Positronium decays with a dark Z and fermionic dark matter

Dong-Won Jung [*]
Department of Physics, Korea University, Seoul 02841, Korea
Department of Physics, Yonsei University, Seoul 03722, Korea

Kang Young Lee [†]
Department of Physics Education & RINS Gyeongsang National University, Jinju 52828, Korea

Chaehyun Yu [‡]
Department of Physics, Korea University, Seoul 02841, Korea
RINS, Gyeongsang National University, Jinju 52828, Korea

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We investigate the invisible decay of positronium to probe the fermionic light dark matter mediated by the dark Z boson. The invisible decay rate of positronium through weak interaction in the standard model is too small to be detected in the experiment. We show that it can be enhanced to be observed in the future if the dark matter is lighter than the electron in the dark Z model. We also compute the relic abundance of such light dark matter and discuss the big bang nucleosynthesis constraint with an alternative thermal history scenario.

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I. INTRODUCTION

Positronium (Ps) is a leptonic atom which consists of an electron-positron bound state and the lightest bound state in the standard model (SM). The wave function of the Ps in the leading order is obtained by solving the Schrödinger equation of the hydrogen atom with the reduced mass equal to the half of the electron mass. The lowest states of Ps are the spin-singlet ($^1S_0$) and the spin-triplet states ($^3S_1$), called parapositronium (p-Ps) and orthopositronium (o-Ps), respectively. The energy spectra and lifetimes of Ps states can be calculated in QED with high accuracy since the theoretical study of Ps is free from hadronic uncertainty. Combined with precise measurements, the study of Ps allows us to test our understanding of bound-state structure of QED [1,2].

The dominant decay channel of the p-Ps is the two-photon decay with lifetime $\tau = 7989.6060(2)^{-1}$ μs [3], while that of the o-Ps is the three-photon decay with lifetime $\tau = (7.0380\text{–}7.0417)^{-1}$ μs [4]. The triplet state o-Ps may decay into neutrino pairs via weak interaction to leave invisible final state in the SM. However the SM invisible decay rate of the o-Ps is extremely small such that the branching ratios $\sim 6.2 \times 10^{-18}$ (for ν_e) and $\sim 9.5 \times 10^{-21}$ (for $\nu_{\mu,\tau}$) [5]. The p-Ps may also decay into neutrino pairs via weak interactions, but its decay rates are much smaller than those of the o-Ps because the weak decay rates into neutrinos for the p-Ps are proportional to the squares of the neutrino masses. Thus, the Ps invisible decays can be a good testing laboratory of the new physics beyond the SM in both o-Ps and p-Ps decays. Actually, sizable invisible decays of o-Ps are predicted in many new physics models, e.g., millicharged particles, paraphotons etc. [6–8].

Experimental searches for invisible decays of Ps have been performed but no signals have been obseved so far. The most stringent upper bound on the o-Ps invisible decay branching ratio is set to be [9]

$$\mathrm{Br}(\text{o-Ps} \to \text{invisible}) < 4.2 \times 10^{-7} \quad (1)$$

at 90% C.L. Recently Vigo *et al.* [10] also set the model-independent upper limit, $\mathrm{Br}(\mathrm{Ps} \to \text{invisible}) < 1.7 \times 10^{-5}$ at 90% C.L. in an alternative experiment. (Note that decays of o-Ps is more manageable in experiments due to its longer lifetime.)

One of the most important challenges in particle physics at present is resolving the nature and origin of nonbaryonic dark matter (DM). Many theoretical models have predicted the weakly interacting massive particles (WIMP) with electroweak scale mass as DM candidates. The WIMPs

[*]dongwonj@korea.ac.kr
[†]kylee.phys@gnu.ac.kr
[‡]chyu@korea.ac.kr

are assumed to be produced by the thermal freeze-out mechanism in the early Universe, and can be fermions, scalars, or vector bosons depending on the model. No signals of WIMP have been observed in high-energy colliders and direct detection experiments; however, much interest is devoted to the possibility of light dark matter (LDM) in the keV–MeV mass range [6].

In this paper, we consider a fermionic LDM model which is suggested in Ref. [11] where the hidden sector is mediated by an additional SU(2) scalar doublet. When we consider singlet fermions as DM candidates, singlet scalars are usually introduced together as a mediator between hidden sector and the SM, and an additional mass scale is also introduced by the vacuum expectation value (VEV) of the singlet scalar [12–16]. However, if we take the scalar doublet as a mediator, neither the singlet scalar nor the new mass scale is required. Instead, the $\mathrm{U}(1)_X$ symmetry is required for the fermionic-DM candidate to couple to the mediator scalar doublet. Thus, the hidden sector in this model is QED-like, which consists of a SM singlet fermion and a hidden $\mathrm{U}(1)_X$ gauge field. Since the mediator scalar is the SU(2) doublet and also carries the hidden $\mathrm{U}(1)_X$ charge, the $\mathrm{U}(1)_X$ symmetry is broken by the electroweak symmetry breaking (EWSB) and the corresponding gauge boson gets its own mass by the electroweak VEV. The new massive gauge boson is mixed with the Z boson and is called the dark Z boson. This model satisfies strong electroweak constraints from low-energy experiments and high-energy collider phenomenology on neutral current (NC) interactions.

It turns out that this model favors rather light dark-Z bosons and fermionic DM. If the DM candidate is lighter than the electron, the o-Ps can annihilate into the DM pair through the dark Z boson and the final state is invisible in this model. Being light, the dark Z boson enables predictions of invisible decay rates of o-Ps into the DM pair to be much enhanced compared to the weak invisible decay rate in the SM, which is a clear signal of the new physics.

On the other hand, LDM with mass less than MeV suffers from tension with cosmological observables when it is in thermal equilibrium with the bath of the SM particles in the early Universe [17–19]. This is because the temperature where the BBN started is affected by extra relativistic degrees of freedom and the predictions of the abundance of light elements would be altered. Recently, Berlin and Blinov reported that sub-MeV LDM is allowed when the equilibrium of the light state with the SM occurs later than the neutrino decoupling [20,21]. We take this scenario to accept that our fermionic LDM candidate is lighter than the electron here.

In this paper we investigate the exotic decays of Ps including invisible decays and single-photon decays when the DM candidate is lighter than the electron in the singlet fermionic DM model with hidden $\mathrm{U}(1)_X$ gauge group and an additional scalar doublet mediator. The outline of this paper is as follows: We briefly describe the model with electroweak constraints in Sec. II. In Sec. III we present the predictions of the positronium decays in this model. The dark matter phenomenology is elaborated on in relation to the positronium decays in Sec. IV. We finally conclude in Sec. V.

II. DARK Z PHENOMENOLOGY

The hidden sector of the model consists of a SM gauge singlet Dirac fermion ψ_X as a DM candidate and a gauge field for a new $\mathrm{U}(1)_X$ gauge symmetry. We assume no kinetic mixing between the hidden $\mathrm{U}(1)_X$, the SM $\mathrm{U}(1)_Y$ and the gauge charge of ψ_X to be $(1,1,0,X)$ based on the $\mathrm{SU}(3)_c\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y\times\mathrm{U}(1)_X$ gauge group. The SM fields do not carry the $\mathrm{U}(1)_X$ gauge charge and do not couple to the hidden sector fermion directly.

We introduce an additional SU(2) scalar doublet H_1 as a mediator field between the hidden sector and the SM sector; the content of Higgs fields is the two Higgs doublet model (2HDM). There are three free parameters; $\mathrm{U}(1)_X$, gauge coupling g_X, and the $\mathrm{U}(1)_X$ charges of H_1 and ψ_X (but they just appear in the form of $g_X X$). Thus, we have freedom to fix only one of the three parameters. We take the $\mathrm{U}(1)_X$ charge of H_1 to be $1/2$ for convenience. Then we let the charge assignments of H_1 and the SM-like Higgs doublet H_2 be $(1,2,\frac{1}{2},\frac{1}{2})$ for H_1, and $(1,2,\frac{1}{2},0)$ for H_2, respectively. Due to the $\mathrm{U}(1)_X$ charge, H_1 does not couple to the SM fermions and the H_2 couplings to the SM fermions are the same as the SM-Yukawa interactions as in the Higgs sector in the 2HDM of type I. The Higgs sector Lagrangian is given by

$$\mathcal{L}_H=(D^\mu H_1)^\dagger D_\mu H_1+(D^\mu H_2)^\dagger D_\mu H_2 - V(H_1,H_2)+\mathcal{L}_\mathrm{Y}(H_2), \tag{2}$$

where $V(H_1,H_2)$ is the Higgs potential, $\mathcal{L}_\mathrm{Y}$ the Yukawa couplings, and the covariant derivative defined by

$$D^\mu=\partial^\mu+igW^{\mu a}T^a+ig'B^\mu Y+ig_X A_X^\mu X, \tag{3}$$

with T^a $(a=1,2,3)$ being the SU(2) generators. Here X is the hidden $\mathrm{U}(1)_X$ charge operator and A_X^μ the corresponding gauge field. The Higgs potential is given by

$$V(H_1,H_2)=\mu_1^2H_1^\dagger H_1+\mu_2^2H_2^\dagger H_2+\lambda_1(H_1^\dagger H_1)^2+\lambda_2(H_2^\dagger H_2)^2 +\lambda_3(H_1^\dagger H_1)(H_2^\dagger H_2)+\lambda_4(H_1^\dagger H_2)(H_2^\dagger H_1), \tag{4}$$

where $\mu_{1,2}^2$ are dimension-two couplings for quadratic terms while $\lambda_{1,2,3,4}$ are the dimensionless quartic couplings. Note that the soft Z_2 symmetry-breaking terms of the $H_1^\dagger H_2$ quadratic term and the quartic term $(H_1^\dagger H_2)^2$ with the λ_5 coupling are forbidden by the $\mathrm{U}(1)_X$ gauge symmetry.

The physical gauge bosons after the EWSB, photon, Z boson, and the extra Z boson (Z') are defined by

$$
\begin{aligned}
A_X &= c_X Z' + s_X Z, \\
W_3 &= -s_X c_W Z' + c_X c_W Z + s_W A, \\
B &= s_X s_W Z' - c_X s_W Z + c_W A,
\end{aligned} \tag{5}
$$

where $s_W = \sin\theta_W = g'/\sqrt{g^2+g'^2}$, $c_W = \cos\theta_W$ is the Weinberg angle, and $s_X = \sin\theta_X$, $c_X = \cos\theta_X$ the $Z - Z'$ mixing defined by

$$
\tan 2\theta_X = \frac{-2g_X g' s_W \cos^2\beta}{g'^2 - g_X^2 s_W^2 \cos^2\beta}. \tag{6}
$$

The VEVs of two Higgs doublets, $\langle H_i \rangle = (0, v_i/\sqrt{2})^T$ with $i = 1, 2$, define $\tan\beta = v_2/v_1$. We have the neutral gauge boson masses

$$
m_{Z,Z'}^2 = \frac{1}{8}\Big(g_X^2 v_1^2 + (g^2+g'^2)v^2 \pm \sqrt{(g_X^2 v_1^2 - (g^2+g'^2)v^2)^2 + 4g_X^2(g^2+g'^2)v_1^4} \Big), \tag{7}
$$

where $v^2 = v_1^2 + v_2^2$. Note that only two mixing angles are required to diagonalize the neutral gauge boson mass matrix in this model.

The NC interactions of Z and Z' bosons are given by

$$
\mathcal{L}_{\rm NC} = (c_X Z^\mu + s_X Z'^\mu)(g_V \bar{f}\gamma_\mu f + g_A \bar{f}\gamma_\mu \gamma^5 f), \tag{8}
$$

where g_V and g_A are the SM Z couplings to the fermions. Since the Z' interactions are same as the SM Z interactions except for the overall suppression by s_X, we call it the dark Z boson.

The Z boson mass is shifted in this model such that

$$
m_Z^2 = \frac{m_W^2}{c_W^2 c_X^2} - m_{Z'}^2 \frac{s_X^2}{c_X^2}, \tag{9}
$$

which leads to the shift of the ρ parameter

$$
\begin{aligned}
\frac{1}{\rho} &= \frac{m_Z^2 c_W^2}{m_W^2} = \frac{1}{c_X^2} - \frac{m_{Z'}^2 c_W^2}{m_W^2}\frac{s_X^2}{c_X^2} \\
&\approx 1 + s_X^2\left(1 - \frac{m_{Z'}^2 c_W^2}{m_W^2}\right) \equiv 1 + \Delta\rho_{Z'}
\end{aligned} \tag{10}
$$

in the leading order of s_X^2. Moreover, there also exist new scalar contributions to the $\Delta\rho$ in this model, given by

$$
\Delta\rho_{\rm NS}^{(1)} = \frac{\alpha}{16\pi m_W^2 s_W^2}\left(m_\pm^2 - \frac{m_H^2 m_\pm^2}{m_H^2 - m_\pm^2}\log\left(\frac{m_H^2}{m_\pm^2}\right)\right) \tag{11}
$$

at one-loop level [22]. Here m_H is the SM-Higgs mass and $m_\pm$ is the charged Higgs mass. Then, $\Delta\rho$ predicted in this model is $\Delta\rho_{\rm New} = \Delta\rho_{Z'} + \Delta\rho_{\rm NS}^{(1)}$. The present limit on $\Delta\rho$ reads from the measurements $\alpha(m_Z)^{-1} = 127.955 \pm 0.010$ and Peskin-Takeuchi T value $T = 0.07 \pm 0.12$ [23] as

$$
-0.00039 < \Delta\rho < 0.001485. \tag{12}
$$

When $m_\pm \geq 120$ GeV, $\Delta\rho_{\rm NS}$ exceeds 0.001485 and no points of $(m_{Z'}, -s_X)$ are allowed. Consequently, 120 GeV is an upper limit on $m_\pm$ in this model. Actually, $\Delta\rho$ is insensitive to $m_\pm$ below 120 GeV.

The precise measurement of the atomic parity violation (APV) also provides a strong constraint on the exotic NC interactions. The dark Z exchange also contributes to the shift of the weak charge

$$
Q_W = Q_W^{\rm SM}\left(1 + \frac{m_Z^2}{m_{Z'}^2}s_X^2\right) \tag{13}
$$

in the leading order of s_X. The SM prediction of the Cs atom is $Q_W^{\rm SM} = -73.16 \pm 0.05$ [24,25], and the present experimental value is $Q_W^{\rm exp} = -73.16 \pm 0.35$ [26], which yields the bound

$$
\frac{m_Z^2}{m_{Z'}^2}s_X^2 \leq 0.006 \tag{14}
$$

at 90% C.L. The $\Delta\rho$ and APV constraints are depicted in Fig. 1 by the red region and red line, respectively.

The light Z' coupled to the electron generically affects the anomalous magnetic moment of the electron through loop diagrams. We calculate the Δa_e contribution from the

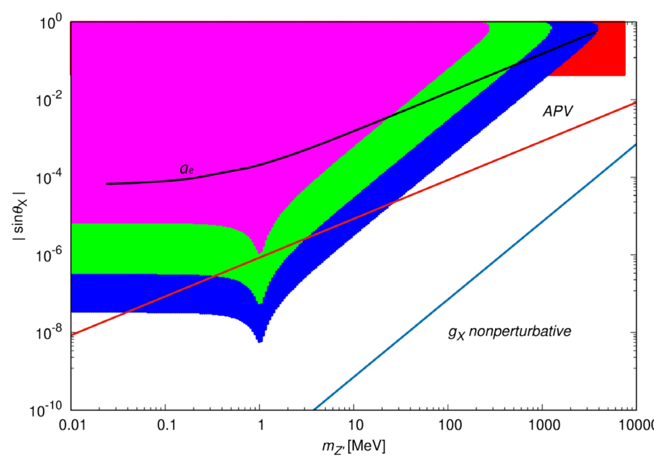

FIG. 1. Exclusion regions by the positronium invisible decays in the plane of the model parameter $(m_{Z'}, |\sin\theta_X|)$. The magenta region is excluded by the current limit Br(o-Ps → invisible) $< 4.2 \times 10^{-7}$, the green region by the future limit $< 10^{-9}$, and the blue region by the future limit $< 10^{-11}$. Electroweak constraints of $\Delta\rho$ is the red region (overlapped by the Ps exclusion regions) and the APV constraints above the red line. The black line is the 1-σ exclusion line by the anomalous magnetic moment of the electron.

dark Z loop and also show the constraints on $(m_{Z'}, |\sin\theta_X|)$ at 1-σ level by the present experimental value of a_e taken from Ref. [23] in Fig. 1.

We assume that the hidden sector lagrangian is QED-like,

$$\mathcal{L}_{\rm hs} = -\frac{1}{4} F_X^{\mu\nu} F_{X\mu\nu} + \bar{\psi}_X i\gamma^\mu D_\mu \psi_X - m_X \bar{\psi}_X \psi_X, \qquad (15)$$

where

$$D^\mu = \partial^\mu + i g_X A_X^\mu X. \qquad (16)$$

After the EWSB, the $U(1)_X$ is broken and ψ_X is connected to the SM through Z and dark Z bosons, given in Eq. (5). Note that the fermion mass m_X is a free parameter in this model. We will set $m_X < m_e$ for the invisible decay of Ps into ψ_X pair in the next section.

III. POSITRONIUM DECAYS

In this model Ps can annihilate into the DM fermion pair through the dark Z boson when the DM fermion is lighter than the electron, $m_X < m_e$. We obtain the dark Z contribution to the invisible decays as

$$\begin{aligned}\Gamma(\text{o-Ps} \to Z' \to \bar{\psi}_X\psi_X) = {} & \frac{1}{12\pi m_e^2} s_X^2 c_X^2 g_V^2 (g_X X)^2 \\ & \times \left[\left(1 - \frac{m_{Z'}^2}{4m_e^2}\right)^2 + \frac{m_{Z'}^2 \Gamma_{Z'}^2}{16 m_e^4}\right]^{-2} \\ & \times \sqrt{1 - \frac{m_X^2}{m_e^2}} \left(1 + \frac{m_X^2}{2m_e^2}\right) |\psi(0)|^2,\end{aligned} \qquad (17)$$

where the square of the Ps wave function at the origin is $|\psi(0)|^2 = m_e^3 \alpha^3 / 8\pi$. The dark Z couplings to the SM particles are suppressed by the mixing angle s_X while not suppressed by the DM fermions. Thus, the decay width of the dark Z boson is dominated by $Z' \to \bar{\psi}_X \psi_X$, where $\Gamma_{Z'} = \Gamma(Z' \to \bar{\psi}_X\psi_X) \sim 10^{-2}$ MeV, when $m_{Z'} > 2m_X$. On the other hand, if $m_{Z'} < 2m_X$, then only the neutrino channels are allowed and $\Gamma_{Z'} = 3\Gamma(Z' \to \nu\bar{\nu}) = 3\Gamma(Z \to \nu\bar{\nu}) (s_X/c_X)^2 (m_{Z'}/m_Z) < 10^{-13}$ MeV at most. Therefore we can neglect $\Gamma_{Z'}$ to calculate the invisible decay rate except for around the resonance region.

The branching ratio of the o-Ps invisible decay is

$$\text{Br}(\text{o-Ps} \to \text{invisible}) = \frac{\Gamma(\text{o-Ps} \to \text{invisible})}{\Gamma_0 + \Gamma(\text{o-Ps} \to \text{invisible})}, \qquad (18)$$

where the SM decay rate Γ_0 is dominated by the three photon decay given by

$$\begin{aligned}\Gamma(\text{o-Ps} \to \gamma\gamma\gamma) &= \frac{2(\pi^2 - 9) m_e \alpha^6}{9\pi} \left(1 - 10.28661 \frac{\alpha}{\pi} + \mathcal{O}(\alpha^2)\right) \\ &\approx 7.0382\,\mu\text{s}^{-1}.\end{aligned} \qquad (19)$$

Figure 1 depicts the exclusive region defined by the invisible positronium decays with the present data of Eq. (1) and Br(o-Ps $\to$ invisible) $< 10^{-9}$ and $< 10^{-11}$ as future experimental reaches [27,28]. This model has two more free parameters on the DM sector, the hidden $U(1)_X$ charge, and the mass of ψ_X. We take $(g_X X)^2 = 2\pi$ and $m_X = m_e/2$ in Fig. 1 as benchmark values which are chosen to probe the parameter region below the APV bound and to accommodate the DM phenomenology, as will be discussed in the next section.

The parapositronium dominantly decays into two photons. In this model, p-Ps can decay into a photon and a dark Z boson, then we will observe the single photon decay process. The decay rate into $\gamma Z'$ is

$$\Gamma(\text{p-Ps} \to \gamma Z') = \frac{2\alpha}{m_e^2} s_X^2 g_V^2 \left(1 - \frac{m_{Z'}^2}{4m_e^2}\right) |\psi(0)|^2. \qquad (20)$$

Meanwhile o-Ps can also decay into the $\gamma Z'$ final state due to the axial coupling of the dark Z such as

$$\Gamma(\text{o-Ps} \to \gamma Z') = \frac{8\alpha}{3m_{Z'}^2} s_X^2 g_A^2 \left(1 - \frac{m_{Z'}^2}{4m_e^2}\right)\left(1 + \frac{m_{Z'}^2}{4m_e^2}\right) |\psi(0)|^2. \qquad (21)$$

We estimate the branching ratios Br(p-Ps $\to \gamma Z'$) $< 10^{-12}$ and Br(o-Ps $\to \gamma Z'$) $< 10^{-13}$ with the allowed values of $(m_{Z'}, |\sin\theta_X|)$ given in Fig. 1, which are smaller than the future experimental reaches considered in Fig. 1. Neither the experimental limits for the single photon decay of p-Ps nor those of o-Ps have been reported yet.

IV. DARK MATTER PHENOMENOLOGY

We calculate the relic abundance $\Omega_{\rm CDM} h^2$ in the thermal freeze-out scenario using the `micrOMEGAs` [29] with the allowed values of parameters $(m_{Z'}, |\sin\theta_X|$ given in the previous section) and show that the model prediction can accommodate the present measurements with high precision [23]

$$\Omega_{\rm CDM} h^2 = 0.1186 \pm 0.0020, \qquad (22)$$

from the anisotropy of the cosmic microwave background (CMB) and of the spatial distribution of galaxies. Since we are interested in the parameter set with which the dark sector has a sizable effect on the positronium physics, we consider a LDM scenario that constrains the DM mass to be smaller than the electron mass. In this case, the model is barely constrained by both the direct [30–33] and indirect [34,35] detection experiments as discussed in Ref. [11]. Recently, a lot of direct searches for very light DM with

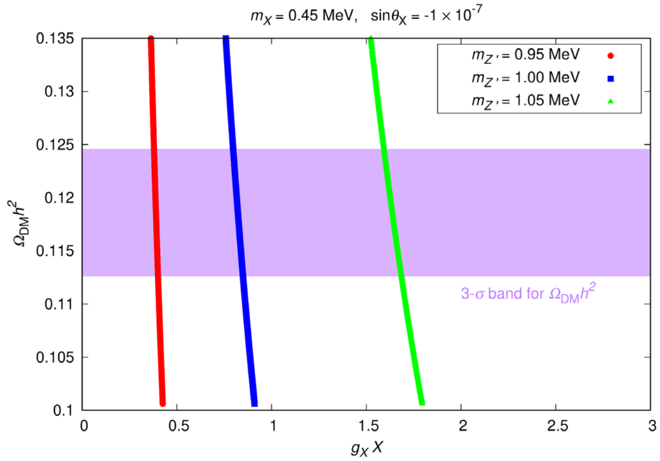

FIG. 2. Behaviors of relic density for a few benchmark points, as functions of the interaction strength between DM pairs and Z' boson.

mass around sub-MeV have being conducted [36–42]. However, constraints on light DM have not been well established for masses less than 0.5 MeV, which is the region of our interest in relation with positronium decays.

We demonstrate generic behaviors of $\Omega_{\rm CDM}h^2$ for some benchmark points near the resonance region of Ps decay in Fig. 2. We note that the DM annihilation process which contributes to the relic abundance arises mainly through the s-channel $X\bar{X} \to Z' \to \nu\bar{\nu}$ at the resonance region where $M_X \sim m_{Z'}/2$, while the dominant channels are the t-channel processes $X\bar{X} \to Z'Z'$ and etc. at the nonresonance region. We find that the relic abundance is very sensitive to the value of $g_X X$ in the resonance region around $m_{Z'} = 1$ MeV.

In Fig. 3 we show the branching ratios of the o-Ps invisible decay for points that accommodate the present relic density with a 3σ range as a function of the Z' mass in unit of MeV. For this analysis, we scan the DM masses less than 0.5 MeV and the Z' masses in the range 0.01 MeV–4 MeV to investigate possible correlations between the DM and positronium physics. As shown in Fig. 3, the branching ratio of o-Ps invisible decay can be enhanced significantly in the resonance region around $m_{Z'} \simeq 1$ MeV. Though relic density depends on various model parameters, it is largely determined by the interaction strength between DM pair and Z' boson, $g_X X$, generically. Perturbativity bound on the size of the coupling between the DM pair and Z' is also imposed. In this figure, two distinct kinds of points are overlapped. One is the case of $m_{Z'} < 2m_X$, where the decay width of Z' is relatively suppressed compared with the opposite case, $m_{Z'} > 2m_X$. The green horizontal line corresponds to the present limit on the branching ratio of the o-Ps invisible decay, 4.2×10^{-7}. On the other hand, the dark matter self-interaction can affect structures of halos at the small scale and it is known that the observation of the 'Bullet Cluster' provides one of the strongest constraint on such dark matter self-interaction. The simulation on the 'Bullet Cluster' yields an upper bound on dark matter self-interaction cross section over masses such as $\sigma/m \lesssim 1\ \mathrm{cm}^2/\mathrm{g}$ [43]. In Fig. 3, the points which satisfy the 'Bullet Cluster' constraint are represented by large squares. At the nonresonance region, the allowed invisible branching ratios are very suppressed compared with the current experimental bound. However, we find that points near the resonance region can reach the current bound and could be tested in near future.

We also compute the branching ratios of o-Ps and p-Ps decays into a photon and a Z' boson, which will be observed as single photon decays, with parameter sets allowed by the electroweak constraints and DM relic density, and show the results in Fig. 4. The points which satisfy the 'Bullet Cluster' constraint are represented by large squares. The predicted branching ratios of o-Ps and p-Ps are at most $O(10^{-12})$ and $O(10^{-13})$ as estimated in the previous section, which are rather far from the reach of precision for the search of the invisible decays of positronium in near future.

A few comments are in order. Generically, DM masses below 1 MeV are disfavored by BBN and CMB data through modifying the effective number of neutrino species when employing conventional thermal freeze-out scenarios [18,44,45]. Recently the authors of Ref. [20,21] suggested an alternative cosmological scenarios that can alleviate the problem for sub-MeV DM. It is dubbed as 'delayed equilibration scenario' in which sub-MeV DM thermalizes with the SM sector below the neutrino-photon decoupling temperature. In this scenario, the SM bath is cooled down by the equilibration and is heated again by the freeze-out of DM.

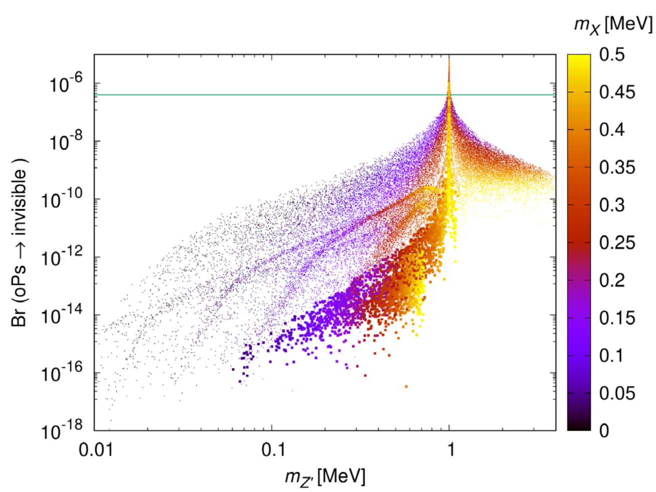

FIG. 3. Branching ratios of the o-Ps invisible decay for points that accommodate the observed relic density of DM within a 3σ range as a function of Z' mass in unit of MeV. The points which satisfy the 'Bullet Cluster' constraint are represented by large squares.

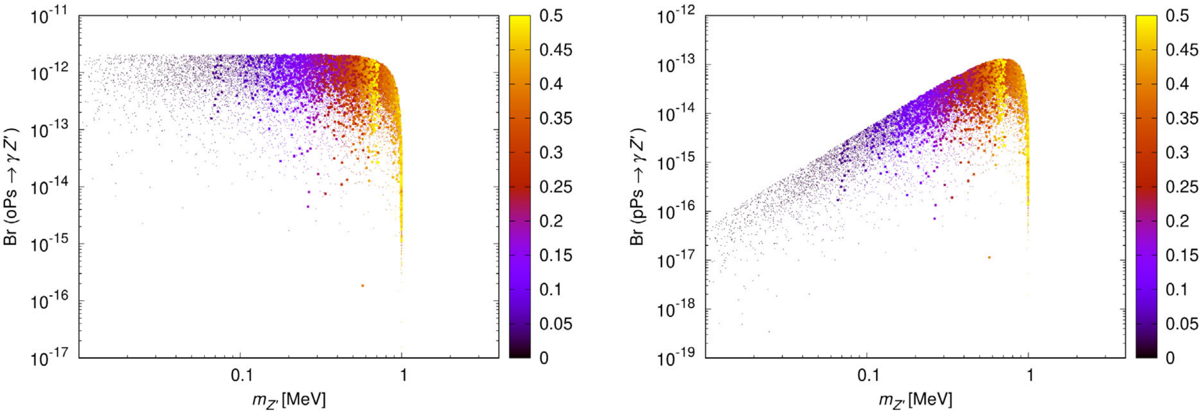

FIG. 4. Branching ratios of o-Ps and p-Ps decay into a photon and a Z' boson for points that accommodate the observed relic density of DM within 3σ range as a function of the Z' mass in unit of MeV. They appear as the conversion of positronium to single photon. The points which satisfy the 'Bullet Cluster' constraint are represented by large squares.

In the conventional scenario where DM enters equilibrium with the SM bath before neutrino-photon decoupling, the effective number of neutrino species, $N_{\rm eff}$, is given by

$$N_{\rm eff} \simeq 3\left(1+\frac{g_*^X}{g_*^\nu}\right), \tag{23}$$

where g_*^i is the effective number of relativistic degrees of freedom in the bath of a light particle i. In the 'delayed equilibration scenario', $N_{\rm eff}$ is modified to [20,21]

$$N_{\rm eff} \simeq 3\left(1+\frac{g_*^X}{g_*^\nu}\right)^{1/3} \tag{24}$$

as long as the initial condition $\xi_X^0 \ll 1$, where ξ_X^0 is defined by the DM temperature normalized to the photon temperature before DM enters into the thermal equilibrium with the SM particles. Since the DM candidate in the dark Z model is a Dirac fermion, we have $g_*^X = (7/8)\times 2\times 2$, which gives rise to $N_{\rm eff}\simeq 3.56$. This effective number moderately agrees with $N_{\rm eff} = 2.85\pm 0.28$ obtained from detailed studies of BBN data [46] with about a 2.5σ deviation, but it is consistent with $N_{\rm eff} = 3.15\pm 0.23$ from the observation of CMB by the Planck Collaboration [47] within 2σ. This can be compared with that in the thermal freeze-out scenario, which is almost excluded with more than 7σ.

In the dark Z model, a DM pair can convert to an electron-positron pair through Z' to be constrained from the supernova. However, after comprehensive analysis about the supernova constraints on the dark photon portal models the authors in Ref. [48] found that such constraints can be evaded for a dark sector with dark fine structure constant $\alpha_D \gtrsim 10^{-7}$. We find that the dark Z model, in which DM coupling to Z' is taken to be large enough to accommodate the present relic density as in Fig. 2, can avoid the supernova constraints. We conclude that the parameter region studied in this paper is still valid.

V. CONCLUDING REMARKS

We study the invisible decay of o-Ps into fermionic DM pair through the dark Z boson when the fermionic DM particle is lighter than the electron. The predictions of the Ps invisible decay rates in the dark Z model are still less than the present experimental reach but are much enhanced compared to the SM predictions through the weak interaction. The fermionic LDM scenario with the light mediator can also satisfy the relic abundance and is not constrained by the present direct detection experiment and indirect observations. We discuss that the LDM model discussed in this work can be accommodated in the recent delayed equilibration scenario. In conclusion, the Ps invisible decay provides attractive phenomenology of the dark Z model with fermionic DM which is independent of collider and dark matter phenomenology.

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