

Landscape of heavy baryons from the perspective of the chiral quark-soliton model

Maxim V. Polyakov¹ and Michał Praszalowicz^{2,*}

¹*Institut für Theoretische Physik II, Ruhr-Universität Bochum, D-44780 Bochum, Germany*

²*Institute of Theoretical Physics, Jagiellonian University, Łojasiewicza 11, 30-348 Kraków, Poland*



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We employ the chiral quark-soliton model and the heavy quark symmetry to describe spectra of charm and beauty baryons. Heavy baryons can be classified according to the SU(3) representations of the light sector. We argue that recently discovered Ξ_b states can be interpreted as negative-parity excited antitriplets or sextets, and the Σ_b states as negative-parity sextets. Consequences of such assignments for the decay patterns are discussed and also predictions of masses of the yet unmeasured sextet members are given.

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I. INTRODUCTION

Recent discoveries of heavy baryons, i.e., baryons with one heavy quark, $Q = c$ or b ,¹ renewed interest in heavy baryon spectroscopy. In 2017 the LHCb Collaboration announced five new excited Ω_c^0 states, two of them of a very small width [3], which were confirmed by the Belle Collaboration [4] in 2018. Further analysis of the decay modes and possible spin assignment of these states has been published recently (Ref. [5]). Furthermore, in 2018 LHCb announced new b baryons; the isospin doublet of $\Xi_b(6227)$ [6], charged members of $\Sigma_b(6097)$ multiplet [7], and two nearly degenerate Λ_b baryons [8] at 6146 MeV/ c^2 and 6152 MeV/ c^2 , which are today interpreted as $1/2^+$ and $3/2^+$ spin states [9]. However, the interpretation of these states as excited Σ_b^0 states cannot be excluded. In 2020 LHCb reported four Ω_b^- excited states [10], which are presently considered to be only one star resonances [9]. If confirmed, they would pose, as we will discuss later, a challenge for theoretical interpretation. Next, LHCb reported a new $\Lambda_b^0(6072)$ state [11], a new $\Xi_b^0(6227)$ state [12], and two other Ξ_b^0 baryons [13] at 6327 MeV/ c^2 and 6333 MeV/ c^2 . Finally, the CMS Collaboration reported a $\Xi_b^-(6100)$ baryon interpreted as, presumably, a $3/2^-$ spin state [14]. All these new baryons, as well as the ones found earlier and included in the PDG [9], are listed in Sec. II for the convenience of the reader.

*michal.praszalowicz@uj.edu.pl

¹Doubly heavy baryons, like Ξ_{cc} [1,2], are beyond scope of this paper.

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There is a wealth of literature devoted to the theoretical description of heavy baryons, for a complete list of references for charmed baryons we refer the reader to a recent review by Hai-Yang Cheng [15]. Likewise, an extensive bibliography for bottom baryons can be found in Ref. [16]. The approach that we will advocate in the present paper is closely related to the approaches based on heavy quark effective theory [17], quark-diquark models [18,19], and the constituent quark model [20–26],

In the present paper we will continue our earlier analysis [27–30] based on the chiral-quark soliton model (χ QSM) [31–34] where baryons are viewed as rotational excitations of the chiral-quark soliton model, which is a good description of the light baryonic system in the large N_c limit [35]. Heavy baryons are constructed by removing one quark from the N_c light quarks and replacing it by a heavy quark Q . In the large N_c limit the light sector is hardly changed by such a replacement, and one can describe both ground state and excited positive parity baryons rather successfully [27–30]. Excitations appear due to the chiral rotation of the soliton and are therefore analogous to the diquark excitations (so-called ρ modes) in the quark language.

At this point it is useful to compare the present model with very popular approach to heavy baryons based on a diquark-heavy quark dynamics (see e.g., Ref. [26]). In such models one distinguishes two types of excitations: inner diquark excitations, referred often to as ρ modes, and diquark-heavy quark excitations referred to as λ modes. From this point of view the soliton model described above corresponds only to the rotational ρ modes. This can be justified by invoking large N_c arguments, see Sec. III, on which the whole approach used here is based. Therefore in the following we in fact test a hypothesis, whether heavy baryon masses can be explained if λ modes are neglected. Further support for neglecting the λ modes comes from the bound-state approach to heavy baryons in the Skyrme model and is

discussed in Sec. VC. Phenomenologically, however, λ modes may of course play a non-negligible role.

Even with λ modes neglected the heavy baryon spectrum is very rich. If, furthermore, λ modes and also radial excitations are included, one predicts a densely populated spectrum in the mass range already scanned by different experiments. Since the experimental evidence is much less abundant, the large N_c counting discussed above may serve as an effective Occam's razor.

The price for such a truncation is, in principle, rather small accuracy of numerical predictions; our approach is certainly more qualitative than quantitative. Nevertheless some predictions are very accurate, for example the sum rule involving Ω_c states—see caption in Table IV and Eq. (22) in Ref. [27].

We show that the recently discovered Ξ_b states can be interpreted as negative-parity excited antitriplets or sextets, and the Σ_b states as sextets. We discuss the consequences of this assignment for the decay patterns and for the charm sector.

The paper is organized as follows. In Sec. II we review the present experimental data situation in heavy baryon sector and summarize phenomenological expectations based on the naive extrapolation of the c sector to the b sector. Next, in Sec. III, we briefly introduce the chiral quark-soliton model (χ QSM) with emphasis on its extension to heavy baryons. In Sec. IV we analyze phenomenological consequences of the χ QSM for the ground states including exotica (Sec. IVA), excited negative parity antitriplets (Sec. IV B), and sextets (Sec. IV C). In the latter case we discuss possible SU(3) assignments of $\Sigma_b(6097)$, $\Xi_b(6327)$, and $\Xi_b(6333)$ to fix model parameters and we discuss consequences of these assignments for the charm sector. Next in Sec. V we try to understand decay patterns and analyze how they impact some of the recent experiments. Finally we conclude in Sec. VI.

II. EXPERIMENT

Ground state heavy baryons can be conveniently classified according to the SU(3) structure of the light subsystems (diquarks), which can be in the spin 0 antitriplet and spin-1 sextet. The same group structure follows from the χ QSM [27]. Adding one heavy quark to the light subsystem one ends up with spin-1/2⁺ antitriplet and two sextets of spin 1/2⁺ and 3/2⁺, which are hyperfine split and the splittings are proportional to the inverse of the heavy quark mass. These structures are fully confirmed experimentally, as can be seen from Tables I and II. In these Tables we also display the first orbitally excited negative parity antitriplets that form two hyperfine split multiplets of spin 1/2⁻ and 3/2⁻. All members of these two antitriplets in the charm sector have been known for a long time. Until recently only Λ_b^0 's

TABLE I. Ground-state and excited-charm baryons in the SU(3) antitriplet. All listed baryons are PDG 3-star resonances [9]. Masses are in MeV/ c^2 . Entries in parenthesis denote isospin averages used for numerical calculations.

S^P	Λ_c^+	$\Xi_c^{+,0}$	
1/2 ⁺	2286.46 ± 0.14	2467.71 ± 0.23 2470.44 ± 0.28	(2469)
1/2 ⁻	2592.25 ± 0.28	2791.90 ± 0.50 2793.90 ± 0.50	(2793)
3/2 ⁻	2628.11 ± 0.19	2816.51 ± 0.25 2819.79 ± 0.60	(2818)

TABLE II. Ground-state and excited-beauty baryons in the SU(3) antitriplet. All listed baryons are PDG 3-star resonances [9], except for Ξ_b^- marked with a star, which has been reported last year by CMS [14] and is not included in 2021 PDG. Masses are in MeV/ c^2 . Entries in parenthesis denote isospin averages used for numerical calculations. Isospin partners not yet measured are marked by dots.

S^P	Λ_b^0	$\Xi_b^{-,0}$	
1/2 ⁺	5619.60 ± 0.17	5797.0 ± 0.6 5791.9 ± 0.5	(5795)
1/2 ⁻	5912.19 ± 0.17	...	
3/2 ⁻	5920.08 ± 0.17	6100.3* ± 0.64 ...	(6100)

have been measured in the b sector. In 2021 one Ξ_b^- state, presumably 3/2⁻, was observed by the CMS Collaboration at CERN [14].

Looking at masses, both in the ground state and the excited antitriplets, we observe with accuracy < 15% equal splittings between Ξ_Q and Λ_Q states. Indeed (in MeV/ c^2),

$$\begin{aligned} \delta_c^{1/2^+}(\Xi - \Lambda) &\approx 182, & \delta_c^{1/2^-}(\Xi - \Lambda) &\approx 201, \\ \delta_c^{3/2^-}(\Xi - \Lambda) &\approx 190, & \delta_b^{1/2^+}(\Xi - \Lambda) &\approx 175. \end{aligned} \quad (1)$$

Equal mass splittings in each parity multiplet, independently of the heavy quarks involved, follow from the SU(3) and heavy quark symmetries. However the equality of splittings in different parity multiplets is not obvious. As we have shown in Ref. [29] this is a prediction of the χ QSM. Since we expect heavy quark symmetry to work better in the b sector, we immediately get predictions for the isospin averaged masses of excited Ξ_b 's,

$$\begin{aligned} \Xi_b^{1/2^-} &= \Lambda_b^{1/2^-} + \delta_b^{1/2^+}(\Xi - \Lambda) \approx 6087 \text{ MeV}/c^2, \\ \Xi_b^{3/2^-} &= \Lambda_b^{3/2^-} + \delta_b^{1/2^+}(\Xi - \Lambda) \approx 6095 \text{ MeV}/c^2. \end{aligned} \quad (2)$$

First equation is a prediction, and the second one is in perfect agreement with recent CMS [14] finding,² see Table II.

Similar pattern is observed for the ground-state sextets, both in charm (in MeV/c^2)

$$\begin{aligned} \delta_c^{1/2^+}(\Xi - \Sigma) &\approx 124, & \delta_c^{1/2^+}(\Omega - \Xi) &\approx 117, \\ \delta_c^{3/2^+}(\Xi - \Sigma) &\approx 128, & \delta_c^{3/2^+}(\Omega - \Xi) &\approx 120, \end{aligned} \quad (3)$$

and in beauty sectors (in MeV/c^2),

$$\begin{aligned} \delta_b^{1/2^+}(\Xi - \Sigma) &\approx 122, & \delta_b^{1/2^+}(\Omega - \Xi) &\approx 111, \\ \delta_b^{3/2^+}(\Xi - \Sigma) &\approx 121. \end{aligned} \quad (4)$$

Again, we see the independence of the splittings from the mass of the heavy quark. Unfortunately, as we shall see in the following, the χ QSM predicts that splittings in the excited sextets should be different [29].

It is interesting to look at the splittings between different heavy quark multiplets. Here the simplest comparison can be made for the ground state triplets

$$\delta(\Lambda_b - \Lambda_c) = 3333, \quad \delta(\Xi_b - \Xi_c) = 3326, \quad (5)$$

which corresponds to the difference $m_b - m_c$ in MeV/c^2 . Notably this is $400 \div 200 \text{ MeV}/c^2$ higher than the PDG [9] value either for the $\overline{\text{MS}}$ or the pole mass, respectively. Similar mass differences for excited antitriplets and for sextets require taking spin effects into account, and we relegate this to the next sections. Spin splittings in the charm sector are of the order of $\pm 17 \text{ MeV}/c^2$ and in the beauty case $\pm 4 \text{ MeV}/c^2$. Neglecting spin effects we expect the b -baryon spectrum to be a copy of c -baryons shifted by approximately $3330 \pm 20 \text{ MeV}/c^2$. Given recent measurements of five Ω_c 's that fall between $3000 \text{ MeV}/c^2$ and $3120 \text{ MeV}/c^2$, one would expect similar structure in the b sector in an interval between $(6330 - 6450) \pm 20 \text{ MeV}/c^2$. One sees indeed four Ω_b candidates (see Table VI) but in a much narrower interval of $35 \text{ MeV}/c^2$ only. One should, however, remember that Ω_b states are one-star resonances and are not listed in the PDG summary listings [9]. For possible interpretation of these states as doubly-strange bss states see Ref. [36] and also [37].

III. CHIRAL QUARK SOLITON MODEL

In this section we recapitulate shortly the chiral quark-soliton model, for more detailed discussion we refer the reader to the original works [31] and to the reviews of

²Note that CMS has measured only Ξ_b^- , so Ξ_b^0 , which we expect to be lower in mass, is still to be found. Therefore, the average isospin mass is expected to be lower than $6100 \text{ MeV}/c^2$.

Refs. [32–34] (and references therein). The χ QSM is based on an old argument by Witten [35], that in the limit of a large number of colors ($N_c \rightarrow \infty$), N_c relativistic valence quarks generate chiral mean fields represented by a distortion of the Dirac sea. Such distortion interacts with the valence quarks changing their wave function, which in turn modifies the sea until stable configuration is reached. This configuration called chiral soliton corresponds to the solution of the Dirac equation for the constituent quarks (with gluons integrated out) in the mean-field approximation where the mean fields respect so called hedgehog symmetry. Hedgehog symmetry follows from the fact that it is impossible to construct a pseudoscalar field that changes sign under inversion of coordinates, which would be compatible with the $\text{SU}(3)_{\text{flav}} \times \text{SO}(3)$ space symmetry. It has been shown that the hedgehog symmetry, which is smaller than $\text{SU}(3)_{\text{flav}} \times \text{SO}(3)$, leads to the correct baryon spectrum (see below).

In vacuum quarks are characterized by two independent $\text{SU}(2)$ symmetries; spin S and isospin T . In the soliton configuration, due to the hedgehog symmetry, neither spin, nor isospin are good quantum numbers. Instead a grand spin $\mathbf{K} = \mathbf{S} + \mathbf{T}$ is a good quantum number. Solutions of the Dirac equations are therefore labeled by K^P quantum numbers (P standing for parity). The ground state configuration corresponds to the fully occupied $K^P = 0^+$ valence level, as shown in Fig. 1(a). Therefore, the soliton does not carry definite quantum numbers except for the baryon number resulting from the valence quarks.

Spin and isospin appear when the rotations in space and flavor are quantized. This procedure results in a collective Hamiltonian analogous to the one of a quantum mechanical symmetric top. There are two conditions that the collective wave functions have to satisfy:

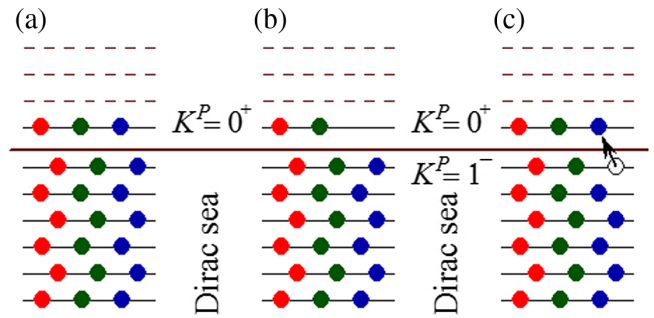


FIG. 1. Schematic pattern of light quark levels in a self-consistent soliton configuration. In the left panel all sea levels are filled and N_c ($= 3$ in the figure) valence quarks occupy the $K^P = 0^+$ lowest positive energy level. Unoccupied positive energy levels are depicted by dashed lines. In the middle panel one valence quark has been stripped off, and the soliton has to be supplemented by a heavy quark not shown in the figure. In the right panel a possible excitation of a sea-level quark, conjectured to be $K^P = 1^-$, to the valence level is shown, and again the soliton has to couple to a heavy quark. (Figure from Ref. [29].)

- (i) allowed SU(3) representations must contain states with hypercharge $Y' = N_c/3$,
- (ii) the isospin T' of the states with $Y' = N_c/3$ couples with the soliton spin J to a singlet: $T' + J = 0$.

Such a configuration has been used to describe octet and decuplet of light baryons, and is not of interest for us here.

Instead, we will focus on the configuration depicted in Fig. 1(b), when one light quark has been removed from the valence level. Since the valence level has grand spin $K = 0$, such configuration does not carry any quantum numbers other than the baryon number that is $2/3$, or strictly speaking $(N_c - 1)/N_c$. Such a soliton has to be supplemented by a heavy quark to form a baryon with baryon number equal one. Following [38], it has been shown in Ref. [27] that such a model satisfactorily fits experimental data.

In the large N_c limit light sector both in light and heavy baryons is described essentially by the same mean field. The only difference is now in the quantization condition:

- (i) allowed SU(3) representations must contain states with hypercharge $Y' = (N_c - 1)/3$,
- (ii) the isospin T' of the states with Y' couples with the soliton spin J to a singlet: $T' + J = 0$.

The lowest allowed SU(3) representations for such configurations are $\bar{3}$ of spin 0 and 6 of spin 1, as in the quark model. They are shown in Fig. 2 together with exotic $\bar{15}$ that corresponds to heavy pentaquarks [29]. Therefore, heavy baryons that are constructed from the soliton and a heavy quark form, an SU(3) antitriplet of spin 1/2, and two sextets of spin 1/2 and 3/2 that are subject to a hyper-fine splitting.

Finally, following Ref. [29] we shall conjecture that the first occupied sea level in Fig. 1 is $K = 1^-$. If we excite one quark from this shell to the free valence level, the soliton will have not only the baryon number but also K corresponding to the hole in the sea. In this case, shown in Fig. 1(c), baryon parity is negative and the quantization condition reads:

- (i) allowed SU(3) representations must contain states with hypercharge $Y' = (N_c - 1)/3$,

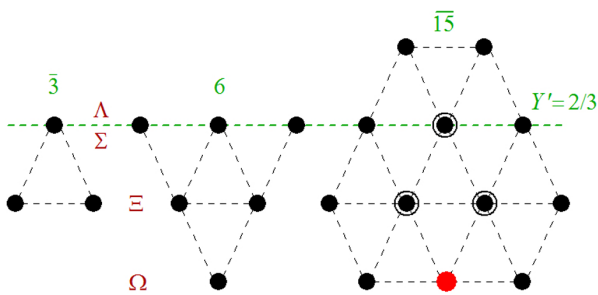


FIG. 2. Rotational band of a soliton with one valence quark stripped off. Soliton spin corresponds to the isospin T' of states on the quantization line $Y' = 2/3$. We show three lowest allowed representations: antitriplet of spin 0, sextet of spin 1 and the lowest exotic representation $\bar{15}$ of spin 1 or 0. Heavy quarks have to be added. Red dot corresponds to the exotic $\Omega_Q(T_3 = 0)$ discussed in [29]. (Figure from Ref. [28].).

- (ii) the isospin T' of the states with $Y' = (N_c - 1)/3$ couples with the soliton spin J as follows: $T' + J = K$, where K is the grand spin of the unoccupied sea level.

Therefore the rotational bands are the same as in Fig. 2 with, however, different spin assignments.

For $\bar{3}$ $T' = 0$, however $K = 1$, so the soliton has spin $J = 1$ and we expect two hyperfine split heavy baryon antitriplets of spin-1/2 and spin-3/2. As discussed in Sec. II this is indeed experimentally the case.

For 6 $T' = 1$, which for $K = 1$ gives the soliton spin $J = 0, 1, 2$ in direct analogy to the total angular momentum of the light subsystem in the quark model. By adding one heavy quark we end up with five possible excitations, which have the following total spin S ; for $J = 0$: $S = 1/2$, for $J = 1$: $S = 1/2$ and $3/2$, and for $J = 2$: $S = 3/2$ and $5/2$. States corresponding to the same J are subject to small hyperfine splittings as depicted in Fig. 3. All these states have negative parity.

The formula for the soliton mass in the chiral limit for the states in the SU(3) representation \mathcal{R} has been derived in Ref. [38] and reads,

$$\begin{aligned} \mathcal{M}^{(K)} = M_{\text{sol}}^{(K)} + \frac{1}{2I_2} \left[C_2(\mathcal{R}) - T'(T' + 1) - \frac{3}{4} Y'^2 \right] \\ + \frac{1}{2I_1} [(1 - a_K)T'(T' + 1) + a_K J(J + 1) \\ - a_K(1 - a_K)K(K + 1)], \end{aligned} \quad (6)$$

where $C_2(\mathcal{R})$ stands for the SU(3) Casimir operator. $M_{\text{sol}}^{(K)} \sim N_c$ denotes classical soliton mass, $I_{1,2}$ represent moments of inertia, and a_K is a parameter that takes into account one-quark excitation. Although all these parameters can, in principle, be calculated in a specific model, we shall follow a so-called model-independent approach introduced in the context of the Skyrme model in Ref. [39], where all the parameters are extracted from the experimental data. This approach has been used in the context of five excited Ω_c states in Ref. [29].

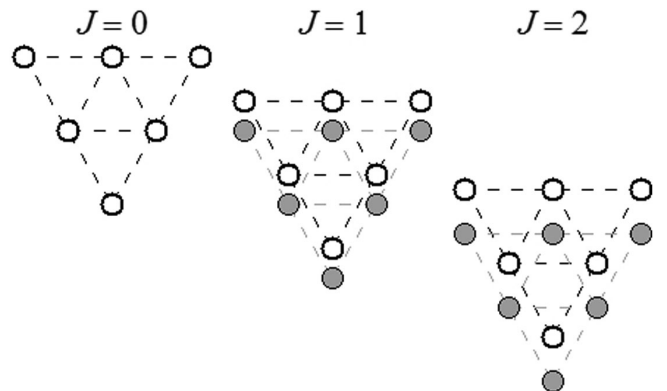


FIG. 3. Excited heavy baryons belonging to the SU(3) sextets.

At this point it is important to note that Hamiltonian (6) corresponds to the excitations of the light sector only, i.e., to the so called ρ modes in the quark model language. Such excitations are dominant in the large N_c limit. Indeed, all parameters in the rotational hamiltonian (6) have definite N_c dependence,

$$M_{\text{sol}}^{(K)}, \quad I_{1,2} \sim \mathcal{O}(N_c), \quad a_K \sim \mathcal{O}(1). \quad (7)$$

Therefore rotational excitations of the soliton (diquark excitations in the quark language) have energy splittings which are parametrically small ($1/I_{1,2} \sim 1/N_c$).

On the other hand, λ modes correspond, in principle, to a color confining soliton- Q (or diquark- Q) interaction that could be taken into account by solving a Schrödinger equation with reduced mass μ . (In a heavy quark limit $\mu \sim N_c$). Excitation energy $\Delta_\lambda E$ dependence on μ is closely related to the potential used and is linear in μ for the Coulomb potential. For the logarithmic potential it does not depend on μ , and for the linear potential it is proportional to $\mu^{-1/3}$. In all these cases mass splittings corresponding to the soliton ρ modes are parametrically smaller than the energy splittings of the λ modes and are therefore dominant. This should be contrasted with the quark models where typically λ modes are dominant (e.g., Refs. [20,26]).

The rotational Hamiltonian (6) has to be supplemented by the SU(3) symmetry-breaking collective Hamiltonian and the hyperfine splitting Hamiltonian, which we parametrize as follows [27]:

$$H_{\text{hf}} = \frac{2}{3} \frac{\kappa}{m_Q} \mathbf{J} \cdot \mathbf{J}_Q, \quad (8)$$

where κ is flavor-indepenent free parameter that may depend on the SU(3) representation and on the soliton grand spin, K . The operators \mathbf{J} and \mathbf{J}_Q represent the spin operators for the soliton and the heavy quark, respectively. As discussed in Sec. VC, such interaction term can be derived from the bound state approach to the Skyrme model.

The symmetry breaking Hamiltonian, which can be found in Ref. [29], is given in terms of the SU(3) D functions, hypercharge Y , \mathbf{T}' isospin operators, and grand

spin \mathbf{K} operator. It is parametrized by four constants: α, β, γ , and δ . When this Hamiltonian is sandwiched between the collective wave functions, one obtains that mass splittings in antitriplets and sextets are proportional to the hypercharge Y , with coefficients given as some combinations of the parameters α, \dots, δ . We refer the reader to Refs. [27,29] for their explicit forms.

It is convenient to introduce the following quantities for the ground state baryons

$$\begin{aligned} \mathcal{M}_{Q\bar{3}} &= m_Q + M_{\text{sol}} + \frac{1}{2I_2}, \\ \mathcal{M}_{Q6} &= \mathcal{M}_{Q\bar{3}} + \frac{1}{I_1}, \end{aligned} \quad (9)$$

and for $K = 1$ excited multiplets

$$\begin{aligned} \mathcal{M}'_{Q\bar{3}} &= m_Q + M'_{\text{sol}} + \frac{1}{2I'_2} + \frac{a_1^2}{I'_1}, \\ \mathcal{M}'_{Q6} &= \mathcal{M}'_{Q\bar{3}} + \frac{1}{I'_1}. \end{aligned} \quad (10)$$

Here primes indicate that both the soliton mass M_{sol} and the moments of inertia $I_{1,2}$ calculated for the excited configuration are numerically different from the ones calculated for the ground state. Also, the soliton mass and moments of inertia for the ground state heavy baryons may differ from the ones in the light sector. These differences are in principle of the order of $1/N_c$ and therefore are negligible for large N_c , but might be important for the real world $N_c = 3$ phenomenology.

With the help of Eq. (9) we can write concise formulas for the ground states,

$$\begin{aligned} M_{QY}^{\bar{3}} &= \mathcal{M}_{Q\bar{3}} + \delta_{\bar{3}} Y, \\ M_{QY}^6 &= \mathcal{M}_{Q6} + \delta_6 Y + \frac{\kappa}{m_Q} \begin{cases} -2/3 & \text{for } S = 1/2 \\ +1/3 & \text{for } S = 3/2 \end{cases}, \end{aligned} \quad (11)$$

and using Eq. (10) for negative parity excited states

$$\begin{aligned} M_{QY}^{\bar{3}} &= \mathcal{M}'_{Q\bar{3}} + \delta_{\bar{3}} Y + \frac{\kappa'}{m_Q} \begin{cases} -2/3 & \text{for } S = 1/2 \\ +1/3 & \text{for } S = 3/2 \end{cases}, \\ M_{QY}^{6^{J=0}} &= \mathcal{M}'_{6Q} - 2 \frac{a_1}{I'_1} + \left(\delta_6 - \frac{3}{10} \delta \right) Y, \\ M_{QY}^{6^{J=1}} &= \mathcal{M}'_{6Q} - \frac{a_1}{I'_1} + \left(\delta_6 - \frac{3}{20} \delta \right) Y + \frac{\kappa'}{m_Q} \begin{cases} -2/3 & \text{for } S = 1/2 \\ +1/3 & \text{for } S = 3/2 \end{cases}, \\ M_{QY}^{6^{J=2}} &= \mathcal{M}'_{6Q} + \frac{a_1}{I'_1} + \left(\delta_6 + \frac{3}{20} \delta \right) Y + \frac{\kappa'}{m_Q} \begin{cases} -1 & \text{for } S = 3/2 \\ +2/3 & \text{for } S = 5/2 \end{cases}. \end{aligned} \quad (12)$$

TABLE III. Ground-state charm baryons in the SU(3) sextet. All listed baryons are PDG 3-star resonances [9]. Masses are in MeV/ c^2 . Entries in parenthesis denote isospin averages used for numerical calculations.

S^P	$\Sigma_c^{+++,0}$	$\Xi_c^{'+,0}$	Ω_c^0
1/2 ⁺	2453.97 ± 0.14	2578.20 ± 0.50	2695.20 ± 1.70
	2452.90 ± 0.40 (2454)	2578.70 ± 0.50 (2578)	
	2453.75 ± 0.14	2578.70 ± 0.50	
3/2 ⁺	2518.41 ± 0.20	2645.10 ± 0.30	2765.90 ± 2.00
	2517.50 ± 2.30 (2518)	2646.16 ± 0.25 (2646)	
	2518.48 ± 0.20	2646.16 ± 0.25	

Here, δ_3 and δ_6 are effective breaking parameters constructed from α , β , and γ , and δ is a new parameter that enters the splittings of the excited sextets only [27]. We see that sextet mass splittings depend on J and, therefore, care must be taken when applying the Eckart-Wigner theorem to these multiplets (see e.g., Ref. [20]). We will come back to this point later.

An immediate consequence of Eqs. (11) and (12) for antitriplets is the equality of splittings in ground and excited states, already mentioned in Sec. II. This property goes beyond the usual Eckart-Wigner type SU(3) relations and is a consequence of the hedgehog symmetry. From Tables I and II, and from Eq. (2) we can estimate average excited antitriplet masses (in MeV/ c^2),

$$\mathcal{M}'_{\bar{3}_c} = 2745, \quad \mathcal{M}'_{\bar{3}_b} = 6034, \quad (13)$$

which give the heavy quark mass difference $m_b - m_c = 3289$ MeV/ c^2 , slightly below (5).

IV. PHENOMENOLOGY

A. Ground state heavy baryons

In this subsection we briefly recapitulate phenomenology of the ground state heavy baryons based on mass formulas (11), discussed initially in [27]. As already mentioned in Sec. II both antitriplet and sextet splittings have little dependence on the heavy quark mass [see Eqs. (1), (3), and (4)]. Throughout this paper in the following analysis we shall use an average value

$$\delta_6 = -118 \text{ MeV}/c^2. \quad (14)$$

Furthermore, the hyperfine splittings for the ground state charm sextet give (see Table III)

$$\frac{\kappa}{m_c} = 64 \div 71 \text{ MeV}/c^2. \quad (15)$$

This value of the hyperfine splitting resulted in assigning the two excited Ω_c states [3] of masses 3050 MeV/ c^2 and 3119 MeV/ c^2 (see Table III) as members of $\bar{\mathbf{15}}$ exotic SU(3) multiplets [29]. Subsequent calculation of the decay widths [30], which for exotic representation $\bar{\mathbf{15}}$ are expected to be small in the large N_c limit [40], strengthened this assignment.

In the beauty sector (see Table IV)

$$\frac{\kappa}{m_b} \approx 20 \text{ MeV}/c^2, \quad (16)$$

in agreement with simple scaling of the hyperfine splitting with the quark masses.

From mass centres of antitriplet and sextet multiplets we can calculate the difference of the heavy quark masses. For $\bar{\mathbf{3}}$ (from Tables I and II) we have (in what follows, to simplify notation, we will use particle symbols for their masses),

$$m_b - m_c = \frac{1}{3}(\Lambda_Q^{1/2+} + 2\Xi_Q^{1/2+})|_{b=c} = 3328 \text{ MeV}/c^2. \quad (17)$$

 TABLE IV. Ground-state beauty baryons in the SU(3) sextet. All listed baryons are PDG 3-star resonances [9], except for $\Omega_b(3/2^+)$ marked with a star which has not been measured; the mass is a prediction from the sum rule derived in Ref. [27]. Masses are in MeV/ c^2 . Entries in parenthesis denote isospin averages used for numerical calculations. Isospin partners not yet measured are marked by dashes.

S^P	$\Sigma_b^{+,0,-}$	$\Xi_b^{\prime-,0}$	Ω_b^-
1/2 ⁺	5810.56 ± 0.25	5935.02 ± 0.05	6046.1 ± 1.7
	... (5813)	... (5935)	
	5815.64 ± 0.27	5952.30 ± 0.60	
3/2 ⁺	5830.32 ± 0.27	5955.33 ± 0.12	6076.8 ± 2.2*
	... (5833)	... (5954)	
	5834.74 ± 0.30	5955.33 ± 0.12	

For 6 we have to average over spin. Using Tables III and IV one gets,

$$m_b - m_c = \frac{1}{6} \left(3 \frac{\Sigma_Q^{1/2+} + 2\Sigma_Q^{3/2+}}{3} + 2 \frac{\Xi_Q^{1/2+} + 2\Xi_Q^{3/2+}}{3} + \frac{\Omega_Q^{1/2+} + 2\Omega_Q^{3/2+}}{3} \right) \Big|_{b-c} = 3327 \text{ MeV}/c^2. \quad (18)$$

We see perfect agreement of both estimates of the difference of heavy quark masses with (5) (see Ref. [41] the discussion of the analogous relation for heavy mesons).

B. Excited antitriplets

As already mentioned splitting parameter $\delta_{\bar{3}}$ [see first equation in (12)] is identical to the one of the ground state, both for charm and beauty with accuracy better than 15%.

Furthermore, for charm we can extract the hyperfine splitting parameter from the following mass differences

$$\begin{aligned} \frac{\kappa'}{m_c} &= \Lambda_c^{3/2-} (2628) - \Lambda_c^{1/2-} (2592) = 36 \text{ MeV}/c^2, \\ \frac{\kappa'}{m_c} &= \Xi_c^{3/2-} (2818) - \Xi_c^{1/2-} (2793) = 25 \text{ MeV}/c^2. \end{aligned} \quad (19)$$

This relatively big difference ($\sim 17\%$) between the two estimates may be attributed to an imperfect heavy quark symmetry for charm.

For beauty we have

$$\frac{\kappa'}{m_b} = \Lambda_b^{3/2-} (5920) - \Lambda_b^{1/2-} (5912) = 8 \text{ MeV}/c^2. \quad (20)$$

This is in good agreement with the charm estimate if we recall that $m_c/m_b \simeq 0.3$.

From spin averaged masses of Λ_Q baryons³

$$\frac{1}{3} (\Lambda_b^{1/2-} + 2\Lambda_b^{3/2-}) = \mathcal{M}_{\bar{3}} + \delta_{\bar{3}} Y, \quad (21)$$

we can calculate

$$m_b - m_c = \frac{1}{3} (\Lambda_Q^{1/2-} + 2\Lambda_Q^{3/2-}) \Big|_{b-c} = 3302 \text{ MeV}/c^2, \quad (22)$$

in agreement with our previous estimates (17) and (18).

Finally, let us observe that the model predicts with no free parameters masses of $\Xi_b^{1/2-}$ and $\Xi_b^{3/2-}$, as already shown in Eq. (2).

³Unlike in the case of the ground state antitriplets, $\Xi_b^{1/2-}$ has not been measured.

C. Excited sextets

There are three excited sextets characterized by the soliton spin $J = 0, 1, 2$. The spin S of the heavy baryon emerges from the heavy quark coupling with the soliton spin. So, we have five sextets: $(J = 0, S = 1/2)$, $(J = 1, S = 1/2)$, $(J = 1, S = 3/2)$, $(J = 2, S = 3/2)$, and $(J = 2, S = 5/2)$ all of parity $P = -$. Therefore we expect 5 isospin multiplets of negative parity Σ_Q 's and Ξ_Q 's (not to mention positive parity radial excitations of the ground state multiplets). As seen from Tables V and VI we have only one Σ_Q isospin multiplet candidate both for charm and for beauty. The situation is somewhat better in the Ξ_Q case; however, here we have to remember that Ξ_Q candidates may belong both to $\bar{3}$ and 6 SU(3) multiplets. The only unambiguous sextet candidates are excited Ω_c^0 states (see Table III)—or at least some of them—reported by LHCb [5]. Before we recall the possible assignments of the of Ω_c^0 's proposed in Ref. [29], let us discuss some general features of the sextet spectra.

It seems that the predictive power of the sextet formulas of Eq. (12) is rather weak since we have two new parameters: δ that makes hypercharge splittings in the excited sextet different from the ones of the ground

TABLE V. Charm baryons with unknown SU(3) assignment. Entries in bold face denote 3-star PDG resonances [9], other entries are 1-star or 2-star resonances omitted from the summary listings, except for $\Omega_c^0(3188)$ taken from [5]. Masses are in MeV/ c^2 . Isospin partners not yet measured are marked by dashes. Entries in parenthesis denote isospin averages used for numerical calculations. Experimental errors are for orientation only (typically the largest PDG error is listed).

S^P	Λ_c^+	S^P	$\Xi_c^{+,0}$	
?	2766.60 ± 2.40	?	...	(2923)
$3/2^+$	2856.10 ± 5.60	?	2923.04 ± 0.25	(2923)
$5/2^+$	2881.63 ± 0.24	?	2942.30 ± 4.40	(2940)
$3/2^-$	2939.60 ± 1.50	?	2938.55 ± 0.22	(2940)
		?	2964.30 ± 1.50	(2966)
		?	2967.10 ± 1.70	(2966)
		?	3055.90 ± 0.40	(3056)
		?	...	
		?	3077.20 ± 0.40	(3079)
		?	3079.90 ± 1.40	(3079)
		?	3122.90 ± 1.30	(3123)
			...	
S^P	Ω_c^0	S^P	$\Sigma_c^{+,+,0}$	
?	3000.41 ± 0.22		$2801. \pm 6.0$	
?	3050.20 ± 0.13	?	$2792. \pm 14.$	(2800)
?	3065.46 ± 0.28		$2806. \pm 7.0$	
?	3090.00 ± 0.50			
?	3119.10 ± 0.90			
?	3188.00 ± 13.0			

TABLE VI. Beauty baryons with unknown SU(3) assignment. Entries in bold face denote 3-star PDG resonances [9], other entries are 1-star or 2-star resonances omitted from the summary listings, except for two Ξ_b^0 resonances marked with a star, which have been reported 2021 by LHCb [13] and are not included in 2021 PDG. Masses are in MeV/ c^2 . Isospin partners not yet measured are marked by dashes. Entries in parenthesis denote isospin averages used for numerical calculations. Experimental errors are for orientation only (typically the largest PDG error is listed).

S^P	Λ_b^0	S^P	$\Xi_b^{-,0}$
?	6072.3 ± 2.9	?	6227.9 ± 0.9
$3/2^+$	6146.2 ± 0.4	?	6226.8 ± 1.5
$5/2^+$	6152.5 ± 0.4	?	...
			$6327.28^* \pm 0.33$
		?	...
			$6332.69^* \pm 0.31$
S^P	Ω_b^-	S^P	$\Sigma_b^{+,0,-}$
?	6315.6 ± 0.58		6095.8 ± 1.7
?	6330.3 ± 0.58	?	...
?	6339.7 ± 0.58		6098.0 ± 1.7
?	6349.8 ± 0.64		

state, and a_1 which is responsible for splittings of different J multiplets. However, as has been observed in Ref. [29], splittings of different J multiplets before hyperfine splitting (i.e., for $\kappa' = 0$)

$$\begin{aligned}\Delta_1(Y) &= (M'_{QY}{}^{6J=1} - M'_{QY}{}^{6J=0})|_{\kappa'=0}, \\ \Delta_2(Y) &= (M'_{QY}{}^{6J=2} - M'_{QY}{}^{6J=1})|_{\kappa'=0},\end{aligned}\quad (23)$$

do not depend on Q and read as follows:

$$\Delta_1(Y) = \frac{a_1}{I_1} + \frac{3}{20} \delta Y, \quad (24)$$

and

$$\Delta_2(Y) = 2\Delta_1(Y), \quad (25)$$

which significantly constrains sextet spectra.

As said previously, the only unambiguous candidates for the excited sextets are five LHCb [5] Ω_c^0 's. However, it is impossible to assign all of them to these five sextets [29] due to the values of the hyperfine splittings and the constraint (25). Moreover, two of the LHCb Ω_c^0 's are very narrow, around 1 MeV, while the remaining three have widths ranging from 3.4 MeV to 8.7 MeV. Such spread of the decay widths would greatly violate SU(3) relations between the decay constants.

Therefore it has been proposed in Ref. [29] to interpret two narrow Ω states, namely $\Omega_c^0(3050)$ and $\Omega_c^0(3119)$, as

the hyperfine split members of the exotic $\overline{15}$. This assignment has been motivated by the fact⁴ that their hyperfine splitting is equal to the one of the ground state sextet (15).

Leaving aside the interpretation of $\Omega_c^0(3050)$ and $\Omega_c^0(3119)$, we are left with three remaining states $\Omega_c^0(3000)$, $\Omega_c^0(3065)$, and $\Omega_c^0(3090)$, which have been interpreted as members of $J = 0$ and $J = 1$ sextets [29]. Indeed, if $\Omega_c^0(3065)$ and $\Omega_c^0(3090)$ are hyperfine split members of $J = 1$ sextet, then their hyperfine splitting should be equal to the one of the excited antitriplet (19), which is indeed the case. From this assignment one obtains,

$$\Delta_1(Y = -4/3) = \frac{a_1}{I_1} - \frac{1}{5} \delta = 82 \text{ MeV}/c^2. \quad (26)$$

In view of relation (25) we should have two other Ω_c^0 states approximately 164 MeV/ c^2 higher. Indeed, in Ref. [29] masses of two hyperfine split members of $J = 2$ sextet have been estimated to be 3222 MeV/ c^2 and 3262 MeV/ c^2 . In this scenario these states have masses above the $\Xi + D$ threshold at 3185 MeV/ c^2 , i.e., they can have rather large widths and may be not clearly seen in the LHCb data.⁵

Recent report of the LHCb Collaboration [13] on $\Xi_b^-(6327)$ and $\Xi_b^-(6333)$ allows for narrowing the model parameter space. Indeed, these—together with $\Xi_b(6227)$ and possibly $\Sigma_b(6097)$ —are the only inputs in the b sector, which can be used. Four Ω_b^- states are only one star resonances and therefore cannot give reliable information to constrain the model. On the contrary $\Xi_b^-(6327)$ and $\Xi_b^-(6333)$ fit very well a hypothesis that they are hyperfine split members of $J = 1$ excited sextet. Hyperfine splitting is in this case ~ 5.5 MeV/ c^2 in accordance with an expectation from the b antitriplet of 8 MeV/ c^2 (20). Note that hyperfine splittings have rather large model uncertainty, see estimates for charm (19).

Therefore we propose the following assignments,

$$\begin{aligned}\Sigma_b(6097) &= \Sigma_b^{1/2^-}(\mathbf{6}', J = 0), \\ \Xi_b(6327) &= \Xi_b^{1/2^-}(\mathbf{6}', J = 1), \\ \Xi_b(6333) &= \Xi_b^{3/2^-}(\mathbf{6}', J = 1),\end{aligned}\quad (27)$$

where we have also used the LHCb [7] $\Sigma_b(6097)$. Other assignments of $\Sigma_b(6097)$ give much worse fits. One might try to assign $\Xi_b(6327)$ and $\Xi_b(6333)$ to $(\mathbf{6}', J = 2)$, however for $J = 2$ the hyperfine splitting is expected to be ~ 13.3 MeV, see Eq. (12), rather than 8 MeV for $J = 1$

⁴Note that the ground state sextet and exotic $\overline{15}$ belong to the same rotational band, and therefore should have approximately the same value of κ .

⁵Note that LHCb sees some wide bumps at 3188 MeV/ c^2 and higher.

(vs experimental 5.5 MeV). Therefore we use (27) in the following fits.

In Fit 0 we fix the unknown model parameters, namely δ and a_1/I'_1 [or equivalently $\Delta_1(-4/3)$, see Eq. (26)] using all masses in Eq. (27) as inputs. The results of this fit are shown in Table VII. The resulting $\Delta_1(-4/3) = 98 \text{ MeV}/c^2$ is a bit higher than the one obtained from the charm sector (26). In order to make contact with the charm spectrum in the three following fits we fix $\Delta_1(-4/3) = 82 \text{ MeV}/c^2$ [which follows from three Ω_c^0 states used as an input (26)] and use different combinations of two inputs from Eq. (27). The results are presented in Tables VIII–X, where also the resulting charm spectrum is shown. Entries in bold face have been used as inputs, underlined entries denote masses that can be attributed to some of the baryons of the unknown SU(3) assignment listed in Tables V and VI; solid line if mass difference is of the order of $12 \text{ MeV}/c^2$ or less, dashed line if the mass difference is $15 \text{ MeV}/c^2$ – $20 \text{ MeV}/c^2$.

We see that although fits 1–3 give rather different values of δ , they confirm, with possible exception of Fit 2 that misses $\Sigma_b(6097)$ by $32 \text{ MeV}/c^2$, the hypothesis of Eq. (27). Moreover, they suggest clear interpretation of four charm states from Table V,

$$\begin{aligned}\Sigma_c(2800) &= \Sigma_c^{1/2^-}(\mathbf{6}', J=1), \\ \Xi_c(2940) &= \Xi_c^{1/2^-}(\mathbf{6}', J=1), \\ \Xi_c(2966) &= \Xi_c^{3/2^-}(\mathbf{6}', J=1), \\ \Xi_c(3123) &= \Xi_c^{3/2^-}(\mathbf{6}', J=2).\end{aligned}\quad (28)$$

Interestingly, a three star $\Xi_c(3056)$ cannot be interpreted as a member of negative parity $\mathbf{6}$, while a two star $\Xi_c(2940)$

TABLE VII. Fit 0: $\delta = 40$ and $\Delta_1(-4/3) = 98$. Inputs are in bold face. All masses are in MeV/c^2 .

J	S	Σ_b	Ξ_b	Ω_b
0	1/2	6097	6227	6357
	1/2	6203	6327	6451
1	3/2	6209	6333	6557
	3/2	6422	6534	6646
2	5/2	6431	6543	6655

TABLE VIII. Fit 1, $\delta = 76$. Inputs are in bold face. Underlined entries can be attributed to some of the known baryons (see text). All masses are in MeV/c^2 .

J	S	Σ_b	Ξ_b	Ω_b	Σ_c	Ξ_c	Ω_c
0	1/2	6097	<u>6238</u>	6378	2719	2859	3000
	1/2	6198	6327	6457	<u>2807</u>	<u>2937</u>	3066
1	3/2	6204	6333	6462	2831	<u>2961</u>	3090
	3/2	6406	6512	6619	3009	<u>3115</u>	3222
2	5/2	6415	6521	6628	3049	3155	3262

seems to be a spin partner of a three star $\Xi_c(2966)$, and one star $\Xi_c(3123)$ is interpreted as $J = 2$ sextet excitation. Moreover, there are no experimental candidates for the lowest $\mathbf{6}$ multiplet of $J = 0$.

With these assignments we can calculate average sextet masses both for charm and bottom baryons (in MeV/c^2),⁶

$$\mathcal{M}'_{6c} = 3007, \quad \mathcal{M}'_{6b} = 6385, \quad (29)$$

which results in $m_b - m_c = 3378 \text{ MeV}/c^2$, i.e., $\sim 70 \text{ MeV}/c^2$ above (5). These differences of $m_b - m_c$ extraction from average masses of excited multiplets (13) and (29) together with perfect agreement of $m_b - m_c$ extraction from the ground states (17) and (18) suggest that we miss some contributions to the overall masses. This can be further exemplified when one realizes that the sextet masses are in principle calculable in the present approach, see Eqs. (9) and (10). Indeed, from the ground state antitriplet and sextet we obtain $1/I_1 = 172 \text{ MeV}/c^2$ and $170 \text{ MeV}/c^2$ for charm and beauty, respectively (see Ref. [27] for discussion). In the excited sector we get $1/I'_1 = 262 \text{ MeV}/c^2$ and $351 \text{ MeV}/c^2$ for charm and beauty, respectively. Two remarks are here in order. We expect some numerical difference between $1/I_1$ and $1/I'_1$, but $90 \text{ MeV}/c^2$ is larger than anticipated. What is more troubling, is the large difference between charm and beauty estimates. A practical solution would be to lower the average sextet mass for beauty, which would also improve $m_b - m_c$. This, however, would require to associate two LHCb states $\Xi_b(6327)$ and $\Xi_b(6333)$ with $J = 2$ sextet, rather with $J = 1$. However, the expected hyperfine splitting in $J = 2$ sextet is almost twice larger than for $J = 1$, and this would be completely incompatible with the charm sector. On the other hand even if we took $1/I'_1 = 1/I_1$ and applied this to the excited sextets, we would underestimate \mathcal{M}'_{6Q} only by $\sim 3\%$ both for charm and beauty.

V. DISCUSSION

Observation of five Ω_c^0 states by the LHCb Collaboration [3] implies that we should expect similar states in the beauty sector. Indeed, the replacement of the c -quark by the b -quark leads only to the overall mass shift of approximately $3330 \text{ MeV}/c^2$ [see Eq. (5)] and to the rescaling of the hyperfine splittings. On general grounds the mass splittings inside the excited multiplets and splittings between various multiplets should not change under the replacement of the c -quark by the b -quark. So, we expect at least five narrow Ω_b 's populating the mass interval $6350 \text{ MeV}/c^2$ – $6650 \text{ MeV}/c^2$ (see Tables VII–X) and the same number of the corresponding Ξ_Q and Σ_Q states in both sectors.

However, we see from the review of present experimental data of Sec. II that only a few candidates have been

⁶We use Fit 1.

TABLE IX. Fit 2, $\delta = 147$. Inputs are in bold face. Underlined entries can be attributed to some of the known baryons (see text). All masses are in MeV/c^2 .

J	S	Σ_b	Ξ_b	Ω_b	Σ_c	Ξ_c	Ω_c
0	1/2	6065	6227	6389	2676	2838	3000
1	1/2	6187	6327	6467	2786	<u>2926</u>	3066
	3/2	6193	6333	6473	<u>2810</u>	<u>2950</u>	3090
2	3/2	6438	6534	6630	3030	<u>3126</u>	3222
	5/2	6447	6543	6639	3070	3166	3262

 TABLE X. Fit 3, $\delta = 40$. Inputs are in bold face. Underlined entries can be attributed to some of the known baryons (see text). All masses are in MeV/c^2 .

J	S	Σ_b	Ξ_b	Ω_b	Σ_c	Ξ_c	Ω_c
0	1/2	6097	6227	6357	2740	2870	3000
1	1/2	6188	<u>6312</u>	6436	<u>2818</u>	<u>2942</u>	3066
	3/2	6193	<u>6317</u>	6441	2842	<u>2966</u>	3090
2	3/2	6374	6486	6598	2998	<u>3110</u>	3222
	5/2	6382	6494	6606	3038	3150	3262

found so far. Why do we not see all the states required by this simple argument? Possible explanations are:

- (i) Some of the $\mathbf{6}'$ states have small couplings to the ground state $\bar{\mathbf{3}}$. This is indeed the case, as illustrated in Table XI.
- (ii) For some reason the observed peak is not one resonance but several ones almost degenerated in mass. Such possibility is admitted e.g., by the LHCb Collaboration [7].
- (iii) It could be that some of expected excited sextet states are very wide and therefore are not seen experimentally. This seems to be the case for the $J = 2$ sextet, where the allowed decays are in D wave (see Table XI).

 TABLE XI. Type of couplings for the decays of excited heavy baryons to ground state multiplets and excited $\bar{\mathbf{3}}$ plus a pseudoscalar meson. Note that in the heavy quark limit c - and b -quarks act as spectators when the soliton decays. If the decay is forbidden by the angular momentum conservation, then the heavy quark spin flip may provide the missing angular momentum, but there is a penalty for the spin flip of $1/m_Q$ in the decay amplitude.

	$\bar{\mathbf{3}}(J^P = 0^+)$	$\mathbf{6}(J^P = 1^+)$	$\bar{\mathbf{3}}'(J^P = 1^-)$
$\bar{\mathbf{3}}'(J^P = 1^-)$	$1/m_Q$	S	...
$\mathbf{6}'(J^P = 0^-)$	S	$1/m_Q$	P
$\mathbf{6}'(J^P = 1^-)$	$1/m_Q$	S	P
$\mathbf{6}'(J^P = 2^-)$	D	D	P

- (iv) Some of the excited states are very narrow and the experimental resolution is not good enough to detect them.

In the following we shall review from this point of view some recent experimental searches of the Σ_Q and Ξ_Q excited states.

A. Where are the missing Σ_Q states?

Let us start with a b sector and the LHCb observation of $\Sigma_b(6097)$ [7] in the distribution of Λ_b^0 and π^\pm mass over the range between $5760 \text{ MeV}/c^2$ – $6360 \text{ MeV}/c^2$. One can clearly see (Fig. 2 in Ref. [7]) two small peaks corresponding to the ground state sextet; $\Sigma_b(5813)$ and $\Sigma_b(5833)$, and the large peak of $\Sigma_b(6097)$. With our interpretation (27) in this energy range at approximately $6200 \text{ MeV}/c^2$ we expect two hyperfine split resonances corresponding to $(\mathbf{6}', J = 1)$ multiplet,⁷ however, there is no sign of any enhancement in the data.⁸ A possible explanation would be, that the decay rate of the $J = 1$ multiplet to the ground state $\bar{\mathbf{3}}$ and a pseudoscalar meson is suppressed by $1/m_b^2$ (see Table XI). This suppression is too small in the charm sector, this is why the excited Ω_c states have been found with small, albeit observable widths. On the other hand p -wave decay to the excited anti-triplet is not suppressed, so $\Sigma_b(6097) \rightarrow \Lambda_b'(5912, 5920) + \pi^\pm$ should be visible. Σ_b states in $\mathbf{6}'(J^P = 2^-)$ are above the energy range scanned by LHCb; however, they are expected to be rather wide d -wave resonances.

We can roughly estimate partial decay width of $\Sigma_b(6097)$, since on general grounds [26] the l -wave decay width of baryon B_1 to B_2 where the soliton is in the $\text{SU}(3)$ representations $\mathbf{R}_{1,2}$ respectively, and a pseudoscalar meson Φ , is proportional to,

$$\Gamma_{B_1 \rightarrow B_2 + \Phi} \sim \frac{p_\Phi^{2l+1}}{F_\Phi^2} \frac{M_2}{M_1} \left[\begin{array}{c} 8 \quad \mathbf{R}_2 \mid \mathbf{R}_1 \\ \Phi \quad B_2 \mid B_1 \end{array} \right]^2. \quad (30)$$

Here $M_{1,2}$ are masses of baryons $B_{1,2}$, F_Φ is Φ meson decay constant, and the square bracket corresponds to the pertinent $\text{SU}(3)$ isoscalar factor. From Eq. (30) we get

$$\frac{\Gamma(\Sigma_b(6097) \rightarrow \Lambda_b(5620) + \pi)}{\Gamma(\Omega_c(3000) \rightarrow \Xi_c(2469) + \bar{K})} \sim 2.25, \quad (31)$$

since both decays are in s -wave, and both Σ_b and Ω_c are assumed to belong to $(\mathbf{6}', J = 0)$. Given rather small width of $\Omega_c(3000)$ of $\sim 4.5 \text{ MeV}$, we get

⁷This energy corresponds to $Q \sim 440 \text{ MeV}/c^2$, which is used in Ref. [6].

⁸A different $\text{SU}(3)$ assignment, e.g., $(\mathbf{6}', J = 1)$ would require a strong peak $\sim 100 \text{ MeV}/c^2$ below $\Sigma_b(6097)$ and two hyperfine split peaks $200 \text{ MeV}/c^2$ above, i.e., within the range scanned by LHCb.

$$\Gamma(\Sigma_b(6097) \rightarrow \Lambda_b(5620) + \pi) \sim 10 \text{ MeV}. \quad (32)$$

LHCb estimated the total width to be 30 MeV, however, theoretical total width will be larger than 10 MeV, as the new channels, like $\Sigma_b(6097) \rightarrow \Lambda'_b + \pi$ open.

In the charm sector the isotriplet of excited baryons decaying into $\Lambda_c^+ + \pi$ was observed in 2005 by the Belle Collaboration [42] with mass of 2800 MeV/ c^2 and is now included in the PDG [9] (see Table V). Originally Belle has tentatively identified these resonances as $S^P = 3/2^-$ but from the $J = 2$ multiplet. In our Fit 1 $\Sigma_b(1800)$ would be $S^P = 1/2^-$, in Fit 2 is $S^P = 3/2^-$ but from $J = 1$ sextet.

Belle has scanned rather large range of $\Lambda_c^+ + \pi$ invariant mass from 2285 MeV/ c^2 to 3085 MeV/ c^2 , where all five Σ_c resonances should be seen. On the other hand the *BABAR* Collaboration in a similar mass region reported a Σ_c state at 2846 MeV/ c^2 [43], which is 3σ from the Belle measurement, or a new resonance. Certainly experimental situation in this region is not clear, moreover one should remember that in *BABAR* charmed baryons are produced from B decays, and therefore their yields depend on the production mechanism. It is therefore possible that B decays to higher-spin baryons are suppressed [43].

B. Where are the missing Ξ_Q states?

In 2018 the LHCb Collaboration reported new $\Xi_b^{0,-}(6227)$ states in the invariant mass distributions of $\Lambda_b \bar{K}$ and $\Xi_b \pi$ over the range 6120–6520 MeV/ c^2 of total width $\Gamma \sim 18.1 \pm 5.7$ MeV [6]. It follows from our discussion in Sec. IV C that $\Xi_b(6227) = \Xi_b(\mathbf{6}', J^P = 0^-)$. In the same energy range we expect two hyperfine split $\Xi_b(6327)$ and $\Xi_b(6333)$, which we have classified as $(\mathbf{6}', J^P = 1^-)$. These states have been recently discovered by the LHCb Collaboration [13] in a three body decay to $\Lambda_b^0 \bar{K}^- \pi^+$, however, LHCb suggested a different interpretation following Refs. [44,45], namely as a 1D doublet of spin $3/2^+$ and $5/2^+$. On the other hand, according to the present interpretation, for these states two-body decays are suppressed as $1/m_b^2$ and therefore are not seen in $\Lambda_b \bar{K}$ and $\Xi_b \pi$ mass distribution. Furthermore, as seen from Tables VII–X, two hyperfine split $(\mathbf{6}', J^P = 2^-)$ states are above the energy range scanned by LHCb.

As in the case of $\Sigma_b(6097)$ we shall try to estimate the decay width of $\Xi_b(6227)$. From Eq. (30) we obtain for s -wave decays

$$\frac{\Gamma(\Xi_b(6227) \rightarrow \Xi_b(5795) + \pi)}{\Gamma(\Xi_b(6227) \rightarrow \Lambda_b(5620) + K)} \sim 2.7. \quad (33)$$

On the other hand

$$\frac{\Gamma(\Xi_b(6227) \rightarrow \Xi_b(5795)\pi)}{\Gamma(\Sigma_b(6097) \rightarrow \Lambda_b(5620) + \pi)} \sim 0.7. \quad (34)$$

Therefore the total width of $\Xi_b(6227)$ can be estimated as

$$\begin{aligned} & \Gamma(\Xi_b(6227) \rightarrow \Xi_b(5795) + \pi) \\ & + \Gamma(\Xi_b(6227) \rightarrow \Lambda_b(5620) + K) \sim 9.6 \text{ MeV}, \end{aligned} \quad (35)$$

where we have used (32). This is almost two times too small, but still within the accuracy of the present model, given large experimental error for the total $\Xi_b(6227)$ width of 5.7 MeV. Note that, unlike in the case of $\Sigma_b(6097)$, $\Xi_b(6227)$ has no open channel to $\bar{\mathbf{3}}^+$ pseudoscalar meson.

C. Comparison with the Skyrme model

The approach to heavy baryons in the soliton models dates back to the late 1980s and was initiated by a seminal paper by Callan and Klebanov [46] (with subsequent improvement in [47]) where strange baryons are viewed as bound states of kaonic fields in the soliton background of the Skyrme model [48,49]. Subsequently, with an advent of heavy quark symmetry [50], soliton bound states with charm or beauty mesons have been used to describe heavy baryons [51].

There are certain similarities and differences between our approach and the one of the Skyrme model. First of all the soliton in our picture is a relativistic quark configuration (see Fig. 1) with valence levels and a fully occupied Dirac sea [31]. The energy of the Dirac sea can be approximated using gradient expansion, yielding expressions analogous to the Skyrme lagrangian [31]. In an artificial limit where the soliton size is increased beyond its physical value that minimizes the aggregate energy of the valence level and the Dirac sea, the valence level sinks into the negative continuum and the model resembles the Skyrme model. The soliton energy in this case can be expressed entirely in terms of the gradient expansion of the chiral field. However, in the realistic situation the quark degrees of freedom are more appropriate for heavy baryon description. It would be interesting to see how in the large soliton limit the hole in the Dirac sea [from the missing valence quark—see Fig. 1(b)] combines with the heavy quark to form an effective heavy meson field.

On phenomenological side it is difficult to make an exact comparison of the bound state Skyrme model results and our predictions, because—to the best of our knowledge—in all Skyrme model calculations strangeness was also included in terms of the kaonic bound states. On the contrary, we treat the strange quark mass perturbatively and rely rather heavily on the underlying SU(3) representation structure, which is missing in the bound state approach.

Nevertheless, some comparison is possible if we confine ourselves only to the SU(2) substructure. The authors of Refs. [52,53] computed rotational and $1/m_Q$ corrections to

the bound state approach that lead to the hyperfine splittings of different spin multiplets. Expanding their [52] formula (28) in terms of a constant $c \sim 1/m_Q$ one recovers rotational energy analogous to our formula (6) and, in the linear order in c , spin-spin interaction identical to our Eq. (8) (with obvious exchange of their unknown parameter c by κ/m_Q).⁹

The bound-state approach boils down to finding a solution to the corresponding Schrödinger equation for a meson-soliton bound state in a potential provided by the soliton background. These modes correspond to the λ modes discussed in the Introduction. Typically the soliton potential is rather weak allowing only for a few such states. For example in Ref. [54] only two states with orbital momentum $l = 0$ have been found and one with $l = 1$. No states with higher l 's have been observed. This observation, although obviously strongly model dependent, reinforces our approach where λ modes have been neglected.

VI. CONCLUSIONS

In this paper we have tried to classify recently discovered heavy baryons in terms of the SU(3) multiplets of the light subsystem. To this end we have used heavy quark symmetry and the chiral quark-soliton model for which one can derive mass formulas both for the ground state baryons and for excited states. The relative heavy quark-soliton excitations, so called λ -modes, have not been taken into account. We have presented arguments that they are parametrically suppressed in the large N_c limit, on which the χ QSM is based. For the purpose of this analysis we have adopted so called model-independent approach, where *a priori* calculable quantities are fitted from a small number of input masses. The remaining masses and splittings can be then predicted.

Although the mass formulas (11) and (12) are the same both for charm and beauty baryons, it is clear that the heavy quark symmetry should work better in the b sector. Therefore as the only input from the charm sector we used the SU(3) assignment [29] of excited Ω_c states discovered by the LHCb Collaboration in 2017 [3]. This assignment has been further reinforced by the study of the decay widths of some of the Ω_c states [30].

⁹Note that $\mathbf{J} \cdot \mathbf{J}_Q = (S(S+1) - J(J+1) - J_Q(J_Q+1))/2$, and $J_Q(J_Q+1) = 3/4$, where S and J denote baryon spin and soliton angular momentum, respectively.

Further input came from the b sector (27) based on the hyperfine splittings and the constraints from the c sector. Our results are best illustrated in Table VIII. We have shown that all known Ξ_b and Σ_b states from Table VI can be interpreted as members of different sextets of negative parity. Unfortunately recently reported Ω_b states pose a problem, especially their mass differences that are much smaller than the Ω_c mass splittings in the charm sector, see however Ref. [36].

In the charm sector our analysis leaves two unassigned states: $\Xi_c(3056)$ and $\Xi_c(3079)$. It has been observed in Refs. [10,20,21] that the following mass splittings are equal: $\Omega_c(3050) - \Xi_c(2923)$, $\Omega_c(3065) - \Xi_c(2939)$, $\Omega_c(3090) - \Xi_c(2965)$, leading to the supposition that all these states belong to the SU(3) sextets. In our case the equality of the first splitting with the two others is rather accidental, as $\Omega_c(3050)$ and $\Xi_c(2923)$ in our model belong to the exotic **15**. Nevertheless the mass difference is numerically approximately equal to the ones in the $J = 1$ sextet (see Table III in Ref. [29]). Moreover, splittings in excited sextets of different J are not equal in the present approach [see Eqs. (11) and (12)], although numerically the difference may be quite small.

With the proposed assignment we have analyzed possible two-body decay patterns giving arguments why some states have not been seen in the two-body mass distributions. We have also predicted masses of yet unmeasured members of the excited sextets.

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