Complementarity-entanglement tradeoff in quantum gravity

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(Received 31 July 2021; accepted 13 September 2021; published 28 April 2022)

The quantization of gravity remains one of the most important—yet extremely elusive—challenges at the heart of modern physics. Any attempt to resolve this long-standing problem seems to be doomed, as the route to any direct empirical evidence (i.e., detecting gravitons) that sheds light on the quantum aspect of gravity is far beyond the current capabilities. Recently, it was discovered that gravitationally induced entanglement, tailored in the interferometric frameworks, can be used to witness the quantum nature of gravity. Even though these schemes offer promising tools for investigating quantum gravity, many fundamental and empirical aspects of the schemes are yet to be discovered. Considering the fact that, besides quantum entanglement, the quantum uncertainty and complementarity principles are the two other foundational aspects of quantum physics, the quantum nature of gravity needs to manifest all of these features. Here, we lay out an interferometric platform for testing these nonclassical aspects of quantum mechanics in the quantum gravity setting, which connects gravity and quantum physics in a broader and deeper context. As we show in this work, all of these fundamental features of quantum gravity can be framed and thoroughly analyzed in an interferometric scheme.

DOI: 10.1103/PhysRevD.105.086024

I. INTRODUCTION

Gravity, as one of the four building-block forces of the Universe, is the most sensible of the forces that we deal with in our everyday lives. However, it is also considered the most controversial force in modern physics. The general theory of relativity for gravitation, which was formulated by Einstein in 1915, aside from its thundering triumphs in explaining the Universe, encounters important issues at high energy levels, such as black hole information paradox and the singularity problem [1,2]. These issues arise from the fact that there is no known way to reconcile gravity with the other three forces of nature. It seems that to rectify these issues, one needs to integrate quantum physics and gravity in a unified framework, which remains one of the most important unsettled issues in physics.

Therefore, there has been an ongoing attempt to establish a quantum theory of gravity, leading to the development of different approaches such as string theory and quantum loop gravity [3–5]. On the other hand, considering the aforementioned challenges, the search for alternative theories such as theories of emergent gravity instead of quantum gravity has also attracted great attention [6,7]. The main hurdle in establishing a quantum theory of gravity is associated with the weakness of the gravitational interaction. In other words, the quantum properties of gravity become sensible only in ranges smaller than the Planck scale, requiring experiments with energies well beyond the scales that are within near-future capabilities. This fact makes it impossible to discern a reliable route to the problem and to discriminate between various developed models of quantum gravity [8]. Therefore, laying out some empirically feasible methods for studying and testing the quantum nature of gravity is of utmost importance.

In recent years, there has been an increasing interest in the quantum-information-theoretic approaches to the study of relativistic and gravitational systems [9–11]. Recently, as a novel approach for probing the quantum nature of gravity, it has been discovered that gravitationally induced entanglement can indeed serve as a witness of the quantumness of gravity [12,13]. As was demonstrated in Refs. [12,13], the detection of entanglement, attained by gravitational interactions of massive particles, can be considered a sufficient criterion for quanta of the gravitational field. As was demonstrated, such an entanglement could be tailored in interferometric schemes, and considering the rapid progress in quantum information and interferometry technologies, the experimental feasibility of these approaches is within sight.

Even though entanglement is central to the Hilbert space structure of composite quantum systems, the quantum feature of the systems is in no way restricted to the entanglement. In fact, in quantum physics, Heisenberg's uncertainty principle [14], Bohr's complementarity principle [15], and quantum correlations in composite quantum systems are the most fundamental aspects of systems, such that the entire fabric of the quantum weirdness can be captured by these fundamental characteristics [16]. Therefore, it is imperative to search for all three of these building blocks of quantum mechanics in the quantum nature of gravity in a feasible experimental setup.

As an other fundamental aspect of quantum physics, Bohr's complementarity principle [15] provides one of the most fundamental aspects of nature, which explains that the two mutually exclusive attributes, such as the waviness and the particleness, can both be imprinted in quantum systems, such that measuring one feature prohibits its dual feature from being exhibited [15,16]. As an example, in the case of a single photon passing through an interferometer, the particleness of the photon is embedded in the path predictability of the photon, while the waviness is encoded in the visibility of the interference pattern on the screen [17]. The quantitative notion of such a scenario was first introduced by Wootters and Zurek in 1979 [18], leading to a mathematical description in the form of an inequality as $\mathcal{P}^2 + \mathcal{V}^2 \leq 1$, where $\mathcal{P}$ represents the predictability of a quantum system, which contains the path information and indicates a quantity for particleness, and $\mathcal{V}$ stands for the visibility of the interference pattern, measuring the waviness of the system [19–22].

In an interesting attempt, it has been shown that the wave-particle duality has a relationship with the entanglement in the system [23–25]. This unification of the duality and entanglement in double-slit (two-qubit) analyses provides an important relation as $\mathcal{P}^2 + \mathcal{V}^2 + \mathcal{C}^2 = 1$ [23–25], in which $\mathcal{C}$ is a measure of entanglement known as concurrence [26]. More recently, an interesting geometrical correspondence between this relation and stereographic projection of S^7 geometry was also formulated [27], which gives a full geometrical proof for the duality-entanglement relation [27].

In this paper, we put forward an experimentally feasible platform that enables testing Heisenberg's uncertainty principle, Bohr's complementarity principle, and quantum entanglement as the nonclassical aspects of quantum mechanics in the quantum gravity framework, connecting gravity and quantum physics in a broader and deeper context. As one important aspect of our study, we show that all three of these fundamental characteristics of quantum gravity can be framed and tested in an interferometric scheme.

The paper is organized as follows. In Sec. II, we introduce the quantum-gravitational interaction potential and discuss its fundamental implications. In Sec. III, we briefly address the entanglement-complementarity relation and its main features. In Sec. IV, we put forward the gravitationally induced complementarity and entanglement analyses in an interferometric quantum superposition scenario. In Sec. V, we briefly analyze Bell inequality which provides a practical way for the detection of entanglement in quantum systems. In Sec. VI, we lay out the discussion of the uncertainty principle in the context of gravitationally induced quantum phases. In Sec. VII, we discuss the experimental feasibility of the gravitationally induced entanglement and its analyses provided in this work. We finally provide a short summary and conclude in Sec. VIII.

II. MAIN ASPECTS OF QUANTUM GRAVITY

Here, we briefly consider some important aspects of the gravitationally induced phase and the significance of the potential that results in such an entanglement. To this end, we start with the action of the gravitational field as [28]

$$S = \int d^4x\sqrt{-g}[R + \mathcal{L}_m], \tag{1}$$

where g is the determinant of metric $g_{\mu\nu}$, and R is the Ricci scalar that depends on the derivations of the metric and is related to the Ricci tensor by $R = g^{\mu\nu}R_{\mu\nu}$. Also, $\mathcal{L}_m$ represents the matter part of the action.

Now, by taking the variation over the metric one can reach the Einstein field equation [28]

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{2}$$

where $T_{\mu\nu}$ is the stress-momentum tensor. To make the quantum nature of gravity manifest, we study the perturbation $h_{\mu\nu}$ on the background metric of Minkowski spacetime $\eta_{\mu\nu}$. Therefore, the entire metric can be written as $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$. Considering the weak-field limit, after expanding the metric and omitting the terms beyond the quadratic terms and choosing the harmonic gauge, we obtain [29]

$$S_{WF} = \frac{1}{2}\int d^4x\Bigg(\Bigg(-\frac{1}{32\pi G} \times \left(\partial_\lambda h^{\mu\nu}\partial^\lambda h_{\mu\nu} - \frac{1}{2}\partial_\lambda h\partial^\lambda h\right) + h^{\mu\nu}T_{\mu\nu}\Bigg). \tag{3}$$

Now, considering h as a field, the first two terms refer to the free field of gravity and the third term indicates the interaction of gravity with matter. The free gravitational field can be written as [29]

$$h^{\mu\nu} = \frac{1}{(2\pi)^3}\int \frac{d^3k}{\sqrt{\omega}}\sum_{\sigma=\pm2}(e^{\mu\nu}_\sigma(\vec{p})a_\sigma(\vec{p})e^{i\vec{p}.\vec{x}} + \mathrm{H.c.}), \tag{4}$$

where σ denotes the spin of the graviton and $e^{\mu\nu}$ is the polarization tensor with the properties $e^{\mu\nu}_\sigma e^{\mu\nu\star}_{\sigma'} = 2\delta_{\sigma\sigma'}$. Also, $\vec{p} = \hbar\vec{k}$ states the relation between the momentum of the system and its wave vector.

As a result, we can calculate the propagator and the scattering process of two masses. To illustrate, in one loop level, after Fourier-transformation of scattering amplitude in momentum space, the potential can be obtained as [30–32]

$$V(r) = -\frac{Gm_1m_2}{r}\left(1 + 3\frac{G(m_1+m_2)}{rc^2} + \frac{41G\hbar}{10\pi r^2c^3} + O(G^2)\right). \quad (5)$$

The second term is the general-relativistic correction, and the third term is related to the quantum gravity correction to the classical potential. These two terms are extremely small compared to the first term. More specifically, the last term is quite negligible compared to the first two terms; therefore, the detection of the quantum nature of gravity does not seem to be feasible if one tries to detect it by using the last term of this equation. This, in fact, shows why the observation of the quantum-mechanical nature of gravity is such a difficult task.

Considering the extremely small contribution of the quantum part in the potential in Eq. (5), the substantially important problem is whether it is possible to observe the quantum nature of gravity through the first term (the Newtonian approximation) of the potential. The answer to this question is, surprisingly, yes. This could be achieved by a purely information-theoretic approach to gravity. In particular, as was recently discovered, an interesting route to this goal is gravity-induced entanglement, which circumvents the challenges of the weak strength of the quantum contribution to the gravitational interactions. The subtleness of the problem lies in the fact that the induced entanglement can emerge even at the low energy limits, where we can approximate the potential with its first term, i.e., the Newtonian approximation [33,34]. Interestingly, this entanglement-assisted analysis of the quantum nature of gravity offers an advantage for the accessibility of the induced quantumness in an empirical setup, compared to the effect that is directly proportional to the Planck length as appears in the last term of the potential in Eq. (5).

The important role of entanglement relies on the fact that if the gravitational interaction can generate entanglement among two masses, the gravitational field itself should be of a quantum nature. In other words, we assume that two masses interact via gravity, in which the interaction generates entanglement. In this setting, gravity acts as a mediator of the entanglement between two masses. The interaction is induced by the exchange of gravitons as the mediator of the entanglement. If the entanglement is generated, then the gravitational field needs to be coupled quantum mechanically to each test mass in order to generate entanglement. This is due to the fact that no classical mediator can generate entanglement, as was shown in Ref. [35]. A similar conclusion can be drawn from the LOCC theorem, which states that no local operation and classical communication (LOCC) can increase the entanglement [36]. Therefore, starting with two disentangled masses, the entire system remains disentangled unless some quantum-mechanical interactions are applied. As an important implication of this fact, detection of entanglement in such a setting is sufficient to conclude that gravity is of quantum nature.

III. TWO-QUBIT COMPLEMENTARITY-ENTANGLEMENT RELATION

The space of a two-qubit state is the product of the Hilbert spaces of each qubit, denoted by $H_1^{\mathbb{C}} \otimes H_2^{\mathbb{C}}$, which gives a four-dimensional Hilbert space for the system. Considering the fact that each Hilbert space, in this setting, can be spanned by two orthogonal bases $\{|0\rangle, |1\rangle\}$, the basis of the two-qubit Hilbert space can be written as $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$, where $|ij\rangle$ denotes the composite basis as $|i\rangle \otimes |j\rangle$, with i, $j = 0, 1$. Therefore, the general form of a pure two-qubit state can be expressed as

$$|\psi\rangle = \alpha_0|00\rangle + \alpha_1|01\rangle + \alpha_2|10\rangle + \alpha_3|11\rangle. \quad (6)$$

We can encode a two-state system, such as the paths in Young double-slit experiments or a Mach-Zehnder interferometer, as a correspondence to a two-qubit system. To characterize the setup for quantum analyses of gravity in this work, we consider two massive particles, each subjected to a Mach-Zehnder interferometer, as depicted in Fig. 1. We can assign to the upper path and the lower path states in the interferometer two orthogonal states $|u\rangle$ and $|d\rangle$, respectively. This encoding is equivalent to $|1\rangle$ and $|0\rangle$ in Eq. (6). Hence, assigning the bases $|u_i\rangle$ and $|d_i\rangle$ for the

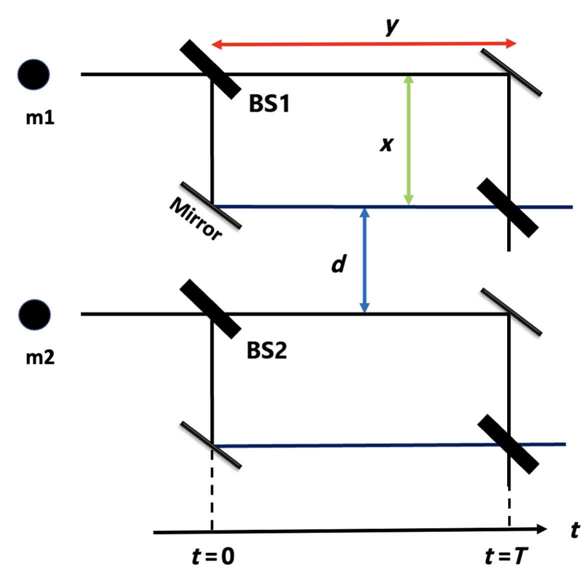

FIG. 1. Experimental setup of the interferometer for gravitationally induced entanglement. The masses m_1 and m_2 undergo independent Mach-Zehnder-type interference, and interact with each other via gravity. BSi ($i = 1, 2$) indicates a beam splitter that is characterized by (r_i, t_i). The second beam splitter in each interferometer is assumed to be a 50:50 beam splitter.

ith interferometer ($i = 1,\ 2$), the system of the two interferometers is described by the state of the form

$$|\psi\rangle = \alpha_0|u_1u_2\rangle + \alpha_1|u_1d_2\rangle + \alpha_2|d_1u_2\rangle + \alpha_3|d_1d_2\rangle. \tag{7}$$

With this description, we can study the duality-entanglement concept in a two-qubit system, as shown in the setup in Fig. 1. In our analyses of the wave-like and particle-like features, without loss of generality, we consider the first interferometer (first mass). Naturally, a similar discussion is also valid for the second interferometer.

Now, the wave-like feature in this system (first mass) is quantified by the visibility, which is determined by [23–25,27]

$$\mathcal{V} = \frac{p_D^{\max} - p_D^{\min}}{p_D^{\max} + p_D^{\min}}, \tag{8}$$

where p_D is the probability of detecting the mass in the first interferometer, and the upper indices indicate the maximum and minimum of the probability. Applying this relation to the quantum state in Eq. (7), the visibility reduces to [27]

$$\mathcal{V} = 2|\alpha_2^*\alpha_0 + \alpha_3^*\alpha_1|. \tag{9}$$

In a similar vein, the predictability is determined as [23–25,27]

$$\mathcal{P} = \frac{|p_u - p_d|}{|p_u + p_d|}, \tag{10}$$

where the parameters p_u and p_d are the probabilities of finding the first particle in each of the chosen paths (i.e., the probability of finding the first mass in the upper or lower arm of its corresponding interferometer). Hence, from this equation, the particleness of the system can be attained as [27]

$$\mathcal{P} = |(|\alpha_0|^2 + |\alpha_1|^2) - (|\alpha_2|^2 + |\alpha_3|^2)|. \tag{11}$$

On the other hand, the amount of entanglement in the above system can be obtained using concurrence, which is defined through the R matrix such that $R = \sqrt{\sqrt{\rho}\bar{\rho}\sqrt{\rho}}$, with $\bar{\rho} = (\sigma_x \otimes \sigma_x)\rho^\star(\sigma_x \otimes \sigma_x)$. By organizing the eigenvalues of the R matrix in decreasing order, the concurrence is defined as [26]

$$\mathcal{C} = \max\{0, \lambda_0 - \lambda_1 - \lambda_2 - \lambda_3\}, \tag{12}$$

in which the eigenvalues of the matrix R are denoted by λ_i in deceasing order. In the case of the state in Eq. (7), the concurrence is given by [27]

$$\mathcal{C} = 2|\alpha_0\alpha_3 - \alpha_1\alpha_2|. \tag{13}$$

Recently, it was realized that concurrence plays a significant role in a quantum complementarity setting, such that [24,25]

$$\mathcal{P}^2 + \mathcal{V}^2 + \mathcal{C}^2 = 1. \tag{14}$$

This relation shows that entanglement can indeed control duality in quantum systems; hence, the full description of the system entails a wave-particle-entanglement triality relation as above, rather than the duality description alone. This relation can also be proven geometrically, as outlined in Ref. [27], where it was shown that the complementarity principle could indeed be analyzed from a completely geometric perspective.

IV. GRAVITATIONALLY INDUCED COMPLEMENTARITY AND ENTANGLEMENT

As we discussed earlier, the ultimate theory of quantum gravity should enable us to express gravity as a superposition of different states [33,34]. This is an important feature of quantum gravity which makes it different from classical theories and can be used in finding experimental approaches to test the theory in table-top experiments. Following this line of thought, it was recently found that the quantum nature of gravity can be tested in light of entanglement generated by gravity [12,13]. In these interesting works, it was shown that an interferometric setup enables testing the existence of the gravitationally induced entanglement between two masses.

Here, we lay out a rather general framework to consider the quantum foundation of gravity. To this end, we assume two massive particles, m_1 and m_2, each subjected to an interferometer as depicted in Fig. 1. We also assume that in the first interferometer the first beam splitter, which particle encounters, is characterized by (r_1, t_1), where r_1 and t_1 represent the reflectivity and transmissivity parameters of the first beam. Similarly, in the second interferometer the first beam splitter is characterized by (r_2, t_2). For simplicity, we assume that the arms which, are perpendicular to the initial direction of motion of the particles in the interferometer, are negligible in comparison to the parallel arms [12,13]. Therefore, the initial state of the two masses after passing through the first beam splitters can be expressed as the product of the states of the first and second particle paths, which is given by

$$|\psi(t=0)\rangle = (t_1|u_1\rangle + r_1|d_1\rangle) \otimes (t_2|u_2\rangle + r_2|d_2\rangle). \tag{15}$$

Note that the setting considered here is much more general compared to the previous studies [12,13] where only 50:50 beam splitters were taken into consideration.

The interaction of a quantum particle with a gravitational field results in an induced phase, which its first experimental demonstration was attained in a famous experiment by Colella, Overhauser, and Werner (which is known as the COW experiment) in 1975 [37]. Now, similar to this experiment, we consider the setting where the gravitational field can indeed induce phase shift on the quantum systems [12,13]. In this setting, gravity induces a phase shift and decouples from the system once the phase shift is attained [12,13]. Therefore, the initial state of the systems given by $|\psi(t=0)\rangle$ evolves into

$$|\psi(t=T)\rangle = r_1 r_2 e^{i\phi_1}|d_1\rangle|d_2\rangle + r_1 t_2 e^{i\phi_2}|d_1\rangle|u_2\rangle + t_1 r_2 e^{i\phi_3}|u_1\rangle|d_2\rangle + t_1 t_2 e^{i\phi_4}|u_1\rangle|u_2\rangle, \quad (16)$$

before entering the second beam splitter in each interferometer. Here, the phases are given by

$$\phi_1 = G\frac{m_1 m_2}{\hbar(d+x)}T, \qquad \phi_2 = G\frac{m_1 m_2}{\hbar d}T,$$
$$\phi_3 = G\frac{m_1 m_2}{\hbar(d+2x)}T, \qquad \phi_4 = \phi_1, \quad (17)$$

in which T is the time duration of the gravitational interaction in each arm and x is the width of the interferometer.

Hence, having this state at hand, we can calculate the entanglement using Eq. (13), from which we obtain

$$\mathcal{C} = 2r_1 r_2 t_1 t_2 |1 - e^{i(2\phi_1-\phi_2-\phi_3)}|. \quad (18)$$

This immediately results in

$$\mathcal{C}^2 = 8(r_1 r_2 t_1 t_2)^2(1-\cos(\phi)), \quad (19)$$

in which $\phi = 2\phi_1 - \phi_2 - \phi_3$. It is easy to check that when $\phi = 2n\pi$ (where n is an integer number) the entanglement is zero, while it becomes maximum when $\phi = n\pi$ (with n being an odd number).

The behavior of this equation is presented in Fig. 2. In Fig. 2(a), the dependence of the concurrence on the gravitationally induced phase and reflectivity of the first beam splitter in the first interferometer is illustrated. In Fig. 2(b), the concurrence as a function of the reflectivity of the first beam splitters in each interferometer is plotted, in which the gravitationally induced phase is set to $\phi = \pi$. In both cases, when we choose the reflectivity of the beam splitters as $r_1 = r_2 = 1/\sqrt{2}$, the entanglement becomes maximum.

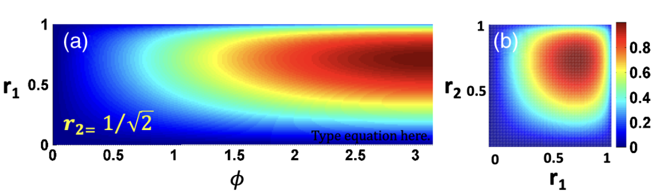

FIG. 2. Concurrence of the system as a function of different parameters in the setup: (a) the dependence of concurrence on the gravitationally induced phase and reflectivity of the first beam splitter in the first interferometer; (b) the concurrence as a function of the reflectivity of the first beam splitters in each interferometer, where the gravitationally induced phase is set to $\phi = \pi$.

We note that the entanglement generated in this setting is indeed induced by the gravitational field. Here, gravity acts as a mediator of the entanglement, and if the entanglement is observed, we can conclude that gravity is of quantum nature. This is due to the fact that no classical mediator can generate entanglement [35].

Next, we consider the visibility of the interference in the first interferometer. Using Eq. (9), we can obtain the visibility of the first particle,

$$\mathcal{V} = 2|(r_1 r_2 e^{i\phi_1})(t_1 r_2 e^{-i\phi_3}) + (t_1 t_2 e^{-i\phi_4})(r_1 t_2 e^{i\phi_2})|. \quad (20)$$

Therefore, the square of the visibility provides

$$\mathcal{V}^2 = 4(r_1 t_1)^2 |r_2^2 + t_2^2 e^{-i\phi}|^2 = 4(r_1 t_1)^2[r_2^4 + t_2^4 + 2(r_2 t_2)^2\cos(\phi)]. \quad (21)$$

As can readily be seen from this result, if the reflectivity in the second interferometer becomes zero ($r_2 = 0$), the gravitational interaction has no effect on the visibility of the first interferometer.

In Fig. 3 the results for the visibility are plotted. Figure 3(a) illustrates the dependence of the visibility on the gravitationally induced phase and reflectivity of the first beam splitter in the first interferometer. In Fig. 3(b), the visibility of the system as a function of the reflectivity of the first beam splitters in each interferometer is depicted, where the gravitationally induced phase is set to $\phi = \pi$. Finally, we attain the predictability of the first mass from Eq. (11) as

$$\mathcal{P}^2 = (r_1^2 - t_1^2)^2. \quad (22)$$

Putting these relations together, the equality in Eq. (14) is fulfilled. As a result, gravity induces a tradeoff between entanglement and complementarity. In Fig. 4 the results for

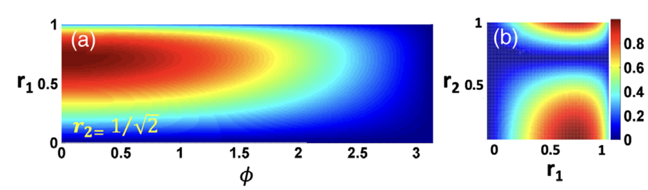

FIG. 3. Visibility of the system as a function of different parameters in the setup: (a) the dependence of visibility on the gravitationally induced phase and reflectivity of the first beam splitter in the first interferometer; (b) visibility as a function of the reflectivity of the first beam splitters in each interferometer, assuming that the gravitationally induced phase is $\phi = \pi$.

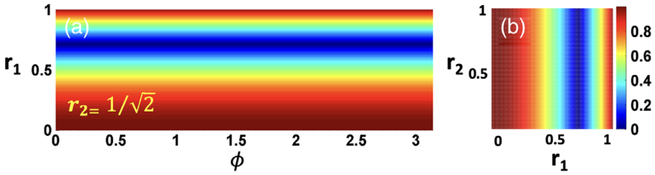

FIG. 4. Predictability of the system as a function of different parameters in the setup: (a) dependence of predictability on the gravitationally induced phase and reflectivity of the first beam splitter in the first interferometer; (b) predictability as a function of the reflectivity of the first beam splitters in each interferometer, where the gravitationally induced phase is fixed to $\phi = \pi$.

the predictability in terms of various involving parameters are illustrated. In Fig. 4(a) the dependence of the predictability on the gravitationally induced phase and reflectivity of the first beam splitter in the first interferometer is illustrated. In Fig. 4(b), the predictability as a function of the reflectivity of the beam splitters in each interferometer is depicted. Here the gravitationally induced phase is fixed to $\phi = \pi$. As we expect from Eq. (22), it only depends on the reflectivity of the first beam splitter, and for $r_1 = 1/\sqrt{2}$ the path information entirely vanishes.

Taking $r_1 = r_2 = 1/\sqrt{2}$, the entanglement becomes maximum, while visibility reduces. Practically, choosing a 50∶50 beam splitter observing the quantum entanglement features of the gravity becomes more feasible. In the special case of choosing the beam splitter of the second interferometer as 50∶50, the relations for the first particle simplify to

$$\mathcal{C}^2 = 2(r_1 t_1)^2(1 - \cos(\phi)),$$
$$\mathcal{V}^2 = 2(r_1 t_1)^2(1 + \cos(\phi)),$$
$$\mathcal{P}^2 = (r_1^2 - t_1^2)^2. \quad (23)$$

This readily provides $\mathcal{P}^2 + \mathcal{V}^2 + \mathcal{C}^2 = 1$. Therefore, the quantum nature of gravity provides complementarity and entanglement, enabling a feasible experimental analysis of quantum gravity and providing important features of the quantum nature of gravity.

V. BELL INEQUALITY AND TESTING QUANTUM GRAVITY

The nonlocal correlation exhibited by quantum entanglement was first addressed in 1935 by Einstein, Podolsky, and Rosen in a famous paper (usually called the EPR paper) [38], which demonstrated the apparently paradoxical context of quantum mechanics. Almost three decades later, Bell found an experimentally manageable and quantitative platform to investigate this controversial feature of composite quantum systems and how to discriminate it from classical descriptions [39]. Here, we discuss the Bell inequality in the context of quantum gravity, which provides an experimentally testable approach for the quantum nature of gravity. The most commonly used Bell inequality is the so-called Bell-CHSH inequality [40,41]. The CHSH operator is given by [40,41]

$$\hat{B} = \vec{a} \cdot \vec{\sigma} \otimes (\vec{b} + \vec{b'}) \cdot \vec{\sigma} + \vec{a'} \cdot \vec{\sigma} \otimes (\vec{b} - \vec{b'}) \cdot \vec{\sigma},$$

where $\vec{a}, \vec{a'}, \vec{b}, \vec{b'}$ are unit vectors. Given the above relation, the Bell-CHSH inequality reads [40,41]

$$|\langle \hat{B} \rangle| = |tr(\rho \hat{B})| \leq 2.$$

Violation of this inequality, i.e., $|\langle \hat{B} \rangle| > 2$, indicates the existence of quantum correlations in a quantum state, and hence it provides an experimentally appealing method for the test of quanta of gravity.

As an interesting connection, there is a relation between entanglement and the maximum violation of the Bell-CHSH inequality, which is given as [42,43]

$$\mathcal{B} = 2(\sqrt{1 + \mathcal{C}^2}). \quad (24)$$

This, in turn, shows that the Bell inequality can be violated by all entangled states ($\mathcal{C} \neq 0$). Since quantum-mechanical nonlocality appears when this parameter exceeds 2, we plot $\mathcal{I} = \mathcal{B} - 2$ in Fig. 5. Accordingly, Fig. 5(a) indicates the violation of the Bell inequality as a function of the reflectivity of the first beam splitter in the first interferometer and the gravitationally induced phase. In Fig. 5(b) we show the Bell parameter $\mathcal{I}$ as a function of the reflectivity of the first beam splitters in each interferometer, where we have assumed that the gravitationally induced phase is $\phi = \pi$. Therefore, the maximum violation of the Bell-CHSH inequality occurs when both of the beam splitters are 50∶50.

To establish an interesting relation between complementarity and the Bell parameter, using Eq. (24) we can obtain a new relation as

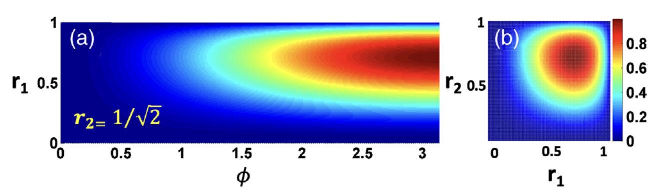

FIG. 5. Violation of the Bell parameter $\mathcal{I} = \mathcal{B} - 2$ as a function of interferometric parameters: (a) the violation of the Bell parameter $\mathcal{I} = \mathcal{B} - 2$ as a function of the reflectivity of the first beam splitter in the first interferometer and the gravitationally induced phase; (b) the violation of the Bell parameter as a function of the reflectivity of the first beam splitters of first interferometers, in which the gravitationally induced phase is set to $\phi = \pi$.

$$4(\mathcal{P}^2 + \mathcal{V}^2) + \mathcal{B}^2 = 8. \tag{25}$$

This relation demonstrates that in order to measure quantum nonlocality, which is a sufficient condition for the quantum nature of gravity, we can simply measure the path information and the visibility. This observation is insightful for the practical detection of nonclassical correlations through gravity.

VI. UNCERTAINTY RELATIONS IN INTERFEROMETRIC QUANTUM GRAVITY

In the previous sections, we have considered entanglement and complementarity as the two fundamental aspects of quantum mechanics. We now address the uncertainty principle in the context of quantum gravity, as the other fundamental concept of quantum mechanics [44]. The concept of quantum uncertainty was first proposed by Heisenberg for the position and momentum operators [14]. Later, Robertson generalized it for any pair of noncommutative operators, such that considering a pair of noncommutative operators A and B, the uncertainty relation reads [45]

$$\Delta A \Delta B \geq \frac{1}{2} |\langle [A, B] \rangle|, \tag{26}$$

where ΔA and ΔB are the standard deviations of these operators and $\langle [A, B] \rangle$ is the expectation value of the commutator of the operators. We could define operators corresponding to the predictability and visibility as follows [46]:

$$\hat{\mathcal{P}} = \sigma_z,$$
$$\hat{\mathcal{V}} = \cos\theta \sigma_x + \sin\theta \sigma_y, \tag{27}$$

where

$$\sigma_z = |u_1\rangle\langle u_1| - |d_1\rangle\langle d_1|,$$
$$\sigma_x = |u_1\rangle\langle d_1| + |d_1\rangle\langle u_1|,$$
$$\sigma_y = -i|u_1\rangle\langle d_1| + i|d_1\rangle\langle u_1|. \tag{28}$$

Also, θ is a free parameter ranging between 0 and 2π, which is set to maximize the absolute expectation value of the visibility. Therefore, assuming the nonzero off-diagonal elements in the density matrix, we obtain

$$\theta = \tan^{-1}\left(\frac{r_2^2 \sin(\phi_1 - \phi_3) + t_2^2 \sin(\phi_1 - \phi_3)}{r_2^2 \cos(\phi_1 - \phi_3) + t_2^2 \cos(\phi_1 - \phi_3)}\right). \tag{29}$$

Since the expectation values of an arbitrary operator O, with density matrix ρ, are defined as $\langle O \rangle = \mathrm{Tr}(\rho O)$, one can write the uncertainty relation for the predictability and the visibility operators as

$$\Delta\mathcal{P}\Delta\mathcal{V} \geq 2(r_1 t_1) \times |r_2^2 \sin(\phi_1 - \phi_3 + \theta) + t_2^2 \sin(\phi_2 - \phi_4 + \theta)|. \tag{30}$$

Obviously, when there is no gravity, the minimum uncertainty bound is zero. As the gravitational interaction induces a phase, the uncertainty can change. To illustrate this, for the case of 50:50 beam splitters, if we set $\phi_1 - \phi_3 = \pi$ and choose $\phi_2 - \phi_4$ as $\pi/6$, $\pi/4$, and $\pi/2$, the minimum uncertainty bound becomes 0.13, 0.27, and 0.71, respectively.

Even though this uncertainty relation is fundamentally significant in the context of quantum mechanics, some tighter (and hence better) uncertainty relations have been considered as alternative uncertainty relations [44,47–49]. One useful alternative uncertainty is defined as the sum of the uncertainties of each operator. As a result, for two Hermitian unitary operators $A = \vec{a}.\vec{\sigma}$ and $B = \vec{b}.\vec{\sigma}$ (where $\vec{a}$ and $\vec{b}$ are the unit vectors) acting simultaneously on the density matrix $\rho = 1/2(I + \vec{r}.\vec{\sigma})$ (where $\vec{r}$ is a unit vector), the uncertainty is defined as [44]

$$\Delta A^2 + \Delta B^2 \geq 1 + |\vec{a}.\vec{b}|^2 - 2|\vec{a}.\vec{b}| \times \sqrt{1 - \Delta A^2}\sqrt{1 - \Delta B^2}|(\vec{a} \times \vec{b}).\vec{r}|^2. \tag{31}$$

Now, considering such an uncertainty in the context of the problem at hand, we obtain

$$\Delta\mathcal{P}^2 + \Delta\mathcal{V}^2 \geq 1 + 4(r_1 t_1)^2 \times |r_2^2 \sin(\phi_1 - \phi_3 + \theta) + t_2^2 \sin(\phi_2 - \phi_4 + \theta)|^2. \tag{32}$$

The minimum of the sum uncertainty is one, which can be obtained when one of the operators has zero uncertainty. The second term on the right-hand side can never exceed one, and interestingly it is the quadrant of the uncertainty relation in Eq. (30). Therefore, the sum uncertainty bound is tighter than the uncertainty relation in Eq. (30).

In an experimental setting, one can control the quantum uncertainty effects of gravity by controlling the gravitationally induced phases. This, in fact, provides a deep connection between quantum mechanics and gravity from the uncertainty perspective.

VII. EXPERIMENTAL CHALLENGES

Experimental investigations of quantum gravity via induced entanglement address one of the most important problems of physics. Therefore, it is important to consider the feasibility of the induced entanglement generations via current technology. In this section, we briefly review the important challenges that such an experiment is subjected to and discuss the feasibility of the study.

One important challenge for generating entanglement in this realm is that one needs to create a large enough induced phase shift that can be measurable in the experiments. Considering the relation for the induced phase, $Gm_1m_2T/\hbar d$, there are three parameters that we could adjust in reaching a measurable phase in the order of unit. The first parameter is the masses of the particles. The product of the masses must be big enough to enhance the entanglement degree. This requires preparing a large system in a quantum superposition. This problem has been intensively considered in the context of quantum mechanics, and many improvements have been made thus far [50–54]. Also, experiments with the aim of applying massive quantum systems interferometry to gravitational measurements and quantum gravity have been investigated in recent years [55,56]. Taking recent achievements into account, a system with a mass of about 10^{-14} kg is a reasonable size for this experiment; possible candidates include massive molecules [57,58], microcrystals [12], or Bose-Einstein condensates [59,60].

The other factor in controlling the induced phase is the distance between the two interferometers. In a realistic setting, if the two objects get closer to each other than a certain limit, the other interactions, such as electrostatic interactions, can dominate the gravitational interaction [12,13]. To overcome this challenge, it has been proposed that by using a conductor in between the two masses (and hence shielding other interactions) we can adjust the distance of the interferometer to close as about $d = 1\ \mu$m [61]. The last parameter that we can control is the time that each particle travels in the interferometer arms. To illustrate this, if we adjust the travel time of particles in the interferometer arms on the order of $T = 0.1$ s, the induced phase could be adjusted in the order of unit, and consequently, the maximum entanglement of two particles through gravitational interaction would be achievable. As another practical setup, one can consider two coupled nanomechanical oscillators with the mass 10^{-12} kg, and the interaction time $T = 10^{-6}$ s that would enable the phase shifts large enough to prepare maximally entangled states with the distance d in the order of a few micrometers [13]. Therefore, on such a time scale, we must protect the system from decoherence from the environment. To eliminate the effect of the environment on the system, we need a high vacuum and a low temperature. The estimated pressure is on the order of 10^{-15} Pa, and the temperature must be about 0.1 K [12], which is an achievable condition with current technologies [62].

VIII. CONCLUSION

The quantization of gravity remains one of the most important yet extremely elusive challenges at the heart of modern physics. It has been argued that direct empirical evidence for the quantum nature of gravity (i.e., detecting gravitons) can shed light on the characteristics of the ultimate theory of quantum gravity. However, such a task is far beyond the current technological capabilities, and it seems not to be achievable in terms of near-future technologies. To overcome this, it was recently shown that gravitationally induced entanglement, tailored in the interferometric frameworks, can be used to detect the quantum nature of gravity. However, many fundamental and empirical aspects of these schemes are yet to be discovered. Considering the fact that, besides quantum entanglement, the quantum uncertainty and complementarity principles are the other two foundational aspects of quantum physics, the quantum nature of gravity needs to manifest all of these features. In this work, we considered an interferometric setup for testing these three nonclassical aspects of quantum mechanics in a quantum gravity setting, which can shed light on the connections between gravity and quantum physics in a broader and deeper discipline. As we showed in this work, all of these fundamental features of quantum gravity can be framed and fully analyzed in an interferometric scheme. We showed the relation between gravitationally induced entanglement and the complementarity principle and investigated its features. We also developed a relation between the violation of the Bell inequality as a sufficient criterion for the quantumness of entanglement and complementarity principle.

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