1$											
	$I=0$			$I=1$			$I=0$			$I=1$			$I=0$			$I=1$		
E_b	Λ_M	Λ_E	Λ_D	Λ_M	Λ_E	Λ_D	Λ_M	Λ_E	Λ_D	Λ_M	Λ_E	Λ_D	Λ_M	Λ_E	Λ_D	Λ_M	Λ_E	Λ_D
−5	861	754	1126	⋯	⋯	⋯	1130	964	1469	⋯	⋯	⋯	871	799	1165	⋯	⋯	⋯
−10	894	817	1195	⋯	⋯	⋯	1151	1019	1522	⋯	⋯	⋯	906	865	1238	⋯	⋯	⋯
−15	920	865	1249	⋯	⋯	⋯	1172	1062	1567	⋯	⋯	⋯	933	914	1291	⋯	⋯	⋯
−20	943	905	1294	⋯	⋯	⋯	1192	1099	1607	⋯	⋯	⋯	955	953	1335	⋯	⋯	⋯
−25	964	940	1335	⋯	⋯	⋯	1210	1131	1642	⋯	⋯	⋯	975	987	1374	⋯	⋯	⋯
−30	984	973	1373	⋯	⋯	⋯	1228	1161	1675	⋯	⋯	⋯	992	1017	1407	⋯	⋯	⋯

$D\bar{D}^*$ threshold or produced by a virtual state which must have a hadronic molecular nature. From the BS approach with quasipotential approximation [18], no bound state was induced from the interactions of $D\bar{D}^*$ in the isovector sector, which also suggested that the molecular state explanation for $Z_c(3900)$ was excluded. Based on the lattice QCD the CLQCD Collaboration [54] studied the low-energy scattering of the $(D\bar{D}^*)^{\pm}$ meson system. It was found that the $D\bar{D}^*$ interaction was weakly repulsive; hence, the results did not support the possibility of a shallow bound state for the two mesons for the pion mass values studied. In the framework of the one-boson exchange model [47], the results showed that the momentum-related corrections was unfavorable for the formation of the molecular state in the $I=0$, $J^{PC}=1^{+-}$ channel in the $D\bar{D}^*$ system. In fact, since the mass of $Z_c(3900)$ is above the threshold of $D\bar{D}^*$ system, we propose to study the possible molecular structure of $Z_c(3900)$ as a resonance or a virtual state. The $X(3872)$ can be a isoscalar $D\bar{D}^*$ bound state with the cutoff $\Lambda = 1043$ MeV, 993 MeV, and 1432 MeV for the monopole, exponential, and dipole form factors in our method, respectively. The properties of $X(3872)$ as a $D\bar{D}^*$ bound state have been studied in our previous work [55].

For the DD^* bound state, the recently experimentally discovered T_{cc}^+ state could be associated with it. In our model, the T_{cc}^+ can be $I=0$ DD^* bound state with the cutoff Λ taking the values of 957 MeV, 885 MeV, and 1292 MeV for the monopole, exponential, and dipole form factors, respectively. Theoretically, the DD^* system has been studied within a variety of approaches and different models. In Ref. [56], the authors studied doubly charmed exotic states by solving the scattering problem of two D mesons. Their results pointed to the existence of a stable isoscalar doubly charmed bound state with the quantum numbers $(I)J^P=(0)1^+$. By solving the coupled channel Schrödinger equations, Ohkoda *et al.* [7] found the DD^* system could be a deeply $(I)J^P=(0)1^+$ bound state, and no isovector bound state could exist. Via solving the single channel BS equation, the authors of Ref. [19] found the $I(J^P)=0(1^+)$ DD^* system could be the double charm tetraquark T_{cc}^+ observed by LHCb observation with a reasonable cutoff regularizing the loop integral. In Ref. [6], the authors studied the DD^* system up to $O(\epsilon^2)$ at the one-loop level within the framework of heavy meson chiral effective field theory. There existed a bound state in the $I=0$ channel as the cutoff is near m_ρ, no bound state was found in the $I=1$ channel within a wide range of the cutoff parameter.

In the bottom sector, only the $Z_b(10610)$ reported by the Belle Collaboration in 2011 is close to the $B\bar{B}^*$ threshold [57]. A later analysis for the same experiment allowed for an amplitude analysis where the quantum numbers $I^G(J^P)=1^+(1^+)$ were strongly favored for $Z_b(10610)$ [58]. In our model, no isovector $B\bar{B}^*$ bound state was found, which disfavors the bound state explanation for the $Z_b(10610)$.

For $Z_c(3900)$ and $Z_b(10610)$ although they cannot exist as bound states in many studies, however they can be interpreted as virtual states by performed heavy-quark flavor symmetry on heavy meson hadronic molecules [59], coupled-channels calculation [60–63], and Lippmann-Schwinger equation [64]. The virtual state which is also below the lowest threshold, and with the pole on the second Riemann sheet. In Ref. [65], the authors discussed in detail the virtual and demonstrated the virtual state should be classified as molecular.

IV. SUMMARY

In this work, we applied the BS equation to systematically study the $D\bar{D}^*/B\bar{B}^*$ and $DD^*/\bar{B}\bar{B}^*$ systems with the ladder approximation and the instantaneous approximation, try to find the possible bound states of these systems. In our calculations, both direct and cross diagrams were considered for the kernel induced by ρ, ω, π, η, and σ exchanges. Since the constituent particles and the exchanged particles in the $D\bar{D}^*/B\bar{B}^*$ or $DD^*/\bar{B}\bar{B}^*$ systems are not pointlike, we introduced three different form factors (monopole, exponential, and dipole form factors) which all contain a cutoff Λ that reflects the effects of the structure of these particles. Since Λ is controlled by nonperturbative QCD and cannot be determined exactly, we let it vary in a reasonable range within which we tried to find possible bound states of the $D\bar{D}^*/B\bar{B}^*$ and $DD^*/\bar{B}\bar{B}^*$ systems.

The results of our studies showed that only the S-wave $D\bar{D}^*/B\bar{B}^*$ and $DD^*/\bar{B}\bar{B}^*$ systems with $I = 0$ could exist as bound states. We also found that for the same binding energy E_b the values of the cutoff Λ in the bottom sector are smaller than those in the charm sector, and that the values of the cutoff Λ is larger for the dipole form factor than those for the exponential and dipole form factors in both charm and bottom sector. From our results, we can see that the experimentally observed $X(3872)$ and T_{cc}^+ can be assigned as $I = 0$ $D\bar{D}^*$ and $I = 0$ DD^* bound states, respectively, corresponding to $\Lambda = 1043$ MeV, 993 MeV, and 1432 MeV and $\Lambda = 957$ MeV, 885 MeV, and 1538 MeV for the monopole, exponential and dipole form factors, respectively. No bound state was found for isovector from S-wave $D\bar{D}^*/B\bar{B}^*$ and $DD^*/\bar{B}\bar{B}^*$ systems, which disfavors the bound state explanation for $Z_c(3900)$ and $Z_b(10610)$.

The possible S-wave $D\bar{D}^*/B\bar{B}^*$ and $DD^*/\bar{B}\bar{B}^*$ bound states studied in our work are helpful in explaining the structure of experimentally discovered exotic states and the discovery of unobserved exotic states. In some cases the theoretical explanations of the structures for the experimentally observed exotic states and the existence of theoretical predictions of the possible molecular states remains controversial. Therefore more precise experimental studies of the exotic states will be needed to check the results of theoretical studies and to improve theoretical model.

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APPENDIX A: THE CALCULATION PROCESS OF ISOSPIN FACTORS c_I^d AND c_I^c

The field operators for D and D^* mesons are

$$D_1(x) = \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{E_D}}(a_{D^+}e^{-ip\cdot x} + a^\dagger_{D^-}e^{ip\cdot x}),$$
$$D_2(x) = \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{E_D}}(a_{D^0}e^{-ip\cdot x} + a^\dagger_{\bar{D}^0}e^{ip\cdot x}), \tag{A1}$$

and

$$D_1^{*\mu}(x) = \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{E_{D^*}}}\sum_{\lambda=0}^{3}(a^{(\lambda)}_{D^{*+}}\epsilon^{(\lambda)\mu}e^{-ip\cdot x} + a^{(\lambda)\dagger}_{D^{*-}}\epsilon^{(\lambda)\mu*}e^{ip\cdot x}),$$
$$D_2^{*\mu}(x) = \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{E_{D^*}}}\sum_{\lambda=0}^{3}(a^{(\lambda)}_{D^{*0}}\epsilon^{(\lambda)\mu}e^{-ip\cdot x} + a^{(\lambda)\dagger}_{\bar{D}^{*0}}\epsilon^{(\lambda)\mu*}e^{ip\cdot x}), \tag{A2}$$

respectively.

By projecting the bound states onto the field operator (A1) and (A2) we get

$$\langle 0|TD_i(x_1)D_j^{*\mu}(x_2)|P\rangle_{I,I_3} = C^{ij}_{(I,I_3)}\chi_P^{\mu(I)}(x_1,x_2), \tag{A3}$$

then we can obtain the corresponding BS equation as

$$C^{i,j}_{I,I_3}\chi_P^{\alpha(I)}(p) = S^{\alpha\lambda}(p_1)\int\frac{d^4q}{(2\pi)^4}\sum_{kl}\bar{K}^{ij,kl}_{\lambda\tau}(P,p,q)\chi_P^{\tau(I)}(q)S(p_2), \tag{A4}$$

In the following we take $|T_{cc}^+\rangle_{I=0}$ as an example. The isospin coefficients $C^{i,j}_{I,I_3}$ for $|T_{cc}^+\rangle_{I=0}$ are

$$C^{12}_{(1,0)} = -C^{21}_{(1,0)} = 1/\sqrt{2}, \quad \text{else} = 0, \tag{A5}$$

After inserting the Eq. (A5) into Eq. (A4), for the $|T_{cc}^+\rangle_{I=0}$ state, we have

$$\begin{aligned}\chi_P^{\alpha(0)}(p) &= S^{\alpha\lambda}(p_1)\int\frac{d^4q}{(2\pi)^4}[\bar{K}^{12,12(d)}_{\lambda\tau}(P,p,q) - \bar{K}^{12,21(d)}_{\lambda\tau}(P,p,q) + \bar{K}^{12,21(c)}_{\lambda\tau}(P,p,q) - \bar{K}^{12,12(c)}_{\lambda\tau}(P,p,q)]\chi_P^{\tau(0)}(q)S(p_2)\\ &= S^{\alpha\lambda}(p_1)\int\frac{d^4q}{(2\pi)^4}\bar{K}^{\text{total}}_{\lambda\tau}(P,p,q)\chi_P^{\tau(0)}(q)S(p_2),\end{aligned} \tag{A6}$$

where the superscripts c and d on $\bar{K}^{ij,kl}_{\lambda\tau}(P,p,q)$ represent kernels from the direct and crossed Feynman diagrams, respectively.

Based on the Lagrangian in Eq. (13), the total kernel $\bar{K}^{\text{total}}_{\lambda\tau}$ of $|T_{cc}^+\rangle_{I=0}$ induced by ρ, ω, σ, π, and η exchange mesons is

$$\begin{aligned}\bar{K}^{\text{total}}_{\lambda\tau} = & -\frac{3}{2}\bar{K}^{\lambda\tau,\rho}_{DD^*-\text{direct}} + \frac{1}{2}\bar{K}^{\lambda\tau,\omega}_{DD^*-\text{direct}} + \bar{K}^{\lambda\tau,\sigma}_{DD^*-\text{direct}} + \frac{3}{2}\bar{K}^{\lambda\tau,\rho}_{DD^*-\text{crossed}} - \frac{1}{2}\bar{K}^{\lambda\tau,\omega}_{DD^*-\text{crossed}} + \frac{3}{2}\bar{K}^{\lambda\tau,\pi}_{DD^*-\text{crossed}} \\ & -\frac{1}{6}\bar{K}^{\lambda\tau,\eta}_{DD^*-\text{crossed}}. \end{aligned} \tag{A7}$$

The $\bar{K}^{\lambda\tau,\phi}_{DD^*-\text{direct}}$ and $\bar{K}^{\lambda\tau,\phi}_{DD^*-\text{direct}}$ have been given in Eq. (17), and the coefficients in front of them are the isospin factors c_I^d and c_I^c listed in Table I.

APPENDIX B: THE BS EQUATION AFTER TAKING THE INSTANTANEOUS APPROXIMATION AND SIMPLIFICATION

$$\begin{aligned} f(p) = & \frac{-i}{[(\lambda_1 M + p_l)^2 - w_1^2 + i\epsilon][(\lambda_2 M - p_l)^2 - w_2^2 + i\epsilon]} \int \frac{d^4 q}{(2\pi)^4} \Bigg\{ 4 c_I^d g_\sigma^2 m_{D^*} m_D \\ & \times \Bigg\{ \frac{[(\lambda_1 M + p_l)^2 - \mathbf{p}_{\mathrm{t}}^2]}{3m_1^2} - \frac{(\lambda_1 M + p_l)[p_l(\lambda_1 M + p_l) - \mathbf{p}_{\mathrm{t}}\cdot\mathbf{q}_{\mathrm{t}}]}{3m_1^2 p_l} - 1 \Bigg\} F^2(\mathbf{k}_{\mathrm{t}}^2, m_\sigma) \\ & - c_I^c g_{DD^*P}^2 \left[\frac{1}{3}(\mathbf{p}_{\mathrm{t}} - \mathbf{q}_{\mathrm{t}})^2 + \frac{(\mathbf{p}_{\mathrm{t}}\cdot\mathbf{q}_{\mathrm{t}} - \mathbf{p}_{\mathrm{t}}^2)^2}{3m_1^2} \right] F^2(\mathbf{k}_{\mathrm{t}}^2, m_P) \\ & + \Bigg\{ c_I^d g_{DDV} g_{D^*D^*V} \Bigg\{ \frac{[(\lambda_1 M + p_l)^2 - \mathbf{p}_{\mathrm{t}}^2][4(\lambda_1 M + p_l)(\lambda_2 M - p_l) - (\mathbf{p}_{\mathrm{t}} - \mathbf{q}_{\mathrm{t}})^2]}{3m_1^2} \\ & - \frac{(\lambda_1 M + p_l)[p_l(\lambda_1 M + p_l) - \mathbf{p}_{\mathrm{t}}\cdot\mathbf{q}_{\mathrm{t}}][4(\lambda_1 M + p_l)(\lambda_2 M - p_l) + (\mathbf{p}_{\mathrm{t}} - \mathbf{q}_{\mathrm{t}})^2]}{3m_1^2 p_l} \\ & - 4(\lambda_1 M + p_l)(\lambda_2 M - p_l) - (\mathbf{p}_{\mathrm{t}} - \mathbf{q}_{\mathrm{t}})^2 - \frac{(\mathbf{p}_{\mathrm{t}}^2 - \mathbf{q}_{\mathrm{t}}^2)^2 (\lambda_1 M + p_l)[p_l(\lambda_1 M + p_l) - \mathbf{p}_{\mathrm{t}}\cdot\mathbf{q}_{\mathrm{t}}]}{3m_1^2 m_V^2 p_l} \\ & + \frac{(\mathbf{p}_{\mathrm{t}}^2 - \mathbf{q}_{\mathrm{t}}^2)^2[(\lambda_1 M + p_l) - \mathbf{p}_{\mathrm{t}}^2]}{3m_1^2 m_V^2} - \frac{(\mathbf{p}_{\mathrm{t}}^2 - \mathbf{q}_{\mathrm{t}}^2)^2}{m_V^2} \Bigg\} \\ & - c_I^d g_{DDV} g'_{D^*D^*V} \Bigg\{ \frac{2(\lambda_2 M - p_l)(\mathbf{p}_{\mathrm{t}}\cdot\mathbf{q}_{\mathrm{t}} - \mathbf{p}_{\mathrm{t}}^2)[p_l(\lambda_1 M + p_l) - \mathbf{p}_{\mathrm{t}}\cdot\mathbf{q}_{\mathrm{t}}]}{3m_1^2 p_l} \\ & - \frac{2(\lambda_2 M - p_l)(\mathbf{q}_{\mathrm{t}}^2 - \mathbf{p}_{\mathrm{t}}\cdot\mathbf{q}_{\mathrm{t}})}{3p_l} \Bigg\} \Bigg\} F^2(\mathbf{k}_{\mathrm{t}}^2, m_V) \\ & - g_{DD^*P}^2 \frac{M^2[2m_1^2(\mathbf{p}_{\mathrm{t}} - \mathbf{q}_{\mathrm{t}})^2 + \mathbf{p}_{\mathrm{t}}^2\mathbf{q}_{\mathrm{t}}^2 - (\mathbf{p}_{\mathrm{t}}\cdot\mathbf{q}_{\mathrm{t}})^2]}{3m_1^2} F^2(\mathbf{k}_{\mathrm{t}}^2, m_P) \Bigg\} f(q), \end{aligned} \tag{B1}$$

where $\mathbf{k}_{\mathrm{t}} = \mathbf{p}_{\mathrm{t}} - \mathbf{q}_{\mathrm{t}}$, and we have made explicit use of the covariant instantaneous approximation (in which the energy exchanged between the constituent particles of the binding system is neglected.).

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