Investigation of $\Delta^0\Delta^0$ dibaryon in QCD

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We study a hypothetical $\Delta^0\Delta^0$ dibaryon state by means of the QCD sum rule method. We construct a scalar interpolating current to extract the mass and decay constant of this state taking into account contributions of various quark, gluon, and mixed vacuum condensates. The predictions for the mass and decay constants are $m_D = 2326^{+114}_{-126}$ MeV and $f_D = 2.94^{+0.30}_{-0.34} \times 10^{-4}$ GeV^7, respectively. We also made an estimation about binding energy and size of the $\Delta^0\Delta^0$ dibaryon. The obtained mass value is below the $\Delta^0\Delta^0$ threshold and may be a good dibaryon candidate for being observed experimentally.

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I. INTRODUCTION

The possibility of multiquark states had been argued in the early days of the quark model [1] in which multiquark states such as tetraquarks ($qq\bar{q}\bar{q}$) and pentaquarks ($qqqq\bar{q}$) were proposed apart from the conventional meson ($q\bar{q}$) and baryon (qqq, $\bar{q}\bar{q}\bar{q}$) states. In the past two decades, experimental observation of the so-called "exotic" hidden-charm XYZ tetraquark and P_c pentaquark states [2–11] opened a new window in the quark model.

Another type of multiquark system is dibaryon or hexaquark. A dibaryon consists of two baryons. Deuteron is a well-known example of dibaryon. A recent dibaryon candidate $d^*(2380)$ was observed at COSY [12–15]. This state was first predicted on the basis of SU(6) symmetry by Dyson and Xuong in 1964 [16], although its existence was challenged by some studies (see for instance Refs. [17,18]). Another dibaryon structure was proposed by Jaffe [19]. This dibaryon was assumed to have $uuddss$ quark content and named dihyperon or the preferred H-dibaryon. The mass and stability of H-dibaryon were investigated in several works followed by Jaffe's paper such as the MIT bag model [20], the chiral model [21], the quark cluster model [22], the solitons in chiral quark model [23], the nonrelativistic quark model [24], nonrelativistic quark cluster model [25,26], QCD sum rule [27,28], and lattice QCD [29]. The results of these studies are controversial; mass results change from 1.1 GeV to 2.2 GeV. Furthermore, some of these studies pointed out that $q_1q_1q_2q_2q_3q_3$ type dibaryons are most favorable for being stable. These dibaryon states were studied by means of stability in [30,31]. Reference [30] used a simple chromomagnetic model to study dibaryon states. No stable dibaryon state is found but it is mentioned that some dibaryons can be seen as resonances in certain channels. On the other hand, a QCD inspired string model calculation showed that the ground state of a six-quark (q^6) state is stable against dissociation into two baryons [31]. In a recent study, spectroscopic parameters of $S = uuddss$ hexaquark were calculated, and its candidacy for dark matter was argued [32]. Reference [33] used the extension of the Gürsey-Radicati mass formula to obtain some nonstrange dibaryon mass spectrum.

The strangest dibaryon $\Omega\Omega$ was studied by lattice QCD and was found to be located near the unitary regime [34]. This lattice QCD simulation triggered further studies. In Ref. [35], momentum correlation functions of $\Omega\Omega$ and $p\Omega$ dibaryons are investigated via constructing correlation function by single-particle distributions. Quark delocalization color screening and chiral quark models are used to reanalyze $\Omega\Omega$ dibaryon states with $J^P = 0^+, 1^-, 2^+, 3^-$ quantum numbers and $J^P = 0^+$ is found to be bound [36]. $\Omega\Omega$ dibaryon states with $J^P = 0^+$ and $J^P = 2^+$ are investigated in a molecular picture by using the QCD sum rule [37]. Evidence is found for a strange $S = -1$ ΣN dibaryon bound state in the ΛN cross section [38]. The $N\Omega$

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system was studied in the $^5S_2$ channel by lattice QCD, and a quasibound state was predicted near unitarity [39]. The possible $N\Omega_{ccc}$ and $N\Omega_{bbb}$ dibaryons are studied in the quark delocalization color screening model [40]. Their results showed that these states are bound. $N\Omega$ systems in $^3S_1$ and $^5S_2$ channels with quantum numbers of $J^P = 1^+$ and $J^P = 2^+$ are studied in the molecular picture by the QCD sum rule [41]. They found that $N\Omega$ dibaryon may exist in the $^5S_2$ channel. In Ref. [42], the S-wave $\Lambda\Lambda$ and $N\Xi$ systems are studied by lattice QCD. It was found that the $\Lambda\Lambda$ interaction at low energies resulted in a weak interaction, whereas the $N\Xi$ interaction leaded the $N\Xi$ system near unitarity.

Dibaryons with two or more heavy quark contents are also being studied. In Ref. [43], the doubly charmed scalar $uuddcc$ hexaquark state is studied by the QCD sum rule method and $6.60^{+0.12}_{-0.09}$ GeV mass value is obtained. The spectra of the hidden-bottom and hidden-charm hexaquark states were investigated in the molecular picture via the QCD sum rule [44]. Their results indicate that 0^{++} and 1^{--} states in the b quark sector are possible baryonium states. The triply charmed $\Xi_{cc}\Sigma_c$ dibaryon states were also studied via the QCD sum rule by considering dibaryon and two-baryon scattering states [45]. $\Sigma_c\Xi_{cc}$, $\Omega_c\Omega_{cc}$, $\Sigma_b\Xi_{bb}$, $\Omega_b\Omega_{bb}$, and $\Omega_{ccb}\Omega_{cbb}$ particles were studied by lattice QCD formalism [46]. Among these states, evidence for the existence of $\Sigma_c\Xi_{cc}$ and $\Sigma_b\Xi_{bb}$ dibaryons could not be obtained, whereas for the other dibaryons their presence is suggested. Triply charmed $\Xi_{cc}\Sigma_c$ dibaryon states were investigated in the one-boson exchange approach [47]. A number of triply charmed and triply bottomed states of isospins $1/2$ and $3/2$ are found to be bound. In Ref. [48], the axial vector triply charmed hexaquark state is studied with the QCD sum rule method. The possibility of the $bbbccc$ dibaryon was investigated by a constituent quark model [49]. They observed that there is no evidence for any stable type of $bbbccc$ dibaryon. The existence of fully heavy dibaryons $\Omega_{ccc}\Omega_{bbb}$, $\Omega_{ccc}\Omega_{ccc}$, and $\Omega_{bbb}\Omega_{bbb}$ with $J = 0, 1, 2, 3$ and $P = 1$ quantum numbers is investigated in the constituent quark model [50]. Their results showed that all-heavy b or c dibaryons of $J^P = 0^+$ are possible to be bound, whereas $cccbbb$ configuration is not. Scattering properties of the $\Omega_{ccc}\Omega_{ccc}$ dibaryon system were studied by the lattice QCD method and found to be located in the unitarity regime [51]. Low-lying doubly heavy dibaryon systems with strangeness $S = 0$ and various quantum numbers were studied in the quark delocalization color screening framework [52]. They found three bound state candidates for doubly charm and doubly bottom dibaryon systems. In Ref. [53], the prediction of the $\Omega_{bbb}\Omega_{bbb}$ dibaryon state was studied in the extended one-boson exchange model and yielded a possible $\Omega_{ccc}\Omega_{ccc}$ dibaryon state.

In addition to dibaryon configurations mentioned above, according to quantum chromodynamics (QCD), many other dibaryon structures are possible. The theory of QCD and its Lagrangian was first proposed by Fritzsch and Gell-Mann in 1972 [54]. In principle, besides the dynamics of quarks and gluons, this Lagrangian should be responsible for hadrons and the determination of their properties. Unfortunately, it is valid only in a limited region. Today, QCD is widely accepted as a true theory of the strong interactions, despite the fact that it is not directly applicable at the low-energy region. Hadron-hadron interactions are prototypes in understanding QCD as a fundamental theory of strong interactions. However, for hadron-hadron interactions and exotic systems, using QCD directly to probe the low-energy region is difficult due to the nonperturbative aspects. Therefore some non-perturbative methods are required. QCD sum rule, lattice QCD, chiral perturbation theory, as well as quark model and effective field theories are in play to study the multi-quark systems such as tetraquarks and pentaquarks from the physical perspective. Among these nonperturbative methods, the QCD sum rule is a powerful nonperturbative tool to study many hadronic phenomena. The QCD sum rule method is based on the QCD Lagrangian which is the main advantage of the method. It is relativistic and does not include any free parameter.

In this present study, we compute mass and decay constant of $\Delta^0\Delta^0$ dibaryon, which is a possible dibaryon candidate in the light quark sector by using the QCD sum rule method. This possible dibaryon state was studied via the extended chiral SU(3) quark model [55], the quark delocalization color screening and chiral quark models [56], and the three-body interaction model [57]. To the best of our knowledge, there is no reported QCD sum rule study for this dibaryon.

This work is arranged as follows: in Sec. II we briefly introduce the QCD sum rule method and obtain sum rules for spectroscopic parameters of the $\Delta^0\Delta^0$ dibaryon. The calculations were done by carrying out various vacuum condensates. In Sec. III, we present numerical results and discussions. Section IV is reserved for our concluding remarks.

II. METHODOLOGY

The QCD sum rule (QCDSR) is a semiphenomenological framework to extract spectroscopic information from the QCD Lagrangian. The method was first proposed for mesons in [58,59] and later generalized to baryons [60]. One can relate the hadron spectrum to the QCD Lagrangian in this method. QCDSR handles the bound state problem by starting to study short distance relations and then moving to large distance relations by including nonperturbative effects of the QCD vacuum. This is done via an appropriate correlation function.

QCDSR formalism handles the corresponding state by starting from short distances (large q^2) where asymptotic freedom makes perturbative calculation doable and moving to large distances (low q^2) where hadronic states are formed due to the nonperturbative interactions. Provided that there is some interval in q, which is the fundamental hypothesis of the QCDSR formalism, where both representations overlap, one can obtain information about hadronic properties. Masses and decay constants (residues) of the hadrons are generally investigated within the two-point correlation functions, whereas the strong coupling constants and form factors of different interactions are studied within three-point or light-cone correlation functions.

In order to obtain sum rules for the mass and decay constant, we study the two-point QCDSR formalism. The method focuses on the correlation function

$$\Pi(p) = i\int d^4x e^{ipx}\langle 0|\mathcal{T}\{J(x)J^\dagger(0)\}|0\rangle \tag{1}$$

and obtains sum rules to compute physical properties of the related state. Here, $\mathcal{T}$ is the time order operator and $J(x)$ is the corresponding interpolating current. To study the $\Delta^0\Delta^0$ dibaryon in the framework of QCDSR, we need to construct the interpolating current which is the main ingredient of the analysis. We construct $\Delta^0\Delta^0$ interpolating current by using Ioffe current for the Δ^0 baryon as

$$\eta_\mu = \frac{1}{\sqrt{3}}\varepsilon^{abc}\left[2(d^{aT}C\gamma_\mu u^b)d^{cT} + (d^{aT}C\gamma_\mu d^b)u^c\right], \tag{2}$$

where C is the charge conjugation operator and a, b, c are color indices. Then, the $J^P = 0^+$ interpolating current for $\Delta^0\Delta^0$ dibaryon can be constructed as

$$J(x) = \eta_\mu \cdot C\gamma_5 \cdot \eta^\mu. \tag{3}$$

There are two paths to calculate the correlation function in the standard prescriptions of the QCDSR method. At the first path which is called the phenomenological side, the correlation function is being calculated by inserting intermediate hadronic states that have the same quantum numbers of interpolating current $J(x)$. At this step, the correlation function is represented by the hadronic degrees of freedom. To do this, we express the correlation function [Eq. (1)] with the help of the $\Delta^0\Delta^0$ dibaryon mass m_D, coupling f_D, and its matrix element

$$\langle 0|J|D\rangle = m_D f_D. \tag{4}$$

The ground state contribution can be isolated from the contributions of higher states and continuum for $\Pi(p)$. Then we obtain

$$\Pi^{\rm Phen}(p) = \frac{\langle 0|J|D(p)\rangle\langle D(p)|J^\dagger|0\rangle}{m_D^2 - p^2} + \cdots. \tag{5}$$

With the definition of the matrix element [Eq. (4)], we can rewrite $\Pi^{\rm Phen}(p)$ as

$$\Pi^{\rm Phen}(p) = \frac{m_D^2 f_D^2}{m_D^2 - p^2} + \cdots, \tag{6}$$

where $\cdots$ represents contributions of higher states and continuum.

In the second step, calculation of the correlation function [Eq. (1)] proceeds by operator product expansion (OPE). This representation is called OPE or the QCD side of the correlation function. We insert the interpolating current $J(x)$ which is composed of the quark fields into the correlation function and contract all the quark fields with the help of Wick theorem. As a result we obtain

$$\begin{aligned}\Pi^{\rm QCD}(p) = \frac{16}{9}\varepsilon^{abc}\varepsilon^{def}\varepsilon^{a'b'c'}\varepsilon^{d'e'f'}\int d^4x e^{ipx}\\ \times\{{\rm Tr}\ [\gamma_\mu S_u^{bb'}(x)\gamma_\nu \tilde{S}_d^{a'a}(-x)]\\ \times {\rm Tr}\ [\gamma_\mu S_u^{ee'}(x)\gamma_\nu \tilde{S}_d^{d'd}(-x)]\\ \times {\rm Tr}\ [\gamma_5 S_d^{ff'}(x)\gamma_5 \tilde{S}_d^{cc'}(-x)]\}\\ + \text{many similar terms},\end{aligned} \tag{7}$$

where $\tilde{S}(x) = CS^T(x)C$. To proceed in the QCD side, the light u and d quarks propagators are used:

$$\begin{aligned}S_q^{ab}(x) = {}& i\frac{\not{x}}{2\pi^2x^4}\delta_{ab} - \frac{m_q}{4\pi^2x^2}\delta_{ab} - \frac{\langle\bar{q}q\rangle}{12}\left(1 - i\frac{m_q}{4}\not{x}\right)\delta_{ab}\\ & - \frac{x^2}{192}\langle\bar{q}g_s\sigma Gq\rangle\left(1 - i\frac{m_q}{6}\not{x}\right)\delta_{ab}\\ & - \frac{ig_sG_{ab}^{\mu\nu}}{32\pi^2x^2}[\not{x}\sigma_{\mu\nu} + \sigma_{\mu\nu}\not{x}] - \frac{\not{x}x^2g_s^2}{7776}\langle\bar{q}q\rangle^2\delta_{ab}\\ & - \frac{x^4\langle\bar{q}q\rangle\langle g_s^2G^2\rangle}{27648}\delta_{ab}\\ & + \frac{m_qg_s}{32\pi^2}G_{ab}^{\mu\nu}\sigma_{\mu\nu}\left[\ln\left(\frac{-x^2\Lambda^2}{4}\right) + 2\gamma_E\right] + \cdots,\end{aligned} \tag{8}$$

where $q = u, d, \gamma_E \simeq 0.577$ is the Euler constant, Λ is a scale parameter, $G_{ab}^{\mu\nu} \equiv G_A^{\mu\nu}t_{ab}^A, A = 1, 2, \ldots, 8,$ and $t^A = \lambda^A/2$, with λ^A being the Gell-Mann matrices.

We shall note that we use the factorization approximation for the higher dimension operators. The central idea behind the factorization hypothesis is replacing vacuum expectation values of higher dimensional operators with the products of the lower dimensional ones. This is one of the key sources of uncertainties in the QCDSR approach. For example the four-quark condensate ($d = 6$) $\langle\bar{q}q\bar{q}q\rangle$ and quark condensate times mixed condensate ($d = 8$) $\langle\bar{q}q\bar{q}g\sigma\cdot Gq\rangle$ can be factorized as

$$\begin{aligned}\langle\bar{q}q\bar{q}q\rangle &= \rho\langle\bar{q}q\rangle^2,\\ \langle\bar{q}q\bar{q}g\sigma\cdot Gq\rangle &= \rho\langle\bar{q}q\rangle\langle\bar{q}g\sigma\cdot Gq\rangle,\end{aligned} \tag{9}$$

where ρ is a parameter and account deviations from the hypothesis. $\rho = 1$ gives the vacuum saturation, and $\rho = 2.1$

indicates the violation of the factorization hypothesis [61–64]. It was shown in Ref. [65] that $d = 8$ condensate under the factorization hypothesis is almost negligible by using a molecular current couple to $D_s^* D_{s0}^*$. The effect of nonfactorized diagrams is found to be small for the doubly hidden 0^{++} molecule and tetraquark states in Ref. [66]. The effect of factorization in a sum rule calculation at leading order (LO) is about 2.2% for the residue (decay constant) and 0.5% for the mass in the XYZ exotic states [67,68].

In order to extract information from the correlation function, the invariant amplitudes $\Pi^{\mathrm{Phen}}(p)$ and $\Pi^{\mathrm{QCD}}(p)$ must be equated. To suppress the contributions of the higher states and continuum, the Borel transformation should be applied to both sides of the obtained sum rule. In this step, the quark-hadron duality assumption is used to perform continuum subtraction. One can reach an expression, after these technical elaborations, in terms of the mass m_D and decay constant f_D of the $\Delta^0\Delta^0$ in the hadronic side and the QCD degrees of freedom such as quark masses and vacuum condensates of different dimensions in the QCD side. After Borel transformation and continuum subtraction the QCD side of the sum rule takes the following form:

$$\tilde{\Pi}^{\mathrm{QCD}} = \int_{\mathcal{M}^2}^{s_0} \rho(s) e^{-s/M^2} ds + \Gamma, \tag{10}$$

where $\mathcal{M} = (4m_d + 2m_u)$, s_0 is the continuum threshold which is the energy characterizing the beginning of the continuum, M^2 is the Borel parameter, and $\rho(s)$ is the spectral density obtained from the imaginary part of the correlation function, $\rho(s) = \mathrm{Im}[\Pi(s)]/\pi$. The explicit expressions of the $\rho(s)$ and Γ under the assumptions of $m_u \to 0$ and $m_d \to 0$ are given as

$$\begin{aligned}\rho(s) = {} & \frac{1873 s^7}{2^{22}3^5 5^2 7^2 \pi^{10}} - m_0^2 \langle \bar{q}q\rangle^4 \left(\frac{3715457 g_s^2}{2^{12}3^8\pi^4} + \frac{61091}{2^{10}3^2\pi^2}\right) + \langle \bar{q}q\rangle^4 s \left(\frac{9365 g_s^4}{2^8 3^{10}\pi^6} + \frac{4771 g_s^2}{2^3 3^8 \pi^4} + \frac{4771}{2^3 3^8 \pi^2}\right) \\ & + \langle \bar{q}q\rangle^2 \left(\frac{243193 m_0^4 s^2}{2^{18}3^3\pi^6} - \frac{1034923 m_0^2 s^3}{2^{18}3^5\pi^6} + \frac{1873 g_s^2 s^4}{2^{14}3^7 5\pi^8} + \frac{4771 s^4}{2^{12}3^5 5\pi^6}\right) \\ & + \langle g^2 G^2\rangle \left(\frac{1159 m_0^4 \langle \bar{q}q\rangle^2}{2^{20}3^5\pi^6} - \frac{2347 m_0^2 \langle \bar{q}q\rangle^2 s}{2^{19}3^5\pi^6} + \frac{g_s^2 \langle \bar{q}q\rangle^2 s^2}{2^{12}3^5\pi^8} + \frac{11\langle \bar{q}q\rangle^2 s^2}{2^{16}3^3\pi^6} + \frac{s^5}{2^{21}3^2 5^2 \pi^1 0}\right),\end{aligned} \tag{11}$$

$$\Gamma = m_0^4 \langle \bar{q}q\rangle^4 \left(\frac{2494081 g_s^2}{2^{14}3^8\pi^4} + \frac{1038769}{2^{12}3^4\pi^2}\right) + \langle g^2 G^2\rangle \langle \bar{q}q\rangle^4 \left(\frac{g_s^4}{2^9 3^8\pi^6} + \frac{11 g_s^2}{2^{11}3^7\pi^4} + \frac{59}{2^{10}3^6\pi^2}\right). \tag{12}$$

Thus, the following sum rule relates the hadronic parameters to the QCD degrees of freedom as well as the auxiliary parameters M^2 and s_0:

$$f_D^2 m_D^2 e^{-\frac{m_D^2}{M^2}} = \tilde{\Pi}^{\mathrm{QCD}}. \tag{13}$$

The mass m_D and residue f_D of the dibaryon are obtained from the above sum rule as

$$m_D^2(s_0, M^2) = \frac{\tilde{\Pi}'^{\mathrm{QCD}}}{\tilde{\Pi}^{\mathrm{QCD}}}, \tag{14}$$

with $\tilde{\Pi}'^{\mathrm{QCD}} = \frac{d}{d(-\frac{1}{M^2})}\tilde{\Pi}^{\mathrm{QCD}}$, and

$$f_D^2(s_0, M^2) = \frac{e^{m_D^2/M^2}}{m_D^2}\tilde{\Pi}^{\mathrm{QCD}}. \tag{15}$$

III. NUMERICAL RESULTS AND DISCUSSION

We numerically analyze the sum rules for the $\Delta^0\Delta^0$ by using the input parameters which are given in Table I.

There are two further parameters which are used in QCDSR analysis: the s_0 continuum threshold parameter which arises after continuum subtraction via using quark-hadron duality and the M^2 Borel parameter which arises after Borel transformation as we also noted before. Any physical quantity should be independent of these parameters. Although these parameters are auxiliary, they are not arbitrary and should verify standard constraints. In order to extract reliable results from QCDSR, we must find the working regions of s_0 and M^2 where the results have s_0 and M^2 stability. So an important problem emerges for the proper choice of Borel M^2 and continuum threshold s_0 parameters.

For Borel parameter M^2, a good OPE convergence and suppression of higher and continuum states are necessary to determine the region. The upper and lower limits of the Borel parameter need an additional examination of the sum rules.

TABLE I. QCD input parameters.

Parameter	Numerical value
$\langle \bar{q}q\rangle$	$(-0.24 \pm 0.01)^3$ GeV^3 [69]
m_0^2	(0.8 ± 0.1) GeV^2 [69]
$\langle \frac{\alpha_s}{\pi} G^2\rangle$	(0.012 ± 0.004) GeV^4 [70]

At the upper limit of the Borel parameter, the pole contribution (PC) should reside fairly in the correlation function, whereas at the lower limit of the Borel parameter it must be a dominant contribution. This examination can be done by defining PC as

$$\mathrm{PC} = \frac{\tilde{\Pi}^{\mathrm{QCD}}(s_0, M^2)}{\tilde{\Pi}^{\mathrm{QCD}}(s_0 = \infty, M^2)}. \tag{16}$$

A balance can be possible between upper limit M^2_{max} and lower limit M^2_{min} of the Borel parameter, which provides both convergence of the series and dominance of the single resonance contribution.

The continuum threshold s_0 isolates the ground state contribution from the continuum states and higher resonances in the correlation function. s_0 characterizes the threshold where continuum states begin. In a standard analysis, s_0 should be just below the energy of the first excited state. The energy that is needed for this excitation reads as $\delta = \sqrt{s_0} - m$, with m being the mass of the ground state. In order to estimate the s_0 value, experimental information on the masses of the ground and first excited states of the channel under study is required. For the conventional hadrons, parameters of excited states can be obtained from either experimental measurements or theoretical studies. For these states δ varies mainly in the interval, $0.3\ \mathrm{GeV} \le \delta \le 0.8\ \mathrm{GeV}$. For multiquark hadrons or, better to say, exotic states, however, there can be a lack of relevant information. In this case s_0 can be obtained by limits imposed on PC and the convergence of OPE. The interval for the s_0 can be chosen as the smallest value which provides a reliable Borel region. One can fix s_0, by satisfying other constraints, to achieve a maximum for PC. Equipped with this it is possible to monitor numerical consistency of analytical calculations. Performed analysis shows that the working windows are as

$$M^2 \in [2.4, 3.0]\ \mathrm{GeV}^2, \qquad s_0 \in [8, 10]\ \mathrm{GeV}^2, \tag{17}$$

which meet all aforementioned restrictions. As it is clear, the working window for s_0 lies much higher than the production threshold of two Δ^0 baryons ($4m^2_{\Delta^0} \simeq 6.07\ \mathrm{GeV}^2$). The obtained interval for the continuum threshold corresponds to $(m_{\Delta^0\Delta^0} + \delta)^2$ with $0.50\ \mathrm{GeV} \le \delta \le 0.84\ \mathrm{GeV}$, which is a reasonable interval of energy to excite a $\Delta^0\Delta^0$ system to its first excited state. Note that in the Δ^0 three-quark baryon channel the energy difference between the ground and the first excited states with the same quantum numbers, $\Delta(1600)$, is about 0.37 GeV.

Using the above working regions for the auxiliary parameters, the pole contribution varies in the interval

$$0.12 \le \mathrm{PC} \le 0.68. \tag{18}$$

In the standard analysis of QCDSR, the pole contribution should be larger than $1/2$ for conventional hadrons (baryons and mesons). In the case of four-quark states, it turns out to be as $\mathrm{PC} \ge 0.2$. In Refs. [37,71], it is pointed out that dibaryon spectral densities led to small PC and therefore failed to specify the Borel parameter.

To extract the mass m_D and decay constant f_D of the $\Delta^0\Delta^0$ dibaryon, we calculate them at different choices of the Borel parameter M^2 and continuum threshold s_0 and find their mean values averaged over the working regions given in Eq. (17). Our results for m_D and f_D read

$$m_D = 2326^{+114}_{-126}\ \mathrm{MeV},$$
$$f_D = 2.94^{+0.30}_{-0.34} \times 10^{-4}\ \mathrm{GeV}^7. \tag{19}$$

The values in Eq. (19) correspond to an average value of lower, middle, and upper values in the working regions. The plus and minus bounds for the mass and decay constant are yielded from the highest and lowest values of m_D and f_D. In Figs. 1 and 2, we plot the mass and decay constant of the $\Delta^0\Delta^0$ dibaryon as functions of M^2 and s_0.

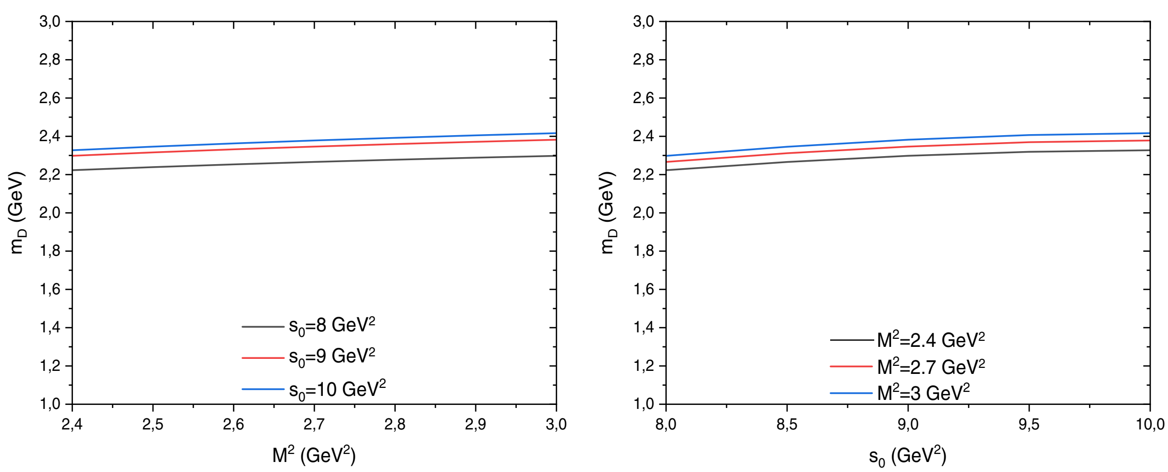

FIG. 1. The mass of the $\Delta^0\Delta^0$ dibaryon as a function of M^2 at fixed s_0 (left panel), and as a function of s_0 at fixed M^2 (right panel).

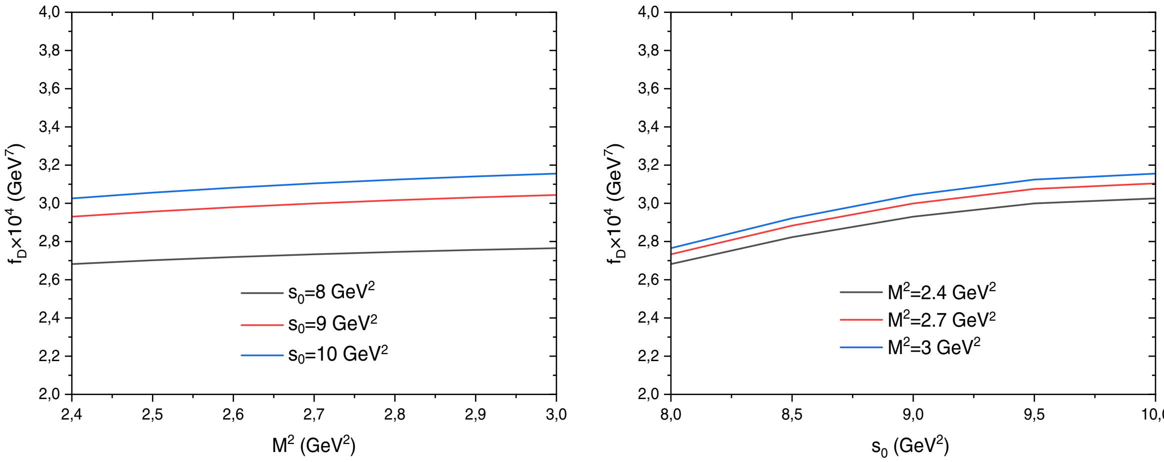

FIG. 2. Same as in Fig. 1, but for the coupling f_D of the $\Delta^0\Delta^0$ dibaryon.

Figure 1 (left panel) shows that the mass is relatively stable in the region for Borel parameter M^2. In the right panel of Fig. 1, the mass has a mild dependence on the region for continuum threshold s_0. In Fig. 2, it can be seen from both left and right panels that there is a residual dependence on the parameters of s_0 and M^2. The dependence on M^2 and s_0 is an essential part of theoretical errors in sum rule calculations. We have also added, on average, 1% and 3% uncertainties to the values of the mass and decay constant, respectively, due to the factorization hypothesis of the higher dimensional operators. Theoretical uncertainty in the extracted mass m_D is about $\pm 5.5\%$, where in the decay constant f_D, it is $\pm 12\%$. These uncertainties are well below the accepted limits in QCDSR calculations. We shall remark that, in the present study, we do not consider the contributions of the radiative corrections. Although the mass, due to the ratios of two sum rules, is known to be less sensitive to these corrections, the numerical value of the residue obtained from one sum rule may be changed considerably. Note that the next-to-leading order (NLO) perturbative corrections to the sum rules of baryons were obtained to be large [72–74]. The NLO α_s corrections to the perturbative term in the OPE of spectral functions were found to be large for light tetraquark currents as well; however, these corrections to the whole sum rules are small because of the dominance of the nonperturbative condensate contributions [75]. In Ref. [76], the NLO corrections to the spectral density of the light pentaquarks are found to be large and spoil momentum space QCD sum rule analysis, suggesting that a sum rule analysis is more reasonable in coordinate space. For pentaquark interpolating currents, the NLO corrections to the correlators were also found to be large in Ref. [77]. However, vacuum condensates at the order of $\mathcal{O}(\alpha_s^k)$ with $3/2 < k \leq 3$ play tiny roles in the sum rules for $\Lambda_c\Lambda_c$ dibaryon according to Ref. [78]. One should examine the contributions of the radiative corrections to the spectroscopic parameters of the light dibaryons.

The binding energy B_E can be obtained via the relation

$$B_E = 2m_{\Delta^0} - m_D, \tag{20}$$

where m_{Δ^0} is the mass of Δ^0 baryon and m_D is the predicted mass. The binding energy of $\Delta^0\Delta^0$ dibaryon is obtained to be $B_E \simeq 138$ MeV where Δ^0 baryon mass is borrowed from Ref. [79]. Finally it will be interesting to look at the size of $\Delta^0\Delta^0$ dibaryon. The following relation [6] can be used for this purpose:

$$r \sim \frac{1}{\sqrt{2\mu B_E}}, \tag{21}$$

where r is the distance between the components and $\mu = m_1 m_2/(m_1 + m_2)$ denotes the reduced mass of the two-hadron system. We obtain the size as $r \simeq 0.48$ fm. This is less than the so-called confinement radius, 1 fm.

IV. SUMMARY AND FINAL NOTES

In the present work, we considered the exotic scalar $\Delta^0\Delta^0$ dibaryon composed of *uddudd* quarks. Using the QCDSR technique and by satisfying its requirements such as the pole dominance and OPE convergence, the values of its mass and decay constant (residue) were extracted as $m_D = 2326^{+114}_{-126}$ MeV and $f_D = 2.94^{+0.30}_{-0.34} \times 10^{-4}$ GeV^7. We obtained the binding energy $B_E \simeq 138$ MeV for the $\Delta^0\Delta^0$ system. We also made an estimation for the size of the $\Delta^0\Delta^0$ dibaryon which is $r \simeq 0.48$ fm. A QCDSR calculation can provide evidence in favor of or against the existence of a state. The obtained result for the binding energy shows that the scalar $\Delta^0\Delta^0$ dibaryon system can be stable and long-lived. We hope that our results will be useful for both the future theoretical and the experimental studies on dibaryons.

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