

Two- and three-body hadronic decays of charmed mesons involving a tensor meson

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We study the quasi-two-body $D \rightarrow TP$ decays and the three-body D decays proceeding through intermediate tensor resonances, where T and P denote tensor and pseudoscalar mesons, respectively. We employ the $D \rightarrow T$ transition form factors based upon light cone sum rules and the covariant light-front quark model to evaluate the decay rates, with the former giving a better agreement with current data. Though the tree amplitudes with the emitted meson being a tensor meson vanish under factorization approximation, contributions proportional to the tensor decay constant f_T can be produced from vertex and hard spectator-scattering corrections. We also investigate the finite-width effects of the tensor mesons and find that, contrary to three-body B decays, the tensor-mediated D decays are more seriously affected and the narrow width approximation has to be corrected. More experimental data are required in order to extract information topological amplitudes associated with quasi-two-body $D \rightarrow TP$ decays. Among the data, the $D^+ \rightarrow f_2\pi^+$ and $D^+ \rightarrow \bar{K}_2^{*0}\pi^+$ branching fractions are not self-consistent and further clarification is called for.

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I. INTRODUCTION

In this paper, we set to study the quasi-two-body $D \rightarrow TP$ decays and the three-body D decays proceeding through intermediate tensor resonances, where T and P denote tensor and pseudoscalar mesons, respectively. The $D \rightarrow TP$ decays have been studied previously in Refs. [1–5]. In Ref. [4], we pointed out that the $D \rightarrow TP$ measurements pose a big problem for theory. It appeared that the predicted branching fractions based on the factorization approach were at least two orders of magnitude smaller than data, even when the decays were free of weak annihilation contributions. Calculations in Refs. [3,4] were based on the Isgur-Scora-Grinstein-Wise (ISGW) model [6] (or its improved version ISGW2 model [7]) and the covariant light-front quark model (CLFQM) [8] for $D \rightarrow T$ transition form factors. Recently, these form factors have been evaluated using light cone sum rules (LCSR) in Ref. [5]. It turns out that form factors obtained from LCSR are much larger than those found in the ISGW

model or CLFQM. Consequently, the discrepancy between theory and experiment gets improved.

As discussed in Refs. [3,4], one generally has two sets of distinct diagrams for each topology in $D \rightarrow TP$ decays. For example, there are two external W -emission and internal W -emission diagrams, depending on whether the emitted particle is an even-parity meson or an odd-parity one. Following the convention in Refs. [3,4], we shall denote the primed amplitudes T' and C' for the case when the emitted meson is a tensor meson. Since the tensor meson cannot be produced from the $V - A$ current, its vector decay constant vanishes identically. Hence, we have set $T' = C' = 0$ before in the naïve factorization approach. Nevertheless, as stressed in Ref. [9], beyond the factorization approximation, contributions proportional to the decay constant f_T defined in Eq. (3.4) below can be produced through vertex and spectator-scattering corrections in the QCD factorization (QCDF) approach [10] for hadronic B decays. Hence, in this work we will generalize QCDF to charmed mesons to estimate the nonfactorizable effects in $D \rightarrow TP$ decays.

There are four $D \rightarrow T$ transition form factors induced from the $(V - A)$ current, A_0, A_1, A_2 , and V parametrized in Refs. [9,11], or $k, b_+, b_-,$ and h defined in the ISGW model [see Eq. (3.8) below]. The latter four form factors were calculated in CLFQM with the results listed in Table VI of

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Ref. [4]. However, as pointed out in Ref. [8], the form factor $k(q^2)$ at zero recoil was problematic as it did not respect heavy quark symmetry in the heavy quark limit. This was the main reason why the calculated branching fractions of $D \rightarrow TP$ decays were at least two orders of magnitude smaller than data. It was advocated in Ref. [8] that one might apply heavy quark symmetry to obtain the form factor $k(q^2)$. In this work we will apply heavy quark symmetry to $D \rightarrow T$ transitions to see any improvement on $k^{DT}(q^2)$.

Very recently, the form factors of $P \rightarrow T$ transition were analyzed in Ref. [12] within the covariant light-front quark model, which we will call CLFQM_b. This time, the four form factors $A_0, A_1, A_2,$ and V were directly evaluated in CLFQM_b in which some issues with the previous study of CLFQM were overcome. We will consider the form factors obtained in this model as a benchmark for comparison.

This paper is organized as follows. In Sec. II, we review the current experimental status of the measurements of three-body charmed meson decays that are relevant to our analysis. We provide the information of flavor SU(3) classification, decay constants, and form factors for the T mesons in Sec. III. Section IV presents the so-called quark-diagram approach to the decays. Each decay mode is decomposed in terms of quark diagrams characterized by their flavor topologies. The goal is to see if current experimental data can be used to infer the magnitude and strong phase associated with each of the amplitudes. In Sec. V, we study the flavor operators $a_{1,2}(M_1M_2)$ for $M_1M_2 = TP$ and PT within the framework of QCDF. Under the factorization assumption, we compute the rate of

each decay mode. We also examine the finite width effects for certain decay modes in Sec. VI. A summary of our findings is given in Sec. VII.

II. EXPERIMENTAL STATUS

It is known that three- and four-body decays of heavy mesons provide a rich laboratory for studying the intermediate state resonances. The Dalitz plot analysis of three- or four-body decays of charmed mesons is a very useful technique for this purpose. We are interested in $D \rightarrow TP$ decays followed by $T \rightarrow P_1P_2$. The results of various experiments are summarized in Table I. To extract the branching fraction for $D \rightarrow TP$, we apply the narrow width approximation (NWA)

$$\Gamma(D \rightarrow TP \rightarrow P_1P_2P) = \Gamma(D \rightarrow TP)_{\text{NWA}} \mathcal{B}(T \rightarrow P_1P_2). \quad (2.1)$$

Since this relation holds only in the $\Gamma_T \rightarrow 0$ limit, we put the subscript NWA to emphasize that $\mathcal{B}(D \rightarrow TP)$ thus obtained is under this limit. Finite width effects in certain decays will be discussed in Sec. VI. To extract the branching fractions of two-body decays of tensor mesons, we shall use [13]

$$\begin{aligned} \mathcal{B}(f_2(1270) \rightarrow \pi\pi) &= (84.2_{-0.9}^{+2.9})\%, \\ \mathcal{B}(f_2(1270) \rightarrow K\bar{K}) &= (4.6_{-0.4}^{+0.5})\%, \\ \mathcal{B}(a_2(1320) \rightarrow K\bar{K}) &= (4.9 \pm 0.8)\%, \\ \mathcal{B}(K_2^*(1430) \rightarrow K\pi) &= (49.9 \pm 1.2)\%. \end{aligned}$$

TABLE I. Experimental branching fractions of various $D \rightarrow TP$ decays. For simplicity and convenience, we have dropped the mass identification for $f_2(1270)$, $a_2(1320)$, and $K_2^*(1430)$. Data are taken from Ref. [13] unless specified otherwise.

$\mathcal{B}(D \rightarrow TP; T \rightarrow P_1P_2)$	$\mathcal{B}(D \rightarrow TP)_{\text{NWA}}$
$\mathcal{B}(D^+ \rightarrow f_2\pi^+; f_2 \rightarrow \pi^+\pi^-) = (5.0 \pm 0.9) \times 10^{-4}$	$\mathcal{B}(D^+ \rightarrow f_2\pi^+) = (8.9 \pm 1.6) \times 10^{-4}$
$\mathcal{B}(D^+ \rightarrow \bar{K}_2^{*0}\pi^+; \bar{K}_2^{*0} \rightarrow K^-\pi^+) = (2.3 \pm 0.7) \times 10^{-4}$	$\mathcal{B}(D^+ \rightarrow \bar{K}_2^{*0}\pi^+) = (6.9 \pm 2.1) \times 10^{-4}$
$\mathcal{B}(D^+ \rightarrow K_2^{*0}\pi^+; K_2^{*0} \rightarrow K^+\pi^-) = (3.9 \pm 2.7) \times 10^{-5}$	$\mathcal{B}(D^+ \rightarrow K_2^{*0}\pi^+) = (1.17 \pm 0.81) \times 10^{-4}$
$\mathcal{B}(D^+ \rightarrow \bar{K}_2^{*0}K^+; \bar{K}_2^{*0} \rightarrow K^-\pi^+) = (1.6_{-0.8}^{+1.2}) \times 10^{-4}$	Prohibited on shell
$\mathcal{B}(D^0 \rightarrow f_2\pi^0; f_2 \rightarrow \pi^+\pi^-) = (1.96 \pm 0.21) \times 10^{-4}$	$\mathcal{B}(D^0 \rightarrow f_2\pi^0) = (3.5 \pm 0.4) \times 10^{-4}$
$\mathcal{B}(D^0 \rightarrow f_2K_S; f_2 \rightarrow \pi^+\pi^-) = (9_{-6}^{+10}) \times 10^{-5}$	$\mathcal{B}(D^0 \rightarrow f_2\bar{K}^0) = (3.2_{-2.1}^{+3.6}) \times 10^{-4}$
$\mathcal{B}(D^0 \rightarrow f_2K_S; f_2 \rightarrow \pi^0\pi^0) = (2.3 \pm 1.1) \times 10^{-4}$	$\mathcal{B}(D^0 \rightarrow f_2\bar{K}^0) = (1.6 \pm 0.8) \times 10^{-3}$
$\mathcal{B}(D^0 \rightarrow K_2^{*-}\pi^+; K_2^{*-} \rightarrow K_S^0\pi^-) = (3.4_{-1.0}^{+1.9}) \times 10^{-4}$	$\mathcal{B}(D^0 \rightarrow K_2^{*-}\pi^+) = (2.0_{-0.6}^{+1.1}) \times 10^{-3}$
$\mathcal{B}(D^0 \rightarrow K_2^{*+}\pi^-; K_2^{*+} \rightarrow K_S^0\pi^+) < 3.4 \times 10^{-5}$	$\mathcal{B}(D^0 \rightarrow K_2^{*+}\pi^-) < 2.0 \times 10^{-4}$
$\mathcal{B}(D^0 \rightarrow a_2^-\pi^+; a_2^- \rightarrow K_S K^-) = (5 \pm 5) \times 10^{-6}$	$\mathcal{B}(D^0 \rightarrow a_2^-\pi^+) = (2.0 \pm 3.9) \times 10^{-4}$
$\mathcal{B}(D^0 \rightarrow a_2^+K^-; a_2^+ \rightarrow K^+K_S) < 1.04 \times 10^{-4}$ ^a	$\mathcal{B}(D^0 \rightarrow a_2^+K^-) < 4.2 \times 10^{-3}$
$\mathcal{B}(D^0 \rightarrow a_2^-K^+; a_2^- \rightarrow K^-K_S) < 0.72 \times 10^{-4}$ ^a	$\mathcal{B}(D^0 \rightarrow a_2^-K^+) < 2.9 \times 10^{-3}$
$\mathcal{B}(D_s^+ \rightarrow f_2\pi^+; f_2 \rightarrow \pi^+\pi^-) = (1.09 \pm 0.20) \times 10^{-3}$	$\mathcal{B}(D_s^+ \rightarrow f_2\pi^+) = (1.94 \pm 0.36) \times 10^{-3}$
$\mathcal{B}(D_s^+ \rightarrow f_2\pi^+; f_2 \rightarrow \pi^0\pi^0) = (0.80 \pm 0.42) \times 10^{-3}$ ^b	$\mathcal{B}(D_s^+ \rightarrow f_2\pi^+) = (2.85 \pm 1.50) \times 10^{-3}$

^aBESIII data taken from Ref. [14].

^bBESIII data taken from Ref. [15].

The extracted branching fractions are shown in Table I under the column $\mathcal{B}(D \rightarrow TP)_{\text{NWA}}$.

Comparing Table I with the experimental data obtained in 2010 as summarized in Table III of Ref. [4], it is clear that only a few new measurements were available since 2010. Many existing measurements need further improvement; for example, the uncertainties of $\mathcal{B}(D^0 \rightarrow f_2 K_S \rightarrow \pi^+ \pi^- K_S)$ and $\mathcal{B}(D^0 \rightarrow a_2^- \pi^+ \rightarrow K_S K^- \pi^+)$ are larger or comparable to their central values. Moreover, as will be discussed in Sec. IV, the existing data of $D^+ \rightarrow f_2 \pi^+$, $\bar{K}_2^{*0} \pi^+$, and $K_2^{*0} \pi^+$ are not self-consistent. In other words, the quality of the data needs to be substantially improved.

III. PHYSICAL PROPERTIES OF TENSOR MESONS

The observed $J^P = 2^+$ tensor mesons $f_2(1270)$, $f_2'(1525)$, $a_2(1320)$, and $K_2^*(1430)$ form an $SU(3)$ 1^3P_2 nonet. The $q\bar{q}$ content for isodoublet and isovector tensor resonances are obvious. Just as the η - η' mixing in the pseudoscalar case, the isoscalar tensor states $f_2(1270)$ and $f_2'(1525)$ also have a mixing, and their wave functions are defined by

$$\begin{aligned} f_2(1270) &= \frac{1}{\sqrt{2}}(f_2^u + f_2^d) \cos \theta_{f_2} + f_2^s \sin \theta_{f_2}, \\ f_2'(1525) &= -\frac{1}{\sqrt{2}}(f_2^u + f_2^d) \sin \theta_{f_2} + f_2^s \cos \theta_{f_2}, \end{aligned} \quad (3.1)$$

with $f_2^q \equiv q\bar{q}$. Since $\pi\pi$ is the dominant decay mode of $f_2(1270)$ whereas $f_2'(1525)$ decays predominantly into $K\bar{K}$ (see Ref. [13]), it is obvious that this mixing angle should be small. It is found that $\theta_{f_2} = 5.6^\circ$ when the quadratic mass formula for the mixing angle is employed [13,16]. Therefore, $f_2(1270)$ is primarily a $(u\bar{u} + d\bar{d})/\sqrt{2}$ state, while $f_2'(1525)$ is dominantly $s\bar{s}$.

The polarization tensor $\varepsilon_{\mu\nu}$ of a 3P_2 tensor meson with $J^{PC} = 2^{++}$ satisfies the relations

$$\varepsilon_{\mu\nu} = \varepsilon_{\nu\mu}, \quad \varepsilon_\mu^\mu = 0, \quad p_\mu \varepsilon^{\mu\nu} = p_\nu \varepsilon^{\mu\nu} = 0, \quad (3.2)$$

where p^μ is the momentum of the tensor meson. Therefore,

$$\langle 0 | (V - A)_\mu | T(\varepsilon, p) \rangle = a \varepsilon_{\mu\nu} p^\nu + b \varepsilon^\nu{}_\nu p_\mu = 0, \quad (3.3)$$

and hence the decay constant of the tensor meson vanishes identically; that is, the tensor meson cannot be produced from the $V - A$ current. Nevertheless, a tensor meson can be created from these local currents involving covariant derivatives [17]

$$\begin{aligned} \langle T(P, \lambda) | J_{\mu\nu}(0) | 0 \rangle &= f_T m_T^2 \varepsilon^*(\lambda)_{\mu\nu}, \\ \langle T(P, \lambda) | J_{\mu\alpha}^\perp(0) | 0 \rangle &= -i f_T^\perp m_T (\varepsilon_{\mu\alpha}^*(\lambda) P_\nu - \varepsilon_{\nu\alpha}^*(\lambda) P_\mu), \end{aligned} \quad (3.4)$$

where λ is the helicity of the tensor meson, and

$$\begin{aligned} J_{\mu\nu}(0) &= \frac{1}{2} (\bar{q}_1(0) \gamma_\mu i \vec{D}_\nu q_2(0) + \bar{q}_1(0) \gamma_\nu i \vec{D}_\mu q_2(0)), \\ J_{\mu\alpha}^\perp(0) &= \bar{q}_1(0) \sigma_{\mu\nu} i \vec{D}_\alpha q_2(0), \end{aligned} \quad (3.5)$$

where $\vec{D}_\mu = \vec{\partial}_\mu - \vec{D}_\mu$ with $\vec{D}_\mu = \vec{\partial}_\mu + i g_s A_\mu^a \lambda^a / 2$ and $\vec{D}_\mu = \vec{\partial}_\mu - i g_s A_\mu^a \lambda^a / 2$. The decay constants f_T and f_T^\perp are scale dependent and they have been evaluated using QCD sum rules at the scale $\mu = 1$ GeV [17]. We list the results of f_T for later convenience (in units of MeV)

$$\begin{aligned} f_T(f_2(1270)) &= 102 \pm 6, & f_T(f_2'(1525)) &= 126 \pm 4, \\ f_T(a_2(1320)) &= 107 \pm 6, & f_T(K_2^*(1430)) &= 118 \pm 5. \end{aligned} \quad (3.6)$$

The general expression for the $D \rightarrow T$ transition has the form [9,11]¹

$$\begin{aligned} \langle T(p, \lambda) | V_\mu | D(p_D) \rangle &= \frac{2}{m_D + m_T} \varepsilon_{\mu\nu\alpha\beta} e_{(\lambda)}^{*\nu} p_D^\alpha p^\beta V^{DT}(q^2), \\ \langle T(p, \lambda) | A_\mu | D(p_D) \rangle &= 2i m_T \frac{e^{(\lambda)*} \cdot p_D}{q^2} q_\mu A_0^{DT}(q^2) + i(m_D + m_T) \left[e_{(\lambda)}^{(\lambda)*} - \frac{e^{(\lambda)*} \cdot p_D}{q^2} q_\mu \right] A_1^{DT}(q^2) \\ &\quad - i \frac{e^{(\lambda)*} \cdot p_D}{m_D + m_T} \left[p_\mu + (p_D)_\mu - \frac{m_D^2 - m_T^2}{q^2} q_\mu \right] A_2^{DT}(q^2), \end{aligned} \quad (3.7)$$

¹The $D \rightarrow T$ transition form factors defined in Refs. [11,9] are different by a factor of i . We shall use the former as they are consistent with the normalization of $D \rightarrow S$ transition given in Ref. [8].

where $q_\mu = (p_D - p)_\mu$ and $e_{(\lambda)}^{*\mu} \equiv \varepsilon^{*\mu\nu}(\lambda)(p_D)_\nu / m_D$. Throughout the paper we will adopt the convention $\varepsilon^{0123} = -1$. In the ISGW model [6], the general expression for the $D \rightarrow T$ transition is parametrized as

$$\begin{aligned}
& \langle T(p, \lambda) | (V - A)_\mu | D(p_D) \rangle \\
& = h(q^2) \epsilon_{\mu\nu\rho\sigma} \epsilon^{*\nu\alpha} p_{D\alpha} (p_D + p)^\rho q^\sigma - ik(q^2) \epsilon_{\mu\nu}^* p_D^\nu \\
& \quad - ib_+(q^2) \epsilon_{\alpha\beta}^* p_D^\alpha p_D^\beta (p_D + p)_\mu - ib_-(q^2) \epsilon_{\alpha\beta}^* p_D^\alpha p_D^\beta q_\mu,
\end{aligned} \tag{3.8}$$

where the form factor k is dimensionless, and the canonical dimension of h, b_+ and b_- is -2 . The relations between these two different sets of form factors are given by

$$\begin{aligned}
V^{DT}(q^2) & = m_D(m_D + m_T)h(q^2), \\
A_1^{DT}(q^2) & = \frac{m_D}{m_D + m_T}k(q^2), \\
A_2^{DT}(q^2) & = -m_D(m_D + m_T)b_+(q^2), \\
A_0^{DT}(q^2) & = \frac{m_D}{2m_T}[k^2(q^2) + (m_D^2 - m_T^2)b_+(q^2) + q^2b_-(q^2)].
\end{aligned} \tag{3.9}$$

The $D \rightarrow T$ transition form factors had been previously evaluated in the ISGW model [6] and its improved version, ISGW2 [7], and the CLFQM [8]. There were two modern investigations: one was based on the light cone sum rule approach [5] and the other on the covariant light-front quark model denoted by CLFQM_b [12]. The four form factors k, b_+, b_- , and h defined in Eq. (3.8) for the $D \rightarrow T$ transition had been studied in CLFQM and shown in Table VI of Ref. [4]. It was pointed out in Ref. [9] that among these four form factors, $k(q^2)$ was particularly sensitive to β_T , a parameter describing the tensor-meson wave function, and that $k(q^2)$ at zero recoil showed a large deviation from the heavy quark symmetry relation. It is possible that the very complicated analytic expression for $k(q^2)$ given in Eq. (3.29) of Ref. [8] is not complete. To overcome this difficulty, it was advocated in Ref. [8] that one might apply the heavy quark symmetry relation to obtain $k(q^2)$ for $P \rightarrow T$ transition [see Eq. (3.40) of Ref. [8]]

$$\begin{aligned}
k(q^2) & = m_P m_T \left(1 + \frac{m_P^2 + m_T^2 - q^2}{2m_P m_T} \right) \\
& \quad \times \left[h(q^2) - \frac{1}{2}b_+(q^2) + \frac{1}{2}b_-(q^2) \right].
\end{aligned} \tag{3.10}$$

In other words, the CLFQM results are obtained by first calculating the form factors $h(q^2), b_+(q^2)$, and $b_-(q^2)$ using the covariant light-front approach [8] and $k(q^2)$ from the heavy quark symmetry relation Eq. (3.10) and then converted them into the form-factor set $V(q^2)$ and $A_{0,1,2}(q^2)$.

Very recently, the $P \rightarrow T$ transition form factors $V(q^2)$ and $A_{0,1,2}(q^2)$ were directly evaluated in CLFQM_b in which the issues with self-consistency and Lorentz covariance of

TABLE II. Form factors $A_0^{DT}(q^2)$ for $D \rightarrow f_2(1270), a_2(1320), K_2^*(1430)$ transitions at $q^2 = 0$ in the ISGW2 model [7], the covariant light-front quark models: CLFQM [8], CLFQM_b [12], and LCSR [5]. The values of $A_0^{DT}(0)$ in the ISGW2 model are readily obtained from Table VI of [4]. The CLFQM results are obtained by first calculating the form factors $h(q^2), b_+(q^2)$, and $b_-(q^2)$ using the covariant light-front approach and $k(q^2)$ from the heavy quark symmetry relation Eq. (3.10) by setting $P = D$ and then converting them into the form-factor set $V(q^2)$ and $A_{0,1,2}(q^2)$.

Transition	ISGW2 [7]	CLFQM [8]	CLFQM _b [12]	LCSR [5]
$D \rightarrow f_2^q$	0.20	1.10	...	1.92
$D \rightarrow K_2^*$	0.27	1.01	$0.68_{-0.08}^{+0.06}$	1.43 ^a
$D \rightarrow a_2$	0.20	0.94	$0.62_{-0.07}^{+0.07}$	1.80
$D_s^+ \rightarrow f_2^s$	0.75	0.90	$0.72_{-0.08}^{+0.07}$	1.20
$D_s^+ \rightarrow K_2^*$	0.84	0.87	$0.58_{-0.08}^{+0.05}$...

^aThe value of 2.98 for $A_0^{DK_2^*}(0)$ given in Table II of [5] is not consistent with that shown in Fig. 4 of the same reference. The correct value should read 1.43 [18].

the covariant light-front approach were carefully examined and resolved [12]. It is clear from Table 5 of Ref. [12] that $B \rightarrow a_2$ and $B \rightarrow K_2^*$ transition form factors obtained in CLFQM and CLFQM_b are consistent with each other, especially for the $B \rightarrow K_2^*$ transition.

Since the relevant form factor is $A_0^{DT}(q^2)$ in the subsequent study of hadronic $D \rightarrow TP$ decays, we exhibit in Table II the values of $A_0^{DT}(0)$ in various models. The CLFQM results are obtained from the form factors $h(q^2), b_+(q^2)$ and $b_-(q^2)$ from Table VI of Ref. [4] and $k(q^2)$ from the heavy quark symmetry relation Eq. (3.10) for the $D \rightarrow P$ transition. Finally, we apply Eq. (3.9) to get $A_0^{DT}(0)$. Unlike the $B \rightarrow P$ transition case, the CLFQM_b results are smaller than CLFQM for various $D \rightarrow T$ transitions. Presumably, this means that the heavy quark symmetry relation Eq. (3.10) has some deviation from the realistic value for $k(q^2)$ as the charm meson is not very heavy. At any rate, we shall take CLFQM_b predictions as the representative values for the covariant light-front approach.

The form-factor q^2 dependence in the CLFQM, CLFQM_b, and LCSR can be found in Refs. [4,12,5], respectively. Evidently, the form factors obtained from LCSR are much larger than those in all the other models. For example, the predicted form factor $A_0^{Da_2}(0)$ in LCSR is larger than that in CLFQM, CLFQM_b, and ISGW2 by a factor of 2, 3, and 9, respectively. This will be tested when we come to the study of $D \rightarrow TP$ decays in Sec. V C.

IV. DIAGRAMMATIC APPROACH

It is known that a least model-dependent analysis of heavy meson decays can be carried out in the so-called

topological diagram approach. In this diagrammatic scenario, all two-body nonleptonic weak decays of heavy mesons can be expressed in terms of six distinct quark diagrams [19–21]: T , the external W -emission tree diagram; C , the internal W -emission; E , the W -exchange; A , the W -annihilation; H , the horizontal W -loop; and V , the vertical W -loop. These diagrams are classified according to the topologies of weak interactions with all strong interaction effects encoded. The one-gluon exchange approximation of the H graph is the so-called penguin diagram. Since given the current data it is premature to consider CP asymmetries in these decays, we ignore both H and V diagrams.

The topological amplitudes for $D \rightarrow TP$ decays have been discussed in Refs. [3,4]. Just as $D \rightarrow VP$ decays, one generally has two sets of distinct diagrams for each topology. For example, there are two external W -emission and internal W -emission diagrams, depending on whether the emitted particle is an even-parity meson or an odd-parity one. Following the convention in Refs. [3,4], we shall denote the primed amplitudes T' and C' for the case when the emitted meson is a tensor meson. For the W -exchange and W -annihilation diagrams with the final state $q_1\bar{q}_2$, the prime amplitude denotes that the even-parity meson contains the quark q_1 . Although T' and C' are usually set to zero in the naïve factorization approach due to the vanishing vector decay constant of the tensor meson induced from the $V - A$ current, they do receive nonfactorizable contributions that will be elucidated in Sec. VA below.

The topological amplitudes for $D \rightarrow TP$ decays are given in Table III. We have 15 independent unknown parameters for the eight topological amplitudes T , C , E , A and T' , C' , E' , A' . It is clear from Table III that we have

only eight items of available data (some of them being redundant) and three upper limits. This means that at present, we have more theory parameters than observables. Moreover, the data for $D^+ \rightarrow TP$ modes appear not self consistent. According to the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements associated with each decay mode and the expectation of $|T| \gg |C'|$, it is expected that $\mathcal{B}(D^+ \rightarrow \bar{K}_2^{*0}\pi^+) > \mathcal{B}(D^+ \rightarrow f_2\pi^+) \gg \mathcal{B}(D^+ \rightarrow K_2^{*0}\pi^+)$. This hierarchy pattern is not respected by the current data.

V. FACTORIZATION APPROACH

The diagrammatic approach has been applied quite successfully to hadronic decays of charmed mesons into PP and VP final states. When generalized to the decay modes involving a tensor meson in the final state, it appears that the current data are still insufficient for us to fully extract the information of all amplitudes. Therefore, we take the naïve factorization formalism as a complementary approach to estimate the rates of these decay modes. In this framework, the W -exchange and -annihilation types of contributions will be neglected.

A. Factorizable and nonfactorizable amplitudes

The factorizable amplitudes for the $D \rightarrow TP$ decays involve the quantities

$$\begin{aligned} X^{(DT,P)} &\equiv \langle P(q) | (V - A)_\mu | 0 \rangle \langle T(p) | (V - A)^\mu | D(p_D) \rangle, \\ X^{(DP,T)} &\equiv \langle T(q) | (V - A)_\mu | 0 \rangle \langle P(p) | (V - A)^\mu | D(p_D) \rangle, \end{aligned} \quad (5.1)$$

with the expression

TABLE III. Topological amplitudes of $D \rightarrow TP$ decays. The experimental branching fractions denoted by \mathcal{B}_{NWA} are taken from Table I.

Decay	Amplitude	\mathcal{B}_{NWA}
$D^+ \rightarrow f_2\pi^+$	$\frac{1}{\sqrt{2}}V_{cd}^*V_{ud}\cos\theta_{f_2}(T + C' + A + A') + V_{cs}^*V_{us}\sin\theta_{f_2}C$	$(8.9 \pm 1.6) \times 10^{-4}$
$D^+ \rightarrow \bar{K}_2^{*0}\pi^+$	$V_{cs}^*V_{ud}(T + C')$	$(6.9 \pm 2.1) \times 10^{-4}$
$D^+ \rightarrow K_2^{*0}\pi^+$	$V_{cd}^*V_{us}(C' + A)$	$(1.2 \pm 0.8) \times 10^{-4}$
$D^+ \rightarrow \bar{K}_2^{*0}K^+$	$V_{cs}^*V_{us}T + V_{cd}^*V_{ud}A$	Prohibited on shell
$D^0 \rightarrow f_2\pi^0$	$\frac{1}{2}V_{cd}^*V_{ud}\cos\theta_{f_2}(C' - C - E' - E) + \frac{1}{\sqrt{2}}V_{cs}^*V_{us}\sin\theta_{f_2}C'$	$(3.5 \pm 0.4) \times 10^{-4}$
$D^0 \rightarrow f_2\bar{K}^0$	$V_{cs}^*V_{ud}[\frac{1}{\sqrt{2}}\cos\theta_{f_2}(C + E) + \sin\theta_{f_2}E']$	$(4.6 \pm 2.7) \times 10^{-4}$ ^a
$D^0 \rightarrow K_2^{*-}\pi^+$	$V_{cs}^*V_{ud}(T + E')$	$(2.0_{-0.6}^{+1.1}) \times 10^{-3}$
$D^0 \rightarrow K_2^{*+}\pi^-$	$V_{cd}^*V_{us}(T' + E)$	$< 2.0 \times 10^{-4}$
$D^0 \rightarrow a_2^-\pi^+$	$V_{cd}^*V_{ud}(T + E')$	$(2.0 \pm 3.9) \times 10^{-4}$
$D^0 \rightarrow a_2^+K^-$	$V_{cs}^*V_{ud}(T' + E)$	$< 4.2 \times 10^{-3}$
$D^0 \rightarrow a_2^-K^+$	$V_{cd}^*V_{us}(T + E')$	$< 2.9 \times 10^{-3}$
$D_s^+ \rightarrow f_2\pi^+$	$V_{cs}^*V_{ud}[\frac{1}{\sqrt{2}}\cos\theta_{f_2}(A + A') + \sin\theta_{f_2}T]$	$(2.0 \pm 0.4) \times 10^{-3}$ ^b

^aTaken from $D^0 \rightarrow f_2K_S \rightarrow \pi^+\pi^-K_S$ and $D^0 \rightarrow f_2K_S \rightarrow \pi^0\pi^0K_S$ of Table I.

^bTaken from $D_s^+ \rightarrow f_2\pi^+ \rightarrow \pi^+\pi^-\pi^+$ and $D_s^+ \rightarrow f_2\pi^+ \rightarrow \pi^0\pi^0\pi^+$ of Table I.

$$X^{(DT,P)} = 2f_P \frac{m_T}{m_D} A_0^{DT}(m_P^2) \epsilon^{*\mu\nu}(0) (p_D)_\mu (p_D)_\nu, \quad (5.2)$$

while $X^{(DP,T)}$ vanishes owing to the fact that the tensor meson cannot be produced through the $V-A$ current. Nevertheless, as shown in Ref. [9], beyond the factorization approximation, nonfactorizable contributions proportional to the decay constant f_T defined in Eq. (3.4) can be produced from vertex and spectator-scattering corrections

$$\bar{X}^{(DP,T)} = \sqrt{6} f_T \frac{m_T^2}{m_D p_c} F_1^{DP}(m_T^2) \epsilon^{*\mu\nu}(0) (p_D)_\mu (p_D)_\nu, \quad (5.3)$$

with p_c being the c.m. momentum of either T or P in the D rest frame.

The amplitudes $X^{(DT,P)}$ and $\bar{X}^{(DP,T)}$ can be further simplified by working in the D rest frame and assuming that T (P) moves along the $-z$ (z) axis [9]. In this case, $p_D^\mu = (m_D, 0, 0, 0)$ and $\epsilon^{*\mu\nu}(0) = \sqrt{2/3} \epsilon^{*\mu}(0) \epsilon^{*\nu}(0)$ with $\epsilon^{*\mu}(0) = (p_c, 0, 0, E_T)^\mu / m_T$ and, consequently,

$$\begin{aligned} X^{(DT,P)} &= 2\sqrt{\frac{2}{3}} f_P \frac{m_D}{m_T} p_c^2 A_0^{DT}(m_P^2), \\ \bar{X}^{(DP,T)} &= 2f_T m_D p_c F_1^{DP}(m_T^2). \end{aligned} \quad (5.4)$$

It is interesting to notice that the expression of $\bar{X}^{(DP,T)}$ has a similar structure as $X^{(DP,V)}$

$$\begin{aligned} X^{(DP,V)} &\equiv \langle V | J'_\mu | 0 \rangle \langle P | J^\mu | D(p_D) \rangle \\ &= 2f_V m_V \epsilon \cdot p_D F_1^{DP}(m_V^2) \\ &\rightarrow 2f_V m_D p_c F_1^{DP}(m_V^2). \end{aligned} \quad (5.5)$$

The color-allowed and color-suppressed tree amplitudes T, T', C , and C' then have the expressions (in units of $G_F/\sqrt{2}$)

$$\begin{aligned} T &= 2a_1(TP) \sqrt{\frac{2}{3}} f_P \frac{m_D}{m_T} p_c^2 A_0^{DT}(m_P^2), \\ C &= 2a_2(TP) \sqrt{\frac{2}{3}} f_P \frac{m_D}{m_T} p_c^2 A_0^{DT}(m_P^2), \\ T' &= 2a_1(PT) f_T m_D p_c F_1^{DP}(m_T^2), \\ C' &= 2a_2(PT) f_T m_D p_c F_1^{DP}(m_T^2), \end{aligned} \quad (5.6)$$

where the nonfactorizable amplitudes are dictated by the tensor decay constant f_T .

The flavor operator coefficients $a_i(M_1 M_2)$ in Eq. (5.6) are basically the Wilson coefficients in conjunction with short-distance nonfactorizable corrections such as vertex corrections and hard spectator interactions. In general, they have the expressions [10,22]

$$\begin{aligned} a_1(M_1 M_2) &= \left(c_1 + \frac{c_2}{N_c} \right) N_1(M_2) + \frac{c_2}{N_c} \frac{C_F \alpha_s}{4\pi} \\ &\quad \times \left[V_1(M_2) + \frac{4\pi^2}{N_c} H_1(M_1 M_2) \right], \\ a_2(M_1 M_2) &= \left(c_2 + \frac{c_1}{N_c} \right) N_2(M_2) + \frac{c_1}{N_c} \frac{C_F \alpha_s}{4\pi} \\ &\quad \times \left[V_2(M_2) + \frac{4\pi^2}{N_c} H_2(M_1 M_2) \right], \end{aligned} \quad (5.7)$$

where c_i are the Wilson coefficients, $C_F = (N_c^2 - 1)/(2N_c)$ with $N_c = 3$, M_2 is the emitted meson and M_1 shares the same spectator quark as the D meson. The quantities $V_i(M_2)$ account for vertex corrections, and $H_i(M_1 M_2)$ for hard spectator interactions with a hard gluon exchange between the emitted meson and the spectator quark of the D meson. The explicit expressions of $V_{1,2}(M)$ and $H_{1,2}(M_1 M_2)$ in QCDF for $B \rightarrow TP$ decays are given in Ref. [9]. The expression of the quantities $N_i(M_2)$, which are relevant to the factorizable amplitudes, reads

$$N_i(P) = 1, \quad N_i(T) = 0. \quad (5.8)$$

It is obvious that $a_{1,2}(PT)$ vanish in the factorization limit and receive nonfactorizable contributions when the strong interactions are turned on. Here we generalize the work of Ref. [9] to $D \rightarrow TP$ decays to obtain the relevant flavor operators. The numerical results for the flavor operators $a_i(M_1 M_2)$ with $M_1 M_2 = TP$ and PT are shown in Table IV.

B. Two- and three-body decays

The decay rate of the $D \rightarrow TP$ decay is given by

$$\Gamma(D \rightarrow TP) = \frac{p_c}{8\pi m_D^2} |A(D \rightarrow TP)|^2, \quad (5.9)$$

where the decay amplitude can be read from Table III directly. The tree amplitudes T, T', C , and C' are given in Eq. (5.6). Sometimes we write $A(D \rightarrow TP) = \mathcal{M}(D \rightarrow TP) \epsilon^{*\mu\nu}(0) (p_D)_\mu (p_D)_\nu$. Then the decay rate is recast to

$$\Gamma(D \rightarrow TP) = \frac{p_c^5}{12\pi m_T^2 m_D^2} |\mathcal{M}(D \rightarrow TP)|^2. \quad (5.10)$$

We consider two different $D \rightarrow T$ transition form-factor models: the CLFQM_b [12] and the LCSR [5].

The calculation of three-body decays mediated by tensor resonances is more complicated because of the angular distribution of a tensor meson decaying into two pseudo-scalar mesons. We shall take the decay $D^+ \rightarrow \bar{K}_2^{*0} \pi^+ \rightarrow K^- \pi^+ \pi^+$ as an example to illustrate the calculation for the

TABLE IV. Numerical values of the flavor operators $a_i^p(M_1M_2)$ for $M_1M_2 = TP$ and PT at the scale $\mu = \bar{m}_c(\bar{m}_c) = 1.3$ GeV.

	$f_2(1270)\pi$	$\pi f_2(1270)$		$f_2(1270)K$	$Kf_2(1270)$
a_1	$1.391 + 0.314i$	$-0.043 + 0.021i$	a_1	$1.599 + 0.928i$	$-0.074 + 0.013i$
a_2	$-0.760 - 0.685i$	$0.098 - 0.055i$	a_2	$-1.209 - 2.006i$	$0.166 - 0.038i$
	$a_2(1320)\pi$	$\pi a_2(1320)$		$a_2(1320)K$	$Ka_2(1320)$
a_1	$1.372 + 0.350i$	$-0.065 + 0.011i$	a_1	$1.664 + 1.576i$	$-0.089 + 0.008i$
a_2	$-0.719 - 0.760i$	$0.099 - 0.055i$	a_2	$-1.347 - 3.402i$	$0.198 - 0.028i$
	$K_2^*(1430)\pi$	$\pi K_2^*(1430)$		$K_2^*(1430)K$	$KK_2^*(1430)$
a_1	$1.254 + 0.403i$	$-0.044 + 0.020i$	a_1	$1.447 - 1.394i$	$-0.039 + 0.098i$
a_2	$-0.467 - 0.877i$	$0.101 - 0.054i$	a_2	$-0.880 + 2.991i$	$0.089 - 0.221i$

three-body rate. Writing $\mathcal{A}_{K_2^*} \equiv A(D^+ \rightarrow \bar{K}_2^{*0}\pi^+ \rightarrow \pi^+(p_1)K^-(p_2)\pi^+(p_3))$ and following Eq. (4.16) of Ref. [23], we have

$$\mathcal{A}_{K_2^*} = g_{\bar{K}_2^{*0} \rightarrow K^-\pi^+} T_{K_2^*}^{\text{BW}}(s_{12}) \frac{q^2}{\sqrt{6}} (1 - 3\cos^2\theta_{13}) \times A(D^+ \rightarrow \bar{K}_2^{*0}(m_{12})\pi^+) + (1 \leftrightarrow 3), \quad (5.11)$$

where the angular distribution of a tensor meson decaying into two spin-zero particles is governed by $(1 - 3\cos^2\theta_{13})$. In general, the angular momentum distribution is described by the Legendre polynomial $P_J(\cos\theta)$. In the above equation, $m_{12} = \sqrt{s_{12}}$ is the invariant mass of K_2^* , θ_{13} is the angle between \vec{p}_1 and \vec{p}_3 measured in the rest frame of the resonance K_2^* , q is the c.m. momentum given by

$$q = |\vec{p}_1| = |\vec{p}_2| = \frac{\sqrt{[s_{12} - (m_1 + m_2)^2][s_{12} - (m_1 - m_2)^2]}}{2m_{12}}, \quad (5.12)$$

and $T_{K_2^*}^{\text{BW}}$ is the relativistic Breit-Wigner line shape for describing the distribution of $K_2^*(1430)$:

$$T_{K_2^*}^{\text{BW}}(s) = \frac{1}{s - m_{K_2^*}^2 + im_{K_2^*}\Gamma_{K_2^*}(s)}, \quad (5.13)$$

with the energy-dependent decay width

$$\Gamma_{K_2^*}(s) = \Gamma_{K_2^*}^0 \left(\frac{q}{q_0}\right)^5 \frac{m_{K_2^*} X_2^2(q)}{\sqrt{s} X_2^2(q_0)}, \quad (5.14)$$

and

$$X_2(z) = \sqrt{\frac{1}{(zr_{\text{BW}})^4 + 3(zr_{\text{BW}})^2 + 9}}, \quad (5.15)$$

with $r_{\text{BW}} \approx 4.0$ GeV $^{-1}$. In Eq. (5.14), q_0 is the value of q when m_{12} is equal to the K_2^* mass and $\Gamma_{K_2^*}^0$ is the normal width of K_2^* .

The explicit expression of $\cos\theta_{13}$ in Eq. (5.11) is given by

$$\cos\theta_{13} = -\frac{1}{4} \frac{Z_1(s_{12}, s_{23})}{|\vec{p}_1||\vec{p}_3|}, \quad (5.16)$$

with the Zemach form [24]

$$Z_1(s_{12}, s_{23}) = s_{23} - s_{13} + \frac{(m_D^2 - m_3^2)(m_1^2 - m_2^2)}{s_{12}}. \quad (5.17)$$

When \bar{K}_2^{*0} is on shell, the decay amplitude of $D^+ \rightarrow \bar{K}_2^{*0}\pi^+$ is given by $V_{cs}^*V_{ud}(T + C')$ (see Table III). Hence,

$$A(D^+ \rightarrow \bar{K}_2^{*0}(m_{12})\pi^+) = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \left[2a_1(K_2^*\pi) \sqrt{\frac{2}{3}} f_\pi \frac{m_D}{m_{12}} \tilde{p}_c^2 A_0^{DK_2^*}(m_\pi^2) + 2a_2(\pi K_2^*) f_{K_2^*} m_D \tilde{p}_c F_1^{D\pi}(s_{12}) \right], \quad (5.18)$$

where the use of Eq. (5.6) has been made and \tilde{p}_c is the c.m. momentum of π^+ or \bar{K}_2^{*0} with the invariant mass m_{12} in the rest frame of D^+

$$\tilde{p}_c = \frac{1}{2m_D} \sqrt{[m_D^2 - (m_{12} + m_3)^2][m_D^2 - (m_{12} - m_3)^2]}. \quad (5.19)$$

Note that, in the rest frame of K_2^* , the momentum of $\pi^+(p_3)$ reads

$$|\vec{p}_3| = \left(\frac{(m_D^2 - m_{12}^2 - m_3^2)^2}{4m_{12}^2} - m_3^2 \right)^{1/2}. \quad (5.20)$$

We see that \tilde{p}_c is related to $|\vec{p}_3|$ through the relation $\tilde{p}_c = (m_{12}/m_D)|\vec{p}_3|$.

Finally, the three-body decay rate reads

$$\begin{aligned} \Gamma(D^+ \rightarrow \bar{K}_2^{*0} \pi^+ \rightarrow \pi^+ K^- \pi^+) &= \frac{1}{2} \frac{1}{(2\pi)^3 32 m_D^3} \int_{(m_K+m_\pi)^2}^{(m_D-m_\pi)^2} ds_{12} \int_{(s_{23})_{\min}}^{(s_{23})_{\max}} ds_{23} |\mathcal{A}_{K_2^*}|^2 \\ &= \frac{1}{2} \frac{1}{(2\pi)^3 32 m_D^3} \int_{(m_K+m_\pi)^2}^{(m_D-m_\pi)^2} ds_{12} \int_{(s_{23})_{\min}}^{(s_{23})_{\max}} ds_{23} \left\{ \frac{|g_{\bar{K}_2^* \rightarrow K^- \pi^+}|^2 F(s_{12}, m_{K_2^*})^2}{(s_{12} - m_{K_2^*}^2)^2 + m_{K_2^*}^2 \Gamma_{K_2^*}^2(s_{12})} \right. \\ &\quad \left. \times \frac{q^4}{6} \left(1 - \frac{3}{16} \frac{Z_1^2(s_{12}, s_{23})}{q^2 \tilde{p}_c^2} \frac{s_{12}}{m_D^2} \right)^2 |A(D^+ \rightarrow \bar{K}_2^{*0}(m_{12})\pi^+)|^2 + (s_{12} \leftrightarrow s_{23}) + \text{interference} \right\}, \end{aligned} \quad (5.21)$$

where the factor of 1/2 accounts for the identical-particle effect. The coupling constant $g_{\bar{K}_2^* \rightarrow K^- \pi^+}$ is determined from the measured width of $\bar{K}_2^*(1430) \rightarrow K^- \pi^+$ through the relation

$$\Gamma_{\bar{K}_2^*(1430) \rightarrow K^- \pi^+} = \frac{q_0^5}{60\pi m_{K_2^*}^2} |g_{\bar{K}_2^* \rightarrow K^- \pi^+}|^2. \quad (5.22)$$

When K_2^* is off the mass shell, especially when s_{12} is approaching the upper bound of $(m_D - m_\pi)^2$, it is necessary to account for the off-shell effect. For this purpose, we have followed Ref. [25] to introduce a form factor $F(s, m_R)$ parametrized as

$$F(s, m_R) = \left(\frac{\Lambda^2 + m_R^2}{\Lambda^2 + s} \right)^n, \quad (5.23)$$

with the cutoff Λ not far from the resonance,

$$\Lambda = m_R + \beta \Lambda_{\text{QCD}}, \quad (5.24)$$

where the parameter β is expected to be of order unity. We shall use $n = 1$, $\Lambda_{\text{QCD}} = 250$ MeV, and $\beta = 1.0 \pm 0.2$ in subsequent calculations.

C. Results and discussion

Branching fractions of two-body $D \rightarrow TP$ and three-body $D \rightarrow TP \rightarrow P_1 P_2 P$ decays are displayed in Tables V and VI, respectively, using the factorization approach with W -exchange and W -annihilation being neglected. Theory predictions are made with two different form-factor models: the CLFQM_b and the LCSR.

It appears from Tables V and VI that form factors A_0^{DT} based on LCSR give a better agreement with experiment, keeping in mind that we have neglected contributions from both W -exchange and W -annihilation. Since the decays $D^+ \rightarrow \bar{K}_2^{*0} \pi^+$ and $D^+ \rightarrow a_2^+ \bar{K}^0$ do not receive W -annihilation contributions, they are ideal for testing the factorization hypothesis. Our prediction of $\mathcal{B}(D^+ \rightarrow \bar{K}_2^{*0} \pi^+) = (1.1-3.0) \times 10^{-3}$ seems to be too large compared to the experimental value of $(6.9 \pm 2.1) \times 10^{-4}$. As discussed in

Sec. IV, the data for $D^+ \rightarrow TP$ modes do not appear to be self consistent. Since the mixing angle θ_{f_2} is small, to a good approximation with negligible W -annihilation compared to external W -emission, it is expected that $\mathcal{B}(D^+ \rightarrow f_2 \pi^+) / \mathcal{B}(D^+ \rightarrow \bar{K}_2^{*0} \pi^+) \approx (\sin \theta_c)^2 / 2 = 0.025$. However, the current data indicate the other way around, $\mathcal{B}(D^+ \rightarrow f_2 \pi^+) \gtrsim \mathcal{B}(D^+ \rightarrow \bar{K}_2^{*0} \pi^+)$. The mode $D^+ \rightarrow K_2^{*0} \pi^+$ is doubly Cabibbo suppressed and hence one would expect $\mathcal{B}(D^+ \rightarrow \bar{K}_2^{*0} \pi^+) \gg \mathcal{B}(D^+ \rightarrow K_2^{*0} \pi^+)$, which is not borne out by current experiments. We hope that the quality of data in the D^+ sector will be improved in the near future.

For $D^0 \rightarrow TP$ decays, LCSR form factors lead to better agreement between theory and experiment. Nevertheless, the role of W -exchange and W -annihilation should be investigated. Especially, external W -emission in

TABLE V. Branching fractions of $D \rightarrow TP$ decays. Theory predictions are made with two different form-factor models: (I) the CLFQM_b and (II) the LCSR, where the mixing angle $\theta_{f_2} = 5.6^\circ$ has been used. For simplicity and convenience, we have dropped the mass identification for $f_2(1270)$, $a_2(1320)$, and $K_2^*(1430)$. Branching fractions denoted by \mathcal{B}_{NWA} are taken from Table I.

Decay	Model I	Model II	\mathcal{B}_{NWA}
$D^+ \rightarrow f_2 \pi^+$	8.9×10^{-5}	6.4×10^{-4}	$(8.9 \pm 1.6) \times 10^{-4}$
$D^+ \rightarrow \bar{K}_2^{*0} \pi^+$	1.1×10^{-3}	3.0×10^{-3}	$(6.9 \pm 2.1) \times 10^{-4}$
$D^+ \rightarrow K_2^{*0} \pi^+$	5.6×10^{-7}	5.6×10^{-7}	$(1.2 \pm 0.8) \times 10^{-4}$
$D^+ \rightarrow a_2^+ \bar{K}^0$	2.9×10^{-5}	1.3×10^{-4}	
$D^0 \rightarrow f_2 \pi^0$	8.3×10^{-6}	6.0×10^{-5}	$(3.5 \pm 0.4) \times 10^{-4}$
$D^0 \rightarrow f_2 \bar{K}^0$	7.8×10^{-5}	5.3×10^{-4}	$(4.6 \pm 2.7) \times 10^{-4}$
$D^0 \rightarrow K_2^{*+} \pi^+$	1.8×10^{-4}	7.7×10^{-4}	$(2.0_{-0.6}^{+1.1}) \times 10^{-3}$
$D^0 \rightarrow K_2^{*+} \pi^-$	3.9×10^{-8}	3.9×10^{-8}	$< 2.0 \times 10^{-4}$
$D^0 \rightarrow a_2^- \pi^+$	3.0×10^{-5}	2.5×10^{-4}	$(2.0 \pm 3.9) \times 10^{-4}$
$D^0 \rightarrow a_2^+ K^-$	6.1×10^{-6}	1.7×10^{-6}	$< 4.2 \times 10^{-3}$
$D^0 \rightarrow a_2^- K^+$	9.8×10^{-8}	2.2×10^{-7}	$< 2.9 \times 10^{-3}$
$D_s^+ \rightarrow f_2 \pi^+$	2.9×10^{-5}	7.7×10^{-4}	$(2.0 \pm 0.4) \times 10^{-3}$
$D_s^+ \rightarrow K_2^{*0} \pi^+$	2.3×10^{-5}		

TABLE VI. Same as Table V except for $D \rightarrow TP \rightarrow P_1 P_2 P$ decays.

$D \rightarrow TP; T \rightarrow P_1 P_2$	Model I	Model II	Experiment
$D^+ \rightarrow f_2 \pi^+; f_2 \rightarrow \pi^+ \pi^-$	5.7×10^{-5}	4.2×10^{-4}	$(5.0 \pm 0.9) \times 10^{-4}$
$D^+ \rightarrow \bar{K}_2^{*0} \pi^+; \bar{K}_2^{*0} \rightarrow K^- \pi^+$	4.2×10^{-4}	1.3×10^{-3}	$(2.3 \pm 0.7) \times 10^{-4}$
$D^+ \rightarrow K_2^{*0} \pi^+; K_2^{*0} \rightarrow K^+ \pi^-$	1.6×10^{-7}	1.6×10^{-7}	$(3.9 \pm 2.7) \times 10^{-5}$
$D^+ \rightarrow \bar{K}_2^{*0} K^+; \bar{K}_2^{*0} \rightarrow K^- \pi^+$	1.4×10^{-6}	8.2×10^{-6}	$(1.6_{-0.8}^{+1.2}) \times 10^{-4}$
$D^+ \rightarrow a_2^+ \bar{K}^0; a_2^+ \rightarrow K^+ \bar{K}^0$	3.6×10^{-5}	2.0×10^{-4}	
$D^+ \rightarrow a_2^+ \bar{K}^0; a_2^+ \rightarrow \eta \pi^+$	1.4×10^{-4}	7.7×10^{-4}	
$D^0 \rightarrow f_2 \pi^0; f_2 \rightarrow \pi^+ \pi^-$	4.3×10^{-6}	3.2×10^{-5}	$(1.96 \pm 0.21) \times 10^{-4}$
$D^0 \rightarrow f_2 \bar{K}^0; f_2 \rightarrow \pi^+ \pi^-$	1.5×10^{-4}	1.0×10^{-3}	$(1.8_{-1.2}^{+2.0}) \times 10^{-4}$
$D^0 \rightarrow f_2 \bar{K}^0; f_2 \rightarrow \pi^0 \pi^0$	7.6×10^{-5}	5.1×10^{-4}	$(4.6 \pm 2.2) \times 10^{-4}$
$D^0 \rightarrow K_2^{*-} \pi^+; K_2^{*-} \rightarrow \bar{K}^0 \pi^-$	6.3×10^{-5}	2.7×10^{-4}	$(6.8_{-2.0}^{+3.8}) \times 10^{-4}$
$D^0 \rightarrow K_2^{*+} \pi^-; K_2^{*+} \rightarrow K^0 \pi^+$	1.2×10^{-8}	1.2×10^{-8}	$< 6.8 \times 10^{-5}$
$D^0 \rightarrow a_2^- \pi^+; a_2^- \rightarrow K^0 K^-$	1.2×10^{-6}	9.5×10^{-6}	$(1 \pm 1) \times 10^{-5}$
$D^0 \rightarrow a_2^+ K^-; a_2^+ \rightarrow K^+ \bar{K}^0$	2.7×10^{-7}	2.7×10^{-7}	$< 2.08 \times 10^{-4}$
$D^0 \rightarrow a_2^- K^+; a_2^- \rightarrow K^- K^0$	9.8×10^{-9}	5.8×10^{-8}	$< 1.44 \times 10^{-4}$
$D_s^+ \rightarrow f_2 \pi^+; f_2 \rightarrow \pi^+ \pi^-$	1.7×10^{-5}	4.7×10^{-5}	$(1.09 \pm 0.20) \times 10^{-3}$
$D_s^+ \rightarrow f_2 \pi^+; f_2 \rightarrow \pi^0 \pi^0$	7.2×10^{-6}	2.0×10^{-5}	$(0.80 \pm 0.42) \times 10^{-3}$

$D_s^+ \rightarrow f_2 \pi^+$ is suppressed by the small mixing angle θ_{f_2} , but W -annihilation is not.

As noticed in passing, the decay $D^+ \rightarrow \bar{K}_2^{*0} K^+$ is prohibited for physical K_2^* and K states. Nevertheless, the three-body decay $D^+ \rightarrow \bar{K}_2^{*0} K^+ \rightarrow K^- \pi^+ K^+$ can proceed owing to the finite width of K_2^* . Our calculation shows that the predicted rate is too small by one to two orders of magnitude compared to experiment (see Table VI).

An inspection of Tables V and VI may lead the reader to wonder why the calculations in these two tables seem to not respect the NWA given by Eq. (2.1). For example, $\mathcal{B}(D^+ \rightarrow \bar{K}_2^{*0} \pi^+) = 3.0 \times 10^{-3}$ and $\mathcal{B}(D^+ \rightarrow \bar{K}_2^{*0} \pi^+ \rightarrow K^- \pi^+ \pi^+) = 1.3 \times 10^{-3}$ obtained using the LCSR form factor do not satisfy the factorization relation

$$\begin{aligned} \mathcal{B}(D^+ \rightarrow \bar{K}_2^{*0} \pi^+ \rightarrow K^- \pi^+ \pi^+) \\ = \mathcal{B}(D^+ \rightarrow \bar{K}_2^{*0} \pi^+) \mathcal{B}(\bar{K}_2^{*0} \rightarrow K^- \pi^+), \end{aligned} \quad (5.25)$$

with $\mathcal{B}(\bar{K}_2^{*0} \rightarrow K^- \pi^+) = \frac{2}{3}(0.499 \pm 0.0012)$. This is ascribed to the finite-width effect that we are going to discuss in the next section.

VI. FINITE WIDTH EFFECTS

The finite-width effect is accounted for by the quantity η_R defined by [23,26]

$$\begin{aligned} \eta_R &\equiv \frac{\Gamma(D \rightarrow RP_3 \rightarrow P_1 P_2 P_3)_{\Gamma_R \rightarrow 0}}{\Gamma(D \rightarrow RP_3 \rightarrow P_1 P_2 P_3)} \\ &= \frac{\Gamma(D \rightarrow RP_3) \mathcal{B}(R \rightarrow P_1 P_2)}{\Gamma(D \rightarrow RP_3 \rightarrow P_1 P_2 P_3)} = 1 + \delta, \end{aligned} \quad (6.1)$$

so that the deviation of η_R from unity measures the degree of departure from the NWA when the resonance width is finite. It is naively expected that the correction δ will be of order Γ_R/m_R . After taking into account the finite-width effect η_R from the resonance, the branching fraction of the quasi-two-body decay reads [23]

$$\begin{aligned} \mathcal{B}(D \rightarrow RP) &= \eta_R \mathcal{B}(D \rightarrow RP)_{\text{NWA}} \\ &= \eta_R \frac{\mathcal{B}(D \rightarrow RP_3 \rightarrow P_1 P_2 P_3)_{\text{expt}}}{\mathcal{B}(R \rightarrow P_1 P_2)_{\text{expt}}}, \end{aligned} \quad (6.2)$$

where $\mathcal{B}(D \rightarrow RP)_{\text{NWA}}$ denotes the branching fraction obtained from Eq. (2.1) valid in the NWA.

We calculate the η_R parameters for tensor resonances produced in the three-body D decays using (I) the CLFQM_b and (II) the LCSR for $D \rightarrow T$ transition form factors. The results are displayed in Table VII. We only consider the D^+ decays as the three-body modes listed in Table VII are not contaminated by the W -annihilation amplitude and hence the calculated finite width effects are more trustworthy. The η_R parameters for various resonances produced in the three-body B decays have been evaluated in Refs. [23,26].

We have checked analytically and numerically that $\eta_R \rightarrow 1$ in the narrow width limit as it should be. To see this, we consider the three-body decay $D^+ \rightarrow \bar{K}_2^{*0} \pi^+ \rightarrow K^- \pi^+ \pi^+$ mentioned in Sec. VB as an illustration. The angular distribution in Eq. (5.21) has the expression [see Eq. (4.25) of Ref. [23]]

TABLE VII. A summary of the η_R parameter for tensor resonances produced in the three-body D decays calculated using (I) the CLFQM_b and (II) the LCSR for $D \rightarrow T$ transition form factors. The masses and widths of tensor resonances are taken from Ref. [13].

Resonance	$D \rightarrow Rh_3 \rightarrow h_1 h_2 h_3$	Γ_R (MeV)	m_R (MeV)	Γ_R/m_R	η_R (I)	η_R (II)
$K_2^*(1430)$	$D^+ \rightarrow \bar{K}_2^{*0} \pi^+ \rightarrow K^- \pi^+ \pi^+$	109 ± 5	1432.4 ± 1.3	0.076 ± 0.002	0.835	0.787
$a_2(1320)$	$D^+ \rightarrow a_2^+ \bar{K}^0 \rightarrow K^+ \bar{K}^0 \bar{K}^0$	107 ± 5	1318.2 ± 0.6	0.081 ± 0.004	0.388	0.370
	$D^+ \rightarrow a_2^+ \bar{K}^0 \rightarrow \eta \pi^+ \bar{K}^0$	107 ± 5	1318.2 ± 0.6	0.081 ± 0.004	0.300	0.280

$$\int_{(s_{23})_{\min}}^{(s_{23})_{\max}} ds_{23} (1 - 3 \cos^2 \theta_{13})^2 = \frac{16 m_D}{5 m_{12}} q \tilde{p}_c. \quad (6.3)$$

In the narrow width limit of $\Gamma_{K_2^*}$, we have

$$\frac{m_{K_2^*} \Gamma_{K_2^*}(s)}{(s - m_{K_2^*}^2)^2 + m_{K_2^*}^2 \Gamma_{K_2^*}^2(s)} \xrightarrow{\Gamma_{K_2^*} \rightarrow 0} \pi \delta(s - m_{K_2^*}^2). \quad (6.4)$$

Under the NWA, $|g_{\bar{K}_2^{*0} \rightarrow K^- \pi^+}|^2 / \Gamma_{K_2^{*0}}$ is finite as it is proportional to the branching fraction $\mathcal{B}(\bar{K}_2^{*0} \rightarrow K^- \pi^+)$. Due to the Dirac δ function in the above equation, we have $s_{12} \rightarrow m_{K_2^*}^2$ in the zero width limit. As a result, $\tilde{p}_c \rightarrow p_c$ and $q \rightarrow q_0$. Likewise, the second term in Eq. (5.21) with the replacement $s_{12} \leftrightarrow s_{23}$ has a similar expression. However, the interference term vanishes in the NWA due to different δ functions. From Eqs. (5.22), (5.9), and (5.21), we are led to the desired factorization relation

$$\Gamma(D^+ \rightarrow \bar{K}_2^{*0} \pi^+ \rightarrow K^- \pi^+ \pi^+) \xrightarrow{\Gamma_{K_2^*} \rightarrow 0} \Gamma(D^+ \rightarrow \bar{K}_2^{*0} \pi^+) \times \mathcal{B}(\bar{K}_2^{*0} \rightarrow K^- \pi^+), \quad (6.5)$$

in the zero width limit.

It is evident from Table VII that the finite-width effect is quite significant in the tensor meson production in D decays, $\eta_{K_2^*} \sim 0.79-0.84$ and $\eta_{a_2} \sim 0.37-0.39$ for $a_2 \rightarrow K\bar{K}$ and of order 0.28–0.30 for $a_2 \rightarrow \eta\pi$. Recall that in three-body B decays, η_R is found to be 0.972 ± 0.001 and $1.004_{-0.002}^{+0.001}$, respectively, for f_2 and K_2^* [23]. This means that the deviation of $\mathcal{B}(D \rightarrow TP)_{\text{NWA}}$ from the realistic branching fraction of quasi-two-body decay $\mathcal{B}(D \rightarrow TP)$ is large and needs to be corrected through Eq. (6.2). If the mass of the D meson were adjusted slightly higher by a few hundred MeV, the value of η_R would quickly go up to values around unity. The same effect would be witnessed if one scaled down the mass and width of the resonance particle.

VII. CONCLUSIONS

In this work we have examined the quasi-two-body $D \rightarrow TP$ decays and the three-body D decays proceeding through intermediate tensor resonances. Our main results are as follows:

- (1) Two new model calculations of $D \rightarrow T$ transition form factors are available very recently: one is based on QCD sum rules (LCSR) and the other on the CLFQM. Our calculations of two- and three-body D decays in the factorization approach are based on these two form-factor models. It appears that form factors based on LCSR give a better agreement with current data.
- (2) Denoting the primed amplitudes T' and C' for the case when the emitted meson is a tensor meson, it is naïvely expected that $T' = C' = 0$ as the tensor meson cannot be produced through the $V-A$ current. Nevertheless, beyond the factorization approximation, contributions proportional to the tensor decay constant f_T can be produced from vertex and hard spectator-scattering corrections.
- (3) We have studied the flavor operator coefficients $a_{1,2}(M_1 M_2)$ for $M_1 M_2 = TP$ and PT within the framework of QCD factorization. It follows that $a_i(PT)$ and $a_i(TP)$ are very different as the former does not receive factorizable contributions.
- (4) We have studied the finite-width effects due to tensor mesons and found it is quite significant in the tensor meson production in D decays, $\eta_{K_2^*} \sim 0.79-0.84$ and $\eta_{a_2} \sim 0.37-0.39$ for $a_2 \rightarrow K\bar{K}$ and of order 0.28–0.30 for $a_2 \rightarrow \eta\pi$, while in three-body B decays, η_R is found to be close to unity for f_2 and K_2^* . This implies that the deviation of $\mathcal{B}(D \rightarrow TP)_{\text{NWA}}$, obtained from narrow width approximation, from the realistic branching fraction of quasi-two-body decay $\mathcal{B}(D \rightarrow TP)$ is large and needs to be corrected by taking η_R into account.
- (5) Although the decay $D^+ \rightarrow \bar{K}_2^{*0} K^+$ is prohibited for physical K_2^* and K states, the three-body decay $D^+ \rightarrow \bar{K}_2^{*0} K^+ \rightarrow K^- \pi^+ K^+$ can proceed owing to the finite width of K_2^* . However, our calculation shows that the predicted rate is too small by one to two orders of magnitude compared to the experiment.
- (6) We have 15 unknown parameters for the 8 topological amplitudes T, C, E, A and T', C', E', A' . However, there are only eight items of available data (some of them being redundant) to fit and three upper limits. At present, we are not able to extract topological amplitudes as we have more theory parameters than observables.
- (7) The current data of $D^+ \rightarrow f_2 \pi^+$ and $D^+ \rightarrow \bar{K}_2^{*0} \pi^+$ are not self-consistent and need to be clarified in the

future. In general, the quality of data for $D \rightarrow TP \rightarrow P_1 P_2 P$ needs to be substantially improved.

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