# *CP*-violating dark photon kinetic mixing and type-III seesaw model

Yu Cheng,[1,2,*] Xiao-Gang He,[1,2,3,†] Michael J. Ramsey-Musolf,[1,2,4,5,‡] and Jin Sun[1,2,§]

[1]*Tsung-Dao Lee Institute and School of Physics and Astronomy, Shanghai Jiao Tong University, Shanghai 200240, China*

[2]*Shanghai Key Laboratory for Particle Physics and Cosmology, Key Laboratory for Particle Astrophysics and Cosmology (MOE), Shanghai Jiao Tong University, Shanghai 200240, China*

[3]*Department of Physics, National Taiwan University, Taipei 10617, Taiwan*

[4]*Amherst Center for Fundamental Interactions, Department of Physics, University of Massachusetts, Amherst, Massachusetts 01003, USA*

[5]*Kellogg Radiation Laboratory, California Institute of Technology, Pasadena, California 91125, USA*

(Received 2 May 2021; revised 16 December 2021; accepted 1 April 2022; published 12 May 2022)

The hypothetical dark photon portal connecting the visible and dark sectors of the Universe has received considerable attention in recent years, with a focus on *CP*-conserving kinetic mixing between the Standard Model hypercharge gauge boson and a new $\mathrm{U}(1)_X$ gauge boson. In the effective field theory context, one may write nonrenormalizable *CP*-violating kinetic mixing interactions involving the $X$ and $\mathrm{SU}(2)_L$ gauge bosons. We construct for the first time a renormalizable model for *CP*-violating kinetic mixing that induces *CP*-violating non-Abelian kinetic mixing at mass dimension 5. The model grows out of the type-III seesaw model, with the lepton triplets containing right-handed neutrinos playing a crucial role in making the model renormalizable and providing a bridge to the origin of the neutrino mass. This scenario also accommodates electron electric dipole moments (EDM) as large as the current experimental bound, making future EDM searches an important probe of this scenario.

DOI: 10.1103/PhysRevD.105.095010

The dark sector in our Universe, for which dark matter and dark energy provide the primary evidence, remains largely unexplained. The dark sector may be well described by simple field content with primarily gravitational interactions. However, there are a plethora of theoretical proposals for a richer dark sector containing multiple particles and new interactions. These possibilities include interactions between the dark sector and the Standard Model (SM) of particle physics, referred to as portals. The most widely considered include the Higgs, axion, neutrino, and dark photon portals, each of which implies distinctive phenomenological consequences. Here we focus on the novel possibility of *CP*-violating (*CPV*) dark photon portal interactions.

To date, most portal studies—dark photon or otherwise—have focused on *CP*-conserving interactions. While of interest in their own right, new *CPV* interactions beyond those of the SM are also needed to explain the cosmic matter-antimatter asymmetry. In the case of the dark photon portal, it is straightforward to construct a *CP*-conserving portal. Indeed, it has long been realized that a dark photon gauge field $X_\mu$ associated with a beyond SM $\mathrm{U}(1)_X$ gauge group can mix with the $\mathrm{U}(1)_Y$ gauge field $B$ in the SM gauge group $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ through a renormalizable kinetic mixing term [1–3]: $X_{\mu\nu}B^{\mu\nu}$. One may write a *CPV* $\tilde{X}_{\mu\nu}B^{\alpha\beta}$ term. Here $X_{\mu\nu} = \partial_\mu X_\nu - \partial_\nu X_\mu$ and $\tilde{X}_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta}X^{\alpha\beta}/2$. However, this interaction has no physical effect at the perturbative level, because it can be written as a total derivative proportional to $\partial_\mu(\epsilon^{\mu\nu\alpha\beta}X_\nu\partial_\alpha B_\beta)$.

In extended models, a dark photon may also mix kinetically with the SM non-Abelian $\mathrm{SU}(2)_L$ gauge bosons [4–8]. In this context, it was recently shown that *CPV* kinetic mixing between a dark photon and SM gauge particles can arise [8]. If one includes a zero hypercharge $\mathrm{SU}(2)_L$ Higgs triplet, one may construct a (nonrenormalizable) dimension 5 operator containing such a term. Several interesting phenomenological consequences follow, notably a new *CPV* source for electric dipole moments (EDMs) of SM fermions, and possible collider signature in jet angular distributions.

However, a renormalizable model realization of this possibility has thus far been lacking. The presence of a nonrenormalizable interaction implies the existence of new

*chengyu@sjtu.edu.cn
†hexg@phys.ntu.edu.tw
‡mjrm@sjtu.edu.cn, mjrm@physics.umass.edu
§019072910096@sjtu.edu.cn

particles and interactions whose detailed nature is not evident from the structure of the low-energy effective operator alone. For example, existence of the well-known dimension 5 neutrino Majorana mass operator would imply nonconservation of the total lepton number at the classical level without revealing its fundamental origin. The construction and phenomenology of models (e.g., the seesaw mechanism) generating this interaction have attracted intensive theoretical interest over the years. In a similar spirit, we construct here for the first time a renormalizable model with $CPV$ kinetic mixing between a dark photon and $SU(2)_L$ gauge bosons and analyze the implications for future EDM searches. We also give general considerations for building such a model which may be realized in other constructions.

Two key minimal ingredients are needed for this purpose: (i) The first is the $SU(2)_L$ Higgs triplet [8] $\Sigma^a: (1,3)(0,0)$, where the brackets denote the transformation properties under $SU(3)_C \times SU(2)_L$ and $U(1)_Y \times U(1)_X$ gauge symmetries, respectively. The neutral component $\Sigma^0$ obtains a nonzero vacuum expectation value (VEV) $\langle \Sigma^0 \rangle = v_\Sigma$.[1] $\Sigma^a$ is needed so that the $SU(2)_L$ index "a" of the gauge triplet field $W^a$ can be contracted to form a gauge group singlet dimension 5 operator

$$\epsilon^{\alpha\beta\mu\nu} X_{\alpha\beta} W^a_{\mu\nu} \Sigma^a, \tag{1}$$

which is nonrenormalizable. (ii) The second ingredient is an introduction of new fields $f$ that, when integrated out, yield the interaction in Eq. (1). $f$ transforms as $(1,n)(0,x_f)$ and cannot be an $SU(2)_L$ singlet $n=1$ in order to mix $W$ and $X$. The source of $CPV$ depends on how $\Sigma$ interacts with $f$. *A priori*, the fields may be fermions or scalars.[2] However, $f$ cannot be a scalar since the tensor $\epsilon^{\alpha\beta\mu\nu}$ cannot arise from tree-level scalar exchanges built from renormalizable interactions or from scalar loops. It can, however, arise from loops containing a chiral fermion $f$ with $\gamma_5$ and a $CPV$ interaction appearing in the $f\bar{f}\Sigma$ couplings (see below for further discussion). Since $f$ is a chiral fermion, one needs to pay attention to make sure the model is gauge anomaly free. The minimal $SU(2)_L$ representation would be $n=2$. Stringent limits from fractionally charged particle searches [9] imply that the components of $f$ must have integer charges. To have integer electric charges for the components in $f$, the hypercharge for $f$ must be a half integer. In this case, the exchange of $f$ in the loop will generate not only $X-W$ but also $Y-X$ and $Y-W$ kinetic mixing terms. The simplest choice is actually $n=3$ with zero hypercharge. The components in $f$ have zero or $\pm 1$ electric charge. Taking the triplet to be right-handed, $f = f_R$, makes it possible to facilitate the type-III seesaw model [10] for neutrino mass generation by identifying the neutral component in $f_R$ as the heavy right-handed neutrino.

If one makes the theory supersymmetric, the lightest supersymmetric particle of the model may also provide a dark matter candidate. Here we will concentrate on the nonsupersymmetric model for purposes of simplicity and illustration. Other choices for the mediator particle content (satisfying the aforementioned criteria) may have distinctive phenomenological consequences and connections to other open problems in particle physics and cosmology. We will later see that our triplet mediator model will produce a fermion electric dipole moment as a unique signature for $CPV$ kinetic mixing.

We will assign one of the $f_R$'s, denoted $f_1$, to transform as $(1,3)(0,x_f)$. Since $f_1$ is a chiral field, one must include more than one such multiplet with different $x_f$ charges in order to ensure anomaly cancellation. To this end, we introduce a second $f_R$, $f_2: (1,3)(0,-x_f)$, whose contribution to the anomaly will cancel that from $f_1$. If future experiments indicate nonzero masses for all three light neutrinos (one massless neutrino is consistent with the present neutrino oscillation data), inclusion of a third $f_R$ would be necessary: $f_3: (1,3)(0,0)$, which does not generate a gauge anomaly. $f_3$ is inessential for our purposes. The component in $f_R$ can be written as

$$f_R = \frac{1}{2}\sigma^a f^a_R = \frac{1}{2}\begin{pmatrix} f^0_R & \sqrt{2} f^+_R \\ \sqrt{2} f^-_R & -f^0_R \end{pmatrix}, \tag{2}$$

and $f_L = f^c_R (f^+_L = (f^-_R)^c, f^0_L = (f^0_R)^c, f^-_L = (f^+_R)^c)$.

After integrating out the $f$ fields, the required dimension 5 operator is given by

$$\mathcal{L}_X = -(\tilde{\beta}_X/\Lambda)\mathrm{Tr}(W_{\mu\nu}\Sigma)\tilde{X}^{\mu\nu}. \tag{3}$$

Expanding it, we have

$$\mathcal{L}_X \to -\frac{\tilde{\beta}_X}{2\Lambda}\tilde{X}^{\mu\nu}[(s_W F_{\mu\nu} + c_W Z_{\mu\nu}) + ig(W^-_\mu W^+_\nu - W^+_\mu W^-_\nu)](v_\Sigma + \Sigma^0). \tag{4}$$

The same loop integral also generates the $CP$-conserving counterpart $-(\beta_X/\Lambda)\mathrm{Tr}(W_{\mu\nu}\Sigma)X^{\mu\nu}$, whose expanded form is obtained by replacing $\tilde{X}^{\mu\nu}$ with $X^{\mu\nu}$ in the above. Here we have normalized the fields as

$$W_\mu = \frac{1}{2}\begin{pmatrix} W^0_\mu & \sqrt{2}W^+_\mu \\ \sqrt{2}W^-_\mu & -W^0_\mu \end{pmatrix}, \quad \Sigma = \frac{1}{2}\begin{pmatrix} \Sigma^0 & \sqrt{2}\Sigma^+ \\ \sqrt{2}\Sigma^- & -\Sigma^0 \end{pmatrix}, \tag{5}$$

[1]By replacing $\Sigma^a$ with a composite triplet, such as $H^\dagger \tau^a H$ from the Higgs doublet H [6], a dimension 6 kinetic mixing operator can also be generated.

[2]One may also want to consider a vector boson running in the loop. Since we are working with renormalizable theory, a vector particle—if not a gauge particle—may complicate the model building. We will not venture into this possibility.

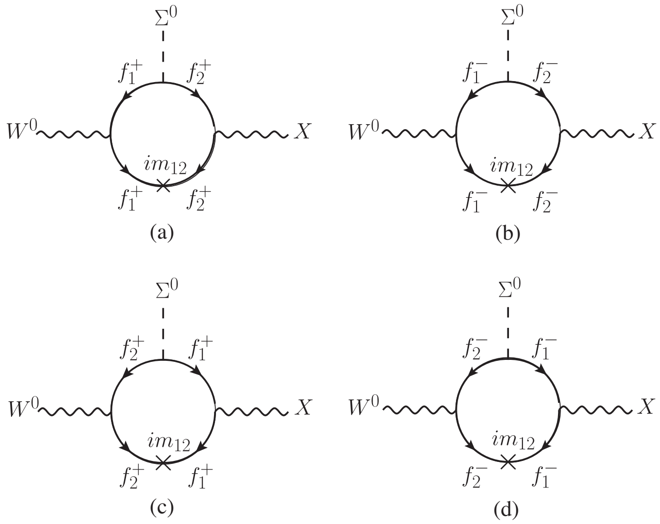


FIG. 1. The one-loop diagrams contributing to kinetic mixing.

where $W^0_\mu$ is a linear combination of the photon $A_\mu$ and the $Z$ field $Z_\mu$ with $W^0_\mu = \sin\theta_W A_\mu + \cos\theta_W Z_\mu$.

To make the dark photon mass $m_X$ nonzero, we introduce a scalar $S_X:(1,1)(0,-2x_f)$ with a VEV$\langle S_X\rangle = v_s/\sqrt{2}$. We obtain $m_X^2 = x_f^2 g_X^2 v_s^2$. It also contributes to the heavy neutrino masses.

We now discuss how to generate a nonzero $\tilde{\beta}_X$. The one-loop Feynman diagrams are shown in Fig. 1. The coupling of $\Sigma$ to $f$ is crucial to the model and is given by

$$4\mathrm{Tr}(\bar{f}^c_{Ri} Y_{f\sigma} \Sigma f_{Rj}) = i Y_{f\sigma} \bar{f}^a_{Li} \Sigma^b f^c_{Rj} \epsilon^{abc}. \tag{6}$$

The appearance of $\epsilon^{abc}$ requires more than one $f$. The couplings between $\Sigma_0$ and $f$ needed in Fig. 1 are given by

$$Y_{f\sigma 12}(\overline{(f^+_{R1})^c} f^+_{R2} - \overline{(f^-_{R1})^c} f^-_{R2})(v_\Sigma + \Sigma^0). \tag{7}$$

The other Yukawa coupling terms for the leptons and quarks responsible for the masses are given by

$$\begin{aligned}
&-\bar{L}_L Y_e \tilde{H} E_R - \bar{L}_L Y_{fL3} \tilde{H} f_{R3} - \bar{f}^c_{R1} Y_{fs1} S_X f_{R1},\\
&-\bar{f}^c_{R2} Y_{fs2} S_X^\dagger f_{R2} - \bar{f}^c_{R1} m_{12} f_{R2} - \bar{f}^c_{R3} m_{33} f_{R3},\\
&-\bar{Q}_L Y_u H U_R - \bar{Q}_L Y_d \tilde{H} D_R.
\end{aligned} \tag{8}$$

Since $f_{1,2}$ do not couple to $H$, the resulting Dirac neutrino mass matrix term $Y_{fL}$ is only of rank 1, which is not acceptable phenomenologically. This problem can be solved by introducing additional scalar $\mathrm{SU}(2)_L$ doublets $H_1':(1,2)(-1/2,-x_f)$ and $H_2':(1,2)(-1/2,x_f)$, whose VEVs are $v_1'/\sqrt{2}$ and $v_2'/\sqrt{2}$, respectively, so that $-\bar{L}_L Y_{fL1} H_1' f_{R1} - \bar{L}_L Y_{fL2} H_2' f_{R2}$ terms can be added. The usual electroweak scale constrains Higgs doublet VEVs since $v^2 + v_1'^2 + v_2'^2 = (246\ \mathrm{GeV})^2$, due to the bound from the $W$ boson mass. Among the VEVs, from a naturalness consideration, the VEV providing the top quark mass should be the largest one; therefore, $v'_{1,2}$ should be smaller than $v$. However, this does not mean that $v'_{1,2}$ need to be very small. They will contribute to neutrino masses, but the small neutrino masses are not due to the smallness of $v_{1,2}$, because the seesaw mechanism is in effect in our model. Lepton mass matrices are given by

$$\begin{aligned}
\mathcal{L}_m = &-\frac{1}{2}(\bar{\nu}_L, \bar{\nu}^c_R)\begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix}\begin{pmatrix} \nu^c_L \\ \nu_R \end{pmatrix}\\
&-(\bar{E}_L, \bar{f}_L)\begin{pmatrix} m_e & \sqrt{2}M_D \\ 0 & M_R \end{pmatrix}\begin{pmatrix} E_R \\ f_R \end{pmatrix},
\end{aligned} \tag{9}$$

where $m_e = Y_e v/\sqrt{2}$ is an arbitrary $3\times 3$ matrix. $M_D$ is a full $3\times 3$ matrix with the three column elements $Y_{fLi1} v_1'/\sqrt{2}$, $Y_{fLi2} v_2'/\sqrt{2}$, and $Y_{fLi3} v/\sqrt{2}$, respectively. $M_R$ is a symmetric $3\times 3$ matrix with nonzero entries $R_{11} = m_1 = Y_{fs1} v_s/\sqrt{2}$, $R_{22} = m_2 = Y_{fs2} v_s/\sqrt{2}$, $R_{12} = m_{12}$, and $R_{33} = m_{33}$. The structure of $M_R$ allows one to separate the seesaw scale represented by $\sim m_{33,12}$ from the dark $\mathrm{U}(1)_X$ breaking scale $\sim v_s$ so that $m_X$ can be smaller than the seesaw scale. Note that all entries in $M_R$ will break the lepton number by two units after $S_X$ develops a nonzero VEV. The largest element in $M_R$ will set the seesaw scale.

Expanding the kinetic Lagrangian terms for $f$, $\Sigma$ and $S_X$, $\mathcal{L}_K = 2\mathrm{Tr}(\bar{f}_{Rj} i\gamma_\mu D^\mu f_{Rj}) + 2\mathrm{Tr}((D_\mu\Sigma)^\dagger(D^\mu\Sigma)) + (D_\mu S_X)^\dagger(D^\mu S_X)$, we have $W^0$ and $X$ couplings with $f_R$ necessary for the remaining vertices in Fig. 1. The interaction Lagrangian $\mathcal{L}_{\mathrm{int}}$ is given by

$$\begin{aligned}
&g_X x_f[\mathrm{Tr}(\bar{f}_{R1}\gamma^\mu X_\mu f_{R1}) - \mathrm{Tr}(\bar{f}_{R2}\gamma^\mu X_\mu f_{R2})]\\
&\quad + g[\mathrm{Tr}(\bar{f}_{R1}\gamma^\mu W_\mu f_{R1}) + \mathrm{Tr}(\bar{f}_{R2}\gamma^\mu W_\mu f_{R2})].
\end{aligned} \tag{10}$$

Evaluating the diagrams in Fig. 1 yields

$$\begin{aligned}
M^{\mu\nu} = &\frac{i}{4\pi^2} g g_X x_f m_{12} Y^*_{f\sigma} \epsilon^{\mu\nu\alpha\beta} p_{X\alpha} p_{W\beta}\\
&\times (f(m_1, m_2, p_W, p_X) + f(m_2, m_1, p_W, p_X)),
\end{aligned} \tag{11}$$

where

$$\begin{aligned}
f(m_1, m_2, p_W, p_X) = &\int_0^1 dx \int_0^{1-x} dy (1-x-y)\\
&\times [D(m_1, m_2, p_W, p_X)\\
&+ D(m_2, m_1, p_X, p_W)],
\end{aligned} \tag{12}$$

with $D(m_1, m_2, p_W, p_X) = (m_1^2/(m_1^2 - m_2^2))/(m_1^2 - y(m_1^2 - m_2^2) - x p_W^2 - y p_X^2 + (x p_W - y p_X)^2)$.

Matching $\mathcal{L}_X$ in Eq. (4), $2\epsilon^{\mu\nu\alpha\beta} p_{X\alpha} p_{W\beta} \epsilon_{W\mu} \epsilon^*_{X\nu} \to -\tilde{X}^{\mu\nu} W^0_{\mu\nu}$ implies

$$\frac{\tilde{\beta}_X}{\Lambda} = \frac{1}{2\pi^2} g g_X x_f \mathrm{Im}(m_{12} Y^*_{f\sigma}) \times [f(m_1, m_2, p_W, p_X) + f(m_2, m_1, p_W, p_X)]. \quad (13)$$

The phase $\delta$ of $m_{12}Y^*_{f\sigma}$ is the $CPV$ source. $\beta_X/\Lambda$ is obtained by replacing $\mathrm{Im}(m_{12}Y^*_{f\sigma})$ in the above with $\mathrm{Re}(m_{12}Y^*_{f\sigma})$. The kinetic mixing in Eq. (4) corresponds to $p_W + p_X = 0$ and $q^2 = (p_W + p_X)^2 = 0$. Note that the presence of the $\epsilon^{\mu\nu\alpha\beta}$ in Eqs. (1) and (3) results from taking the trace of the fermion loop with a $\gamma_5$ type of coupling for $\Sigma$ to $f$ and the mass mixing term $m_{12}$. Loops containing scalars or vectorlike fermions would not yield this structure.

The $CP$-conserving kinetic mixing terms cause mixing of the gauge fields. We have the following relevant Lagrangian $\mathcal{L}_{KM}$ terms before writing the gauge fields in canonical form:

$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \frac{1}{2}\epsilon_{AX}F_{\mu\nu}X^{\mu\nu}, \\ -\frac{1}{2}\epsilon_{ZX}Z_{\mu\nu}X^{\mu\nu} + \frac{1}{2}m_Z^2 Z_\mu Z^\mu + \frac{1}{2}m_X^2 X_\mu X^\mu, \quad (14)$$

where $\epsilon_{AX} = \alpha_{XY}c_W + \beta_X s_W v_\Sigma/\Lambda$ and $\epsilon_{ZX} = -\alpha_{XY}s_W + \beta_X c_W v_\Sigma/\Lambda$, with $\alpha_{XY}$ defined by a possible $\mathrm{U}(1)_Y$ and $\mathrm{U}(1)_X$ kinetic mixing term $-(1/2)\alpha_{XY}X^{\mu\nu}B_{\mu\nu}$ which is independent of $\beta_X$. Here we use the $\mathrm{U}(1)_Y$ gauge field $B_\mu = \cos\theta_W A_\mu - \sin\theta_W Z_\mu$.

The mass eigenstates, the photon $A^m$, the $Z$ boson $Z^m$, and the dark photon $X^m$, to the leading order in small mixing parameters $\epsilon_{AX}$ and $\epsilon_{ZX}$ are related to the original gauge fields $A$, $Z$, and $X$ by

$$\begin{pmatrix} A \\ Z \\ X \end{pmatrix} = \begin{pmatrix} 1 & 0 & -\epsilon_{AX} \\ 0 & 1 & -\xi - \epsilon_{ZX} \\ 0 & \xi & 1 \end{pmatrix} \begin{pmatrix} A^m \\ Z^m \\ X^m \end{pmatrix}, \quad (15)$$

where $\xi$ is the angle describing $X$ and $Z$ mass mixing, $\xi \approx -m_Z^2\epsilon_{ZX}/(m_Z^2 - m_X^2)$. There is an enhancement for $\xi$ when $m_X$ is close to $m_Z$. The modifications for the interaction terms $J^\mu_{em}A_\mu$, $J^\mu_Z Z_\mu$, and $J^\mu_X X_\mu$ can be obtained by replacing the fields according to Eq. (15). Later we will drop the superscript $m$ on the mass eigenstate gauge fields.

We now discuss some related consequences. Before doing so, let us estimate how large $\tilde{\beta}_X$ might be from the data. For simplicity, we consider the nearly degenerate case $m_1 \approx m_2 \approx |m_{12}| \approx m \gg m_{W,X}$. In this limit, we have $\tilde{\beta}_X/\Lambda \approx g g_X x_f |Y^*_{f\sigma}| \sin\delta/6\pi^2 m$. The size of $\tilde{\beta}_X/\Lambda$ is governed by the seesaw mass scale represented by elements in $M_R$. The size of $v_\Sigma$ is also important, as can be seen in Eq. (4), which is constrained by $\rho = 1.00038 \pm 0.00020$ from electroweak precision tests [9]. A nonzero $v_\Sigma$ would modify $\rho$ from 1 in SM to $1 + 4v_\Sigma^2/v^2$. Therefore, the data imply that $v_\Sigma < 3$ GeV at the $2\sigma$ level. Both the ATLAS and CMS experiments at the LHC have carried searches for heavy fermions in the type-III seesaw model and found the mass to be larger than 790 GeV [11] (880 GeV [12]) at 95% C.L. The seesaw scale should be above this limit. For the purpose of illustration, allowing both $g_X x_f$ and $Y_{f\sigma}$ to be as large as their perturbative unitarity bound with approximately $\sqrt{4\pi}$, we obtain at 95% C.L. an upper bound $5\sin\delta \times 10^{-4}$ for $\tilde{\beta}_X v_\Sigma/\Lambda$. The bound for $\beta_X$ is obtained by replacing $\sin\delta$ with $\cos\delta$.

In Ref. [8] two interesting $CP$-violating effects due to $CP$-violating kinetic mixing were identified, the EDMs of SM fermions and a collider signature in dijet angular distribution asymmetry. The latter has a large background and is difficult to observe. We find that the jet analysis is again difficult to probe in our model, but the effects on electron EDM can be dramatic and provide a good test for the model [13–15].

The EDM $d_F$ for a SM fermion $F$ may be induced by the three different contributions shown in Fig. 2. The contribution from Fig. 2(a) arises in all model constructions since it contains the contribution from the dimension 5 operator [Eq. (1)]. This was studied in Ref. [8]. In the present context, it is actually a two-loop contribution since $\tilde{\beta}_X$ is generated at the one-loop level. Figure 2(b) first generates an $X$ EDM (by replacing external $A$ with $X$). As seen from Eq. (15) there is no mixing for $X$ to have the final mass eigenstate of the photon $A$ for $q^2 = 0$, so no ordinary EDM for $F$ can be generated. But a weak EDM $d_F^W$ for a fermion through $X$ mixing with $Z$ will be produced. This, however, provides only a very weak constraint [9]. The third contribution is the Barr-Zee diagram [16] shown in Fig. 2(c). In the diagram, the loop with two photons attached is obtained by replacing $X$ with $W^0$ in Fig. 1. One finds that the contributions from Figs. 1(a) and 1(c) are canceled by those from Figs. 1(b) and 1(d) due to antisymmetric $\Sigma$-$f$-$f$ coupling in fermion space. This is a generic property of this type of model for higher representations which produce a antisymmetric $\Sigma$-$f$-$f$ coupling. Had one chosen a different representation such as $n = 2$ for $f$, the contribution from Fig. 2(c) would not vanish. Its effect would dominate fermion EDM, as shown in Ref. [16], making the test of the $CPV$ kinetic mixing signature difficult. This makes our model with $f$ as a triplet unique from the experimental testing of $CP$ violation in kinetic mixing.

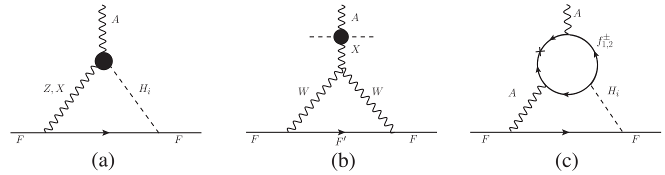


FIG. 2. Different contributions to fermion EDMs.

Therefore, the dominant contribution for $d_F$ is from Fig. 2(a). This expression was given in Eq. (11) of Ref. [8]. Since there are several Higgs doublets $H_i$, one should replace the mixing $s_\theta c_\theta f(r_{ZH_1}, r_{ZH_2})$ and $s_\theta c_\theta f(r_{XH_1}, r_{XH_2})$ by $\Sigma_{i=[1,N-1]} V_{\Sigma i} V_{hi} f(r_{ZH_i}, r_{ZH_\Sigma})$ and $\Sigma_{i=[1,N-1]} V_{\Sigma i} V_{hi} f(r_{XH_i}, r_{XH_\Sigma})$, with $N = 5$. Here the summation sums over all mass eigenstates $H_i$ where the original scalar states $\Sigma^0$ and the neutral real component $h$ in $H$ are expressed as linear combinations of $H_i$, $\Sigma^0 = \sum_i V_{\Sigma i} H_i$, and $h = \sum_i V_{hi} H_i$. The formula in Eq. (11) of Ref. [8] assumed that the *CPV* source comes from $\tilde{\beta}$, which is proportional to the invariant phase from $m_{12} Y^*_{f\sigma}$. One may also wonder whether *CPV* exists in the vertex $H_i$ to $F$, that is, if $V_{\Sigma i} V_{hi}$, which depends on whether the *CPV* phase in the Higgs potential mixes different neutral components from $H_i$, $\Sigma$, and $S_X$. We have checked in detail to see that there is no *CPV* in the Higgs potential with the quantum numbers assigned to the Higgs bosons.

The electron EDM $d_e$ prediction is constrained directly from experimental bounds, assuming a "sole source" analysis of the polar molecule system [15]. For the neutron EDM, we use $d_n = -0.233 d_u + 0.774 d_d + 0.008 d_s$, which was obtained in Ref. [17]. For illustration, we assume that the $V_{\Sigma 1} V_{h1}$ term dominates the contribution with some benchmark values for the heavy $m_{H_\Sigma}$ and $m_X = 60$ GeV and kinetic mixing parameters. In the model, $m_{H_\Sigma}$ should be below the largest scale—the seesaw scale—whose lower limit set by the LHC [11,12] is about 900 GeV. The 13 TeV LHC also excludes a real triplet lighter than 275 (248) GeV for a range of parameter space [18]. We therefore take a benchmark range of 300–600 GeV for $m_{H_\Sigma}$. The dependence on $m_X$ is weak except near $m_Z$. Since $m_{H_1}$ is the SM-like Higgs boson, its mass is assumed to be 125 GeV. For the mixing parameters, we consider two cases to show the details. In case I, the kinetic mixing parameters are all generated from our loop calculations, that is, $\alpha_{XY} = 0$. To maximize the contribution to the EDMs, we choose $\sin\delta = \cos\delta = 1/\sqrt{2}$. In case II, we choose a large but allowed $\alpha_{XY}$ to be $10^{-2}$ and set $\tilde{\beta}_X/\Lambda$ to the maximal allowed value.

We show the results in Fig. 3. The splittings between $d_e$ and $d_n$ in cases I and II behave differently because, in case II, the addition of $\alpha_{XY}$ in $\beta$ modifies $d_e$ and $d_n$ differently. Note that for both cases I and II, the EDM is larger for a larger $m_{H_\Sigma}$ because the first term in $f(x, y)$ defined in Ref. [8] has an $\ln(m^2_{H_\Sigma}/m^2_h)$ term increasing with $m_{H_\Sigma}$. However, it will not increase indefinitely with $m_{H_\Sigma}$, because the mixing parameter $V_{\Sigma 1} V_{h1}$ will vanish in that limit. The current neutron EDM does not constrain the parameters in either case. But with improved sensitivity, such as that of the nEDM experiment at the Spallation Neutron Source [19] or the n2EDM experiment at the Paul Scherrer Institute [20], where a sensitivity of $10^{-28}$ $e$ cm can be reached, case II can be tested. The test of case I is more challenging. But a proposed measurement for proton EDM $d_p$ using a proton storage ring can reach a sensitivity of $10^{-29}$ $e$ cm [21] which can start to put constraints on case I, whereas for $d_e$, the current limit already excludes a certain parameter space. Even for a $V_{\Sigma 1} V_{h1}$ as small as $10^{-2}$, case II can reach a current bound. For case I, to reach a current limit of $d_e$, $V_{\Sigma 1} V_{h1}$ needs to be close to maximal, which is unlikely. But proposed new experiments will further improve the sensitivity to $O(10^{-30})$ $e$ cm [22]. The model can be very well tested.

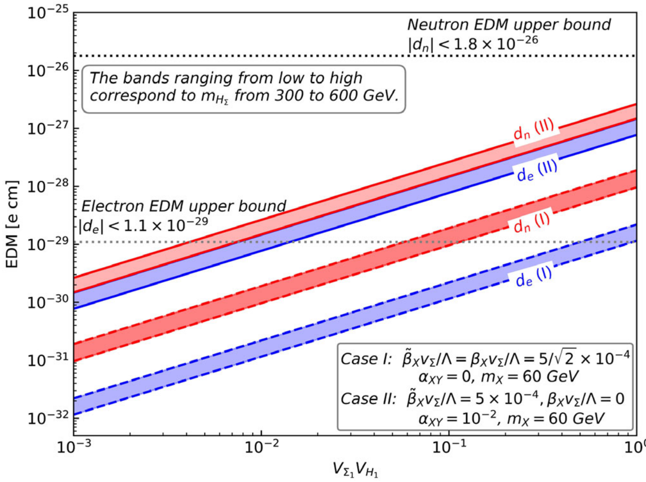


FIG. 3. Ranges for $d_n$ and $d_e$ for cases I and II. The 90% C.L. upper limits for electron and neutron EDMs are from Refs. [23,24], respectively.

As discussed in Ref. [7], collider studies may probe several other aspects of the model since the new particle masses are $\mathcal{O}$(TeV). A particularly interesting signature involves production of one or two triplet scalars, leading ultimately to a pair of displaced lepton jets in conjunction with one or more prompt objects. The specific final states and corresponding branching ratios can provide information on $\beta_X/\Lambda$, whereas the EDM is sensitive to $\tilde{\beta}_X/\Lambda$. Direct production of the $f$ particles, together with a determination of $m_X$ (e.g., via measurement of the lepton jet invariant mass) and the decay length can provide complementary information. Discovery of these properties, in combination with a large electron EDM, would provide strong evidence for the triplet $f$ model. We would also like to point out that the model's new *CPV* phase $\delta$, together with the extended scalar sector potential that could accommodate a first order electroweak phase transition [25–29], may provide the ingredients needed to generate the cosmic baryon asymmetry via electroweak baryogenesis. We will investigate these possibilities in future work.

This work was supported in part by the Key Laboratory for Particle Physics, Astrophysics and Cosmology, the Ministry of Education, and the Shanghai Key Laboratory for Particle Physics and Cosmology (Grant

No. 15DZ2272100), and in part by the NSFC (Grants No. 11735010, No. 11975149, No. 12090064, and No. 19Z103010239). X.-G. H. was supported in part by the MOST (Grant No. MOST 106-2112-M-002-003-MY3). M. J. R.-M. was also supported in part by U.S. Department of Energy Contract No. DE-SC0011095.

[1] P. Galison and A. Manohar, Phys. Lett. **136B**, 279 (1984).
[2] B. Holdom, Phys. Lett. **166B**, 196 (1986).
[3] R. Foot and X. G. He, Phys. Lett. B **267**, 509 (1991).
[4] F. Chen, J. M. Cline, and A. R. Frey, Phys. Rev. D **79**, 063530 (2009).
[5] F. Chen, J. M. Cline, and A. R. Frey, Phys. Rev. D **80**, 083516 (2009).
[6] G. Barello, S. Chang, and C. A. Newby, Phys. Rev. D **94**, 055018 (2016).
[7] C. A. Argüelles, X. G. He, G. Ovanesyan, T. Peng, and M. J. Ramsey-Musolf, Phys. Lett. B **770**, 101 (2017).
[8] K. Fuyuto, X. G. He, G. Li, and M. Ramsey-Musolf, Phys. Rev. D **101**, 075016 (2020).
[9] P. A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020).
[10] R. Foot, H. Lew, X.-G. He, and G. Joshi, Z. Phys. C **44**, 441 (1989).
[11] G. Aad *et al.* (ATLAS Collaboration), Eur. Phys. J. C **81**, 218 (2021).
[12] A. M. Sirunyan *et al.* (CMS Collaboration), J. High Energy Phys. 03 (2020) 051.
[13] X-G He, B. McKellar, and S. Pakvasa, Int. J. Mod. Phys. A **04**, 5011 (1989); **06**, 1063(E) (1991).
[14] J. Engel, M. J. Ramsey-Musolf, and U. van Kolck, Prog. Part. Nucl. Phys. **71**, 21 (2013).
[15] T. E. Chupp, P. Fierlinger, M. J. Ramsey-Musolf, and J. T. Singh, Rev. Mod. Phys. **91**, 015001 (2019).
[16] S. M. Barr and A. Zee, Phys. Rev. Lett. **65**, 21 (1990).
[17] T. Bhattacharya, V. Cirigliano, S. D. Cohen, R. Gupta, A. Joseph, H. W. Lin, and B. Yoon, Phys. Rev. D **92**, 094511 (2015).
[18] C. W. Chiang, G. Cottin, Y. Du, K. Fuyuto, and M. J. Ramsey-Musolf, J. High Energy Phys. 01 (2021) 198.
[19] K. K. H. Leung *et al.*, EPJ Web Conf. **219**, 02005 (2019).
[20] N. J. Ayres *et al.* (n2EDM Collaboration), Eur. Phys. J. C **81**, 512 (2021).
[21] V. Anastassopoulos *et al.*, Rev. Sci. Instrum. **87**, 115116 (2016).
[22] D. M. Kara, I. J. Smallman, J. J. Hudson, B. E. Sauer, M. R. Tarbutt, and E. A. Hinds, New J. Phys. **14**, 103051 (2012).
[23] V. Andreev *et al.* (ACME Collaboration), Nature (London) **562**, 355 (2018).
[24] C. Abel *et al.* (nEDM Collaboration), Phys. Rev. Lett. **124**, 081803 (2020).
[25] H. H. Patel and M. J. Ramsey-Musolf, Phys. Rev. D **88**, 035013 (2013).
[26] N. Blinov, J. Kozaczuk, D. E. Morrissey, and C. Tamarit, Phys. Rev. D **92**, 035012 (2015).
[27] S. Inoue, G. Ovanesyan, and M. J. Ramsey-Musolf, Phys. Rev. D **93**, 015013 (2016).
[28] L. Niemi, H. H. Patel, M. J. Ramsey-Musolf, T. V. I. Tenkanen, and D. J. Weir, Phys. Rev. D **100**, 035002 (2019).
[29] L. Niemi, M. J. Ramsey-Musolf, T. V. I. Tenkanen, and D. J. Weir, Phys. Rev. Lett. **126**, 171802 (2021).