Quantum theory of synchrotron radiation in (2 + 1)-dimensional electrodynamics

P. A. Eminov*

National Research University Higher School of Economics, Myasnitskaya Ulitsa 20, Moscow 101000, Russia

(Received 12 February 2022; accepted 28 March 2022; published 13 May 2022)

In this paper we develop a quantum theory of synchrotron radiation of massive and massless fermions in $(2+1)$-dimensional electrodynamics with a doubled fermionic representation based on an exact formula for the radiation shift of electron energy in a constant magnetic field. Analytical formulas describing the dependences of the spectral distribution and the total radiation power of a massive fermion on the dynamic parameter of synchrotron radiation and the spin quantum number of the initial electron are obtained. The power of synchrotron radiation of a massless charged fermion is calculated in the case of large values of the main quantum number and a relatively weak magnetic field. It was shown that the radiation power of a massless fermion is described by a formula that coincides with the main term of the asymptotic expansion of the radiation power of a massive electron in the ultraquantum case.

DOI: 10.1103/PhysRevD.105.095014

I. INTRODUCTION

The study of radiation effects in an intense external field with the participation of two-dimensional charged fermions, both massive and massless, is one of the topical problems in modern physics.

It should be noted that synchrotron radiation of a two-dimensional charged fermion is one of the main effects associated with the propagation of a fermion in an external magnetic field, the theory of which has not been fully created in $(2+1)$-dimensional quantum electrodynamics. For example, in Ref. [1], within the framework of the classical theory, the power of synchrotron radiation of a massive charged particle in space-time of any odd dimension was calculated, including the case of $(2+1)$-dimensional classical electrodynamics, but the corresponding quantum theory of synchrotron radiation in $(2+1)$-dimensional space-time has not yet been created.

Massless quantum electrodynamics in an external magnetic field is of great interest in connection with the prediction of magnetic catalysis of chiral symmetry breaking, in the physics of solids and low-dimensional systems, as well as in cosmology [2–4]. In graphene and in a number of other planar structures, the dynamics of electronic excitations is described by the effective two-dimensional Dirac equation for both massless and massive charged fermions. The effective electron mass can arise not only as a result of dynamic generation due to electron-electron and other interactions in graphene [5–8], but also due to radiative effects with the participation of two-dimensional electrons in an external magnetic field [2,4,9–11].

In the classical theory, many authors have considered the problem of radiation of a massless charged particle (see, for example, [12,13]). In $(3+1)$-dimensional space-time, the electromagnetic field created by a massless charged particle in the eikonal approximation is described in Ref. [14], and the emission of a photon by an electron in a constant electric field in the Dirac model of graphene is considered in Ref. [15]. The report on the discovery of a massless charged quasiparticle is given in the work [16].

The problem of synchrotron radiation of a massless charged particle in $(3+1)$-dimensional scalar and spinor electrodynamics was first solved within the framework of the quantum theory of synchrotron radiation in Ref. [4]. It was shown that massless electrodynamics and the zero-mass limit in the initially massive quantum theory give coinciding results when calculating the spectral distribution and total power of synchrotron radiation. In Ref. [10], using the amplitude of elastic scattering of a fermion in a constant magnetic field calculated in Ref. [9], the probability of synchrotron radiation of a massless charged fermion in the reduced QED_{3+1} with a two-component fermion was obtained. Note that in this model fermions remain on the plane during their motion, while the carriers of electromagnetic interaction propagate in three-dimensional space [2,9,10,17,18].

The photon polarization operator was calculated in the low-energy effective model of graphene in a relatively weak external magnetic field based on the massless reduced QED_{3+1} [19]. The result of these investigations was used in Refs. [19,20] to study the absorption of light passing

*peminov@mail.ru

through a graphene membrane. Analytical dependences of the transmitted light intensity and the polarization rotation angle on the frequency of the incident plane electromagnetic wave were calculated at different values of the magnetic field strength. Induced bremsstrahlung, as indicated in Ref. [20], may be the main mechanism of perpendicular light reflection in the case when the incident monochromatic wave is perpendicular to the graphene layer.

The purpose of this work is to develop, taking into account the results obtained earlier in Refs. [4,9–11,21,22], the quantum theory of synchrotron radiation of both massive and massless charged fermions in the $(2+1)$-dimensional model of quantum electrodynamics with doubled fermionic representation.

In Sec. II, exact formulas are obtained that describe the spectral distribution and total power of synchrotron radiation in $(2+1)$-dimensional quantum electrodynamics. The calculation is carried out by the method of exact solutions of wave equations in a magnetic field. The chosen calculation method allows one to calculate the exact spectral power density of the radiation for a massive and massless fermion in a unified manner.

In Sec. III, the spectral distribution of the synchrotron radiation power of a massive relativistic electron in QED_{2+1} in a relatively weak magnetic field is calculated. Analytical results are obtained that describe the dependences of the radiation power on the dynamic parameter of synchrotron radiation in the classical approximation and in the ultraquantum limit. The dependence of the synchrotron radiation power on the electron spin in the initial state is also investigated.

In Sec. IV, the power of synchrotron radiation of a massless charged fermion in $(2+1)$-dimensional QED is calculated in the case of high excited states of a fermion in a relatively weak magnetic field. The calculation is carried out both directly based on the exact formula for the spectral distribution of the radiation power, obtained in the massless case in Sec. II, and using formula (3.3) from Sec. III for the emission of a massive electron, which makes it possible to pass to the limit of zero fermion mass.

In Sec. V, we discuss the results of the work.

We shall adopt the units where $c=\hbar=1$.

II. SPECTRAL DISTRIBUTION OF RADIATION POWER

To determine the power of synchrotron radiation, we will use a method based on calculating the total radiative shift of the electron energy in a constant magnetic field.

We consider the four-component fermions in QED_{2+1}, connected with a four-dimensional reducible representation of Dirac's matrices [21]:

$$\gamma^0=\begin{pmatrix}\sigma_3 & 0\\ 0 & -\sigma_3\end{pmatrix},\qquad \gamma^{1,2}=\begin{pmatrix}i\sigma_{1,2} & 0\\ 0 & -i\sigma_{1,2}\end{pmatrix},$$

where $\sigma_{1,2,3}$ are the Pauli matrices.

In a magnetic field given by the potential

$$A^{\mu}_{\mathrm{ext}}=(0,0,xH),\qquad (2.1)$$

the energy of a two-dimensional electron with charge $-e<0$ and mass m is determined by the formula

$$E_n=\sqrt{m^2+p_\perp^2}=\sqrt{m^2+2eHn},\qquad (2.2)$$

where $n=0,1,2,\ldots$ is the principle quantum number.

In the one-loop approximation, the electron mass operator in the QED_{2+1} with the doubled fermion representation is described by the formula [21]

$$\Sigma(x,x')=-ig^2\gamma^\mu S_c(H,x,x')\gamma^\nu D_{\mu\nu}(x-x').\qquad (2.3)$$

We note that in QED_{2+1} theory g^2 in (2.3) has the dimensions of mass.

Here the photon propagator in the Landau gauge

$$D_{\mu\nu}(x-x')=-ig_{\mu\nu}\int\frac{d^3p}{(2\pi)^3}\frac{\exp[-ip(x-x')]}{{p_0}^2-\vec{p}^{\,2}+i0},\qquad (2.4)$$

and for the causal Green's function of an electron in a constant magnetic field, we use the representation [23]

$$S_c(H;x,x')=-\frac{1}{2\pi i}\int_{-\infty}^{+\infty}d\omega\exp[i\omega(t-t')]\times\sum_{s,\varepsilon=\pm1}\frac{\Psi_s^\varepsilon(\vec{x})\bar{\Psi}_s^\varepsilon(\overrightarrow{x'})}{\omega+\varepsilon E_s(1-i\delta)},\qquad (2.5)$$

where the summation is carried out over all quantum numbers $s=\{n',p'_y,\zeta'\}$ of positive frequency $(\varepsilon=+1)$ and negative frequency $(\varepsilon=-1)$ solutions of the Dirac equation.

In the QED_{2+1} model with a doubled fermionic representation, the wave function of the stationary state of a two-dimensional electron in a constant magnetic field is determined by the formula [11,21]

$$\Psi_{\varepsilon=+1}=\frac{(eH)^{\frac{1}{4}}}{\sqrt{2E_n}}\exp[-iE_nt+iyp_y]\left[D_1\begin{pmatrix}\sqrt{E_n+m}U_{n-1}(\eta)\\ \sqrt{E_n-m}U_n(\eta)\\ 0\\ 0\end{pmatrix}+D_{-1}\begin{pmatrix}0\\ 0\\ \sqrt{E_n-m}U_{n-1}(\eta)\\ \sqrt{E_n+m}U_n(\eta)\end{pmatrix}\right].\qquad (2.6)$$

Here the electron energy is determined by formula (2.2), the Hermite function is expressed in terms of the Hermite polynomials by the formula

$$U_n(\eta) = \frac{(eH)^{\frac{1}{4}}}{\sqrt{2^n n! \sqrt{\pi}}} e^{-\frac{\eta^2}{2}} H_n(\eta),$$
$$H_n(\eta) = (-1)^n e^{\eta^2} \frac{d^n}{d\eta^n} e^{-\eta^2}, \tag{2.7}$$

and the argument of these functions

$$\eta = \sqrt{eH}\left(x + \frac{p_y}{eH}\right). \tag{2.8}$$

Positive- and negative-frequency solutions of the Dirac equation $\varepsilon = \pm 1$ obey the additional condition

$$\hat{A}\Psi = \zeta\Psi, \qquad \hat{A} = i\varepsilon \frac{E_n}{m} \gamma^0 \gamma^1 \gamma^2,$$

where the quantum number $\zeta = \pm 1$ has the meaning of the projection of the electron spin on the direction of the magnetic field. In formula (2.6), at $\zeta = +1$ should be set $D_1 = 1, D_{-1} = 0$ (the spin is directed along the field), and at $\zeta = -1$, on the contrary, $D_1 = 0, D_{-1} = 1$ (the spin is directed against the field).

In the one-loop approximation, the mass operator $\Sigma(x, x')$ determines the radiative correction to the electron energy in the form

$$\Delta E_n = -ig^2 \frac{1}{T} \int d^3x d^3x' \overline{\Psi}_{n,q_y,\zeta}(x) \gamma^\mu \times S_c(H; x, x') \gamma^\nu D_{\mu\nu}(x - x') \Psi_{n,q_y,\zeta}(x'), \tag{2.9}$$

where T is the interaction time.

Omitting the details of the calculations, we present the exact expression in the one-loop approximation for the radiative shift of the energy of a two-dimensional electron in a constant magnetic field, obtained in Ref. [11]:

$$\Delta E_n = -\frac{m^2 g^2}{16\pi^{\frac{3}{2}} E_n} e^{i\frac{\pi}{4}} \int_0^1 \frac{du}{\sqrt{u}} \int_0^\infty \frac{dy}{\sqrt{y}} \frac{1}{F} \times \exp[-ip_\perp^2 y(u-1) - 2in \arctan\lambda - im^2 uy] \times \left(\zeta \frac{E_n}{m} \Omega_1 + \Omega_2\right), \tag{2.10}$$

where the notations are accepted

$$\Omega_1 = (2 - u + 2ue^{-2iz}) - (1 - \delta_{0,n}) e^{2i \arctan\lambda} [2u + (2 - u) e^{-2iz}], \tag{2.11}$$

$$\begin{aligned} \Omega_2 = {} & -(2 - u + 2ue^{-2iz}) \\ & - (1 - \delta_{0,n}) e^{2i \arctan\lambda} [2u + (2 - u) e^{-2iz}] \\ & + 2i \left(\frac{p_\perp}{m}\right)^2 (u - 1) \sin z e^{-iz} [1 - e^{2i \arctan\lambda}] \\ & - \left(\frac{p_\perp}{m}\right)^2 (u - 1) [e^{-2iz} + e^{2i \arctan\lambda}] \\ & + 2 \left(\frac{p_\perp}{m}\right)^2 \frac{u - 1}{F} e^{2i \arctan\lambda - 2iz}, \end{aligned} \tag{2.12}$$

$$\lambda = \frac{tgz}{1 + \frac{u}{1-u} \frac{tgz}{z}}, \qquad F = 1 - u + ue^{-iz} \frac{\sin z}{z},$$
$$z = eHy, \qquad p_\perp^2 = 2eHn. \tag{2.13}$$

Note that in formula (2.10), after averaging over the electron spin, the term proportional to $\zeta\Omega_1$ vanishes. This term is also equal to zero in the case of a massless electron, which also directly follows from the obtained formulas (2.10) and (2.11).

As in standard QED_{3+1}, it is necessary to renormalize the electron mass by subtracting from the nonrenormalized value in formula (2.10) its value in a zero external magnetic field [24,25]. According to the optical theorem, the imaginary part of the radiative correction to the electron energy determines the total probability of the electron radiative transition from the initial quantum state in the given external field:

$$w = -2\mathrm{Im}(\Delta E_n). \tag{2.14}$$

As a result, for the power of synchrotron radiation in QED_{2+1}, averaged over the spin states of the electron, we obtain the following representation:

$$W = -2\mathrm{Im}\left\{-\frac{m^2 g^2}{16\pi^{\frac{3}{2}}} e^{i\frac{\pi}{4}} \int_0^1 \sqrt{u} du \int_0^\infty \frac{\Omega}{\sqrt{y} F} \times \exp[-ip_\perp^2 y(u-1) - 2in \arctan\lambda - im^2 uy] dy\right\}, \tag{2.15}$$

where

$$\begin{aligned} \Omega = {} & -(2 - u + 2ue^{-2iz}) \\ & - e^{2i \arctan\lambda} [2u + (2 - u) e^{-2iz}] + 2(u + 2) \\ & + 2i \left(\frac{p_\perp}{m}\right)^2 (u - 1) \sin z e^{-iz} [1 - e^{2i \arctan\lambda}] \\ & - \left(\frac{p_\perp}{m}\right)^2 (u - 1) [e^{-2iz} + e^{2i \arctan\lambda}] \\ & + 2 \left(\frac{p_\perp}{m}\right)^2 \frac{u - 1}{F} e^{2i \arctan\lambda - 2iz}. \end{aligned} \tag{2.16}$$

In the next sections, we consider some limiting cases that follow from formulas (2.10)–(2.16) and are of the greatest physical interest.

III. POWER OF SYNCHROTRON RADIATION OF A MASSIVE ELECTRON IN QED_{2+1}

Let us consider the most interesting case of relativistic values of the electron energy in a weak magnetic field, when the conditions fulfilled

$$\frac{H}{H_0} \ll 1, \qquad \frac{p_\perp}{m} \gg 1, \qquad \frac{p'_\perp}{m} \gg 1, \tag{3.1}$$

where $H_0 = \frac{m^2}{e} \simeq 4$ and 41×10^{13} G is the Schwinger value of the magnetic field strength.

The dynamic parameter of synchrotron radiation is defined by the formula

$$\chi = \frac{e}{m^3}\sqrt{-(F_{\mu\nu}p^\nu)^2} = \frac{H}{H_0}\cdot\frac{p_\perp}{m} \tag{3.2}$$

and can take any values within the framework of the applicability of the one-loop approximation.

When conditions (3.1) are satisfied, the main contribution to the power of the synchrotron radiation is made by the region in which the condition $z = eHy \ll 1$ is satisfied. Carrying out the corresponding semiclassical expansions, from formulas (2.15) and (2.16) for the power of synchrotron radiation of a massive electron in $(2+1)$-dimensional quantum electrodynamics, we obtain the following expression:

$$W = -\frac{mg^2}{4\pi^{\frac{3}{2}}}\mathrm{Im}\left\{e^{i\frac{\pi}{4}}\int_0^\infty \frac{\sqrt{z_0}dv}{(v+1)^2} \times \left[\frac{3v+2}{v+1}g_1(z_0) + \frac{g'(z_0)}{z_0}\left(4+\frac{2v}{3}\right)\right]\right\},$$
$$z_0 = \left(\frac{v}{\chi}\right)^{\frac{2}{3}}, \tag{3.3}$$

where function $g'(z_0)$ is the derivative of function

$$g(z_0) = i\int_0^\infty \sqrt{t}\exp\left[-i\left(z_0 t + \frac{t^3}{3}\right)\right]dt, \tag{3.4}$$

which is an analog of the Hardy-Stokes function in $(3+1)$-dimensional quantum electrodynamics of an intense external field [25,26], and the function $g_1(z_0)$ is defined by the formula

$$g_1(z_0) = \int_0^\infty \frac{dt}{\sqrt{t}}e^{-iz_0 t}\left(e^{-i\frac{t^3}{3}} - 1\right). \tag{3.5}$$

The spectral variables in formula (2.15) and in formula (3.3) are related to the energy of the emitted photon ω and the energy of the initial electron E by the formulas

$$v = \frac{u}{1-u} = \frac{\omega}{E-\omega}. \tag{3.6}$$

Note that the integrand in formula (3.3) is the spectral distribution of the synchrotron radiation power of a two-dimensional electron in QED_{2+1}.

First, let us find the classical formula for the intensity of synchrotron radiation of an electron in QED_{2+1}. For this, in formula (3.3), we pass to integration with respect to the variable

$$y = \left(\frac{v}{\chi}\right)^{\frac{2}{3}}$$

and further in the integrand we put $\chi = 0$. As a result, we get

$$W^{cl} = \frac{3mg^2}{4\pi^{\frac{3}{2}}}\chi\mathrm{Im}(e^{i\frac{\pi}{4}}I), \tag{3.7}$$

where the number I is determined by the integral

$$I = \int_0^\infty y g_1(y)dy - 2g(0). \tag{3.8}$$

Let us show that integral (3.8) is calculated exactly. Using the formula [27]

$$\int_0^\infty x^{\mu-1}e^{-\beta x}\begin{pmatrix}\cos\delta x\\ \sin\delta x\end{pmatrix}dx = \frac{\Gamma(\mu)}{(\beta^2+\delta^2)^{\frac{\mu}{2}}} \times \begin{pmatrix}\cos[\mu\times\arctan(\frac{\delta}{\beta})], & \mathrm{Re}\mu > 0, \mathrm{Re}\beta > |\mathrm{Im}\delta|,\\ \sin[\mu\times\arctan(\frac{\delta}{\beta})], & \mathrm{Re}\mu > -1, \mathrm{Re}\beta > |\mathrm{Im}\delta|,\end{pmatrix}, \tag{3.9}$$

we get

$$g(0) = i\int_0^\infty \sqrt{t}e^{-i\frac{t^3}{3}}dt = \sqrt{\frac{\pi}{3}}e^{i\frac{\pi}{4}}. \tag{3.10}$$

The remaining integral is transformed to the form

$$\int_0^\infty ydy\left(\int_0^\infty \frac{dt}{\sqrt{t}}e^{-iyt}(e^{-i\frac{t^3}{3}} - 1)\right) = \frac{1}{3\sqrt{3}}(A+iB), \tag{3.11}$$

whence, taking into account (3.9), we obtain

$$B = \int_0^\infty x^{-\frac{3}{2}} \sin x dx = \sqrt{2\pi},$$

and it remains to calculate

$$A = \sqrt{2} \int_0^\infty x^{-\frac{3}{2}} \sin^2 x dx. \quad (3.12)$$

This integral is also tabular [27]:

$$\int_0^\infty t^{\mu-1} \sin^2 t dt = -\frac{\Gamma(\mu)}{2^{\mu+1}} \cos\frac{\pi\mu}{2}, \quad -2 < \mathrm{Re}\mu < 0. \quad (3.13)$$

In our case $\mu = -\frac{1}{2}$, and, taking into account also that $\Gamma(-\frac{1}{2}) = -2\sqrt{\pi}$, we find

$$A = B = \sqrt{2\pi}. \quad (3.14)$$

Thus, taking into account formulas (3.7), (3.8), (3.10)–(3.12), and (3.14) we have

$$\mathrm{Im}(e^{i\frac{\pi}{4}} I) = -\frac{4}{3}\sqrt{\frac{\pi}{3}}, \quad (3.15)$$

and the power of synchrotron radiation, which we obtained as the limiting case of formula (3.3), is determined by the exact formula in the classical approximation

$$W^{cl}_{2+1} = \frac{mg^2}{\pi\sqrt{3}}\chi. \quad (3.16)$$

In another limiting case, when $\chi \gg 1$, the main contribution to integral (3.3), as in standard QED_{3+1}, comes from the region $z_0 = (\frac{v}{\chi})^{\frac{2}{3}} \ll 1$.

In this region, the function $g'(z_0)$ in the leading approximation can represented in the form

$$g'(z_0) \simeq g'(0) = \lim_{\varepsilon\to 0} \int_0^\infty t^{\frac{3}{2}} e^{-i\frac{t^3}{3} - \varepsilon t} dt, \quad (3.17)$$

where the integral is tabular and we find

$$g'(z_0) = 3^{-\frac{1}{6}} \Gamma\left(\frac{5}{6}\right) e^{-i\frac{5\pi}{12}}. \quad (3.18)$$

Let us now find the asymptotic expression for the function $\mathrm{Im}(e^{i\frac{\pi}{4}} g_1(y))$ in the formula (3.3), when the argument $y \to 0$.

To do this, note that the function $g_1(y)$ defined by formula (3.5) is a solution of the differential equation

$$g_1'(y) = -g(y) + \frac{\sqrt{\pi}}{2y^{\frac{3}{2}}} e^{-i\frac{\pi}{4}}, \quad (3.19)$$

and it admits the following exact representation:

$$e^{i\frac{\pi}{4}} g_1(y) = e^{i\frac{\pi}{4}} \int_y^\infty g(t) dt - \sqrt{\frac{\pi}{y}}. \quad (3.20)$$

Thus, from formula (3.20) for the function of interest to us, it follows that

$$\mathrm{Im}(e^{i\frac{\pi}{4}} g_1(y)) = \mathrm{Im}\left(e^{i\frac{\pi}{4}} \int_y^\infty g(t) dt\right). \quad (3.21)$$

Using further integral representation (3.4) for the function $g(t)$, we obtain

$$\begin{aligned}\int_0^\infty g(t) dt &= i \lim_{\varepsilon\to 0} \int_0^\infty \sqrt{x} e^{-i\frac{x^3}{3}} dx \left(\int_0^\infty e^{-it(x-i\varepsilon)} dt\right) \\ &= i \int_0^\infty \sqrt{x} e^{-i\frac{x^3}{3}} \left[\pi\delta(x) - i\frac{1}{x}\right] dx \\ &= \int_0^\infty \frac{dx}{\sqrt{x}} e^{-i\frac{x^3}{3}} = 3^{-\frac{5}{6}} \Gamma\left(\frac{1}{6}\right) e^{-i\frac{\pi}{12}}, \end{aligned} \quad (3.22)$$

where the Dirac delta function $\delta(x)$ is introduced and the last integral in (3.22) is calculated using formula (3.9).

Thus, for $z_0 \to 0$, as follows from formulas (3.21) and (3.22), the leading term of the asymptotics of the function $\mathrm{Im}(e^{-i\frac{\pi}{4}} g_1(z_0))$ is determined by the formula

$$\mathrm{Im}(e^{-i\frac{\pi}{4}} g_1(z_0)) \simeq \frac{1}{2} 3^{-\frac{5}{6}} \Gamma\left(\frac{1}{6}\right). \quad (3.23)$$

This means that, in formula (3.3) in the ultraquantum limit, the main term of the asymptotics of the first term $\sim\chi^{-\frac{1}{3}}$, and the main contribution to the total radiation power is made by the second term:

$$W \simeq \frac{mg^2}{4\pi^{\frac{3}{2}}} 3^{-\frac{1}{6}} \Gamma\left(\frac{5}{6}\right) \int_0^1 u^{-\frac{2}{3}} (1-u)^{-\frac{5}{6}} \left[2(1-u) + \frac{u}{3}\right] du. \quad (3.24)$$

The remaining integral over the spectral variable is calculated using the Euler beta function

$$B(x,y) = \int_0^1 u^{\mu-1}(1-u)^{\nu-1} du = \frac{\Gamma(\mu)\Gamma(\nu)}{\Gamma(\mu+\nu)}, \quad \mathrm{Re}\mu > 0, \quad \mathrm{Re}\nu > 0.$$

As a result, we find that in the ultraquantum limit the main term of the asymptotics of the power of synchrotron radiation of a massive electron in QED_{2+1} is determined by the formula

$$W = \frac{4mg^2}{9\sqrt{3}\pi} 3^{-\frac{1}{6}} \Gamma\left(\frac{5}{6}\right) \chi^{\frac{1}{3}} = \frac{4g^2}{9\sqrt{3}\pi} 3^{-\frac{1}{6}} \Gamma\left(\frac{5}{6}\right) (eHp_\perp)^{\frac{1}{3}}, \qquad \chi \gg 1. \qquad (3.25)$$

It follows from (3.25) that the result obtained does not depend on the electron mass. We also note that in the classical approximation the contributions of the first and second terms in the square bracket of the formula (3.3) to the total radiation power of a massive electron are proportional to the same parameter equal to χ.

Let us also calculate that part of the radiation power of an electron in P-even QED, which explicitly depends on the spin quantum number of the initial electron.

From formulas (2.10)–(2.14) under conditions (3.1) for that part of the total radiative shift of the electron energy, which depends on the orientation of the spin of the initial electron, we obtain the following formula:

$$\Delta E_n(\zeta) = \zeta \frac{mg^2}{8\pi^{\frac{3}{2}} p_\perp} e^{i\frac{\pi}{4}} \int_0^1 \frac{u-2}{1-u} g(z_0) du, \qquad (3.26)$$

where the function $g(z_0)$ is defined by formula (3.4). Using the Mellin transformation with respect to the parameter $a = \frac{1}{\chi}$ for the function $g(z_0)$, we find the integral representation for the part of the synchrotron radiation power that depends on the spin of the initial electron:

$$W(\zeta) = \zeta \frac{mg^2}{16\pi^{\frac{3}{2}}} \mathrm{Im}\left[e^{-i\frac{\pi}{2}} \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} 3^{-\frac{s}{2}+\frac{1}{2}} a^{-s} e^{-i\frac{\pi s}{2}} (s+2) \times \Gamma\left(\frac{3s}{2}\right) \Gamma\left(-\frac{s}{2}+\frac{1}{2}\right) \Gamma(s) \Gamma(2-s) ds \right],$$
$$0 < \gamma < \frac{2}{3}, \qquad (3.27)$$

where $\Gamma(z)$ is Euler's gamma function.

Closing the integration contour in formula (3.27) at $\chi \ll 1$ in the right half-plane, we obtain the following asymptotics for the quantity $W(\zeta)$ in the quasiquantum approximation:

$$W(\zeta) = -\zeta \frac{mg^2}{\pi\sqrt{3}} \chi^2, \qquad \chi \ll 1. \qquad (3.28)$$

IV. SYNCHROTRON RADIATION OF A MASSLESS CHARGED FERMION IN QED_{2+1}

The exact expression for the power of synchrotron radiation of a massless charged fermion is determined by the formula following from (2.15) and (2.16) at $m = 0$:

$$W(m=0) = -2\mathrm{Im}\left[\frac{g^2 p_\perp^2}{16\pi^{\frac{3}{2}}} e^{i\frac{\pi}{4}} \int_0^1 \sqrt{u}(1-u) du \int_0^\infty \frac{\Omega_0}{\sqrt{y} F} \times \exp[-ip_\perp^2 y(u-1) - 2in \arctan\lambda] dy \right], \qquad (4.1)$$

where

$$\Omega_0 = 2i \sin z e^{-iz} [1 - e^{2i\arctan\lambda}] - [e^{-2iz} + e^{2i\arctan\lambda}] + \frac{2}{F} e^{2i\arctan\lambda - 2iz}. \qquad (4.2)$$

Let us consider the case of strongly excited states of a fermion in the initial and final states ($n \gg 1, n' \gg 1, n - n' \gg 1$) and in a relatively weak magnetic field ($H \ll H_0$), when the integrals in formula (4.1) can be calculated by expanding the trigonometric functions in the integrand, depending on the quantity $z = eHy$, in power series $z \ll 1$.

As a result, we obtain the following formula for the power of synchrotron radiation of a massless charged fermion in QED_{2+1}:

$$W(m=0) = -\mathrm{Im}\left\{ \frac{g^2 p_\perp^2 e^{i\frac{\pi}{4}}}{3\pi^{\frac{3}{2}}\sqrt{eH}} \left(\frac{3}{2n}\right)^{\frac{5}{6}} \times \int_0^1 \frac{(1-\frac{2u}{3}) du}{u^{\frac{1}{3}}(1-u)^{\frac{2}{3}}} \int_0^\infty x^{-\frac{1}{6}} e^{-ix} dx \right\}. \qquad (4.3)$$

Further calculating the integral over the variable x using formula (3.9)

$$\int_0^\infty x^{-\frac{1}{6}} e^{-ix} dx = \Gamma\left(\frac{5}{6}\right) e^{-i\frac{5\pi}{12}}, \qquad (4.4)$$

we transform formula (4.3) to the form

$$W(m=0) = \frac{g^2 p_\perp^2}{6\pi^{\frac{3}{2}}\sqrt{eH}} \left(\frac{3}{2n}\right)^{\frac{5}{6}} \Gamma\left(\frac{5}{6}\right) \int_0^1 \frac{(1-\frac{2u}{3}) du}{u^{\frac{1}{3}}(1-u)^{\frac{2}{3}}}. \qquad (4.5)$$

The integral over the spectral variable has already been calculated in Sec. III and is equal to $\frac{8\pi}{9\sqrt{3}}$. Thus, the asymptotics of the total power of synchrotron radiation of a massless charged fermion in QED_{2+1} is defined by the formula

$$W_{2+1}(m=0) = \frac{4g^2}{9\sqrt{3}\pi} 3^{-\frac{1}{6}} \Gamma\left(\frac{5}{6}\right) (eHp_\perp)^{\frac{1}{3}}. \qquad (4.6)$$

This result coincides with the main term of the asymptotic expansion of the total power of synchrotron radiation of a massive relativistic fermion under conditions (3.1).

Formula (4.6) can be obtained from formula (3.3), as a result of the passage to the limit to zero mass of the fermion. Indeed, the passage to the limit $m \to 0$ corresponds to $\chi \sim \frac{1}{m^3} \to \infty, z_0 \sim m^2 = 0$; i.e., in the case of a massless fermion in (3.3), one should put

$$\frac{m}{\sqrt{z_0}} = \frac{(eHp_\perp)^{\frac{1}{3}}}{v^{\frac{1}{3}}}, \qquad g'(z_0) = g'(0),$$

$$m\sqrt{z_0}\mathrm{Im}(e^{i\frac{\pi}{4}}g_1(z)) \sim m^2\mathrm{Im}\left(e^{i\frac{\pi}{4}}\int_0^\infty g(t)dt\right) = 0. \qquad (4.7)$$

Using also formula (3.18), we arrive at the result (4.5) obtained directly from the exact formula (4.1).

V. CONCLUSION

In this paper, the influence of quantum effects on the intensity of synchrotron radiation of a massive fermion in P-even $(2+1)$-dimensional quantum electrodynamics with a doubled fermion representation is investigated for the first time. It was shown that, in $(2+1)$-dimensional spacetime, quantum theory predicts the finite values of the synchrotron radiation power of massive and massless charged fermions. Analytical formulas are obtained describing the dependences of the spectral density and total power of synchrotron radiation of a massive electron on the magnetic field strength, fermion energy and electron spin in the initial state.

The asymptotics (3.4), (3.5), and (3.23) of the functions $g(z_0)$, $g'(z_0)$, and $g_1(z_0)$ are found, which determine the dependence of the synchrotron radiation power of a massive relativistic electron at large and small values of the dynamic parameter. In the classical approximation, our formula (3.16) for the radiation power of a massive relativistic electron does not contain divergence and coincides with the results (4.21) and (3.15) in Refs. [28] and [1], respectively, previously obtained by various methods within the framework of two-dimensional classical electrodynamics.

The main term of the asymptotics of the radiation power of a massive relativistic electron in the ultraquantum limit determined by the formula (3.25) and increases with an increase in the dynamic parameter of synchrotron radiation proportionally to $\chi^{\frac{1}{3}}$. Note that in the standard, as well as in the reduced QED_{3+1}, at $\chi \gg 1$ the power of synchrotron radiation of scalar particles and electrons is proportional to $\chi^{\frac{2}{3}}$ [11,24,25].

The power of synchrotron radiation of a massless charged fermion is calculated in the case of large values of the main quantum number and a relatively weak magnetic field. The calculation was carried out based on the exact formula (4.1) and (4.2) for the synchrotron radiation power of a massless fermion, as well as on the basis of formula (3.3), obtained for a massive electron in the semiclassical approximation with subsequent transition to the zero fermion mass limit. It was shown that the main term of the asymptotic expansion of the radiation power of a massless charged fermion coincides with the leading term of the asymptotic expansion of the synchrotron radiation power of a massive electron in the ultraquantum case, which does not depend on the mass of the electron.

ACKNOWLEDGMENTS

The author expresses his gratitude to V. Ch. Zhukovskii, A. V. Borisov, and A. A. Zhukov for discussions of this study and to G. V. Kitaeva for assistance in preparing the manuscript.

[1] D. V. Gal'tsov and M. Khlopunov, Synchrotron radiation in odd dimensions, Phys. Rev. D **101**, 084054 (2020).
[2] V. A. Miransky and I. A. Shovkovy, Quantum field theory in a magnetic field: From quantum chromodynamics to graphene and Dirac semimetals, Phys. Rep. **576**, 1 (2015).
[3] E. J. Ferrer, V. de la Insera, and A. Sanchez, Non-perturbative Euler-Heisenberg Lagrangian and paraelectricity in magnetized massless QED, Nucl. Phys. **B864**, 469 (2012).
[4] D. V. Gal'tsov, Synchrotron radiation from massless charge, Phys. Lett. B **747**, 400 (2015).
[5] A. Neto, F. Guinea, N. M. Peres, K. S. Novoselov, and A. K. Geim, The electronic properties of graphene, Rev. Mod. Phys. **81**, 109 (2009).
[6] M. O. Goerbig, Electronic properties of graphene in a strong magnetic field, Rev. Mod. Phys. **83**, 1193 (2011).
[7] V. N. Kotov, B. Uchoa, V. M. Pereira, F. Guinea, and A. Neto, Electron-electron interactions in graphene: Current status and perspectives, Rev. Mod. Phys. **84**, 1067 (2012).
[8] N. M. R. Peres, Colloquium: The transport properties of graphene: An introduction, Rev. Mod. Phys. **82**, 2673 (2010).
[9] V. R. Khalilov, Radiative dynamical mass of planar charged fermion in a constant homogeneous magnetic field, Eur. Phys. J. C **79**, 196 (2019).
[10] V. R. Khalilov, On elastic scattering amplitude of planar charged fermions in a constant magnetic field, Int. J. Mod. Phys. A **34**, 1950189 (2019).
[11] P. A. Eminov, Radiative effects in two-dimensional models of quantum electrodynamics in a constant magnetic field, Phys. Rev. D **104**, 115020 (2021).

[12] F. Azzurli and K. Lechner, Electromagnetic fields and potentials generated by massless charged particles, Ann. Phys. (Amsterdam) **349**, 1 (2014).

[13] B. Kosyakov, Electromagnetic and gravitational radiation from massless particles, J. Phys. A **41**, 465401 (2008).

[14] R. Jackiw, D. Kabat, and M. Ortiz, Electromagnetic fields of a massless particle and the eikonal, Phys. Lett. B **277**, 148 (1992).

[15] N. Yokomizo, Radiation from electrons in graphene in strong electric field, Ann. Phys. (Amsterdam) **351**, 166 (2014).

[16] S.-Y. Xu, Discovery of a Weyl fermion semimetal and topological Fermi arcs, Science **349**, 613 (2015).

[17] E. S. Marino, A mass term for three-dimensional gauge fields, Nucl. Phys. **B408**, 551 (1993).

[18] E. V. Gorbar, V. P. Gusynin, and V. A. Miransky, Dynamical chiral symmetry breaking on a brane in reduced QED, Phys. Rev. D **64**, 105028 (2001).

[19] D. Valenzuela, S. Hernández-Ortiz, M. Loewe, and A. Raya, Graphene transparency in weak magnetic fields, J. Phys. A **48**, 065402 (2015).

[20] C. Villavicencio and A. Raya, Induced Bremsstrahlung by light in graphene, Rev. Mex. Fis. **64**, 132 (2018).

[21] P. A. Eminov, Anomalous magnetic moment of an electron in a magnetized plasma of topologically massive two-dimensional electrodynamics, Phys. Rev. D **95**, 075029 (2017).

[22] P. A. Eminov, Anomalous magnetic moment of an electron in a constant magnetic field in $(2+1)$-dimensional quantum electrodynamics, Phys. Rev. D **97**, 095019 (2018).

[23] I. M. Ternov, V. Ch. Zhukovskii, P. A. Eminov, and P. G. Midodashvili, Anomalous magnetic moment of the electron at finite temperature, Yad. Fiz. **43**, 764 (1986) [Sov. J. Nucl. Phys. **43**, 485 (1986)].

[24] A. A. Sokolov and I. M. Ternov, *Relativistic Electron* (Nauka, Moscow, 1983).

[25] V. I. Ritus, Problems of quantum electrodynamics of intense field, in *Proceedings of the P. N. Lebedev Physics Institute* (Nauka, Moscow, 1986), Vol. 168, pp. 52–120.

[26] K. V. Zhukovsky and P. A. Eminov, Polarization operator and amplitude of elastic photon scattering in $(2+1)$-dimensional QED in a constant magnetic field, Yad. Fiz. **59**, 1265 (1996) [Phys. At. Nucl. **59**, 1208 (1996)].

[27] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products*, edited by A. Jeffrey and D. Zwillinger (Academic Press, New York, 2013).

[28] E. Shuryak, H.-U. Yee, and I. Zahed, Self-force and synchrotron radiation in odd space-time dimensions, Phys. Rev. D **85**, 104007 (2012).