Next-to-tribimaximal mixing against *CP* violation and baryon asymmetry signs

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Four next-to-tribimaximal (NTBM) mixing patterns are widely considered as the feasible candidates for the observed leptonic mixing structure. With the recent measurements from T2K, as well as intensive global fits, the interval of $\sin\delta_{CP} > 0$ for the Dirac *CP*-violating phase is persistently small in the normal ordering while $\sin\delta_{CP} < 0$ is successively obtained up to 3σ level in the inverted ordering. In this paper, we advocate the fitting results of Dirac *CP*-violating phase as a constraint, and show that it can basically rule out half of the regions allowed by the constraint from three mixing angles only. In addition, given the small $\sin\delta_{CP} > 0$ interval, we find that a unique NTBM pattern can be selected out of the four candidates within 3σ uncertainty of the latest NuFIT5.1, when the patterns are exposed to a Yukawa texture-independent leptogenesis for the baryon asymmetry of the Universe. For the surviving pattern, we find an interesting correlation between the Dirac *CP*-violating phase and the octant of atmospheric angle θ_{23}, where $\theta_{23} > 45°$ can be predicted once $5\pi/6 < \delta_{CP} < \pi$ in the normal ordering can be confirmed in the future measurements.

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I. INTRODUCTION

Neutrino oscillation experiments have undoubtedly unveiled nonzero neutrino masses and nontrivial leptonic mixing. Thus far, the intention to explain neutrino masses and mixing, or the so-called flavor puzzle, has triggered a great deal of theoretical investigations beyond the standard model. The flavor symmetry-induced neutrino mixing was prevailing in the first decade of the twenty-first century (we refer to the recent reviews [1,2] in which early studies can be found). In particular, the so-called tribimaximal (TBM) mixing [3],

$$V_{\rm TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}, \tag{1}$$

was once considered to be the promising candidate for explaining the observed Pontecorvo-Maki-Nakawaga-Sakata (PMNS) matrix, which in the standard parametrization is given by [4]

$$U_P = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & e^{-i\delta_{CP}}s_{13} \\ -s_{12}c_{23} - e^{i\delta_{CP}}c_{12}s_{13}s_{23} & c_{12}c_{23} - e^{i\delta_{CP}}s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{12}s_{23} - e^{i\delta_{CP}}c_{12}s_{13}c_{23} & -c_{12}s_{23} - e^{i\delta_{CP}}s_{12}s_{13}c_{23} & c_{13}c_{23} \end{pmatrix}. \tag{2}$$

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Here, $s_{ij} \equiv \sin\theta_{ij}$, $c_{ij} \equiv \cos\theta_{ij}$, δ_{CP} is the Dirac CP-violating phase, and we have neglected the possible Majorana phases as they do not appear in neutrino oscillations. After the discovery of nonzero θ_{13} [5–9], the TBM mixing that predicts $\theta_{13} = 0$ is ruled out in the neutrino mixing sector where the charged-lepton Yukawa matrix is diagonal. Later on, TBM variants were extensively investigated to accommodate $\theta_{13} \neq 0$ and reproduce the PMNS matrix. Among them, the minimal corrections to TBM mixing, or the next-to-TBM (NTBM) [10–12], which are characterized by a two-parameter family, have brought us certain predictions and correlations among the mixing angles $\theta_{12,13,23}$ and phase δ_{CP}, and have been shown to be able to reproduce the PMNS matrix in model-independent analyses (see, e.g., Refs. [11,13–19] and references therein).

In recent years, experiments from T2K [20,21] and the global fits in the years 2018–2021 [22–26] have persistently indicated a small parameter region for $\sin\delta_{CP} > 0$ in the normal ordering (NO) while $\sin\delta_{CP} < 0$ is successively obtained in the inverted ordering (IO) within 3σ level of uncertainty. This is further confirmed by the latest NuFIT5.1 [27,28]. Nevertheless, the true value of δ_{CP} is still unknown, and this ambiguity partly allows earlier analyses of TBM variants to take δ_{CP} as a *prediction* rather than a *constraint* [16,18,29–33]. Given the persistent results, in this paper we will instead take δ_{CP} as a constraint, and show in a model-independent way, that the viable parameter space of the NTBM patterns significantly shrinks.

While imposing the constraint of leptonic CP violation would cut down nearly half of the parameter space, none of the four NTBM patterns can be uniquely selected out, which is also a common situation in various model-independent analyses. To pin down a unique NTBM, we find that only one pattern survives if the sign of leptonic CP violation matches $\sin\delta_{CP} > 0$. Noticeably, the sign of leptonic CP violation can be closely related to the sign of baryon asymmetry in the Universe (BAU) via leptogenesis [34]. In fact, it has been shown in a Yukawa texture-independent Dirac leptogenesis [35] that a positive BAU requires $\sin\delta_{CP} > 0$.

The motivation for applying Yukawa texture-independent leptogenesis as a selection criterion goes as follows. Lepton Yukawa matrices are the building blocks for the generic leptogenesis mechanism (see e.g., the review [36]), and meanwhile account for the observed lepton masses and PMNS matrix. For Yukawa texture-independent leptogenesis, there exists a direct link between the BAU and PMNS matrix such that the resulting baryon asymmetry is directly attributed to the PMNS structure. Then, for a broad class of theoretical PMNS candidates, such as the NTBM patterns, the BAU selection criterion can potentially provide an additional constraint on the PMNS structures.

A direct BAU-PMNS connection is, nevertheless, still under extensive investigations at present day. A generic way is to invoke some assumptions or parametrizations of the lepton Yukawa structures, as widely considered in Refs. [37–47]. However, the corresponding BAU selection criterion would then crucially depend on the assumptions and parametrizations, or even be irrelevant to the Dirac CP-violating phase [48–50]. Therefore, to build a robust BAU selection criterion, the prediction of baryon asymmetry should not depend on nontrivial Yukawa textures. A purely thermal Dirac leptogenesis considered in Ref. [35] is a sound candidate in this respect. The corresponding mechanism formulates the baryon asymmetry in terms of charged-lepton and Dirac neutrino masses and the PMNS matrix, such that the sign of baryon asymmetry uniquely depends on the sign of $\sin\delta_{CP}$. Besides, the mechanism is free from any underlying flavor theory that can explain the lepton mass spectrum and PMNS matrix. In this respect, the Yukawa texture-independent leptogenesis can also play a significant role in guiding the flavor model buildings, especially when the BAU criterion can uniquely select a theoretical PMNS candidate.

It will be shown that the Yukawa texture-independent Dirac leptogenesis as a BAU criterion can finally select a unique NTBM with a narrow parameter space in the NO spectrum, and disfavors all the patterns in the IO spectrum. Certainly, the approach presented here is not only valid for the NTBM patterns, but also applicable for other two-parameter family candidates, such as the generalization to the bimaximal mixing [51]. Besides, the suggestion of taking the sign of leptonic CP violation as a constraint should be taken seriously if the upcoming measurements and global fits further confirm previous results, and it would become more significant, when being assisted with some Yukawa texture-independent leptogenesis, to phase out most of, or even all of, the theoretical PMNS candidates.

The BAU criterion can help to reveal the three unknowns in current neutrino oscillations, i.e., NO versus IO, the CP violation, and $\theta_{23} < 45°$ versus $\theta_{23} > 45°$ [22]. In addition to favoring an NO spectrum and $\sin\delta_{CP} > 0$, the unique NTBM pattern selected from the BAU criterion further exhibits an intriguing correlation between the Dirac CP-violating phase and the octant of atmospheric angle θ_{23}. It will be shown that if $5\pi/6 < \delta_{CP} < \pi$ in the NO spectrum can be confirmed in the future measurements, $\theta_{23} > 45°$ will be predicted.

After this Introduction, we firstly review in Sec. II the four NTBM patterns and formulate the angles and phase entirely in terms of the two free parameters. In Sec. III, we impose, in a model-independent way, the constraints of mixing angles and the Dirac CP-violating phase on the four NTBM patterns, and then expose the survived parameter space to the Yukawa texture-independent leptogenesis in Sec. IV. The correlation between δ_{CP} and θ_{23} will be analyzed in Sec. V. Finally, we present our conclusions in Sec. VI.

II. TWO-PARAMETER FAMILY OF NTBM PATTERNS

There are six NTBM patterns defined by multiplying Eq. (1) by a unitary rotation matrix on the left or right.

However, the patterns $U_P = V_{\rm TBM}R_{12}$ and $U_P = R_{23}V_{\rm TBM}$ are already excluded since they predict $\theta_{13} = 0$. The remaining four patterns are denoted as

$$\begin{aligned} &\text{TBM}_1: U_P = V_{\rm TBM}R_{23}, \quad \text{TBM}_2: U_P = V_{\rm TBM}R_{13}, \\ &\text{TBM}^2: U_P = R_{13}V_{\rm TBM}, \quad \text{TBM}^3: U_P = R_{12}V_{\rm TBM}. \end{aligned} \tag{3}$$

Here, the rotation matrices are defined as

$$\begin{aligned} R_{12} &= \begin{pmatrix} c_\alpha & s_\alpha e^{i\varphi} & 0 \\ -s_\alpha e^{-i\varphi} & c_\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}, \\ R_{13} &= \begin{pmatrix} c_\alpha & 0 & s_\alpha e^{i\varphi} \\ 0 & 1 & 0 \\ -s_\alpha e^{-i\varphi} & 0 & c_\alpha \end{pmatrix}, \\ R_{23} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\alpha & s_\alpha e^{i\varphi} \\ 0 & -s_\alpha e^{-i\varphi} & c_\alpha \end{pmatrix}, \end{aligned} \tag{4}$$

where $c_\alpha \equiv \cos\alpha$ and $s_\alpha \equiv \sin\alpha$, with $0 \le \alpha \le \pi$ and $0 \le \varphi < 2\pi$. The free parameters α, φ characterize the two-parameter family of NTBM patterns. Note that U_P defined above has taken into account both the charged-lepton and neutrino mixing. In this respect, the pattern, e.g., TBM^2 may be interpreted as TBM neutrino mixing corrected by charged-lepton e-τ mixing [1,2]. In the following, we will give the formulas of mixing angles and CP-violating phase in terms of α, φ for the remaining four NTBM patterns, respectively.

A. TBM_1

The desired formulas can be derived by using the two simplest rephasing invariant observables, namely, the moduli of PMNS matrix, $|U_P|$, and the Jarlskog invariant, $\mathcal{J}_{CP}$ [52]. Explicitly, the mixing angles can be directly obtained by observing the module ratios of PMNS matrix elements via

$$\begin{aligned} \tan\theta_{23} &= \frac{|U_{P,23}|}{|U_{P,33}|} = \left|\frac{3\sqrt{2}c_\alpha + 2\sqrt{3}e^{i\varphi}s_\alpha}{3\sqrt{2}c_\alpha - 2\sqrt{3}e^{i\varphi}s_\alpha}\right|, \\ \tan\theta_{12} &= \frac{|U_{P,12}|}{|U_{P,11}|} = \frac{|c_\alpha|}{\sqrt{2}}, \quad \sin\theta_{13} = |U_{P,13}| = \frac{|s_\alpha|}{\sqrt{3}}. \end{aligned} \tag{5}$$

For the Dirac CP-violating phase, we can equate the $\mathcal{J}_{CP}$ of U_P, which is given by [4]

$$\mathcal{J}_{CP} = \frac{1}{8}\cos\theta_{13}\sin(2\theta_{12})\sin(2\theta_{23})\sin(2\theta_{13})\sin\delta_{CP}, \tag{6}$$

with the $\mathcal{J}_{CP}$ of TBM_1: $\mathcal{J}_{CP} = -s_{2\alpha}s_\varphi/6\sqrt{6}$, leading to

$$\sin\delta_{CP} = -\frac{\text{sgn}(s_{2\alpha})s_\varphi(c_{2\alpha}+5)}{[(c_{2\alpha}+5)^2 - (2\sqrt{6}s_{2\alpha}c_\varphi)^2]^{1/2}}, \tag{7}$$

where $\text{sgn}(s_{2\alpha})$ is the sign function of $s_{2\alpha} \equiv \sin(2\alpha)$. It should be pointed out that, in deriving Eq. (7), we have simply replaced the mixing angles by using Eq. (5) in the first quadrant, e.g., $\theta_{13} = \sin^{-1}(|s_\alpha|/\sqrt{3})$, since the solution in the second quadrant $\theta_{13} = \pi - \sin^{-1}(|s_\alpha|/\sqrt{3})$ leads to the same result in Eq. (7) due to the periodicity of θ_{13} function in Eq. (6), i.e., $\cos\theta_{13}\sin(2\theta_{13}) = \cos(\pi - \theta_{13})\sin[2(\pi - \theta_{13})]$. Similar conclusions also hold for θ_{12} and θ_{23} functions.

B. TBM_2

Similar to the above logic, it is straightforward to obtain the formulas for the TBM_2 pattern. The angles are given by

$$\begin{aligned} \tan\theta_{23} &= \left|\frac{\sqrt{3}e^{i\varphi}s_\alpha - 3c_\alpha}{3c_\alpha + \sqrt{3}e^{i\varphi}s_\alpha}\right|, \\ \tan\theta_{12} &= \frac{1}{\sqrt{2}}\frac{1}{|c_\alpha|}, \qquad \sin\theta_{13} = \sqrt{\frac{2}{3}}|s_\alpha|, \end{aligned} \tag{8}$$

and the CP-violating phase can be simplified as

$$\sin\delta_{CP} = -\frac{\text{sgn}(s_{2\alpha})(c_{2\alpha}+2)s_\varphi}{[(2c_\alpha^2+1)^2 - 3s_{2\alpha}^2c_\varphi^2]^{1/2}}, \tag{9}$$

which is derived by using the $\mathcal{J}_{CP}$ of TBM_2: $\mathcal{J}_{CP} = -s_{2\alpha}s_\varphi/6\sqrt{3}$.

C. TBM^2

For the TBM^2 pattern, it can be shown that

$$\tan\theta_{23} = \frac{1}{|c_\alpha|}, \quad \frac{\tan\theta_{12}}{\sqrt{2}} = \left|\frac{c_\alpha + e^{i\varphi}s_\alpha}{2c_\alpha - e^{i\varphi}s_\alpha}\right|, \quad \sin\theta_{13} = \frac{|s_\alpha|}{\sqrt{2}}, \tag{10}$$

for the mixing angles. Equating the $\mathcal{J}_{CP}$ in TBM^2: $\mathcal{J}_{CP} = s_{2\alpha}s_\varphi/12$ with Eq. (6) gives

$$\sin\delta_{CP} = \frac{\text{sgn}(s_{2\alpha})(c_{2\alpha}+3)s_\varphi}{2[(-2s_{2\alpha}c_\varphi + 3c_\alpha^2 + 1)(1 + s_{2\alpha}c_\varphi)]^{1/2}}. \tag{11}$$

D. TBM^3

Finally, for the TBM^3 pattern, the functions for mixing angles are given by

$$\tan\theta_{23} = |c_\alpha|, \quad \frac{\tan\theta_{12}}{\sqrt{2}} = \left|\frac{c_\alpha + e^{i\varphi}s_\alpha}{2c_\alpha - e^{i\varphi}s_\alpha}\right|, \quad \sin\theta_{13} = \frac{|s_\alpha|}{\sqrt{2}}, \tag{12}$$

where θ_{12} and θ_{13} have the same expressions as in TBM^2. The function for phase is

$$\sin\delta_{CP} = -\frac{\mathrm{sgn}(s_{2\alpha})(c_{2\alpha}+3)s_\varphi}{2[(-2s_{2\alpha}c_\varphi + 3c_\alpha^2 + 1)(1 + s_{2\alpha}c_\varphi)]^{1/2}}, \quad (13)$$

which has a different sign from Eq. (11), as the $\mathcal{J}_{CP}$ in the TBM^3 pattern, $\mathcal{J}_{CP} = -s_{2\alpha}s_\varphi/12$, is opposite to that in TBM^2.

With these formulas, we can use the two free parameters α, φ to fully describe the mixing angles and Dirac CP-violating phase extracted from neutrino oscillation experiments.

III. LEPTONIC CP VIOLATION AS A CONSTRAINT

Concerning the Dirac CP-violating phase, it has been shown over the past few years that the global fits for δ_{CP} have exhibited a rather persistent tendency in a sense that the region of $\sin\delta_{CP} > 0$ is small in the NO spectrum while $\sin\delta_{CP} < 0$ is stably inferred in the IO spectrum. We depict in Fig. 1 the δ_{CP} results obtained from T2K measurement [21] and the global fits obtained from 2018 to present day [22–26]. It is seen that for the NO spectrum, only a small interval allows $\sin\delta_{CP} > 0$ within the 3σ uncertainty, while for the IO, $\sin\delta_{CP} < 0$ is persistently obtained up to 3σ uncertainty. Taking these results as constraints, we would like to show how $\sin\delta_{CP}$ restricts the parameter space of the NTBM patterns. To this end, we apply the formulas of angles and phase obtained in Sec. II to reproduce the oscillation data at 3σ level. For concreteness, we apply the latest NuFIT5.1 result [27,28], while a comparison between different results will be made whenever necessary.

We show in Fig. 2 the survived parameter space for each NTBM pattern under the 3σ-allowed mixing angles (red bands) and CP-violating phase in the NO spectrum. It can be seen that except for TBM$_2$, the constraint from CP-violating phase (regions within blue contour) excludes half of the survived parameter space for other three patterns. For the IO spectrum, on the other hand, the mixing-angle data are similar to the NO spectrum, and hence the red bands basically coincide. However, since $\sin\delta_{CP} < 0$ is maintained within 3σ uncertainty in the IO spectrum, only the red bands outside the magenta lines are allowed.

The conclusion thus far is that, with the global results obtained in recent years, the allowed parameter space for the four NTBM patterns is already quite small, even though the four patterns are still viable if 3σ uncertainties are adopted. The constraint from CP-violating phase is strong and can exclude nearly half of the already allowed parameter space where only three mixing angles are considered. The survived regions are quite small, especially predicting a rotation angle $0.1 < \alpha < 0.3$ (as well as its periodic correspondence $0.1 < \pi - \alpha < 0.3$) for all the four NTBM patterns.

Due to the currently observed correlation between the mass hierarchy and the sign of $\sin\delta_{CP}$, it can be seen from Fig. 2 that if the IO spectrum is confirmed in the future and the data of mixing angles do not change significantly, then none of the four NTBM patterns can be uniquely selected out. However, if the NO spectrum is confirmed, then only the TBM$_2$ pattern will survive provided that $\sin\delta_{CP} > 0$ in the NO spectrum still exists. In fact, $\sin\delta_{CP} > 0$ can be supported in some leptogenesis scenarios, and particularly in a lepton Yukawa texture-independent leptogenesis.

IV. YUKAWA TEXTURE-INDEPENDENT BAU AS A CRITERION

Baryogenesis through leptogenesis [34] is the mechanism to account for the observed BAU today [53]:

$$Y_B^{\mathrm{exp}} \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} \approx 8.75 \times 10^{-11} > 0, \quad (14)$$

where $n_B(n_{\bar{B}})$ is the baryon (antibaryon) number density, which is normalized to photon density n_γ. The generic

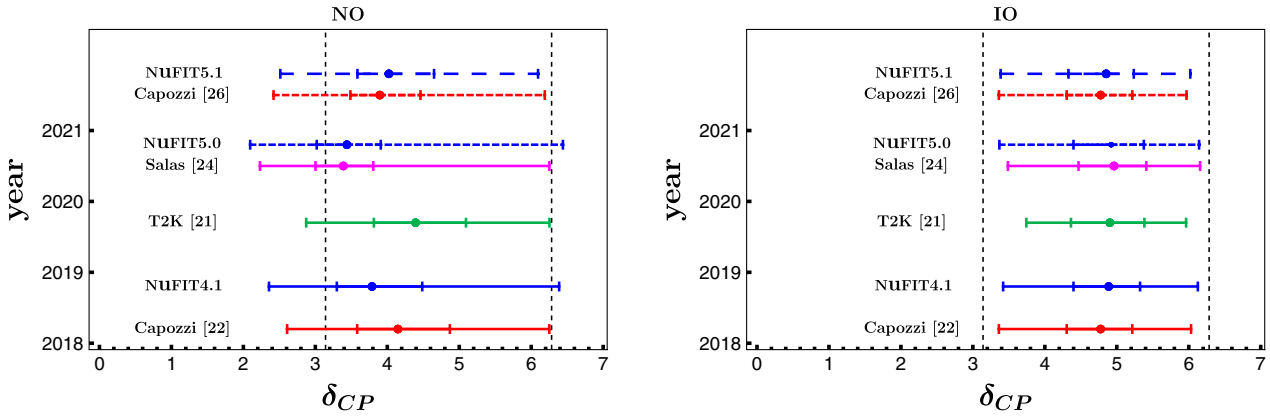

FIG. 1. The results of Dirac CP-violating phase δ_{CP} obtained in the years 2018–2021 within 1σ and 3σ uncertainties [22–28]. The two vertical lines represent $\delta_{CP} = \pi, 2\pi$, respectively.

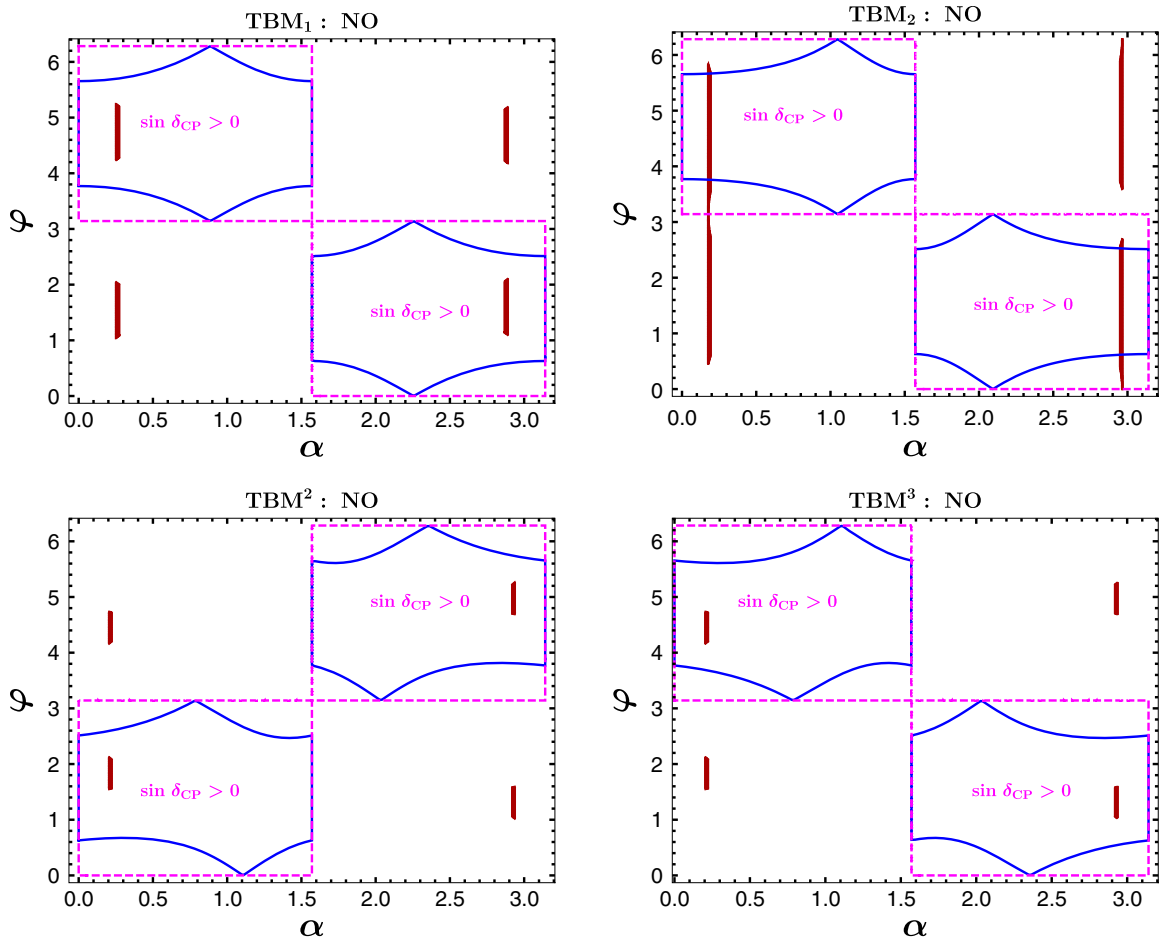

FIG. 2. Constraints on the two free parameters α, φ in the NO spectrum. The 3σ-allowed mixing angles are shown in narrow red bands and the regions within the blue contours are excluded by $\sin\delta_{CP}$ at 3σ level from NuFIT5.1 [27,28]. For clearness, the regions of $\sin\delta_{CP} > 0$ are enclosed by magenta lines.

building block to generate baryon asymmetry in leptogenesis is the lepton Yukawa matrices. It is known that the lepton Yukawa textures are responsible for the nontrivial PMNS mixing observed in neutrino oscillation experiments. Therefore, if the Yukawa textures appearing in leptogenesis can be entirely formulated by the PMNS matrix as well as the physical lepton masses, then, in order to generate the correct amount of Eq. (14), and in particular, the positive sign of $Y_B^{\rm exp}$, the PMNS structure will be constrained in the BAU context. In this respect, the BAU criterion may help to select the theoretical PMNS candidates from a broad class of flavor models [54,55] before the underlying flavor theory is found.

To apply the BAU selection criterion, we consider the mechanism presented in Ref. [35]. The baryon asymmetry obtained therein has a simple dependence on the Yukawa matrix. For our purpose here, it suffices to parametrize the result as

$$Y_B = \sum_{i,j,k,i\neq k} \frac{\mathrm{Im}[Y^*_{\nu,i1} Y_{\nu,k1} (Y_\nu Y_\nu^\dagger)_{ik}]}{|Y_{\nu,j1}|^2} \mathcal{F}_{ijk}, \qquad (15)$$

where $\mathcal{F}_{ijk}$ is a Yukawa-independent scalar function, Y_ν is the Dirac neutrino Yukawa matrix, and the index 1 denotes the fact that the *CP* asymmetry is generated by a scalar decaying into the lightest neutrino (also dubbed as ν_1 leptogenesis). By performing equivalently unitary basis transformations, rather than imposing particular structures, on the charged-lepton and neutrino Yukawa matrices, the leptonic mixing can be encoded in Y_ν, such that

$$Y_\nu \propto U_P \hat{m}_\nu \qquad (16)$$

exists in the basis where the charged-lepton Yukawa matrix is diagonal. Here $\hat{m}_\nu$ denotes the diagonal Dirac neutrino mass matrix.

To see the corresponding constraint, we apply Eqs. (15) and (16) with the PMNS matrix now being the TBM$_2$ pattern. Putting in the explicit Yukawa-independent scalar function $\mathcal{F}_{ijk}$ [35], we are led to the following relation:

$$Y_B^{\rm TBM_2} = -k s_{2\alpha} s_\varphi, \qquad (17)$$

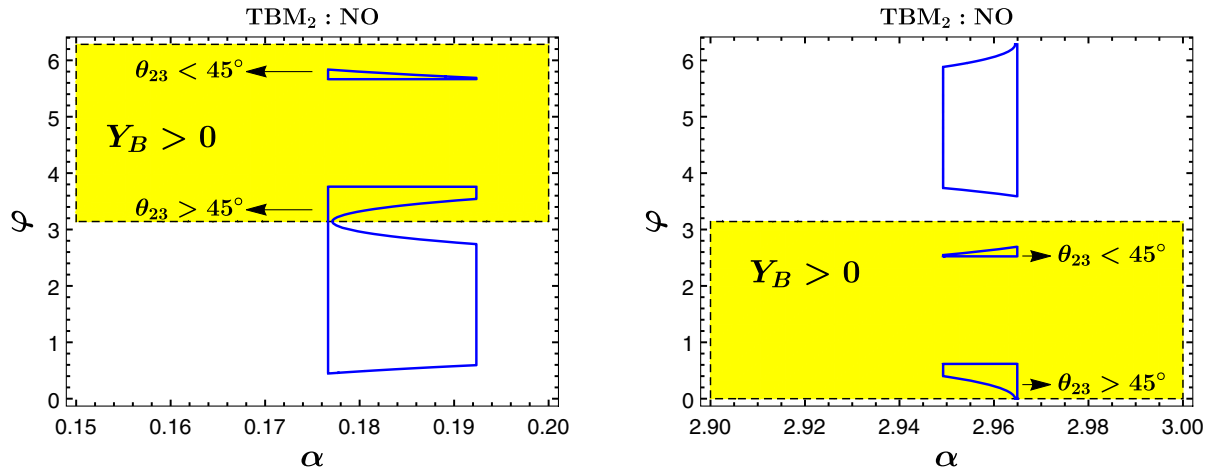

FIG. 3. The TBM$_2$ pattern survives from the BAU selection criterion. Regions for $Y_B > 0$ are shown in yellow band, and the survived parameter space from constraints of mixing angles and Dirac CP-violating phase is shown within the blue contour. For visibility, we have shown the results in two panels with α in the first and second quadrants, respectively. The asymmetric contours in the $Y_B > 0$ region correspond, respectively, to $\theta_{23} < 45°$ and $\theta_{23} > 45°$ octant.

where k is a calculable positive parameter, which is independent of the free parameters α, φ. From Eq. (17), we can see that the BAU criterion can at most constrain the α-φ plane up to the sign of baryon asymmetry, since a predicted value of Y_B further depends on the parameter k. We show in Fig. 3 the regions for $Y_B > 0$ in the α-φ plane and compare with the survived parameter space under the constraint discussed in Sec. III. It can be seen clearly that the BAU criterion sets limits on φ, the region of which is equivalent to requiring $\sin\delta_{CP} > 0$ in Fig. 2. An intriguing situation arises in the $Y_B > 0$ region, where two asymmetric contours coexist. We will show in the next section that the two asymmetric contours correspond to $\theta_{23} < 45°$ and $\theta_{23} > 45°$ octant, respectively, which are closely related to the Dirac CP-violating phase.

It is worth mentioning that the current status of $\sin\delta_{CP} > 0$ is the crucial function that renders the BAU criterion from Ref. [35] available and powerful to select the unique TBM$_2$ pattern. It should also be emphasized that if the interval for $\sin\delta_{CP} > 0$ in the NO spectrum vanishes persistently in the future, the BAU criterion from Ref. [35] would become inapplicable as the leptogenesis scenario therein is ruled out. For $\sin\delta_{CP} < 0$ in the IO spectrum, however, the logic presented here can be generalized to other Yukawa texture-independent leptogenesis, especially those support $\sin\delta_{CP} < 0$, and the selection criterion can then follow the procedure presented above.

V. THREE UNKNOWNS PREDICTED BY THE BAU CRITERION

Currently, there are three unknowns in neutrino oscillation experiments, namely, the neutrino mass ordering, the CP violation, and the octant of atmospheric θ_{23} angle, i.e., whether $\theta_{23} \leq 45°$ or $\theta_{23} > 45°$ [22]. It has been shown above that applying the BAU criterion favors an NO spectrum and $\sin\delta_{CP} > 0$, and hence it can help to reveal the first two unknowns. In addition, since we have also applied the BAU criterion to obtain a quite narrow parameter space of the two parameters in NTBM patterns, it would be a compelling feature if the BAU criterion can further determine the third unknown.

We have observed from Fig. 3 that, there are two asymmetric contours in the $Y_B > 0$ region. By considering the octant of θ_{23}, we find that the small (large) contour corresponds to $\theta_{23} < 45°(\theta_{23} > 45°)$. It can be inferred from Fig. 2 that when δ_{CP} approaches π in the NO spectrum, the allowed parameter space in the TBM$_2$ pattern would further shrink. However, the reduction speed is different in the two asymmetric contours. To visualize this, we depict in Fig. 4 the constraints under NuFIT5.0 [25] ($\delta_{CP} > 120°$, blue curve), Capozzi 2021 [26] ($\delta_{CP} > 0.77\pi$, orange curve) and NuFIT5.1 [27,28] ($\delta_{CP} > 144°$, magenta curve), as well as $\delta_{CP} > 154°$ (black curve), where the 3σ upper limit of $\sin\delta_{CP}$ has stably decreased. It can be seen that when $\delta > 5\pi/6$, the allowed parameter space (red band) in $\theta_{23} < 45°$ octant will be completely ruled out, while a smaller region corresponding to $\theta_{23} > 45°$ octant survives. As a consequence, the TBM$_2$ pattern under the BAU criterion predicts $\theta_{23} > 45°$ for $5\pi/6 < \delta_{CP} < \pi$.

The predictions of NO neutrino mass spectrum and the correlation between the Dirac CP-violating phase and the octant of θ_{23} are what we can expect from the TBM$_2$ pattern selected from the BAU criterion, which is, however, not attainable under the constraints of mixing angles and Dirac CP-violating phase only. Besides, the predictions of three unknowns can be readily tested in the upcoming neutrino oscillation experiments.

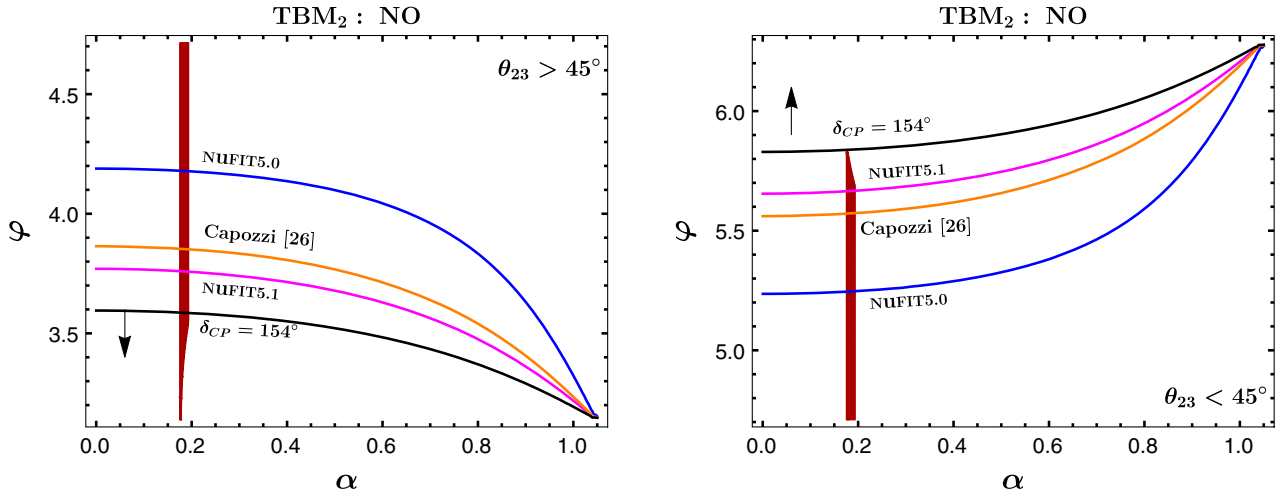

FIG. 4. Correlation of δ_{CP} and θ_{23} under the survived TBM$_2$ pattern. The left (right) panel corresponds to the octant of $\theta_{23} > 45°(\theta_{23} < 45°)$. The red band is allowed by NuFIT5.1 [27,28] within 3σ uncertainties, and the regions above (below) different curves are excluded by Dirac *CP*-violating phase limits in the left (right) panel. The arrow in both panels denotes the increase of δ_{CP} towards π. Note that similar results hold for the periodic region of α in the second quadrant.

VI. CONCLUSION

We have applied two considerable constraints—the leptonic *CP* violation and the BAU criterion—to the NTBM patterns. After imposing the constraint from Dirac *CP*-violating phase, basically half of the existing parameter space is ruled out. Besides, the survived regions for the rotation angle are already small even taking the 3σ uncertainties. For the 3σ-allowed parameter space, we have further applied a Yukawa texture-independent leptogenesis as a BAU criterion to select the unique NTBM pattern. It is found that if the interval $\sin\delta_{CP} > 0$ in the NO spectrum keeps existing but becomes persistently smaller, the BAU criterion can phase out three of the four patterns, rendering the TBM$_2$ pattern as the only viable candidate to explain the PMNS structure.

The BAU criterion can help to uncover the three unknowns in current neutrino oscillation experiments. In addition to favoring the NO spectrum of neutrino masses and a positive $\sin\delta_{CP}$, the unique NTBM pattern under the BAU criterion can further predict a clear octant of $\theta_{23} > 45°$, which is correlated to the Dirac *CP*-violating phase in the range $5\pi/6 < \delta_{CP} < \pi$. All these predictions regarding the three unknowns can be fully tested in the upcoming neutrino oscillation experiments.

The combined selection criterion from the leptonic *CP* violation and BAU suggested in this paper can also be applied to other theoretical PMNS candidates, provided that the theories for flavor puzzle and the BAU generation are independent of each other.

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