Electromagnetic and gravitational form factors of Δ resonance in a covariant quark-diquark approach

Dongyan Fu[1,2,*] Bao-Dong Sun[3,4,†] and Yubing Dong[1,2,‡]

[1]*Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China*
[2]*School of Physical Sciences, University of Chinese Academy of Sciences, Beijing 101408, China*
[3]*Guangdong Provincial Key Laboratory of Nuclear Science, Institute of Quantum Matter,*
South China Normal University, Guangzhou 510006, China
[4]*Guangdong-Hong Kong Joint Laboratory of Quantum Matter, Southern Nuclear Science Computing Center,*
South China Normal University, Guangzhou 510006, China

In this work, the electromagnetic and gravitational form factors of a spin-3/2 particle, Δ resonance, are simultaneously calculated with the help of a relativistic covariant quark-diquark approach. The two kinds of form factors are separately extracted from the matrix elements of the electromagnetic current and of the energy-momentum tensor of the system. Our numerical results show that the approach can reproduce the electromagnetic monopole, dipole, quadrupole, and octupole form factors well compared to the lattice calculations. Our obtained electromagnetic moments are also comparable with some other approaches. Moreover, the obtained gravitational form factors, which give the mechanical properties of the system (like the mass and spin distributions) are also given for the Δ isobar. In addition, some discussions of the sign and the interpretation of the D term are given.

I. INTRODUCTION

It is well known that the electromagnetic form factors (EMFFs) are the indispensable physical quantities which reveal the internal structure of a complicated system. The electromagnetic form factors of hadrons, like π-mesons and nucleons, can tell us the charge or magnetic distributions of the systems. They also illustrate the charge and magnetic radii, which can be extracted by the slopes of the charge and magnetic distributions of the systems at $q^2 = 0$ (with q being the momentum transfer) [1–4]. Furthermore, for a spin-1 system (for instance a deuteron or a vector meson of ρ) its charge, magnetic, and quadrupole form factors can also embody its intrinsic structures such as its charge, magnetic distributions, and quadrupole deformation, (see Refs. [5–10] for the deuteron, and Refs. [11–15] for the ρ meson, respectively). Consequently, EMFFs can provide discriminating information for studying the inner structures of hadrons.

[*]fudongyan@ihep.ac.cn
[†]bao-dong.sun@m.scnu.edu.cn
[‡]dongyb@ihep.ac.cn

There are many studies devoted to the understanding of the electromagnetic form factors of the nucleon, its excitations N^*, and the well-known $N - \Delta$ transitions in the literature. The constituent quark model is one of the successful approaches. In those quark-model calculations, the nucleon or its excitation is regarded as a three-quark system and the electromagnetic current probes each quark instantaneously [16–20]. Then, the form factors are obtained by the calculation of the three-quark contributions to the matrix element by using the wave function of the hadron. Relativistic corrections to the wave function of the nucleon or its excitations, as well as to the electromagnetic interaction operator may also be taken into account in those quantum mechanical calculations. Reasonable results, compared to the experimental measurement, can be obtained. It should be mentioned that, in contrast to those calculations (with some relativistic corrections), the relativistic covariant quark-diquark approach is also employed to study the electromagnetic form factors of nucleons [21–26]. In those relativistic covariant field theory studies, the diquark contribution, as well as the quark one, are simultaneously and explicitly considered. Their results are also very consistent with the available experimental data.

In addition to the electromagnetic form factors of hadrons, the gravitational form factors (GFFs) are also expected to embody the fundamental information of the spatial distributions, like the energy, spin, and strong forces [27] of systems. Those GFFs are defined through the matrix

element of the symmetric energy-momentum tensor (EMT). More details about GFFs can be found in Refs. [27–32]. Clearly, GFFs describe the interaction between the gravitation as an external field, and the matter fields in which the scattering off the graviton is a natural but impractical probe for GFFs. Luckily, because of the similar structures of EMT and electromagnetic current operators [28], hard-exclusive reactions, like deeply virtual Compton scattering (DVCS) and vector-meson electroproduction, provide a realistic way to access the GFFs of hadrons through the generalized parton distributions (GPDs) [33,34] and through the generalized distribution amplitudes (GDAs) [35]. It is expected that the nucleon GPDs will be measured at some facilities, such as Jefferson Lab (JLab.), the future Electron Ion Collider (EIC) [36], and the Electron-Ion Collider in China (EicC) [37].

One reason why GFFs are extremely important partially comes from their connections to the GPDs and GDAs. It is believed that GPDs are an important metric of the three-dimensional hadron structure, and they can be loosely described as amplitudes for removing a parton from a hadron and replacing it with one with a different momentum. In addition, the moments of GPDs are related not only to EMFFs but also to GFFs—and one of the GFFs describes the total angular momentum carried by the partons. It is accepted that the finding the contribution to the sum of the spin- and orbital-angular momenta from specific components of hadrons is of great importance [28,29,38,39]. In particular, it is discussed that there is a very important quantity D term [40], which is closely related to the matrix element of EMT T^{ij} components. As with the energy and angular momentum, the D term also corresponds to the values of GFFs at zero-momentum transfer. Therefore, the D term is considered as the "last global unknown property", which is believed to characterize the spatial deformations as well as other mechanical properties of hadrons [27].

For the GFFs of hadrons with spin 0, 1/2, and 1, much work has been already done [35,41–46]. The common approaches of the chiral quark model, LQCD calculation, the effective chiral theory, the SU(2) Skyrme model, the bag model, the QCD sum rule, and the AdS/CFT correspondence [42,43,45,47–52] have all been employed. Although there are some approaches devoted for the GFFs of a spin-3/2 Δ resonance [53–55], simultaneous discussions and calculations of the EMFFs and GFFs for Δ are still missing. In contrast to hadrons with spin-0, 1/2, and 1, $\Delta(1232)$ is a low-lying baryon resonance with spin-3/2, the study of its EMFFs and GFFs can give more information about the internal structures of this high-spin particle and can be further applied for the transition EMFFs and GFFs of $N - \Delta$ process [56]. Therefore, such a study is of great interest. In this work, we employ the relativistic and covariant quark-diquark approach to simultaneously calculate the EMFFs and GFFs of the spin-3/2 Δ particle. We know that the baryon Δ can be simply regarded as a

three-quark system, and here we treat it as a system of a quark plus an axial-vector diquark. Consequently, the estimated form factors are given by the sum of quark and diquark contributions.

This paper is organized as follows. In Sec. II, the definitions of EMFFs and GFFs for a spin-3/2 particle are given. Section III shows the corresponding matrix elements of the quark and diquark for the electromagnetic and gravitational probes in the covariant quark-diquark approach. In Sec. IV, the model parameters are firstly determined comparing to the Lattice calculations for the EMFFs of Δ^+. Then, our numerical calculations for the electric monopole, magnetic dipole, electric quadrupole, and magnetic octupole form factors are given. Finally, we display our calculated GFFs of Δ, such as its mass and spin distributions, and we particularly address the issues of the sign and the interpretation of the D term. Section V is devoted to a summary.

II. FORM FACTORS OF A SPIN-3/2 PARTICLE

A. Electromagnetic form factors

It is well known that in the one-photon approximation a composite particle with spin-S has $(2S + 1)$-independent electromagnetic form factors due to symmetries and conservations, like parity and time reversal. For the spin-3/2 particle, the matrix element of the electromagnetic current is expressed as [57]

$$\langle p', \lambda' | \hat{J}_a^\mu(0) | p, \lambda \rangle$$

$$= -\bar{u}_{\alpha'}(p', \lambda') \left[\frac{P^\mu}{M} \left(g^{\alpha'\alpha} F_{1,0}^{V,a}(t) - \frac{q^{\alpha'} q^\alpha}{2M^2} F_{1,1}^{V,a}(t) \right) \right.$$

$$\left. + \frac{i\sigma^{\mu\nu} q_\nu}{2M} \left(g^{\alpha'\alpha} F_{2,0}^{V,a}(t) - \frac{q^{\alpha'} q^\alpha}{2M^2} F_{2,1}^{V,a}(t) \right) \right] u_\alpha(p, \lambda), \quad (1)$$

where $u_\alpha(p, \lambda)$ is the known Rarita-Schwinger spinor for a spin-3/2 particle. In general, the index a in Eq. (1) runs from a gluon to a quark flavor and the total form factors $F_{i,j} = \Sigma_a F_{i,j}^{V,a}$. In the present work, we only consider the constituent quark (and diquark) degrees of freedom (d.o.f.) and do not take the gluon contribution into account. In this work, we introduce the kinematical variables $P^\mu = (p^\mu + p'^\mu)/2$, $q^\mu = p'^\mu - p^\mu$, and $q^2 = t$ (which stands for the squared momentum transfer), where $p(p')$ is the initial (final) momentum. The normalization of the Rarita-Schwinger spinor is taken to be $\bar{u}_{\sigma'}(p) u_\sigma(p) = -2M\delta_{\sigma'\sigma}$.

In the nonrelativistic approximation, the EMFFs can be further expressed in terms of $F_{i,0(1)}^V (i = 1, 2)$, according to Eq. (1) and Ref. [58]. In the Breit frame, the average of the baryon momenta and the momentum transfer are respectively defined by $P^\mu = (E, 0, 0, 0)$ and $q^\mu = (0, \boldsymbol{q})$. Thus, $q^2 = -\boldsymbol{q}^2 = t = 4(M^2 - E^2)$ with the Δ isobar mass being M. Then,

$$G_{E0}(t) = \left(1 + \frac{2}{3}\tau\right)[F_{2,0}^V(t) + (1+\tau)(F_{1,0}^V(t) - F_{2,0}^V(t))]$$
$$+ \frac{2}{3}\tau(1+\tau)[F_{2,1}^V(t) + (1+\tau)(F_{1,1}^V(t) - F_{2,1}^V(t))], \tag{2a}$$

$$G_{E2}(t) = [F_{2,0}^V(t) + (1+\tau)(F_{1,0}^V(t) - F_{2,0}^V(t))]$$
$$+ (1+\tau)[F_{2,1}^V(t) + (1+\tau)(F_{1,1}^V(t) - F_{2,1}^V(t))], \tag{2b}$$

$$G_{M1}(t) = \left(1 + \frac{4}{5}\tau\right)F_{2,0}^V(t) + \frac{4}{5}\tau(\tau+1)F_{2,1}^V(t), \tag{2c}$$

$$G_{M3}(t) = F_{2,0}^V(t) + (\tau+1)F_{2,1}^V(t), \tag{2d}$$

where $\tau = -t/4M^2 (\geq 0)$. In Eq. (2), G_{E0}, G_{E2}, G_{M1}, and G_{M3} are the charge, electric-quadrupole, magnetic-

dipole and magnetic-octupole form factors, respectively. When the momentum-transfer squared goes to zero, namely $t = -q^2 \to 0$, we get the charge, magnetic-dipole, electric-quadrupole, and magnetic-octupole moments. Moreover, the slope of electric monopole form factor shows the corresponding charge radius of the system. According to Ref. [59], we have

$$\langle r^2 \rangle_E = 6 \frac{d}{dt} \tilde{G}_{E0}(t)|_{t=0}, \tag{3}$$

in which $\tilde{G}_{E0}$ has been normalized ($\tilde{G}_{E0} = \frac{G_{E0}}{Q_e}$), and Q_e is the charge quantum number carried by the particle.

B. Gravitational form factors

The GFFs for a spin-3/2 particle are defined through the matrix element of its EMT tensor as [53,57]

$$\langle p', \lambda' | \hat{T}^{\mu\nu}(0) | p, \lambda \rangle = -\bar{u}_{\alpha'}(p', \lambda') \left[\frac{P^\mu P^\nu}{M} \left(g^{\alpha'\alpha}F_{1,0}^T(t) - \frac{q^{\alpha'}q^\alpha}{2M^2}F_{1,1}^T(t) \right) + \frac{(q^\mu q^\nu - g^{\mu\nu}q^2)}{4M} \left(g^{\alpha'\alpha}F_{2,0}^T(t) - \frac{q^{\alpha'}q^\alpha}{2M^2}F_{2,1}^T(t) \right) \right.$$
$$+ Mg^{\mu\nu}\left(g^{\alpha'\alpha}F_{3,0}^T(t) - \frac{q^{\alpha'}q^\alpha}{2M^2}F_{3,1}^T(t) \right) + \frac{iP^{\{\mu}\sigma^{\nu\}\rho}q_\rho}{2M}\left(g^{\alpha'\alpha}F_{4,0}^T(t) - \frac{q^{\alpha'}q^\alpha}{2M^2}F_{4,1}^T(t) \right)$$
$$\left. - \frac{1}{M}(q^{\{\mu}g^{\nu\}\{\alpha'}q^{\alpha\}} - 2q^{\alpha'}q^\alpha g^{\mu\nu} - g^{\alpha'\{\mu}g^{\nu\}\alpha}q^2)F_{5,0}^T(t) + Mg^{\alpha'\{\mu}g^{\nu\}\alpha}F_{6,0}^T(t) \right] u_\alpha(p, \lambda). \tag{4}$$

The above definition is for the total GFFs of the system. One can also define the contributions of the quark and gluon individually. Here, $F_{3,0}^T, F_{3,1}^T$, and $F_{6,0}^T$ are nonconserving terms, and they should vanish if we consider the total EMT. Since only the quark contributions are taken into account in our present approach, $F_{3,0}^T, F_{3,1}^T$, and $F_{6,0}^T$

are simply ignored. Moreover, the convention $a^{\{\mu}b^{\nu\}} = a^\mu b^\nu + a^\nu b^\mu$ and $a^{[\mu}b^{\nu]} = a^\mu b^\nu - a^\nu b^\mu$ are adopted.

In the Breit frame, the gravitational multipole form factors (GMFFs) are derived from the matrix element of the EMT current [53,57]. Here we summarize the results as follows:

$$\langle p', \sigma' | \hat{T}^{00}(0) | p, \sigma \rangle = 2ME\left[\varepsilon_0(t)\delta_{\sigma'\sigma} + \left(\frac{\sqrt{-t}}{M}\right)^2 \hat{Q}_{\sigma'\sigma}^{kl} Y_2^{kl} \varepsilon_2(t) \right],$$

$$\langle p', \sigma' | \hat{T}^{0i}(0) | p, \sigma \rangle = 2ME\left[\frac{\sqrt{-t}}{3M} i\epsilon^{ikl} Y_1^l \hat{S}_{\sigma'\sigma}^k \mathcal{J}_1(t) + \left(\frac{\sqrt{-t}}{M}\right)^3 i\epsilon^{ikl} Y_3^{lmn} \hat{O}_{\sigma'\sigma}^{kmn} \mathcal{J}_3(t) \right],$$

$$\langle p', \sigma' | \hat{T}^{ij}(0) | p, \sigma \rangle = 2ME\left[\frac{1}{4M^2}(\Delta^i\Delta^j + \delta^{ij}\Delta^2)D_0(t)\delta_{\sigma'\sigma} + \frac{1}{4M^4}\hat{Q}_{\sigma'\sigma}^{kl}(\Delta^i\Delta^j + \delta^{ij}\Delta^2)\Delta^k\Delta^l D_3(t) \right.$$
$$\left. + \frac{1}{2M^2}(\hat{Q}_{\sigma'\sigma}^{ik}\Delta^j\Delta^k + \hat{Q}_{\sigma'\sigma}^{jk}\Delta^i\Delta^k + \hat{Q}_{\sigma'\sigma}^{ij}\Delta^2 - \delta^{ij}\hat{Q}_{\sigma'\sigma}^{kl}\Delta^k\Delta^l)D_2(t) \right], \tag{5}$$

where the spin-3/2 quadrupole- and octupole-spin operators $\hat{Q}^{ij}$ and $\hat{O}^{ijk}$ are respectively defined as

$$\hat{Q}^{ij} = \frac{1}{2}\left(\hat{S}^i\hat{S}^j + \hat{S}^j\hat{S}^i - \frac{2}{3}S(S+1)\delta^{ij} \right),$$

$$\hat{O}^{ijk} = \frac{1}{6}\left(\hat{S}^i\hat{S}^j\hat{S}^k + \hat{S}^j\hat{S}^i\hat{S}^k + \hat{S}^k\hat{S}^j\hat{S}^i + \hat{S}^j\hat{S}^k\hat{S}^i + \hat{S}^i\hat{S}^k\hat{S}^j + \hat{S}^k\hat{S}^i\hat{S}^j - \frac{6S(S+1)-2}{5}(\delta^{ij}\hat{S}^k + \delta^{ik}\hat{S}^j + \delta^{kj}\hat{S}^i) \right), \tag{6}$$

with $i, j, k = 1, 2, 3$. The spin operators can be expressed in terms of the SU(2) Clebsch-Gordan coefficients in the spherical basis as

$$\hat{S}^a_{\sigma'\sigma} = \sqrt{S(S+1)} C^{S\sigma'}_{S\sigma 1a} \quad \text{with} \quad (a = 0, \pm 1. \sigma, \sigma' = 0, \cdots, \pm S). \tag{7}$$

Obviously, the GMFFs $\varepsilon_{0,1}(t)$ and $\mathcal{J}_{0,1}(t)$, respectively relate to the matrix elements of T^{00} and $T^{i0,0i}$, and $D_{0,2,3}(t)$ to the ones of T^{ij}. They show the fundamental mechanical properties of the system. $\varepsilon_{0,1}(t)$ and $\mathcal{J}_{0,1}(t)$ display the energy- and angular-momentum distributions, and $D_{0,2,3}(t)$ are interpreted as the essential quantities for characterizing the distributions of strong forces inside the system.

The relations among the GMFFs and GFFs are

$$\varepsilon_0(t) = F^T_{1,0}(t) + \frac{t}{6M^2}\left[-\frac{5}{2}F^T_{1,0}(t) - F^T_{1,1}(t) - \frac{3}{2}F^T_{2,0}(t) + 4F^T_{5,0}(t) + 3F^T_{4,0}\right]$$
$$+ \frac{t^2}{12M^4}\left[\frac{1}{2}F^T_{1,0}(t) + F^T_{1,1}(t) + \frac{1}{2}F^T_{2,0}(t) + \frac{1}{2}F^T_{2,1}(t) - 4F^T_{5,0}(t) - F^T_{4,0}(t) - F^T_{4,1}(t)\right]$$
$$+ \frac{t^3}{48M^6}\left[-\frac{1}{2}F^T_{1,1}(t) - \frac{1}{2}F^T_{2,1}(t) + F^T_{4,1}(t)\right], \tag{8a}$$

$$\varepsilon_2(t) = -\frac{1}{6}[F^T_{1,0}(t) + F^T_{1,1}(t) - 4F^T_{5,0}(t)]$$
$$+ \frac{t}{12M^2}\left[\frac{1}{2}F^T_{1,0}(t) + F^T_{1,1}(t) + \frac{1}{2}F^T_{2,0}(t) + \frac{1}{2}F^T_{2,1}(t) - 4F^T_{5,0}(t) - F^T_{4,0} - F^T_{4,1}(t)\right]$$
$$+ \frac{t^2}{48M^4}\left[-\frac{1}{2}F^T_{1,1}(t) - \frac{1}{2}F^T_{2,1}(t) + F^T_{4,1}(t)\right], \tag{8b}$$

$$\mathcal{J}_1(t) = F^T_{4,0}(t) - \frac{t}{5M^2}[F^T_{4,0}(t) + F^T_{4,1}(t) + 5F^T_{5,0}(t)] + \frac{t^2}{20M^4}F^T_{4,1}(t), \tag{8c}$$

$$\mathcal{J}_3(t) = -\frac{1}{6}[F^T_{4,0}(t) + F^T_{4,1}(t)] + \frac{t}{24M^2}F^T_{4,1}(t), \tag{8d}$$

$$D_0(t) = F^T_{2,0}(t) - \frac{16}{3}F^T_{5,0}(t) - \frac{t}{6M^2}[F^T_{2,0}(t) + F^T_{2,1}(t) - 4F^T_{5,0}(t)] + \frac{t^2}{24M^4}F^T_{2,1}(t), \tag{8e}$$

$$D_2(t) = \frac{4}{3}F^T_{5,0}(t), \tag{8f}$$

$$D_3(t) = \frac{1}{6}[-F^T_{2,0}(t) - F^T_{2,1}(t) + 4F^T_{5,0}(t)] + \frac{t}{24M^2}F^T_{2,1}(t). \tag{8g}$$

One can also proceed by calculating the Fourier transformations of GMFFs to get the monopole and quadrupole densities [53]

$$\mathcal{E}_0(r) = M\tilde{\varepsilon}_0(r), \qquad \mathcal{E}_2(r) = -\frac{1}{M}r\frac{d}{dr}\frac{1}{r}\frac{d}{dr}\tilde{\varepsilon}_2(r), \tag{9}$$

with $\tilde{\varepsilon}_{0,2}(r) = \int \frac{d^3q}{(2\pi)^3} e^{-iq\cdot r}\varepsilon_{0,2}(t)$ being the densities in r-space.

The mass radius of Δ is an important property, and it can be derived as [53]

$$\langle r^2 \rangle_M = 6\frac{d}{dt}\varepsilon_0(t)|_{t=0}. \tag{10}$$

Moreover, if one interprets the static $T^{ij}(\mathbf{r})$ connecting to the pressure $p(r)$ and shear force $s(r)$ of the system like classical mechanics, these two physical quantities relate to the D term as [31]

$$p(r) = \frac{1}{6M}\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr}\tilde{D}_0(r),$$
$$s(r) = -\frac{1}{4M}r\frac{d}{dr}\frac{1}{r}\frac{d}{dr}\tilde{D}_0(r),$$
$$\tilde{D}_0(r) = \int \frac{d^3q}{(2\pi)^3} e^{-iq\cdot r}D_0(t). \tag{11}$$

According to Ref. [60], the force on an infinitesimal piece of area dS^j at the distance r for the system has the form $F^i(\mathbf{r}) = T^{ij}(\mathbf{r})dS^j = [\frac{2}{3}s(r) + p(r)]dS^i$ where $dS^j = dSr^j/r$. The corresponding force must be directed outwards for the mechanical stability of the system. Therefore, the local criterion for the mechanical stability can be formulated as [60]

$$p(r) + \frac{2}{3}s(r) > 0. \tag{12}$$

Here we can express the D term, $D = D_0(0)$, by $p(r)$ and $s(r)$ as

$$D = M \int d^3 r r^2 p(r) = -\frac{4M}{15} \int d^3 r r^2 s(r). \tag{13}$$

So,

$$M \int d^3 r r^2 p(r) + \frac{2M}{3} \int d^3 r r^2 s(r)$$
$$= -\frac{3}{2}D = M \int d^3 r r^2 \left(p(r) + \frac{2}{3}s(r) \right) > 0, \tag{14}$$

which implies $D < 0$ for any stable system.

III. COVARIANT QUARK-DIQUARK APPROACH

It is believed that the Δ isobar is composed of three light quarks, u and d quarks. Since it has $I(J^p) = 3/2(3/2^+)$, the total antisymmetry makes the isospin and spin of each pair of quarks equal to 1. Here we treat two of them as a diquark. Therefore, the matrix element of the electromagnetic current is the sum of the contributions of the quark and diquark. For example, Δ^+ contains two u quarks and one d quark. So we can treat a (ud) or (uu) pair as a diquark. If we consider the probability of the two cases, we naively conclude that the probability of (ud) as a diquark is two times that of (uu) as a diquark. It should be stressed that we also explicitly take the internal quark structure of the axial-vector diquark into account. This treatment is different from the nonrelativistic quark model calculations for the nucleon EMFFs and for the $N - \Delta$ transition

amplitudes, where the total contribution is simply regarded as three times that of the single quark contribution although the bound-state wave function is employed [17–20]. The present approach is consistent with the other relativistic and covariant quark-diquark approaches [22,23].

A. EMFFs of Δ contributed by quark

Here, we give the details for the calculation of the EMFFs of Δ in our approach. The electromagnetic current attached to Δ is represented by the Feynman diagrams illustrated in Figs. 1(a) and 1(b) and its matrix element is expressed as the sum of the quark and diquark contributions (labeled by the subscripts of q and D, respectively) as

$$\langle p', \lambda' | \hat{J}^\mu(0) | p, \lambda \rangle = \langle p', \lambda' | \hat{J}_q^\mu(0) | p, \lambda \rangle + \langle p', \lambda' | \hat{J}_D^\mu(0) | p, \lambda \rangle. \tag{15}$$

In the present work, we neglect the longitudinal part $k^\mu k^\nu / m_V^2$ of the vector propagator in order to have finite results [61]. So the quark contribution is

$$\langle p', \lambda' | \hat{J}_q^\mu(0) | p, \lambda \rangle$$
$$= -Q_q^e e \bar{u}_{\alpha'}(p', \lambda')(-i) \int \frac{d^4 l}{(2\pi)^4} \frac{1}{\tilde{\mathfrak{D}}} \tilde{\Gamma}^{\alpha'\beta'} \left(\slashed{l} + \frac{\slashed{q}}{2} + m_q \right)$$
$$\times g_{\beta'\beta} \gamma^\mu \left(\slashed{l} - \frac{\slashed{q}}{2} + m_q \right) \tilde{\Gamma}^{\beta\alpha} u_\alpha(p, \lambda), \tag{16}$$

where Q_q^e is the charge quantum number carried by the active quark, and $\tilde{\mathfrak{D}}$ stands for all the propagator denominators as

$$\tilde{\mathfrak{D}} = \left[\left(l + \frac{q}{2} \right)^2 - m_q^2 + i\epsilon \right] \left[\left(l - \frac{q}{2} \right)^2 - m_q^2 + i\epsilon \right]$$
$$\times [(l - P)^2 - m_D^2 + i\epsilon]. \tag{17}$$

The vertex of Δ with its quark and diquark constituents in Eq. (16) is expressed as $\tilde{\Gamma}^{\alpha\beta} = \Gamma^{\alpha\beta}\Xi$. According to Ref. [62], the Lorentz structure of the vertex $\Gamma^{\alpha\beta}$ is

$$\Gamma^{\alpha\beta} = c_1[g^{\alpha\beta} + g_2\gamma^\beta\Lambda^\alpha + g_3\Lambda^\beta\Lambda^\alpha], \tag{18}$$
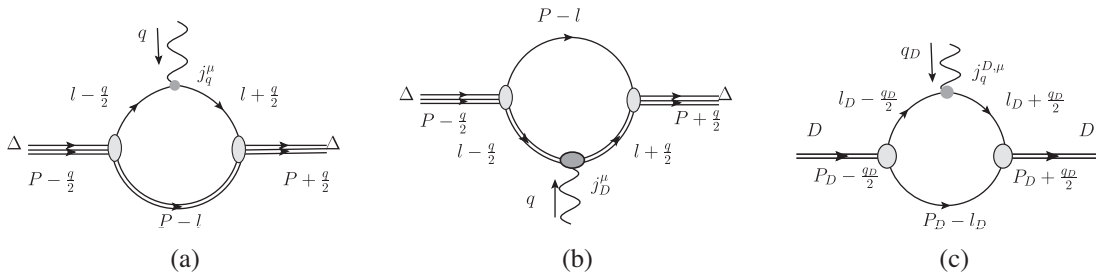

FIG. 1. Feynman diagrams for the electromagnetic current of the Δ resonance, (a) and (b), and of the diquark (c). The left and middle panels stand for the contributions of quark (single line) and diquark (double line) to Δ.

with Λ being the relative momentum between the quark and diquark. The couplings of c_1, g_2, and g_3 in Eq. (18) can be determined by fitting to the experimental data of EMFFs or to the lattice calculation. The superscript β stands for the index of the spin-1 particle. It should be addressed that the vertex $\Gamma^{\alpha\beta}$ contains high-order momentum terms, and they can make the loop integral divergent. To avoid this problem we simply consider an additional scalar function Ξ to simulate the bound state problem of the Δ resonance. In general, this scalar function should be obtained from a dynamical calculation of the system, like solving the Bethe-Salpeter equation. Here, we simply take an ansatz for the scalar function Ξ as [63]

$$\Xi(p_1, p_2) = \frac{c}{[p_1^2 - m_R^2 + i\epsilon][p_2^2 - m_R^2 + i\epsilon]}, \quad (19)$$

where m_R is a cutoff mass parameter, and we find that our numerical results are not sensitive to m_R within a certain range. As discussed already in Refs. [45,64], regularization done by introducing the momentum-dependent form factors breaks the gauge invariance and the electromagnetic Ward-Takahashi identities. For the present work, it leads to the EMFFs and GFFs not being normalized at the same time. Despite of the deficiency, we adopt the function Ξ to simplify the calculation and it turns out the deviation in normalization is small as shown in the later content. Then, Eq. (16) goes to

$$\langle p', \lambda' | \hat{J}_q^\mu(0) | p, \lambda \rangle = -Q_q^e e \bar{u}_{\alpha'}(p', \lambda')(-i\tilde{C}^2)$$
$$\times \int \frac{d^4l}{(2\pi)^4} \frac{1}{\mathfrak{D}} \Gamma^{\alpha'\beta'} \left(\slashed{l} + \frac{\slashed{q}}{2} + m_q \right)$$
$$\times g_{\beta'\beta} \gamma^\mu \left(\slashed{l} - \frac{\slashed{q}}{2} + m_q \right) \Gamma^{\beta\alpha} u_\alpha(p, \lambda), \quad (20)$$

where $\tilde{C} = cc_1$ and denominator is modified to be

$$\mathfrak{D} = \tilde{\mathfrak{D}}[(l-P)^2 - m_R^2 + i\epsilon]^2 \left[\left(l - \frac{q}{2} \right)^2 - m_R^2 + i\epsilon \right]$$
$$\times \left[\left(l + \frac{q}{2} \right)^2 - m_R^2 + i\epsilon \right]. \quad (21)$$

B. EMFFs of Δ contributed by the diquark

In the same way, the diquark contribution to the EMFFs of Δ is

$$\langle p', \lambda' | \hat{J}_D^\mu(0) | p, \lambda \rangle$$
$$= -Q_D^e e \bar{u}_{\alpha'}(p', \lambda') i\tilde{C}^2 \int \frac{d^4l}{(2\pi)^4} \frac{1}{\mathfrak{D}'} \Gamma^{\alpha'}_{\beta'}(\slashed{P} - \slashed{l} + m_q)$$
$$\times j_D^{\mu, \beta'\beta} \Gamma^\alpha_\beta u_\alpha(p, \lambda), \quad (22)$$

where the Q_D^e is the charge quantum number carried by the diquark. It should be mentioned that the diquark is an axial vector (1^+) bound state of two quarks. Here, we adopt the same vertex in Eqs. (18) and (19). Thus, in the above equation,

$$\mathfrak{D}' = \left[\left(l + \frac{q}{2} \right)^2 - m_D^2 + i\epsilon \right] \left[\left(l - \frac{q}{2} \right)^2 - m_D^2 + i\epsilon \right]$$
$$\times [(l-P)^2 - m_q^2 + i\epsilon][(l-P)^2 - m_R^2 + i\epsilon]^2$$
$$\times \left[\left(l - \frac{q}{2} \right)^2 - m_R^2 + i\epsilon \right] \left[\left(l + \frac{q}{2} \right)^2 - m_R^2 + i\epsilon \right]. \quad (23)$$

Figure 1(c) gives the explicit contribution of diquark with its quark structure. The effective Lagrangian for the diquark is [23]

$$\mathcal{L}_{D \to qq} = c_D \Psi_q^T C^{-1} \gamma^\mu \Psi_q \epsilon_{\mu, D}(p_D, \lambda) \Xi_D + \text{H.c.}, \quad (24)$$

where Ψ_q^T stands for the charge conjugate of quark field and $C = i\gamma^2\gamma^0$. The correlation function attached to the vertex in the above Lagrangian and Fig. 1(c) is assumed to be the same as in Eq. (19) for simplicity, and have the same cutoff mass m_R in order to reduce the number of parameters. According to Fig. 1(c), we get

$$\langle p', \lambda' | \hat{J}_D^\mu(0) | p, \lambda \rangle = \Sigma_q \langle p', \lambda' | \hat{J}_q^\mu(0) | p, \lambda \rangle$$
$$= -\epsilon_{\beta'}^*(p'_D, \lambda') j_D^{\mu, \beta'\beta} \epsilon_\beta(p_D, \lambda), \quad (25)$$

where $\epsilon_\beta(p_D, \lambda)$ represents the spin-1 diquark field, and $j_D^{\mu, \beta\beta'}$ represents the effective electromagnetic current of spin-1 diquark. Here, we introduce kinematical variables $P_D^\mu = (p_D^\mu + p_D'^\mu)/2$, $q_D^\mu = p_D'^\mu - p_D^\mu = q^\mu$ and $q_D^2 = -t_D = -t$ (since transfer momentum is all on the diquark). Then

$$\langle p'_D, \lambda' | \hat{J}_q^\mu(0) | p_D, \lambda \rangle$$
$$= -Q_q^e e \epsilon_{\beta'}^*(p'_D, \lambda') i\tilde{C}_D^2$$
$$\times \int \frac{d^4l_D}{(2\pi)^4} \frac{1}{\mathfrak{D}_D} \gamma^{\beta'} \left(\slashed{l}_D + \frac{\slashed{q}}{2} + m_q \right) \gamma^\mu$$
$$\times \left(\slashed{l}_D - \frac{\slashed{q}}{2} + m_q \right) \gamma^\beta (\slashed{l}_D - \slashed{P}_D + m_q) \epsilon_\beta(p_D, \lambda), \quad (26)$$

where the constant $\tilde{C}$ in Eq. (20) is replaced by $\tilde{C}_D = c_1 c_D$. In addition,

$$\mathfrak{D}_D = \left[\left(l_D + \frac{q}{2}\right)^2 - m_q^2 + i\epsilon \right] \left[\left(l_D - \frac{q}{2}\right)^2 - m_q^2 + i\epsilon \right]$$
$$\times \left[(l_D - P_D)^2 - m_q^2 + i\epsilon \right] [(l_D - P_D)^2 - m_R^2 + i\epsilon]^2$$
$$\times \left[\left(l_D - \frac{q}{2}\right)^2 - m_R^2 + i\epsilon \right] \left[\left(l_D + \frac{q}{2}\right)^2 - m_R^2 + i\epsilon \right].$$

$$(27)$$

Finally, the electromagnetic current of the diquark in Eq. (25) can be written as

$$j_D^{\mu,\beta'\beta} = \left[g^{\beta'\beta} F_{D;1}^V(t) - \frac{q^{\beta'} q^{\beta}}{2m_D^2} F_{D;2}^V(t) \right] (p_D' + p_D)^{\mu}$$
$$- (q^{\beta'} g^{\mu\beta} - q^{\beta} g^{\mu\beta'}) F_{D;3}^V(t),$$

$$(28)$$

where $F_{D;1,2,3}^V(t)$ stand for the three form factors of the spin-one particle contributed by quarks and by the loop integral. They contain the binding effect. The expression of this effective current in Eq. (28) is standard for a free spin-1 particle. Moreover, in reproducing the effective electromagnetic current, the normalization of the diquark charge is also employed.

C. GFFs of the Δ contributed by the quark

One may also calculate the matrix elements of energy-momentum tensor for the Δ system by summing the contributions of the quark and the diquark,

$$T^{\mu\nu} = T_q^{\mu\nu} + T_D^{\mu\nu}. \tag{29}$$

The Feynman diagrams for the process are shown in Figs. 2(a) and 2(b).

According to the Lagrangian for a quark with mass m_q

$$\mathcal{L}_q = \frac{i}{2} \bar{\psi}_q \gamma^\mu \overleftrightarrow{\partial}_\mu \psi_q - m_q \bar{\psi}_q \psi_q, \quad \text{with} \quad \overleftrightarrow{\partial}_\mu = \overrightarrow{\partial}_\mu - \overleftarrow{\partial}_\mu, \tag{30}$$

then, the symmetric EMT is defined as

$$T_q^{\mu\nu} = \frac{i}{4} \bar{\psi}_q \gamma^\mu \overleftrightarrow{\partial}^\nu \psi_q + \frac{i}{4} \bar{\psi}_q \gamma^\nu \overleftrightarrow{\partial}^\mu \psi_q. \tag{31}$$

In our covariant quark-diquark approach, the matrix element of EMT current from the contribution of quark is

$$\langle p', \lambda' | \hat{T}_q^{\mu\nu}(0) | p, \lambda \rangle$$
$$= -\bar{u}_{\alpha'}(p', \lambda') \frac{-i\tilde{C}^2}{2} \int \frac{d^4 l}{(2\pi)^4} \frac{1}{\mathfrak{D}} \Gamma^{\alpha'\beta'} \left(\slashed{l} + \frac{\slashed{q}}{2} + m_q \right)$$
$$\times g_{\beta'\beta}(\gamma^\mu l^\nu + \gamma^\nu l^\mu) \left(\slashed{l} - \frac{\slashed{q}}{2} + m_q \right) \Gamma^{\alpha\beta} u_\alpha(p, \lambda), \quad (32)$$

where $\mathfrak{D}$ has been given in Eq. (21).

D. GFFs of the Δ contributed by the diquark

The EMT of the diquark can be obtained from the Proca Lagrangian if we consider it as a structureless particle. Here, we treat the diquark contribution to the EMT matrix element of Δ by considering explicitly its quark contents as we have discussed in Sec. III B. The matrix element of EMT current from the contribution of diquark is

$$\langle p', \lambda' | \hat{T}_D^{\mu\nu}(0) | p, \lambda \rangle$$
$$= -\bar{u}_{\alpha'}(p', \lambda') i\tilde{C}^2 \int \frac{d^4 l}{(2\pi)^4} \frac{1}{\mathfrak{D}'} \Gamma^{\alpha'\beta'} \left(\slashed{l} + \frac{\slashed{q}}{2} + m_q \right)$$
$$\times X_{\beta'\beta}^{\mu\nu} \left(\slashed{l} - \frac{\slashed{q}}{2} + m_q \right) \Gamma^{\beta\alpha} u_\alpha(p, \lambda), \quad (33)$$

where $\mathfrak{D}'$ is shown in Eq. (23), and $X_{\beta'\beta}^{\mu\nu}$ stands for the effective energy-momentum tensor of the diquark.

According to Fig. 2(c), the matrix element of the EMT current of the diquark, due to its two quark structure, is [33,65]
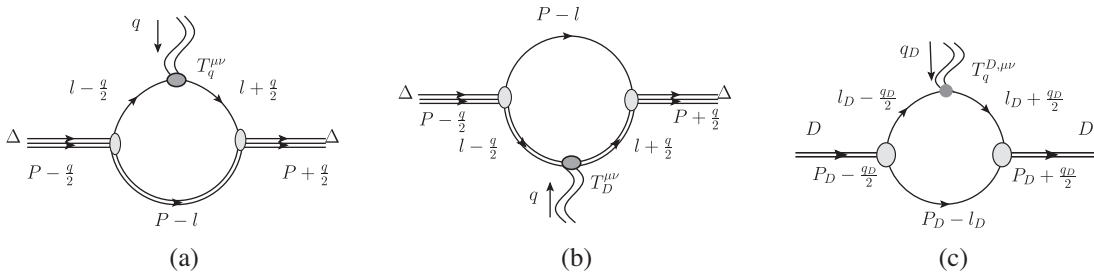

FIG. 2. Feynman diagrams for the GFFs of Δ, contributed to by quark (a), by diquark (b), and the GFFs of the diquark (c).

$$
\begin{aligned}
\langle p'_D, \lambda' | \hat{T}^{\mu\nu}_D(0) | p_D, \lambda \rangle &= 2\langle p'_D, \lambda' | \hat{T}^{\mu\nu}_q(0) | p_D, \lambda \rangle = -\epsilon^*_{\beta'}(p'_D, \lambda') X^{\beta'\beta\mu\nu} \epsilon_\beta(p_D, \lambda) \\
&= -\epsilon^*_{\beta'}(p'_D, \lambda') \left[\frac{P^\mu_D P^\nu_D}{m_D} \left(g^{\beta\beta'} F^T_{D;1,0}(t) - \frac{q^\beta q^{\beta'}}{2m^2_D} F^T_{D;1,1}(t) \right) \right. \\
&\quad + \frac{(q^\mu q^\nu - g^{\mu\nu} q^2)}{4m_D} \left(g^{\beta\beta'} F^T_{D;2,0}(t) - \frac{q^\beta q^{\beta'}}{2m^2_D} F^T_{D;2,1}(t) \right) \\
&\quad \left. - \frac{P^{\{\mu}_D g^{\nu\}[\beta'} q^{\beta]}}{m_D} F^T_{D;4,0}(t) - \frac{1}{m_D} (q^{\{\mu} g^{\nu\}\{\beta'} q^{\beta\}} - 2q^\beta q^{\beta'} g^{\mu\nu} - g^{\beta'\{\mu} g^{\nu\}\beta} q^2) F^T_{D;5,0}(t) \right] \epsilon_\beta(p_D, \lambda), \quad (34)
\end{aligned}
$$

where the nonconserving form factors are ignored. Finally, we get the matrix element of EMT tensor contributed by the diquark

$$
\begin{aligned}
&\langle p'_D, \lambda' | \hat{T}^{\mu\nu}_D(0) | p_D, \lambda \rangle \\
&= -\epsilon^*_{\beta'}(p'_D, \lambda') i\tilde{C}^2_D \int \frac{d^4 l_D}{(2\pi)^4} \frac{1}{\mathfrak{D}_D} \gamma^{\beta'} \left(\slashed{l}_D + \frac{\slashed{q}}{2} + m_q \right) \\
&\quad \times (\gamma^\mu l^\nu_D + \gamma^\nu l^\mu_D) \left(\slashed{l}_D - \frac{\slashed{q}}{2} + m_q \right) \gamma^\beta (\slashed{l}_D - \slashed{P}_D + m_q) \\
&\quad \times \epsilon_\beta(p_D, \lambda). \quad (35)
\end{aligned}
$$

To summarize this section, we employ the relativistic covariant quark-diquark approach to compute the EMFFs and GFFs of the spin-3/2 Δ resonance. In the above formulas the quark structure of the diquark (1^+) is explicitly taken into account by introducing the correlation function and by the loop integrals of Eqs. (26) and (35). In particular, only the fundamental electromagnetic current and the EMT of the quark are involved.

IV. NUMERICAL RESULTS

A. Determination of model parameters

In the present approach, we need to numerically calculate the loop integrals sandwiched between the two Rarita-Schwinger spinors. The on shell identities which have been explicitly proven in Ref. [57] for the Rarita-Schwinger spinors are employed. They are listed in Appendix A. Moreover, Appendix B gives the Feynman parametrizations for the necessary loop integrals.

We also need to input the masses of the Δ resonance M, quark m_q, diquark m_D, and the cutoff m_R in the calculation. Here, we simple choose $M = 1.085$ GeV. It is the average of the masses of nucleon and Δ resonance, and this selection means that we do not consider the mass splitting between the Δ and nucleon. Moreover, we assume $m_q = 0.4$ GeV according to Ref. [3]. Our M and m_q indicate that $M < 3m_q$. Furthermore, we choose $m_D \sim 0.76$ GeV [3], it implies that the diquark is a bound state of two quarks as well. Finally, we simply borrow $m_R \sim 1.6$ GeV from Ref. [15].

It should be addressed that due to the normalization of the charge form factor of Δ at $t^2 = 0$, the overall factor $\tilde{C} = cc_1$ can be fixed. However, g_2 and g_3 in Eq. (20) are still free. They describe the D-wave coupling of the Δ resonance to the quark and the axial-vector diquark in our approach, and they provide an essential effect on the high-order multipoles. To determine these two parameters, the EMFFs calculated by the lattice QCD (LQCD) of Ref. [66] are employed as constraints. Comparing to the LQCD results, we select $g_2 = 0.703$ GeV^{-1} and $g_3 = 0.412$ GeV^{-2}. All the parameters in our calculation are listed in Table I, and the obtained four EMFFs are plotted in Fig. 3 for Δ^+. Figure 3 shows that our results are consistent with the LQCD calculation, at least qualitatively. In the figure, the lines are our calculations with different cutoff masses and the dots are the results from LQCD with different pion masses. We also conclude that our results are not sensitive to the cutoff parameter m_R. In the present work, the units of parameters in figures have been omitted and are consistent with Table I.

To show a more detailed analysis of our model parameters, we first check the impact of g_2 and g_3 on our EMFFs. Figure 4 displays their effect. We find that g_2 and g_3 have a remarkable influence on the electric-quadrupole and magnetic-octupole form factors, and they even change the signs of G_{E2} and G_{M3}. However, they have a little impact on the electric monopole and magnetic-dipole form factors. This conclusion is reasonable since the couplings $g_{2,3}$ stand for the high-partial waves, and they manifest themselves in the high-order multipoles, like quadrupole and octupole form factors.

B. Results for the moments of EMFFs

When the squared momentum transfer goes to zero $t = 0$, the form factors give the moments of the magnetic dipole $\mu_\Delta = G_{M1}(0)\frac{e}{2M}$, of the electric quadrupole $\mathcal{Q}_\Delta = G_{E2}(0)\frac{e}{M^2}$, and of the magnetic octupole $\mathcal{O}_\Delta = G_{M3}(0)\frac{e}{2M^3}$, where e is the electric charge [67].

TABLE I. The parameters used in our approach.

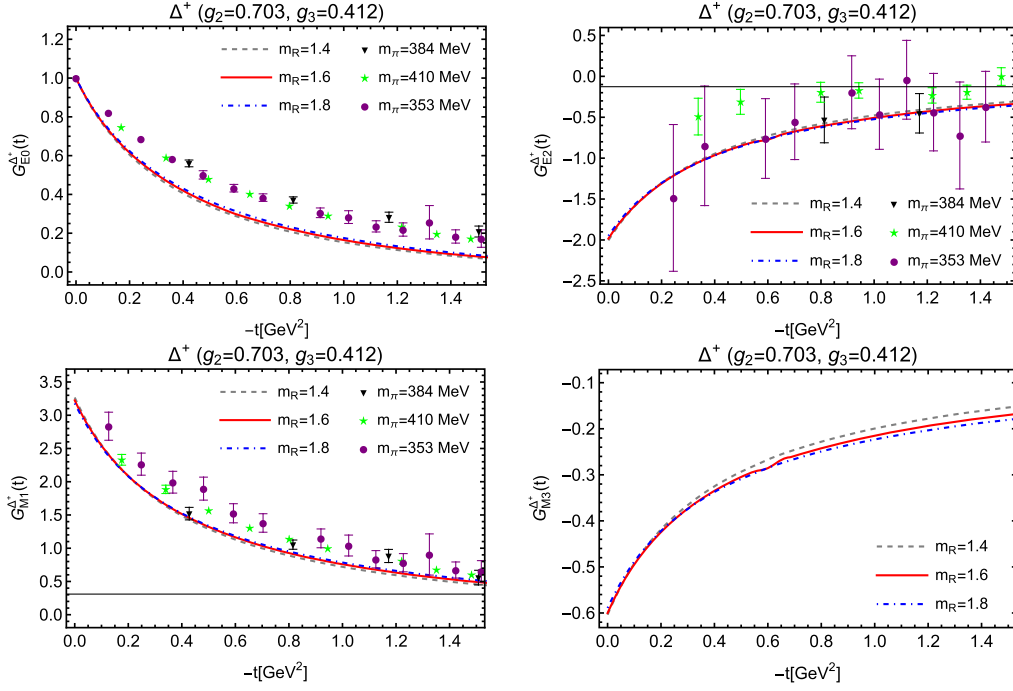| M/GeV	m_q/GeV	m_D/GeV	m_R/GeV	g_2/GeV^{-1}	g_3/GeV^{-2}
1.085	0.4	0.76	1.6	0.703	0.412

FIG. 3. Calculated four EMFFs of Δ^+ comparing to the lattice QCD calculations. The dashed, solid and the dotted-dashed curves represent the results with $m_R = 1.4$ GeV, 1.6 GeV, and 1.8 GeV, respectively. $g_2 = 0.412$ GeV^{-1} and $g_3 = 0.703$ GeV^{-2} are used.
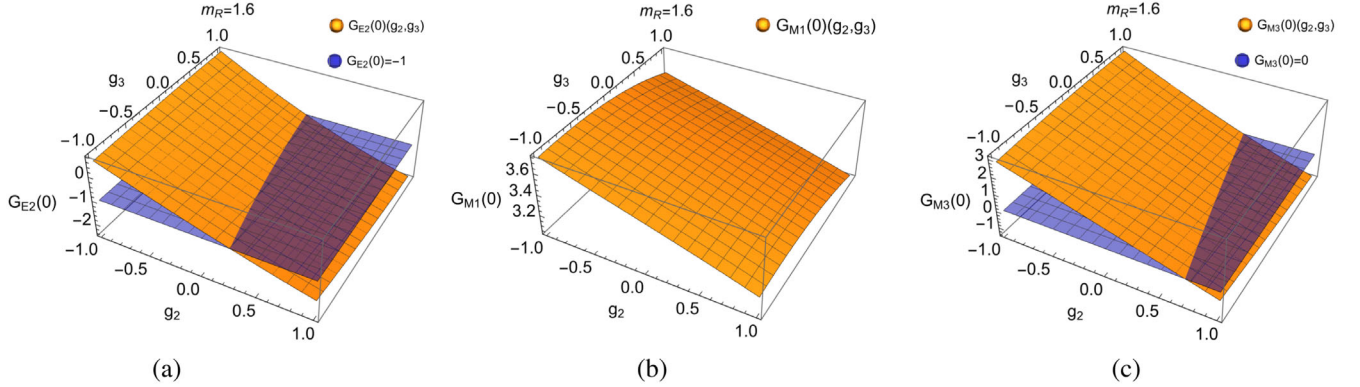

FIG. 4. The parameter g_2 and g_3 dependences of $G_{E2}(0)$ (a), $G_{M1}(0)$ (b), and $G_{M3}(0)$ (c).

We can compare the obtained magnetic-dipole, electric-quadrupole and magnetic-octupole moments to the results of different model calculations, such as nonrelativistic quark model (NQM) [68–70], relativistic quark model (RQM) [71], QCD sum rules (QCDSR) [72–75], light cone QCD sum rules (LCQSR) [76], Large N_c [77–79], chiral-quark model with meson-exchange currents (χQMEC) [80,81], QCD quark model (QCDQM) [82,83], chiral bag model (CBM) [84], general parameterization QCD (GPQCD) [85], chiral quark-soliton model (χQSM) [86], effective mass and screened charge scheme (EMS) [87,88], chiral perturbation theory (χPT) [89–91], lattice QCD (LQCD) [92–94], and chiral constituent quark model (χCQM) [95]. Tables II, III, and IV list the

comparisons of our magnetic, quadrupole, and octupole moments with other model calculations, respectively.

For $G_{M1}(0)$ of Δ^{++}, the results of other model calculations are in the range of $[4.4 \sim 6.93]$, and the minimum value 4.4 ± 0.8 predicted by the LCQSR and the maximum value 6.93 by the χQMEC as shown in Table II. Our result 6.04 is much closer to the one given by the large N_c [77–79,96]. For $G_{E2}(0)$ of Δ^{++} displayed in Table III, there are the minimum value -3.82 in the NQM, and the maximum value -0.0452 ± 0.0113 in the QCDSR. Our result -3.86 is slightly smaller than the results given by other models. The negative sign for $G_{E2}(0)$ is consistent with most of model calculations and indicates that Δ is oblate deformed. For $G_{M3}(0)$ listed in Table IV, we see that

TABLE II.   A comparison of our magnetic-dipole moment with other models.

$G_{M1}(0)$	Δ^{++}	Δ^+	Δ^0	Δ^-
This work	6.04	3.02	0.00	−3.02
NQM [68]	5.56	2.73	−0.09	−2.92
RQM [71]	4.76	2.38	0.00	−2.38
QCDSR [72–74]	4.39 ± 1.00	2.19 ± 0.50	0.00	-2.19 ± 0.50
LCQSR [76]	4.4 ± 0.8	2.2 ± 0.4	0.0	-2.2 ± 0.4
Large N_c [77–79]	5.9(4)	2.9(2)	$\cdots$	−2.9(2)
χQMEC[80,81]	6.93	3.47	0.00	−3.47
QCDQM [82,83]	5.689	2.778	−0.134	−3.045
CBM [84]	4.52	2.12	−0.29	−2.69
EMS [87,88]	4.56	2.28	0	−2.28
χPT [89,90]	5.390	2.383	−0.625	−3.632
LQCD [92–94]	4.91 ± 0.61	2.46 ± 0.31	0.00	-2.46 ± 0.31
χCQM[95]	5.82 ± 0.08	2.63 ± 0.06	-0.56 ± 0.09	-3.75 ± 0.08

TABLE III.   A comparison of our electric-quadrupole moment with other models.

$G_{E2}(0)$	Δ^{++}	Δ^+	Δ^0	Δ^-
This work	−3.86	−1.93	0.00	1.93
NQM [69]	−3.82	−1.91	0	1.91
NQM [70]	−3.63	−1.79	0	1.79
χPT [91]	-3.12 ± 1.95	-1.17 ± 0.78	0.47 ± 0.20	2.34 ± 1.17
χQSM [86]	$\cdots$	−2.15	$\cdots$	$\cdots$
QCDSR [75]	-0.0452 ± 0.0113	-0.0226 ± 0.0057	0	0.0226 ± 0.0057

TABLE IV.   A comparison of our magnetic-octupole moment with other model calculations.

$G_{M3}(0)$	Δ^{++}	Δ^+	Δ^0	Δ^-
This work	−1.12	−0.56	0.00	0.56
GPQCD [85]	−11.68	−5.84	0	5.84
QCDSR [75]	-0.0925 ± 0.0234	-0.0462 ± 0.0117	0	0.0462 ± 0.0117

the results from the two different models vary widely and our result is −1.12 for Δ^{++}. Future measurements for the Δ isobar deformation are expected to discriminate different approaches. From these three qualitative comparisons we conclude that our results are comparable to most of the models. In addition, in our numerical calculations, we do not consider the small mass difference between the u and d quarks, and the different moments for the isospin partners of Δ, displayed in Tables II, III, and IV, due to their charge difference.

Figure 5 gives the individual contributions from the quark and diquark to the EMFFs of Δ^{++}. As shown in the figure, the ratio of the contribution to EMFFs by diquark and quark is close to 2 as $-t$ goes to zero. It means that when the momentum transfer is small the electromagnetic interaction probes the diquark as a pointlike particle. This is consistent with the physical intuition and the constituent quark model calculations [16–18,70]. However, when the momentum transfer increases, the EM current probes much

more inside the diquark such that the effects of the binding and its quark structure become remarkable. It should be noted that, in the nonrelativistic constituent quark model calculation, the coupling of each quark to the electromagnetic probe is considered to be the same (for simplicity) and the total result is the three times that of the quark contribution [16–18,20,70] although the nonrelativistic wave function contains ρ and λ excitations.

Finally, we estimate the root mean squared radius of the Δ resonance according to our electric form factor $G_{E0}(t)$. It is

$$\langle r^2 \rangle_E = 0.665 \text{ fm}^2, \tag{36}$$

for the charge distribution. It should be mentioned that the obtained charged rms radii of the three charged isospin partners Δ are the same since we do not consider the slight mass difference between the u and d quarks.
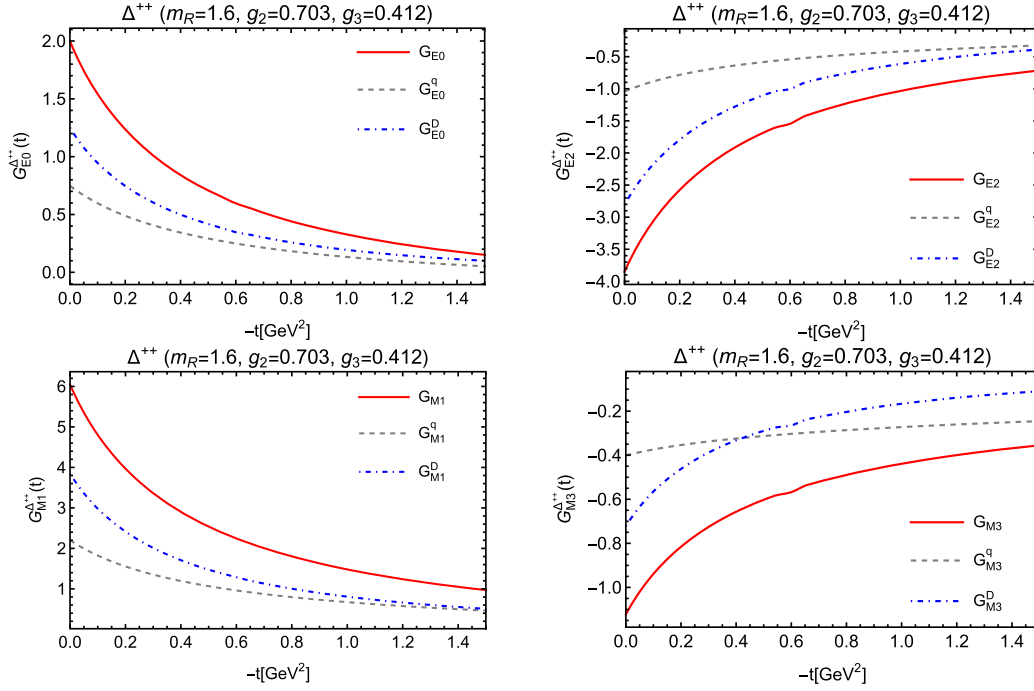
FIG. 5. The calculated four EMFFs of Δ^{++}, the gray dashed, blue dashed-dotted, and red solid curves stand for the contributions from quark and diquark, and their sum, respectively.

C. The results of the matrix elements of EMT and GFFs of Δ

1. EMT of T^{00} and $T^{0i,i0}$

Our relativistic covariant quark-diquark approach can be also applied for the calculations of the matrix element of the energy-momentum tensor for the Δ spin-3/2 system according to the Secs. III C and III D. Here we show our results for GFFs as the functions of $-t$ in Fig. 6, where the same normalization condition and model parameters are adopted as for the case of EMFFs. By comparing our results with Ref. [53] where the Skyrme model is applied, we find that our $F^{T}_{1,(0,1)}$, $F^{T}_{4,(0,1)}$, and $F^{T}_{5,0}$ are consistent with each other. However, our estimated $F^{T}_{2,(0,1)}$ have a big difference. This issue is closely related to the understanding of matrix elements of T^{ij} and it will be discussed later in detail. Then, we can reproduce the physical GMFFs from Eq. (8) for the energy- and angular-momentum distributions. The results are displayed in Figs. 7 and 8.

As shown in Fig. 7 our $\varepsilon^{\Delta}_{0}(0) = 0.97 \sim 1$ which correspond to the normalization condition of Δ mass. This result indicates that the condition is not exactly preserved. This deviation, as discussed after Eq. (19), is because of the momentum-dependent regularization violating the gauge invariance. The ratio of the contribution from diquarks and quarks is also close to 2 when $t = 0$, similar to the case of EMFFs. Furthermore, as shown in Fig. 8, our estimated spin for Δ is $J^{\Delta}_{1}(0) \sim 1.5$ which just corresponds to the total spin of Δ carried out by its two constituents.

The mass radius from Fig. 7 is

$$\langle r^2 \rangle_M = 0.529 \text{ fm}^2, \qquad (37)$$

which is near but smaller than $\langle r^2 \rangle_E$. This number is close to 0.54 fm^2 of Ref. [53].

Furthermore, the quantities, such as the energy densities and angular moment density can be obtained with the results given in Fig. 9 by performing the Fourier transformations as shown in Eq. (9). We know that the Fourier transformation of a plane wave is not well defined, and the transformations of our obtained GFFs, which are the functions of $-t$, cannot be done due to the divergence. Thus, we add a Gaussian-like wave packet $e^{\frac{t}{\lambda^2}}$ [97,98] to guarantee the convergence when $|t|$ increases. Here, the model-parameter λ represents the size of the hadron with $\lambda \sim 1$ GeV. The inclusion of this additional factor is reasonable because of the locality of the particle and the validity of the perturbative field theory. This issue has been discussed explicitly in Refs. [99,100]. Our results for the densities in r space are shown in Fig. 9. We find that the energy densities converge quickly to zero when $r > 1$ fm and when 0.5 GeV $< \lambda < 1.1$ GeV. Moreover, the Compton wavelength corresponding to λ is about $2 \sim 4$ times the obtained radius of the Δ isobar.

2. The matrix elements of T^{ij} and the D term

It should be reiterated that our results for the GFFs of $F^{\Delta}_{20}(t)$ and $F^{\Delta}_{21}(t)$ shown in Fig. 6 are different in sign from
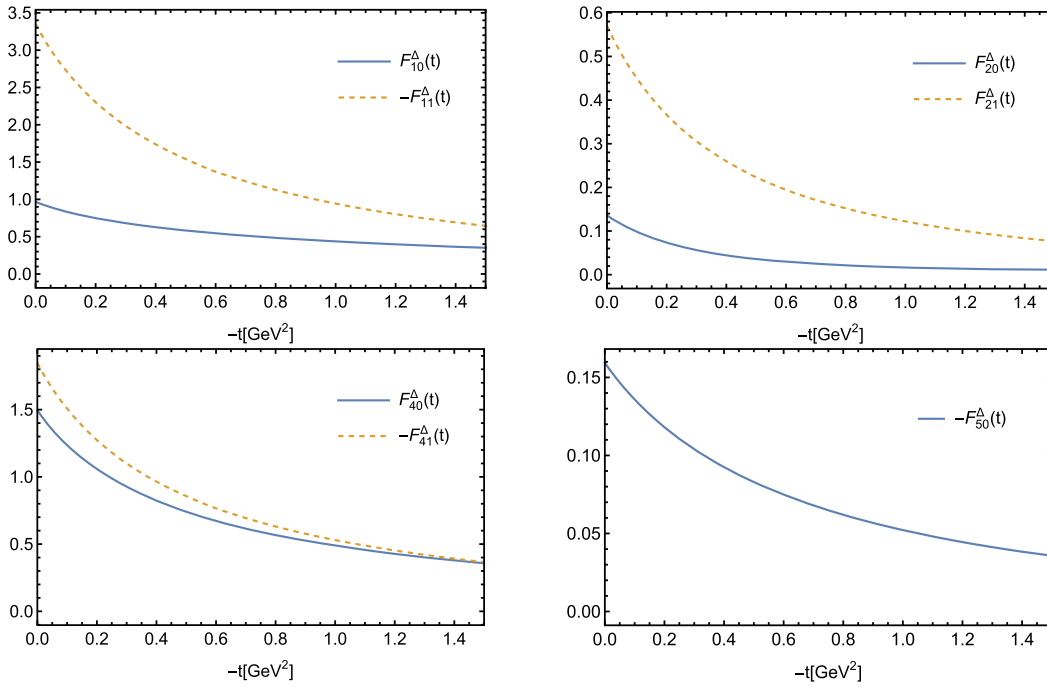
FIG. 6.   Calculated GFFs of $F^T_{10,11,20,21,40,41,50}$ as functions of $-t$ for Δ.
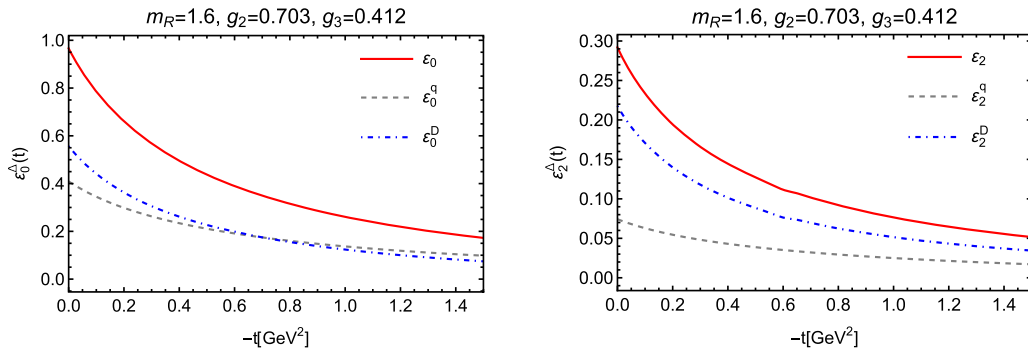
FIG. 7.   The calculated energy-monopole form factor of the Δ as a function of $-t$ (left panel) and the energy quadrupole (right panel). The dashed, dashed-dotted, and solid curves stand for the contributions from quark, diquark, and their sum.
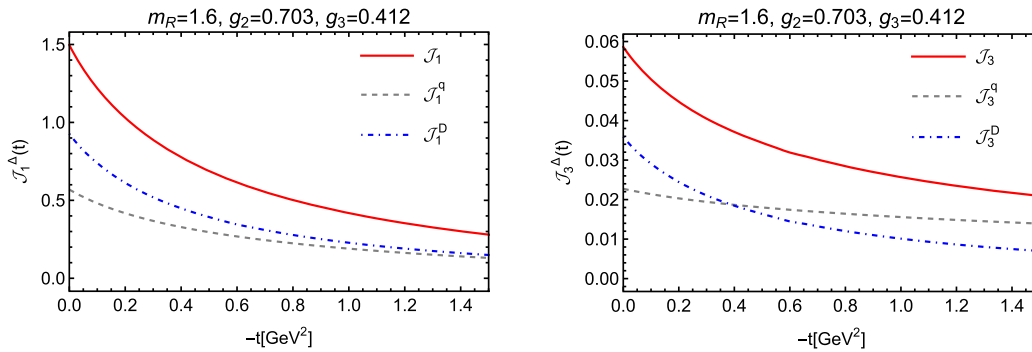
FIG. 8.   The angular-momentum form factor of the Δ as a function of $-t$ (left panel), and the octupole-angular momentum form factor (the right panel). The solid, dashed, and dashed-dotted curves represent the total result, and the contributions of quark and diquark, respectively.
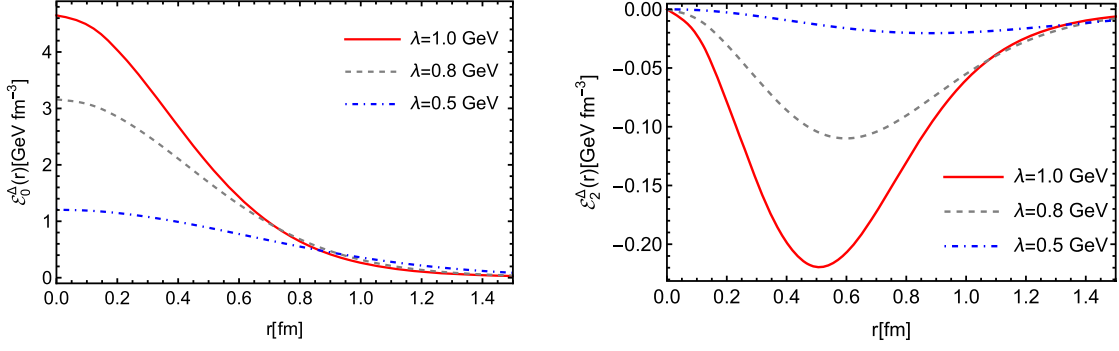
FIG. 9. The calculated energy-monopole density of Δ as a function of r (left panel) and energy-quadrupole density (right panel). The Gaussian wave packet $e^{-\frac{t}{\lambda^2}}$ has been included with $\lambda = 1$ GeV (solid curve), $\lambda = 0.8$ GeV (dashed curve), and $\lambda = 0.5$ GeV (dashed-dotted curve).

the result of Ref. [53]. These two form factors and $F_{50}^{\Delta}(t)$ relate to the matrix element of T^{ij} [see Eqs. (5) and (8)]. In the classical mechanics of continuum media, the energy-momentum tensor T^{ij} is interpreted as the pressure and shear force in the continuum media approximation. When one discusses quantum field theory problems, in analogy to the classical mechanics for the continuum media, one expects that the matrix element of T^{ij} gives information of the pressure and shear force of the system. According to the relations between T^{ij} and the D term and, furthermore, by

considering the stability of the system, which implies that the corresponding pressure is positive, one concludes that the D term should be negative as $D_0(t = 0) < 0$ from Eq. (14).

Actually, the negativity of the D term has been discussed extensively. Ref. [60] explicitly proves this issue by discussing a scalar hadron, which is assumed to be composed by two scalar fields. Under this circumstance, $D_0(t)$ is expressed as (assuming the two constituents have same mass m, and the hadron has the mass $M = 2m - B$ with B being the binding energy)

$$\langle p'|T^{12}|p\rangle = \frac{1}{2}q^1 q^2 D_0(t) = 2ig^2 \int \frac{d^4k}{(2\pi)^4} \frac{(k^1 - q^1/2)(k^2 + q^2/2) + (k^2 - q^1/2)(k^1 + q^1/2)}{[(k-P)^2 - m^2][(k+q/2)^2 - m^2][(k-q/2)^2 - m^2]}, \qquad (38)$$

and

$$D_0(0) \overset{B \to 0}{=} -\frac{11}{3} + \frac{32}{3\pi}\sqrt{\frac{B}{2M}} - \mathcal{O}\left(\frac{B}{2M}\right), \qquad (39)$$

where the numerator in the first equation is due to the energy-momentum tensor of a scalar particle. It is clearly seen that the expected D-term results from the sum of the numerators of $k^1 k^2$ and $-q^1 q^2/4$. The first one has a positive contribution while the second attributes a dominant negative value. Therefore, their sum gives $-\frac{11}{3}$. When the binding B increases, the calculated $D_0(0)$ reduces.

Inspired by the above analysis, the treatment of the hadrons, like Δ in this approach, is carried out in the

following. Instead of considering two spinless constituents, we take the fermion propagators, which is more realistic. Consequently, the matrix element of $T^{\mu\nu}$ shown in Eqs. (32) and (33) are much more complicated and much different from the one of scalar hadrons with two scalar constituents.

In order to address the calculated matrix element of $T^{\mu\nu}$ more transparently and analytically, we simplify the Eq. (32) by replacing the $\Gamma^{\alpha\beta}$ and $\Gamma^{\alpha'\beta'}$ with $c_1 g^{\alpha\beta}$ and $c_1 g^{\alpha'\beta'}$, and by replacing the scalar function $[(l-P)^2 - m_R^2 + i\epsilon]^2[(l - \frac{q}{2})^2 - m_R^2 + i\epsilon][(l + \frac{q}{2})^2 - m_R^2 + i\epsilon]$ in the denominator by $(l - P)^2 - m_R^2 + i\epsilon$. And we omit $i\epsilon$ in writing for brevity. We expect these replacements do not change the qualitative properties of our loop integrals. Then,

$$\langle p', \lambda'|\hat{T}_q^{\mu\nu}(0)|p, \lambda\rangle = -\bar{u}_{\alpha'}(p', \lambda') \frac{-i\tilde{C}^2}{2} \int \frac{d^4l}{(2\pi)^4} \frac{g^{\alpha'\beta'}(\slashed{l} + \frac{\slashed{q}}{2} + m_q)g_{\beta'\beta}(\gamma^\mu l^\nu + \gamma^\nu l^\mu)(\slashed{l} - \frac{\slashed{q}}{2} + m_q)g^{\beta\alpha}}{[(l-P)^2 - m_D^2][(l - \frac{q}{2})^2 - m_q^2][(l + \frac{q}{2})^2 - m_q^2][(l-P)^2 - m_R^2]} u_\alpha(p, \lambda)$$

$$= -\bar{u}_\alpha(p', \lambda')(-i\tilde{C}^2) \int \frac{d^4l}{(2\pi)^4} \frac{\tilde{T}^{\mu\nu}}{[(l-P)^2 - m_D^2][(l - \frac{q}{2})^2 - m_q^2][(l + \frac{q}{2})^2 - m_q^2][(l-P)^2 - m_R^2]} u^\alpha(p, \lambda).$$

$$(40)$$

According to Eq. (4) and Appendix C where $\tilde{T}^{\mu\nu} = \sum_i \tilde{T}_i^{\mu\nu}$, the $\frac{P^\mu P^\nu}{M}$ and $\frac{q^\mu q^\nu}{4M}$ terms can be yielded by the standard Feynman parametrizations (see Appendix B),

$$
\begin{aligned}
&- i\tilde{C}^2 \int \frac{d^4l}{(2\pi)^4} \frac{1}{2M} \frac{[-l^2(l^\nu P^\mu + l^\mu P^\nu) + 4l^\mu l^\nu(l \cdot P) + 4m_q M l^\mu l^\nu + m_q^2(l^\nu P^\mu + l^\mu P^\nu)]}{[(l-P)^2 - m_D^2][(l-\frac{q}{2})^2 - m_q^2][(l+\frac{q}{2})^2 - m_q^2][(l-P)^2 - m_R^2]} \\
&= \frac{P^\mu P^\nu}{M} \tilde{\mathcal{A}} \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \int_0^{1-x_1-x_2} dx_3 \frac{M^2(x_1 + x_2)^3 + 2m_q M(x_1 + x_2)^2 + (\mathcal{M} + m_q^2)(x_1 + x_2)}{\mathcal{M}^2} \\
&\quad + \text{other Lorentz structures} \\
&= \frac{P^\mu P^\nu}{M} F_{10}^T(0) + \text{other Lorentz structures},
\end{aligned}
\tag{41}
$$

and

$$
\begin{aligned}
&- i\tilde{C}^2 \int \frac{d^4l}{(2\pi)^4} \frac{2}{M} \frac{l^\mu l^\nu(l \cdot P) + m_q M l^\mu l^\nu}{[(l-P)^2 - m_D^2][(l-\frac{q}{2})^2 - m_q^2][(l+\frac{q}{2})^2 - m_q^2][(l-P)^2 - m_R^2]} \\
&= \frac{q^\mu q^\nu}{4M} \tilde{\mathcal{A}} \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \int_0^{1-x_1-x_2} dx_3 \frac{2[M^2(x_1 + x_2) + m_q M](2x_3 + x_1 + x_2 - 1)^2}{\mathcal{M}^2} \\
&\quad + \text{other Lorentz structures} \\
&= \frac{q^\mu q^\nu}{4M} F_{20}^T(0) + \text{other Lorentz structures},
\end{aligned}
\tag{42}
$$

where

$$
\begin{aligned}
\tilde{\mathcal{A}} &= \frac{\tilde{C}^2}{(4\pi)^2} > 0, \\
\mathcal{M} &= (x_1 + x_2)^2 M^2 - (x_1 + x_2)M^2 + x_1 m_D^2 + (1 - x_1 - x_2)m_q^2 + x_2 m_R^2 > 0.
\end{aligned}
\tag{43}
$$

Moreover, $F_{50}^T(0) = 0$ in Eq. (40) for this simplified model. Then our $\varepsilon_0(0) = F_{10}^T(0)$ and $D_0(0) = F_{20}^T(0)$, and both of the Feynman integrals are positive obviously. Therefore, we conclude that the sign of the D term in our calculation is the same as for ε_0. The $D_0(0) > 0$ can also be obtained from Eq. (8) and Fig. 6 in our complete model, and the von Laue condition [101] $\int_0^\infty r^2 p(r) dr = 0$ is still satisfied.

We believe that our above conclusion is because of the Fermion properties and the realistic consideration of the Δ isobar. This sign problem also occurs in Ref. [102] when the hydrogen atom is considered. The controversial sign problem of the D term is still open. More realistic calculations for hadrons like nucleons are necessary to check if this problem indeed exists. It has been argued that the analogy to the pressure in classical mechanics of T^{ij} and the constraint of negativity of D term may not be necessary. Instead, the momentum current might be suitable to interpret the matrix element of T^{ij} of a quantum system as argued in Ref. [102].

V. SUMMARY AND CONCLUSIONS

In this work we calculate the electromagnetic and gravitational form factors of the spin-3/2 Δ with the help of relativistic covariant quark-diquark approach. The internal quark structures of Δ as well as of the axial-vector diquark are explicitly considered. In order to simulate the bound-state properties of Δ and diquark, we simply employ an ansatz for the vertex scalar function, and the coupling of Δ to the quark and diquark, given by Ref. [62]. Although the vertex scalar function in Eq. (19) breaks the gauge invariance, it turns out the deviation in the normalization is small. We take the lattice QCD calculations for EMFFs as the constraints to fit our model parameters.

It should be stressed that we simplify the three-body problem into a two-body problem by considering two quarks as a diquark. To get more accurate results, we calculate the GFFs of the diquark as a two-body problem instead of just taking it as a point particle. In Sec. III, we find that our results of EMFFs and electromagnetic moments are reasonable within acceptable region of t. For the EMFFs of Δ, the ratio of the contributions from the diquark and the quark is close to 2 when $t = 0$. That is because they are mainly determined by the number of charges, and the charge ratio of the diquark to the quark is 2. Similarly, because the mass ratio of the diquark to the quark is close to 2, the contribution to GFFs from the diquark is also close to 2 times that of the corresponding

one of the quark when $t = 0$. Finally, we also reasonably reproduce the mass and spin distributions of Δ.

However, we point out that there is a sign difference of our calculated D term from the argument of its negativity. This is because of our realistic consideration of the quark and diquark structures of Δ, as we have shown in the detailed analyses of the matrix elements of T^{ij} and of the Feynman loop integrals. It is argued that the D term must be negative if a system satisfies the local stability criterion, otherwise if this was not the case, the system would collapse [46,60]. This argument originated from the interpretation of the stress tensor T^{ij} as the momentum flux and the normal force is expected to be outward. Our obtained positive D term illustrates that its negativity might not be necessary. Instead, the momentum current interpretation for the matrix element of T^{ij} still might be suitable. More realistic studies for hadrons are needed to clarify this question. Finally, the present relativistic covariant quark-diquark approach will be employed for further studies of the GPDs of the Δ resonance and of the $N - \Delta$ transition form factors.

APPENDIX A: SOME USEFUL ON SHELL IDENTITIES

To compute the matrix element of EMT current and electromagnetic current, some identities explicitly given in Ref. [103] are employed. These identities are satisfied for the Rarita-Schwinger spinors. In terms of the variables $P = (p' + p)/2$ and $q = p' - p$,

$$P^\alpha \doteq \frac{q^\alpha}{2}, \qquad P^{\alpha'} \doteq -\frac{q^{\alpha'}}{2}, \qquad (A1)$$

where $\doteq$ means on shell equality, and we reserve the indices α_i and α'_i. There are some on shell relations derived from the Gordon identity and the Schouten identity,

$$\bar{u}(p',\lambda')\gamma^\mu u(p,\lambda) = \bar{u}(p',\lambda')\left[\frac{P^\mu}{M} + \frac{i\sigma^{\mu\nu}q_\nu}{2M}\right]u(p,\lambda), \quad (A2)$$

$$i\epsilon^{\mu\nu\rho\sigma}g^{\tau\lambda} + i\epsilon^{\nu\rho\sigma\tau}g^{\mu\lambda} + i\epsilon^{\rho\sigma\tau\mu}g^{\nu\lambda} + i\epsilon^{\sigma\tau\mu\nu}g^{\rho\lambda} + i\epsilon^{\tau\mu\nu\rho}g^{\sigma\lambda} = 0. \quad (A3)$$

We can rewrite the Gordon identity using on shell equality

$$\gamma^\mu \doteq \frac{P^\mu}{M} + \frac{i\sigma^{\mu\nu}q_\nu}{2M}. \qquad (A4)$$

The other on shell relations used in our work read [103]

$$1 \doteq \frac{\slashed{P}}{M}, \qquad 0 \doteq \slashed{q}, \qquad (A5a)$$

$$\gamma_5 \doteq \frac{\slashed{q}\gamma_5}{2M}, \qquad 0 \doteq \slashed{P}\gamma_5, \qquad (A5b)$$

$$\gamma^\mu \doteq \frac{P^\mu}{M} + \frac{i\sigma^{\mu q}}{2M}, \qquad 0 \doteq \frac{q^\mu}{2} + i\sigma^{\mu P}, \qquad (A5c)$$

$$\gamma^\mu\gamma_5 \doteq \frac{q^\mu\gamma_5}{2M} + \frac{i\sigma^{\mu P}}{M}, \qquad 0 \doteq P^\mu\gamma_5 + \frac{i\sigma^{\mu q}\gamma_5}{2}, \qquad (A5d)$$

$$i\sigma^{\mu\nu} \doteq -\frac{q^{[\mu}\gamma^{\nu]}}{2M} + \frac{i\epsilon^{\mu\nu P\lambda}\gamma_\lambda\gamma_5}{M}, \qquad 0 \doteq -P^{[\mu}\gamma^{\nu]} + \frac{i\epsilon^{\mu\nu q\lambda}\gamma_\lambda\gamma_5}{2}, \qquad (A5e)$$

$$i\sigma^{\mu\nu}\gamma_5 \doteq -\frac{P^{[\mu}\gamma^{\nu]}\gamma_5}{M} + \frac{i\epsilon^{\mu\nu q\lambda}\gamma_\lambda}{2M}, \qquad 0 \doteq -\frac{q^{[\mu}\gamma^{\nu]}\gamma_5}{2} + i\epsilon^{\mu\nu P\lambda}\gamma_\lambda, \qquad (A5f)$$

where $\sigma^{\mu P} \equiv \sigma^{\mu\nu}P_\nu$, $\epsilon^{\mu\nu P\lambda} \equiv \epsilon^{\mu\nu\rho\lambda}P_\rho$.

The Rarita-Schwinger spinors satisfy this relation,

$$\gamma^{\alpha_i}u_{\alpha_1\ldots\alpha_n}(p,\lambda) = 0, \quad \bar{u}_{\alpha'_1\ldots\alpha'_n}(p',\lambda')\gamma^{\alpha'_i} = 0, \quad i \in \{1,\ldots,n\}. \quad (A6)$$

Combining Eqs. (A1), (A4), and (A6), we can get these on shell identities,

$$i\sigma^{\alpha'\mu} \doteq g^{\alpha'\mu}, \qquad i\sigma^{\nu\alpha} \doteq g^{\nu\alpha}. \qquad (A7)$$

Some important on shell identities we used are derived from the product of three and four Dirac matrices,

$$\gamma^\rho\gamma^\mu\gamma^\sigma = g^{\rho\mu}\gamma^\sigma - g^{\rho\sigma}\gamma^\mu + g^{\mu\sigma}\gamma^\rho - i\epsilon^{\rho\mu\sigma\lambda}\gamma_\lambda\gamma_5, \qquad (A8a)$$

$$\gamma^\rho\gamma^\mu\gamma^\sigma\gamma_5 = g^{\rho\mu}\gamma^\sigma\gamma_5 - g^{\rho\sigma}\gamma^\mu\gamma_5 + g^{\mu\sigma}\gamma^\rho\gamma_5 - i\epsilon^{\rho\mu\sigma\lambda}\gamma_\lambda, \quad (A8b)$$

$$\gamma^\rho \gamma^\mu \gamma^\nu \gamma^\sigma = g^{\rho\mu} g^{\nu\sigma} - g^{\rho\nu} g^{\mu\sigma} + g^{\rho\sigma} g^{\mu\nu} + i\epsilon^{\rho\mu\nu\sigma} \gamma_5 - g^{\rho\mu} i\sigma^{\nu\sigma}$$
$$+ g^{\rho\nu} i\sigma^{\mu\sigma} - g^{\rho\sigma} i\sigma^{\mu\nu} - g^{\nu\sigma} i\sigma^{\rho\mu} + g^{\mu\sigma} i\sigma^{\rho\nu}$$
$$- g^{\mu\nu} i\sigma^{\rho\sigma}. \tag{A9}$$

The nontrivial relation obtained using Eqs. (A4), (A8), and (A9) [58],

$$q^{\alpha'} g^{\mu\alpha} - q^\alpha g^{\mu\alpha'} \doteq 2M\left(1 - \frac{q^2}{4M^2}\right) g^{\alpha'\alpha} \gamma^\mu - 2g^{\alpha'\alpha} P^\mu$$
$$+ \frac{1}{M} q^{\alpha'} q^\alpha \gamma^\mu. \tag{A10}$$

Because of Eqs. (A4) and (A1) this identity can be derived [57],

$$\frac{q^2}{2} q^{[\alpha'} g^{\alpha][\mu} P^{\nu]} \doteq -q^{\alpha'} q^\alpha P^{[\mu} i\sigma^{\nu]q} + P^2 q^{[\alpha'} g^{\alpha][\mu} i\sigma^{\nu]q}. \tag{A11}$$

Another nontrivial relation was derived from Ref. [57],

$$q^2 g^{\mu\nu} g^{\alpha'\alpha} - 2g^{\mu\nu} q^{\alpha'} q^\alpha - g^{\alpha'\alpha} P^{\{\mu} i\sigma^{\nu\}q}$$
$$+ q^{[\alpha'} g^{\alpha]\{\mu} P^{\nu\}} - g^{\alpha'\alpha} q^\mu q^\nu + \frac{1}{2} q^{\{\alpha'} g^{\alpha\}\{\mu} q^{\nu\}}$$
$$\doteq \frac{1}{2} q^{[\alpha'} g^{\alpha]\{\mu} i\sigma^{\nu\}q} - \frac{1}{2} q^{\{\alpha'} g^{\alpha\}\{\mu} q^{\nu\}} + q^2 g^{\alpha'\{\mu} g^{\nu\}\alpha}. \tag{A12}$$

And combining Eqs. (A11) and (A12), we can obtain

$$q^{[\alpha'} g^{\alpha]\mu} i\sigma^{\nu q} \doteq q^2 g^{\mu\nu} g^{\alpha'\alpha} - 2g^{\mu\nu} q^{\alpha'} q^\alpha - g^{\alpha'\alpha} P^{\{\mu} i\sigma^{\nu\}q}$$
$$+ q^{[\alpha'} g^{\alpha]\{\mu} P^{\nu\}} - g^{\alpha'\alpha} q^\mu q^\nu + q^{\{\alpha'} g^{\alpha\}\{\mu} q^{\nu\}}$$
$$- q^2 g^{\alpha'\{\mu} g^{\nu\}\alpha} + \frac{q^2}{4P^2} q^{[\alpha'} g^{\alpha][\mu} P^{\nu]}$$
$$+ \frac{1}{2P^2} q^{\alpha'} q^\alpha P^{[\mu} i\sigma^{\nu]q}, \tag{A13}$$

$$q^{[\alpha'} g^{\alpha]\nu} i\sigma^{\mu q} \doteq q^2 g^{\mu\nu} g^{\alpha'\alpha} - 2g^{\mu\nu} q^{\alpha'} q^\alpha - g^{\alpha'\alpha} P^{\{\mu} i\sigma^{\nu\}q}$$
$$+ q^{[\alpha'} g^{\alpha]\{\mu} P^{\nu\}} - g^{\alpha'\alpha} q^\mu q^\nu + q^{\{\alpha'} g^{\alpha\}\{\mu} q^{\nu\}}$$
$$- q^2 g^{\alpha'\{\mu} g^{\nu\}\alpha} - \frac{q^2}{4P^2} q^{[\alpha'} g^{\alpha][\mu} P^{\nu]}$$
$$- \frac{1}{2P^2} q^{\alpha'} q^\alpha P^{[\mu} i\sigma^{\nu]q}. \tag{A14}$$

There are still some more identities, see Ref. [57].

APPENDIX B: FEYNMAN PARAMETRIZATION AND LOOP INTEGRALS

In our calculation, we use the Feynman parametrization. Some integrals are listed as follows:

$$-i \int \frac{d^4l}{(2\pi)^4} \frac{1}{\mathfrak{D}} = A_{00}, \tag{B1a}$$

$$-i \int \frac{d^4l}{(2\pi)^4} \frac{l^\mu}{\mathfrak{D}} = A_{11} P^\mu, \tag{B1b}$$

$$-i \int \frac{d^4l}{(2\pi)^4} \frac{l^\mu l^\nu}{\mathfrak{D}} = A_{21} g^{\mu\nu} + A_{22} P^\mu P^\nu + A_{23} q^\mu q^\nu, \tag{B1c}$$

$$-i \int \frac{d^4l}{(2\pi)^4} \frac{l^\mu l^\nu l^\gamma}{\mathfrak{D}} = A_{31}(P^\gamma g^{\mu\nu} + P^\mu g^{\gamma\nu} + P^\nu g^{\gamma\mu})$$
$$+ A_{32}(q^\mu q^\nu P^\gamma + q^\gamma q^\nu P^\mu + q^\gamma q^\mu P^\nu)$$
$$+ A_{33} P^\gamma P^\mu P^\nu, \tag{B1d}$$

$$-i \int \frac{d^4l}{(2\pi)^4} \frac{l^\mu l^\nu l^\gamma l^\rho}{\mathfrak{D}} = \sum_{\substack{(i,j,m,n) \\ \in(\mu,\nu,\gamma,\rho)}} \left[\frac{1}{8} A_{41} g^{ij} g^{mn} + \frac{1}{8} A_{42} P^i P^j g^{mn} \right.$$
$$+ \frac{1}{8} A_{43} q^i q^j g^{mn} + \frac{1}{24} A_{44} P^i P^j P^m P^n$$
$$\left. + \frac{1}{24} A_{45} q^i q^j q^m q^n + \frac{1}{8} A_{46} q^i q^j P^m P^n \right], \tag{B1e}$$

where $A_{n_1 n_2}$ stand for the structural integrals. The symmetric properties of the denominator $\mathfrak{D}$ with respect to q, as shown in Eq. (21), is considered.

APPENDIX C: CALCULATION DETAILS ABOUT D TERM

According to Eq. (40)

$$\langle p', \lambda' | \hat{T}_q^{\mu\nu}(0) | p, \lambda \rangle = -\bar{u}_{\alpha'}(p', \lambda') \frac{-i\tilde{C}^2}{2} \int \frac{d^4l}{(2\pi)^4} \frac{g^{\alpha'\beta'}(\slashed{l} + \frac{\slashed{q}}{2} + m_q) g_{\beta\beta'} \gamma^{\{\mu} l^{\nu\}}(\slashed{l} - \frac{\slashed{q}}{2} + m_q) g^{\alpha\beta}}{[(l-P)^2 - m_D^2][(l-\frac{q}{2})^2 - m_q^2][(l+\frac{q}{2})^2 - m_q^2][(l-P)^2 - m_R^2]} u_\alpha(p, \lambda)$$
$$= -\bar{u}_\alpha(p', \lambda')(-i\tilde{C}^2) \int \frac{d^4l}{(2\pi)^4} \frac{\tilde{T}^{\mu\nu}}{[(l-P)^2 - m_D^2][(l-\frac{q}{2})^2 - m_q^2][(l+\frac{q}{2})^2 - m_q^2][(l-P)^2 - m_R^2]} u^\alpha(p, \lambda), \quad \text{(C1)}$$

where

$$\tilde{T}^{\mu\nu} = \frac{1}{2}\left(\slashed{l} + \frac{\slashed{q}}{2} + m_q\right)\gamma^{\{\mu}l^{\nu\}}\left(\slashed{l} - \frac{\slashed{q}}{2} + m_q\right). \tag{C2}$$

Here $\tilde{T}^{\mu\nu}$ contains the information of fermions and can be divided into nine parts $\tilde{T}^{\mu\nu} = \sum_{i=1}^{9}\tilde{T}_i^{\mu\nu}$. Using the on shell identities given in Appendix A, we get

$$\tilde{T}_1^{\mu\nu} = \frac{1}{2}\slashed{l}\gamma^{\{\mu}l^{\nu\}}\slashed{l} \doteq \frac{il^{\mu}l^{\nu}\sigma^{lq}}{M} - \frac{il^2 l^{\{\mu}\sigma^{\nu\}q}}{4M} - \frac{l^2 l^{\{\mu}P^{\nu\}}}{2M} + \frac{2l^{\mu}l^{\nu}(l\cdot P)}{M},$$

$$\tilde{T}_2^{\mu\nu} = \frac{1}{4}\slashed{q}\gamma^{\{\mu}l^{\nu\}}\slashed{l} \doteq \frac{iq^{\{\mu}l^{\nu\}}\sigma^{lq}}{8M} - \frac{il^{\{\mu}\sigma^{\nu\}q}(l\cdot q)}{8M} - \frac{il^{\{\mu}P^{\nu\}}\sigma^{lq}}{4M} + \frac{il^{\{\mu}\sigma^{\nu\}q}(l\cdot P)}{4M} \tag{C3a}$$

$$+ \frac{q^{\{\mu}l^{\nu\}}(l\cdot P)}{4M} - \frac{l^{\{\mu}P^{\nu\}}(l\cdot q)}{4M}, \tag{C3b}$$

$$\tilde{T}_3^{\mu\nu} = \frac{1}{2}m_q\gamma^{\{\mu}l^{\nu\}}\slashed{l} \doteq -\frac{1}{2}im_q l^{\{\mu}\sigma^{\nu\}l} + m_q l^{\mu}l^{\nu}, \tag{C3c}$$

$$\tilde{T}_4^{\mu\nu} = -\frac{1}{4}\slashed{l}\gamma^{\{\mu}l^{\nu\}}\slashed{q} \doteq -\frac{iq^{\{\mu}l^{\nu\}}\sigma^{lq}}{8M} + \frac{il^{\{\mu}\sigma^{\nu\}q}(l\cdot q)}{8M} - \frac{il^{\{\mu}P^{\nu\}}\sigma^{lq}}{4M} + \frac{il^{\{\mu}\sigma^{\nu\}q}(l\cdot P)}{4M} + \frac{l^{\{\mu}P^{\nu\}}(l\cdot q)}{4M} - \frac{q^{\{\mu}l^{\nu\}}(l\cdot P)}{4M}, \tag{C3d}$$

$$\tilde{T}_5^{\mu\nu} = -\frac{1}{8}\slashed{q}\gamma^{\{\mu}l^{\nu\}}\slashed{q} \doteq \frac{iq^2 l^{\{\mu}\sigma^{\nu\}q}}{16M} + \frac{q^2 l^{\{\mu}P^{\nu\}}}{8M}, \tag{C3e}$$

$$\tilde{T}_6^{\mu\nu} = -\frac{1}{4}m_q\gamma^{\{\mu}l^{\nu\}}\slashed{q} \doteq \frac{1}{4}im_q l^{\{\mu}\sigma^{\nu\}q} - \frac{1}{4}m_q q^{\{\mu}l^{\nu\}}, \tag{C3f}$$

$$\tilde{T}_7^{\mu\nu} = \frac{1}{2}m_q\slashed{l}\gamma^{\{\mu}l^{\nu\}} \doteq \frac{1}{2}im_q l^{\{\mu}\sigma^{\nu\}l} + m_q l^{\mu}l^{\nu}, \tag{C3g}$$

$$\tilde{T}_8^{\mu\nu} = \frac{1}{4}m_q\slashed{q}\gamma^{\{\mu}l^{\nu\}} \doteq \frac{1}{4}im_q l^{\{\mu}\sigma^{\nu\}q} + \frac{1}{4}m_q q^{\{\mu}l^{\nu\}}, \tag{C3h}$$

$$\tilde{T}_9^{\mu\nu} = \frac{1}{2}m_q^2\gamma^{\{\mu}l^{\nu\}} \doteq \frac{im_q^2 l^{\{\mu}\sigma^{\nu\}q}}{4M} + \frac{m_q^2 l^{\{\mu}P^{\nu\}}}{2M}. \tag{C3i}$$

According to Appendix B, we see that the $\frac{P^{\mu}P^{\nu}}{M}$ term comes from the loop integrals of $\tilde{T}_1^{\mu\nu}$, $\tilde{T}_3^{\mu\nu}$, $\tilde{T}_7^{\mu\nu}$, and $\tilde{T}_9^{\mu\nu}$, and the $\frac{q^{\mu}q^{\nu}}{4M}$ term, which contributes to D term, results from the ones of $\tilde{T}_1^{\mu\nu}$, $\tilde{T}_3^{\mu\nu}$, and $\tilde{T}_7^{\mu\nu}$.

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