Three-point functions of twist-two operators at two loops

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(Received 14 December 2021; accepted 9 February 2022; published 12 April 2022)

I consider three-point functions of twist-two operators in $\mathcal{N}=4$ SYM, two of which endowed with spin. I supply perturbative data up to twelve units of spins and second perturbative order at weak coupling.

DOI: 10.1103/PhysRevD.105.086007

I. INTRODUCTION

As a conformal field theory, $\mathcal{N}=4$ SYM is defined by the spectrum of its operators and their structure constants. Both problems can be attacked, in principle, with integrability [1,2]. This note focuses on structure constants. In particular, it addresses the computation of three-point functions involving more than one operator with spin. From the perspective of the operator-product expansion (OPE) [3,4], such coefficients emerge from a multiple OPE of a higher-point function of protected operators, such as a five-point correlator. Beyond one-loop order, the computation of such correlators is not as well developed as for four operators, and mostly limited to the work [5]. Recently, the large spin of such higher-point correlators has been analyzed [6,7]. This establishes the exact behavior of structure constants with various spinning operators at all loops, in the large spins limit, and a duality with null Wilson loops. The computation presented here lies in the opposite regime, that is small spins. The calculation is performed perturbatively at weak coupling. In this setting, high values of the operators spins are a nuisance rather than a blessing (ten units of spin already means high in this article). In fact, they are the main source of complexity and computational bottlenecks, which this work aims to attack. Being the simplest, twist-two operators are considered. Their spectrum is well known from both explicit computation [8,9] and integrability [10–13]. Their three-point functions with two protected operators are also known to vertiginous perturbative precision [14–19]. Such results have been derived mostly from the OPE expansion of four-point correlation functions of protected operators. Less is known for structure constants involving more than one operator with spin. This paper intends to address such a paucity of data, focusing on two-loop corrections. The problem has already been considered in [20], demonstrating that it can be attacked with the technology used in the present work. However, the computation in [20] stopped short at rather disappointingly low values of the spins, namely six. This was due to computational complexity, the main bottleneck being integral reduction. Yet, the limitation stemmed mostly from the naivety of the approach taken for completing such a reduction: easy to implement, but too inefficient for attacking a complicated problem. The aim of this paper is to advance further in the two-loop computation and work out an efficient algorithm for deriving a reasonable amount of perturbative data with modest computational resources and time. Thanks to various technical optimizations with respect to [20], a few-weeks computation on an ordinary laptop (plus some aid from a cluster) produced results up to twelve units of operators spins, which are presented here. Such data are too scarce to conjecture a general in spin formula analogous to that encountered at one loop [7,20], even at low values of polarizations, where at one loop simplifications occur. Still, this note provides new solid results, which can be prospectively compared with alternative methods, such as OPE of higher-point functions and integrability.

II. THE PERTURBATIVE COMPUTATION

The setting of the calculation is the same as in [20] and I refer to that for further details. The basic definitions of operators are reported in the Appendix. This note focuses on three-point functions with two spinning operators. In a conformal field theory their form is fixed [21]

$$\langle \hat{\mathcal{O}}_{j_1}(x_1)\mathcal{O}_0(x_2)\hat{\mathcal{O}}_{j_2}(x_3)\rangle = \sum_{l=0}^{\min(j_1,j_2)} \mathcal{C}_l \frac{\hat{Y}_{32,1}^{j_1-l}\hat{Y}_{12,3}^{j_2-l}}{|x_{13}|^{2l}} I_{13}^l \times |x_{12}|^{j_1-j_2-\Delta_{12,3}}|x_{23}|^{j_2-j_1-\Delta_{23,1}}|x_{13}|^{j_1+j_2-\Delta_{31,2}}, \tag{1}$$

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where j_1 and j_2 are the operators spins. The quantities Y and I appearing on the right-hand side are certain invariants whose precise form is spelled out in the Appendix. The powers of squared distances are combinations of the conformal dimensions of the operators. The hats symbols denote contractions of the tensor structures with two sets of light-cone vectors $z_{1,2}$. The index l labels the $0 \le l \le \min(j_1, j_2)$ polarizations of the three-point functions and their corresponding independent structure constants. The objective is their two-loop determination.

They are extracted integrating both sides of (1) on one of the integration points of the operators [22]. The left-hand-side is expanded perturbatively in Feynman diagrams. The integration translates into a soft limit in momentum space. This collapses the three-point function onto a two-point function. The latter, being much simpler than the original three-point problem, can be calculated efficiently, leveraging vast literature and techniques for dealing with two-point integrals. The soft limit introduces additional powers of some propagators, conversely, spins translate to powers of momenta. Both occurrences are dealt with through integration-by-parts identities (IBP) [23,24], reducing all integrals to a finite set of known master integrals. The integration over the insertion point of an operator on the right-hand side of (1) can be easily evaluated, allowing for the extraction of the desired coefficients $\mathcal{C}$, by comparison.

A few subtleties arise, due to the necessity of regularizing various sources of divergent intermediate quantities. First, UV singularities appear, which are eventually renormalized away. Second, the soft limit introduces independent IR divergences, combining with the former. Third, individual Feynman diagrams possess spurious divergences, canceling off in the final result. A regulator is needed for completing the computation. Dimensional regularization is used. In dimensional regularization, IR and UV poles mix and multiply. After renormalization, only IR divergences survive, arising from the soft limit. Depending on the conformal data of operators, these can exhibit two qualitatively different behaviors: increasing pole orders, or a fixed lower bound for ϵ powers at each perturbative order. The same pattern appears integrating the conformal form of the three-point function on the right-hand-side of (1). For consistency, that must be regularized with the same method as in the perturbative expansion of the left-hand-side: dimensional regularization. If the maximal order of poles is fixed for all perturbative orders, their coefficient emerges from the $d = 4$ limit of the integrated three-point function. Any subleading in ϵ corrections can be safely discarded. The extraction of the structure constant is then correct. Conversely, for increasing order of the divergences, there is no sensible limit to $d = 4$ which can be taken on the three-point function. The only consistent comparison between the sides of (1) involves three-point functions in $d = 4 - 2\epsilon$ dimensions, whose form is unknown in general. This impedes a sensible structure constant extraction. In the problem at hand, integrating over the insertion point of the protected operator yields a constant ϵ^{-1} pole and the structure constant can be determined. Integrating a spinning operator produces increasing pole orders, preventing the extraction.

More in general, regulating both sides of (1) introduces an order-of-limits issue. On the right-hand-side, using the conformal expression of three-point functions entails an initial $d \to 4$ limit. Then dimensional regularization is adopted to regulate the integration over an insertion point. On the left-hand side, the soft limit is taken initially on Feynman diagrams, where dimensional regularization is applied. Only at the end of the perturbative computation the limit $d \to 4$ is enforced. The two limits do not commute, as shown explicitly in [25]. Luckily, it so happens that the present computation seems not to be affected by this issue. In fact, the method reproduces correctly the known results for three-point functions with one spinning operator [26]. Since there are no qualitatively new diagrams and integrals associated to the computation with two spinning operators, I assume the order-of-limits issue to be absent and proceed.

III. INTEGRALS TREATMENT

The problem described above boils down to the evaluation of various two-point function integral topologies. These are most efficiently dealt with in momentum space, where loop momenta are denoted by k_α and the external momentum by p. Operator spins translate into numerator powers of loop momenta contracted with two sets of null vectors $z_{1,2}$. Loop integration gives rise to polynomials in $z_1 \cdot z_2 \equiv z_{12}$, with degree up to the minimum power between z_1 and z_2 contractions. Contractions of the external momentum p with the null momenta are fixed by dimensional analysis. Two-loop perturbative order requires three-loop momentum integrals.

(a) *IBP reduction.* The reduction can be performed in various ways. The integrals can be IBP reduced including z_1 and z_2 contractions. This introduces new external momenta, with their respective IBP identities. The advantage of such an approach consists in being directly implementable on IBP reducers on the market. I used FIRE6 [27,28] with LiteRed solved rules [29,30]. The reduction works fine and fast for two-loop integrals and for three-loop integrals, up to certain powers. The disadvantage is that the reduction fails (at least on my PC) for high enough powers of numerator momenta of some three-loop integrals. The onset of this behavior happens for lower powers of momenta, when including z_1 and z_2 dependence and their additional IBP rules, than the case with no such external momenta contractions. In particular, the reduction fails for some integrals involved in the three-point function with two spin-6 operators. This stopped short the earlier computation of [20] around this complexity level.

(b) *Tensor reduction* An alternative approach consists in performing a reduction of tensor integrals to scalar ones, later projecting the numerator onto null vectors. This process generates scalar integrals involving only products with the external momentum p of the two-point functions, which are more rapidly reduced, than those featuring additional external momenta. In this case another bottleneck is the tensor reduction itself. The computational load can be alleviated considering the symmetries of both numerator momenta and the resulting contraction onto null vectors. This is however a case-by-case analysis.

In order to speed up the reduction I used the following method. Each numerator structure of momenta is defined by a set of n_1 indices to be contracted with z_1 and of n_2 indices projected onto z_2. The corresponding indices of numerator loop momenta (distinguished by α and β) are symmetrized and traceless by construction

$$k^{\{\mu_1}_{(\alpha_1)}\dots k^{\mu_{n_1}\}}_{(\alpha_{n_1})} k^{\{\nu_1}_{(\beta_1)}\dots k^{\nu_{n_2}\}}_{(\beta_{n_2})}. \tag{2}$$

A generic ansatz for the right-hand side of such a tensor integral displays the form

$$\int k^{\{\mu_1}_{(\alpha_1)}\dots k^{\mu_{n_1}\}}_{(\alpha_{n_1})} k^{\{\nu_1}_{(\beta_1)}\dots k^{\nu_{n_2}\}}_{(\beta_{n_2})} \prod \text{propagators} = \sum_{i=0}^{\lfloor\frac{n_1}{2}\rfloor}\sum_{j=0}^{\lfloor\frac{n_2}{2}\rfloor}\sum_{k=0}^{\lfloor\frac{n_1-2i+n_2-2j}{2}\rfloor} c(n_1,n_2,\{\alpha\},\{\beta\})_{i,j,k}$$
$$\times \left(g^{\mu_1\mu_2}\dots g^{\mu_{i-1}\mu_i} g^{\nu_1\nu_2}\dots g^{\nu_{j-1}\nu_j} g^{\mu_{i+1}\nu_{j+1}}\dots g^{\mu_{i+k}\nu_{j+k}} p^{\mu_{i+k+1}}\dots p^{\mu_{n_1}} p^{\nu_{j+k+1}}\dots p^{\nu_{n_2}} + \mu,\nu \text{ perms}\right) \tag{3}$$

with identical coefficients for various tensor structures, thanks to symmetry. The permutations evaluate to the same result after null vectors contractions, producing combinatorial factors. The crucial information resides in the coefficients c, depending on the particular integral. They are indexed according to the number of possible contractions among z_1- and z_2-to-be-contracted indices and mixed ones, labeled by i, j, and k in the above formula. Only the coefficients $c(n_1,n_2)_{0,0,k}$ survive the contraction with null vectors. The system can be inverted, in principle, after multiplying both sides by suitable tensor structures. The inversion coefficients are polynomials in z_{12} and rational functions of d, multiplying powers of external momenta and metric tensors of similar structure. Each coefficient c reads

$$c(n_1,n_2,\{\alpha\},\{\beta\})_{i,j,k} = \sum_{l=0}^{\lfloor\frac{n_1}{2}\rfloor}\sum_{m=0}^{\lfloor\frac{n_2}{2}\rfloor}\sum_{n=0}^{\lfloor\frac{n_1-2i+n_2-2j}{2}\rfloor} d(n_1,n_2)^{l,m,n}_{i,j,k}\left(g^{\rho_1\rho_2}\dots g^{\rho_{l-1}\rho_l} g^{\sigma_1\sigma_2}\dots g^{\sigma_{m-1}\sigma_m} g^{\rho_{l+1}\sigma_{m+1}}\dots g^{\rho_{l+n}\sigma_{m+n}}\right.$$
$$\left.\times\, p^{\rho_{l+n+1}}\dots p^{\rho_{n_1}} p^{\sigma_{m+n+1}}\dots p^{\sigma_{n_2}} + \rho,\sigma \text{ perms}\right)\int k^{\rho_1}_{(\alpha_1)}\dots k^{\rho_{n_1}}_{(\alpha_{n_1})} k^{\sigma_1}_{(\beta_1)}\dots k^{\sigma_{n_2}}_{(\beta_{n_2})} \times \text{propagators}\,, \tag{4}$$

where permutations take care of symmetries of numerator indices. The inversion coefficients $d(n_1,n_2)^{l,m,n}_{i,j,k}$ multiply tensor structures contracting indices in the numerator of to-be-reduced integrals. These are generic for given n_1 and n_2 and do not depend on the specific numerator or integral topology. Therefore they can be computed once and recycled into other integrals with same n_1 and n_2. The inversion process can turn costly for high powers of numerators. Moreover, all solutions for the coefficients c have to be derived, though only the limited subset $c(n_1,n_2)_{0,0,k}$ is relevant for the computation.

In practice, I took an alternative route. I fixed the relevant inversion coefficients $d(n_1,n_2)^{l,m,n}_{0,0,k}$ heuristically, by comparing a sufficient number of independent reduced tensor integrals with a direct IBP reduction including z_1 and z_2 scalar products. Since the inversion coefficients must not depend on the particular integral, just on n_1 and n_2, I evaluated a test reduction on the simplest three-loop integral topology, the triple bubble, with a bunch of different numerator combinations. I compared the result with a direct FIRE6 reduction. This is feasible and fast, also for high momentum powers, thanks to the simplicity of the integral topology. The two sets of results must coincide, independently for each z_{12} power, producing a (potentially highly overdetermined) linear system of equations. For each z_{12} power the inversion coefficients are expected to be rational functions of the dimension d. With some experimentation, a rough upper bound estimate of the maximal powers of d in such rational functions can be established. Then, it suffices to fix the coefficients of such d polynomials. For this, the relevant systems of equations can be evaluated at n independent integer or rational values of d, where the system does not lose rank, speeding up the inversion. The IBP reduction can be performed with rational, instead of generic d, if that produces a faster process. With one trial, a sufficient set of integrals to be IBP reduced can be identified, supplying enough independent equations to invert the system. This eliminates the redundancy, mentioned above. For larger spins, it becomes more efficient evaluating inversion coefficients at the relevant integer dimension $d = 4$ and solve for ϵ corrections perturbatively. The needed order at two loops is ϵ^2, given the maximal poles of the integrals emerging from Feynman diagrams. In a few situations the inversion coefficients can be determined analytically, for generic values of the parameters, for instance when only one null momenta is present. A heuristic formula for such inversion coefficients was derived for generic powers, in terms of combinatorial factors.

Whence all relevant inversion coefficients are known for a numerator with powers n_1 and n_2, the explicit tensor reduction of a generic integral can be performed. This step generates larger and larger amounts of scalar integrals, for higher and higher powers of numerator momenta. A first optimization consists in leveraging the further symmetries of the reduction owing to the particular numerators. Numerators contain repeated powers of loop momenta k_α, introducing subsets of symmetric indices. This symmetry can be used, reducing the number of independent contractions in (4). The output (containing scalar integrals yet to be IBP reduced) can still be bulky. At that stage the remaining scalar integrals are reduced, expanded in ϵ up to the required order and substituted.

A *Mathematica* implementation of this algorithm turns too slow starting at spin 10. Hence I performed most of the simplifications in FORM [31], which efficiently deals with large expressions. For IBP reducing scalar integrals I used FIRE6, until it failed around the level of complexity of a three-point function with two spin-10 operators, in the nonplanar topology. I switched to Mincer [32] for higher spins. The process described above is rather roundabout, but it works far more efficiently than the approach in [20] and allows to push the computation to higher values of the spins. At spin 12 a few hundred thousands integrals were reduced, which required deployment on a small cluster of around 300 cores for completing the process in a reasonable span of time.

l	$\bar{C}_{0,0,l}$	$\bar{C}_{0,2,l}$	$\bar{C}_{0,4,l}$	$\bar{C}_{0,6,l}$	$\bar{C}_{0,8,l}$	$\bar{C}_{0,10,l}$	$\bar{C}_{0,12,l}$
0	0	66	$\frac{3532955}{31752}$	$\frac{189088963}{1306800}$	$\frac{29113728110377}{169682857344}$	$\frac{21581306350157590607 89}{11109996916822440000}$	$\frac{435771907729880824453812721}{203663703930213952656 0000}$

l	$\bar{C}_{2,2,l}$	$\bar{C}_{2,4,l}$	$\bar{C}_{2,6,l}$	$\bar{C}_{2,8,l}$	$\bar{C}_{2,10,l}$	$\bar{C}_{2,12,l}$
0	147	$\frac{2712265}{15876}$	$\frac{246135733}{1306800}$	$\frac{857381969298607}{4242071433600}$	$\frac{593007433738882813411}{2777499229205610000}$	$\frac{454698118039581082150556101}{2036637039302139526560000}$
1	$\frac{111}{2}$	$\frac{2718197}{31752}$	$\frac{673255007}{6534000}$	$\frac{4874130059289013}{42420714336000}$	$\frac{1375252520779004546629}{11109996916822440000}$	$\frac{266509253777609629958381701}{2036637039302139526560000}$
2	-87	$-\frac{474107}{15876}$	$-\frac{219497639}{32670000}$	$\frac{158091242862743}{21210357168000}$	$\frac{975270424131169349 57}{5554998458411220000}$	$\frac{517686532104945806140209 01}{2036637039302139526560000}$

IV. RESULTS AND CONCLUSIONS

In the following tables the results are collected for two-loop corrections to three-point functions of twist-two operators with two spinning ones spins j_1 and j_2 and polarization l. Primary operators possess even integer spins j_1 and j_2. Such structure constants are normalized by the two-point functions of the corresponding operators, i.e., they form an orthonormal basis (A3). The ratio of quantum corrections $C^{(2)}_{j_1,j_2,l}$ with the tree-level value $C^{(0)}_{j_1,j_2,l}$ is reported. The two-loop corrections include a transcendental contribution proportional to $\zeta(3) \equiv \sum_{n=1}^{\infty} \frac{1}{n^3}$, reading $24|S_1(j_1) - S_1(j_2)|\zeta(3)$, where $S_1(j) \equiv \sum_{n=1}^{j} \frac{1}{n}$ denote harmonic numbers. Such a contribution is independent of the polarization index. It would be interesting to ascertain whether this pattern persists at higher loops for the highest transcendentality part. Such parts are removed from the results below, representing the quantity

$$\bar{C}_{j_1,j_2,l} \equiv \frac{C^{(2)}_{j_1,j_2,l}}{C^{(0)}_{j_1,j_2,l}} - 24|S_1(j_1) - S_1(j_2)|\zeta(3). \qquad (5)$$

The results are available in a file attached to the submission [33]. The table with one scalar operator reproduces structure constants with a single spinning operator, whose general form was derived in [26] in terms of harmonic sums. This provides a test of the correctness of the calculation. The result in [26] was obtained via OPE from a four-point function of protected operators. For structure constants with two spinning operators, a five-point function of protected operators would be needed. An integrand expression was derived in [5]. It should in principle be possible to reproduce and outperform the results presented here, from that angle. This would constitute an important consistency check of the data presented here. A first test I have implemented is the gauge invariance of the underlying Feynman diagrams, inherited from [20]. Second, the three-point functions renormalize correctly, through the known anomalous dimensions, providing a further consistency check, as does the transcendental part. In conclusion, several indications point at the correctness of the results. Still, an additional independent computation is desirable. The extension of the present computation to the next perturbative order (restricted to lower values of the spins) constitutes an interesting development, especially so since no information on higher-point functions of protected operators is presently available at three loops.

l	$\bar{C}_{4,4,l}$	$\bar{C}_{4,6,l}$	$\bar{C}_{4,8,l}$	$\bar{C}_{4,10,l}$	$\bar{C}_{4,12,l}$
0	$\frac{14378795}{63504}$	$\frac{2916214006}{12006225}$	$\frac{325587235460813}{1272621430080}$	$\frac{5922079528930347198953}{22219993833644880000}$	$\frac{561365563443718032464257591}{20366370393021395265600000}$
1	$\frac{20988115}{127008}$	$\frac{368404919191}{1920996000}$	$\frac{26680050546883301}{127262143008000}$	$\frac{4952650041620262784703}{22219993833644880000}$	$\frac{118798880329263503401350229}{509159259825534881640000}$
2	$\frac{9858115}{127008}$	$\frac{55429699999}{480249000}$	$\frac{8793803296085377}{63631071504000}$	$\frac{3415365961547396145553}{22219993833644880000}$	$\frac{336218394887834544824734241}{20366370393021395265600000}$
3	$-\frac{9823085}{127008}$	$-\frac{38694354697}{3841992000}$	$\frac{3075995211223931}{127262143008000}$	$\frac{167602787595330815863}{3703332305607480000}$	$\frac{60686065453702053232240433}{10183185196510697632800000}$
4	$-\frac{33859255}{63504}$	$-\frac{15231046612}{60031125}$	$-\frac{5281918551104911}{31815535752000}$	$-\frac{886064155194573792349}{7406664611214960000}$	$-\frac{183755638365180822400932709}{20366370393021395265600000}$

l	$\bar{C}_{6,6,l}$	$\bar{C}_{6,8,l}$	$\bar{C}_{6,10,l}$	$\bar{C}_{6,12,l}$
0	$\frac{1830754919}{6534000}$	$\frac{7767142939238407}{26512946460000}$	$\frac{163201313679479030946391}{5377238507742060960000}$	$\frac{127291297917344130207411287}{407327407860427905312000}$
1	$\frac{4801454329}{19602000}$	$\frac{56532609025579481}{212103571680000}$	$\frac{759639819436841718481109}{2688619253871030480000}$	$\frac{40073620608915066190184749}{135775802620142635104000}$
2	$\frac{91122352181}{490050000}$	$\frac{11480219641330889}{53025892920000}$	$\frac{9588187464549482960255129}{40329288808065457200000}$	$\frac{516552348613652745354815891}{20366370393021395265600000}$
3	$\frac{5936713867}{65340000}$	$\frac{132612893302991}{981960980000}$	$\frac{2200778638960949186122999}{13443096269355152400000}$	$\frac{937088520682121221186763101}{5091592598255348816400000}$
4	$-\frac{33451386031}{490050000}$	$\frac{10709771810119}{1767529764000}$	$\frac{662923727291126768354903}{13443096269355152400000}$	$\frac{794400384020675231059986731}{10183185196510697632800000}$
5	$-\frac{40877954761}{98010000}$	$-\frac{1327269952072459}{5891765880000}$	$-\frac{1119484730526088189240969}{8065857761613091440000}$	$-\frac{179097466961413259473566071}{20366370393021395265600000}$
6	$-\frac{78834756497}{32670000}$	$-\frac{22289653409130383}{26512946460000}$	$-\frac{273824872195577586641357}{5377238507742060960000}$	$-\frac{740555199209788799579842721}{20366370393021395265600000}$

l	$\bar{C}_{8,8,l}$	$\bar{C}_{8,10,l}$	$\bar{C}_{8,12,l}$
0	$\frac{22704102808747603}{70701190560000}$	$\frac{1783416528737094358181653}{5377238507742060960000}$	$\frac{7328208103787158045948346317}{21511978727628848749290000}$
1	$\frac{86566002046549997}{282804762240000}$	$\frac{696844256510622981986857}{2150895403096824384000}$	$\frac{465432258097017446899761936313}{1376766638568246319954560000}$
2	$\frac{265016748937304477}{989816667840000}$	$\frac{224952776474139284568427}{768176929677437280000}$	$\frac{214598301789441142089475922519}{688383319284123159977280000}$
3	$\frac{1200614119876978627}{5938900007040000}$	$\frac{5085355659157099863815449}{21508954030968243840000}$	$\frac{359608758568933328334171885613}{1376766638568246319954560000}$
4	$\frac{42838323556844129}{424207143360000}$	$\frac{56335962578633828807766253}{376406695541944267200000}$	$\frac{31514397982642390372514409227}{172095829821030789994320000}$
5	$-\frac{357059515124454179}{5938900007040000}$	$\frac{2693116666499226374227159}{150562678216777706880000}$	$\frac{185234376440809030468641673751}{2753533277136492639909120000}$
6	$-\frac{362711133602400899}{989816667840000}$	$-\frac{3033712676511763517679131}{15056267821677770688000}$	$-\frac{156150020012746868378186296217}{1376766638568246319954560000}$
7	$-\frac{42825105265219741}{314227513600000}$	$-\frac{14770837508550764976004499}{21508954030968243840000}$	$-\frac{1236799833779991423347461560529}{2753533277136492639909120000}$
8	$-\frac{994642692067652671}{70701190560000}$	$-\frac{15194325613374482381168069}{5377238507742060960000}$	$-\frac{28822588898998607305184907023}{21511978727628848749290000}$

l	$\bar{C}_{10,10,l}$	$\bar{C}_{10,12,l}$
0	$\frac{1574921544648209552 6927}{444399876672897600 00}$	$\frac{32917278967140929076026532 31}{905767525373846263128 0000}$
1	$\frac{7897647650615056830585 1}{222199938336448800000}$	$\frac{16784128050700595588662524 41}{452883762686923131564 0000}$
2	$\frac{2219366175875683715387 21}{6665998150093464000 00}$	$\frac{96204087530081668220773953 53}{271730257612153878938 40000}$
3	$\frac{3826829917427234401006 73}{13331996300186928000 00}$	$\frac{11415290241596258730012194 0297}{362307010149538505251 200000}$
4	$\frac{2869817617436417720756 09}{13331996300186928000 00}$	$\frac{34321789080047915144640561 253}{135865128806076939469 200000}$
5	$\frac{3236476264324378209167}{296266584448598400 00}$	$\frac{10515010882721731452455676 5713}{652152618269169309452 160000}$
6	$-\frac{6986474428622313882709 1}{13331996300186928000 00}$	$\frac{14932116341568607688438285 03}{543460515224307757876 80000}$
7	$-\frac{2234040185252157839641 81}{6665998150093464000 00}$	$-\frac{88889569910532527420617057 87}{483076013532718007001 60000}$
8	$-\frac{6926166266829326422247 029}{6665998150093464000 00}$	$-\frac{24245096576388127676391203 3011}{407595386418230818407 600000}$
9	$-\frac{1127489007711542666224 859}{222199938336448800000}$	$-\frac{17236311320207441039240351 4227}{905767525373846263128 00000}$
10	$-\frac{4865338120964131771262 467}{444399876672897600 00}$	$-\frac{11248744962245258577781572 6041}{905767525373846263128 0000}$

l	$\bar{C}_{12,12,l}$
0	$\frac{77908762040019854799669565 7}{203663703930213952656 0000}$
1	$\frac{80791895062277055072795130 7}{203663703930213952656 0000}$
2	$\frac{86679607978968341080217965 27}{224030074323235347921 60000}$
3	$\frac{15937355877134580591515157 41}{448060148646470695843 2000}$
4	$\frac{61128677209271089728045411 923}{201627066890911813129 440000}$
5	$\frac{45594882715350651705471365 527}{201627066890911813129 440000}$
6	$\frac{47402836982718173298223566 1}{407327407860427905312 0000}$
7	$-\frac{18282063762698606798756009 831}{403254133781823626258 880000}$
8	$-\frac{62843808176804166978683449 193}{201627066890911813129 440000}$
9	$-\frac{19774098758064379277379327 887}{224030074323235347921 60000}$
10	$-\frac{67981469157471204246987687 547}{224030074323235347921 60000}$
11	$-\frac{17273833425891636232934216 909}{678879013100713175520 000}$
12	$-\frac{20787750423105590623279535 61427}{203663703930213952656 0000}$

ACKNOWLEDGMENTS

This work was supported by Dirección de Investigación, Universidad Austral de Chile, Fondo de Investigación VIDCA 2020, Grant No. S-2020-06 and Fondo Nacional de Desarrollo Científico y Tecnológico, FONDECYT 1220240. I also acknowledge computing at the Magi cluster at ICFM, Universidad Austral de Chile and thank its administrator Simón Poblete for assistance and maintenance after several crashes. I thank the authors of [6] for a conversation which stimulated this work.

APPENDIX: SETTING

The focus of this paper is three-point functions with two spinning operators, of spins j_1 and j_2, and a third scalar operator, in $\mathcal{N}=4$ SYM with $SU(N)$ gauge group. Their three-point function is constrained to the general form (1) where the invariants read

$$\hat{Y}_{ij,k} \equiv Y^{\mu}_{ij,k} z_{\mu}, \qquad Y^{\mu}_{ij,k} \equiv x^{\mu}_{ik}|x_{ik}|^{-2} - x^{\mu}_{jk}|x_{jk}|^{-2}$$
$$I_{ij} \equiv I^{\mu\nu}_{ij} z_{1\mu} z_{2\nu} = z_{12} - 2(x_{ij}\cdot z_1)(x_{ij}\cdot z_2)|x_{ij}|^{-2}$$
$$x_{ij} \equiv x_i - x_j, \qquad \Delta_{ij,k} \equiv \Delta_i + \Delta_j - \Delta_k.$$

The tensor structure of operators is projected onto null vectors z_i ($z_i^2 = 0$), automatically forming symmetric traceless combinations, attaining the correct representation. Two null vectors are used for the two spinning operators, giving rise to a nontrivial invariant $z_{12} \equiv z_1 \cdot z_2$, parametrizing three-point functions polarizations. The coefficients $\mathcal{C}_l$ are $\min(j_1, j_2) + 1$ independent structure constants. They are functions of the coupling constant g^2 and the gauge group rank N, via the t' Hooft coupling $\lambda = \frac{g^2 N}{16\pi^2}$, since no color-subleading corrections appear up to two loops. Their perturbative expansion reads

$$\mathcal{C}_l = \mathcal{C}_l^{(0)} + \mathcal{C}_l^{(1)}\lambda + \mathcal{C}_l^{(2)}\lambda^2 + \ldots \tag{A1}$$

A space-time (dimensionally regulated) integral is taken of (1), streamlining structure constants extraction. Their normalization is fixed so that the relevant operators two-point functions be orthonormal, thereby forming a CFT canonical basis. This note considers three-point functions of twist-two operators $\mathcal{O}_j$ of spin j, whose expression reads schematically

$$\hat{O}^j \equiv \mathrm{Tr}((z_i\cdot D)^k X (z_i \cdot D)^{j-k} X) + \ldots \tag{A2}$$

X being a complex scalar of $\mathcal{N}=4$ SYM, D covariant derivatives and z_i light-cone vectors. Flavors are selected so that the three-point function is nonvanishing. Twist-two operators mix nontrivially under renormalization in the closed $sl(2)$ sector, with descendants $\partial^k O^{j-k}$ of same spin j. After renormalization, an eigenbasis of the dilatation operator is selected, forming the conformal operators appearing in (1). Their two-point function is fixed by conformal symmetry

$$\langle \hat{\mathcal{O}}^j(x_1)\hat{\mathcal{O}}^k(x_2)\rangle = C(g^2, N)\delta^{jk} I_{12}^j |x_{12}|^{-2\Delta}. \tag{A3}$$

Δ being the conformal dimension of the operator. The tensor structure is encapsulated in the quantity I. The normalization coefficient C depends on the perturbative definition of the operators. This is chosen canonically in such a way that the operators form an orthonormal basis, including subleading in ϵ corrections, to the required order.

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