Black to white transition of a charged black hole

Antoine Rignon-Bret[1] and Carlo Rovelli[2,3,4]

[1]École Normale Supérieure, 45 rue d'Ulm, F-75230 Paris, France
[2]Aix-Marseille University, Université de Toulon, CPT-CNRS, F-13288 Marseille, France
[3]Department of Philosophy and the Rotman Institute of Philosophy, 1151 Richmond Street N, London, Ontario N6A5B7, Canada
[4]Perimeter Institute, 31 Caroline Street N, Waterloo, Ontario N2L2Y5, Canada

(Received 27 October 2021; accepted 13 March 2022; published 8 April 2022)

We present an exact solution of the Maxwell-Einstein equations, which describes the exterior of a charged spherical mass collapsing into its own trapping horizon and then bouncing back from an antitrapping horizon at the same space location of the same asymptotic region. The solution is locally but not globally isometric to the maximally extended Reissner-Nordström metric and depends on seven parameters. It is regular and defined everywhere except for a small region, where quantum tunneling is expected. This region lies outside the mass: The mass bounce and its near exterior are governed by classical general relativity. We discuss the relevance of this result for the fate of realistic black holes. We comment on the possible effects of the classical instabilities and the Hawking radiation.

DOI: 10.1103/PhysRevD.105.086003

I. INTRODUCTION

Contrary to what is sometimes assumed, the long-term evolution of the exterior of a black hole is likely to be affected by quantum gravitational effects. This is because the backreaction of the Hawking radiation makes the curvature grow to Planckian values outside the horizon too. At this point, and its immediate future, the Einstein equations are likely to be violated by quantum gravity effects, in particular, by conventional quantum tunneling. Elsewhere, it is reasonable to expect the evolution of spacetime to be governed by the classical Einstein equations. What do these permit for the long-term evolution of a black hole?

A surprisingly result obtained in [1] is that the Einstein equations admit a spherically symmetric vacuum solution that describes a black hole that tunnels into a white hole. This spacetime does not contradict Birkhoff's theorem, because it is locally—but not globally—isometric to the Kruskal spacetime. The result reveals an interesting scenario: (i) Black hole horizons are event horizons only in the classical limit; (ii) before the end of the evaporation, the trapping horizon tunnels into an antitrapping horizon; (iii) the black hole interior tunnels into a white hole interior; (iv) the collapsed matter bounces out. This scenario [2–10], its possible astrophysical implications [11–22], and its relevance for the black hole information paradox [23,24] are currently under intense investigation.

So far, the literature has focused on the nonrotating and noncharged case. Realistic black holes rotate and are approximated by the Kerr-Newman metric, whose maximal extension has a markedly different structure from the Kruskal spacetime. Is the intriguing black to white transition a peculiarity of the Schwarzschild metric, or is it possible in general?

The lack of spherical symmetry makes the analysis of the Kerr-Newman case harder, but there is an interesting intermediate case that has the same global structure as the extended Kerr metric (the Carter-Penrose diagrams of their maximal extension are similar) and yet has spherical symmetry: the Reissner-Nordström metric. This is the solution of the Maxwell-Einstein equations around a spherical symmetric charged mass. In this paper, we extend the result of [1] to this case. We show that there is an exact solution of the Einstein-Maxwell equations that describes the exterior of a charged spherical mass that collapses into its own trapping horizon and then bounces back from an antitrapping horizon at the same spatial location of the same asymptotic region. The solution is locally but not globally isometric to the maximally extended Reissner-Nordström metric, is regular, and is everywhere defined except for a compact finite region, where a quantum gravitational tunneling transition can be expected.

What we find is surprising. Unlike the noncharged case, the quantum region does not continue inside the hole all the way to the collapsing and bouncing matter. The bounce of the collapsing matter and its surrounding evolve classically without ever entering the quantum region. This is comprehensible, as the global structure of Reissner-Nordström metric (and Kerr metric) allows a timelike geodesic to enter a black hole, traverse it, and exit from a white hole without encountering singularities or high curvature regions. In the maximally extended metric, the white hole is in a different

 © 2022 American Physical Society

asymptotic region; here we show (following [1]) that the white hole can be in the same asymptotic region as the black hole in its immediate future.

This result makes the charged case transition (and presumably the rotating case as well) easier to understand and treat than the Schwarzschild case. In a sense, the spacetime region which needs to be described by quantum gravity is smaller than in the Schwarzschild case: Some part of the mechanism of the black to white transition is already contained in the classical solution. Quantum effects are not needed for the charged mass to bounce, nor for the black hole interior to evolve into a white hole exterior. Only the horizon area undergoes a quantum transition when it reaches Planckian curvature.

We recall the main features of the black to white transition in the Schwarzschild case in Sec. II and the causal structure of the maximally extended Reissner-Nordström metric in Sec. III. The solution of the Maxwell-Einstein equations that describes the black to white transition of the charged black hole is built in Sec. IV, and the reason a quantum tunneling is to be expected is discussed in Sec. V. Then, we comment on the possible effects of the classical instabilities [25–27] and the Hawking evaporation in Secs. VI and VII. We use units where $G = c = \hbar = 1$.

II. SCHWARZSCHILD METRIC

The Carter-Penrose diagram of a (classical) Schwarzschild black hole created from a gravitational collapse is depicted in Fig. 1. The dark gray region is where the classical theory becomes unreliable due to quantum gravitational effects. We expect this to happen when the curvature becomes Planckian, for instance, when the Kretschmann scalar

$$K^2 = R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} = 48\frac{M^2}{r^6} \tag{1}$$

becomes of order 1. Here, M is the black hole mass and r is the Schwarzschild radius. This happens before the $r = 0$ singularity inside the black hole. Just outside the horizon, $K \sim \frac{1}{M^2}$. The Hawking evaporation steadily decreases the mass M of an isolated black hole, bringing it down to Planckian values; hence, the quantum region extends outside the horizon. General arguments and some specific calculations [5,28] indicate that the transition probability P from black holes to white holes is proportional to

$$P \sim e^{-M^2}, \tag{2}$$

thus becoming dominant at the end of the evaporation, where $M \sim 1$. It is also possible that quantum effects could appear earlier [1,29] at a time of order M^2 after the collapse.

Here, we are not directly concerned with these estimates. We only retain the fact that the quantum region extends outside the horizon. This region can be organized into three subregions [9] (see Figs. 1 and 2):

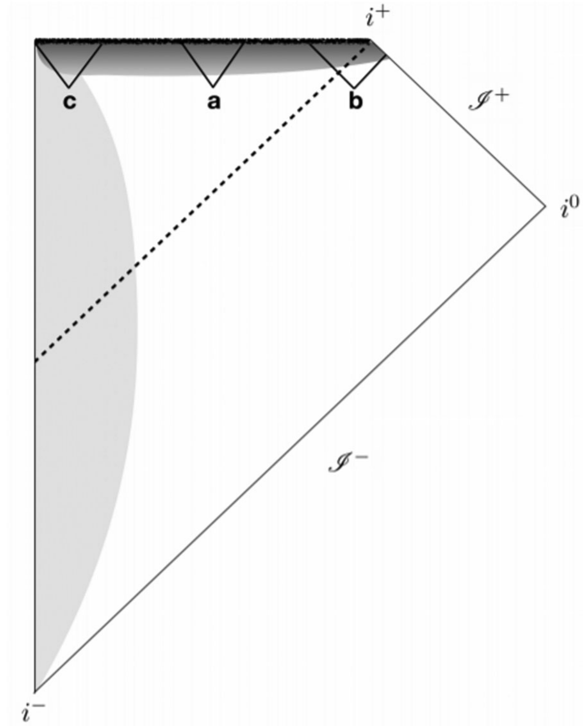

FIG. 1. The Carter-Penrose diagram of a Schwarzschild black hole until the onset of quantum gravity. The light gray region is the collapsing star. The dark gray region is where quantum gravity becomes relevant.

(i) Region B: the horizon region
(ii) Region C: the collapsing star region
(iii) Region A: the region which is neither directly causally connected to the horizon nor to the collapsing star

It is shown in [9] that the phenomena in these three regions can be considered causally disconnected, as they are separated by a large spacelike distance, which at the end of the evaporation grows to

$$L \simeq M^{\frac{10}{3}}, \tag{3}$$

which is huge for a macroscopic black hole. To understand the physics of the end of a black hole, we have to understand the quantum evolution of these three regions of spacetime.

The Carter-Penrose diagram of the classical metric describing the black to white bounce [1] is depicted in Fig. 2. The white region is locally but not globally isomorphic to the Kruskal metric, the maximal extension of the Schwarzshild metric. There is a trapped as well as a later antitrapped region (see also Fig. 8 in Ref. [30]) separated by a small compact region where the Einstein equations are violated by quantum gravity effects. The horizon of the black hole is not an event horizon.

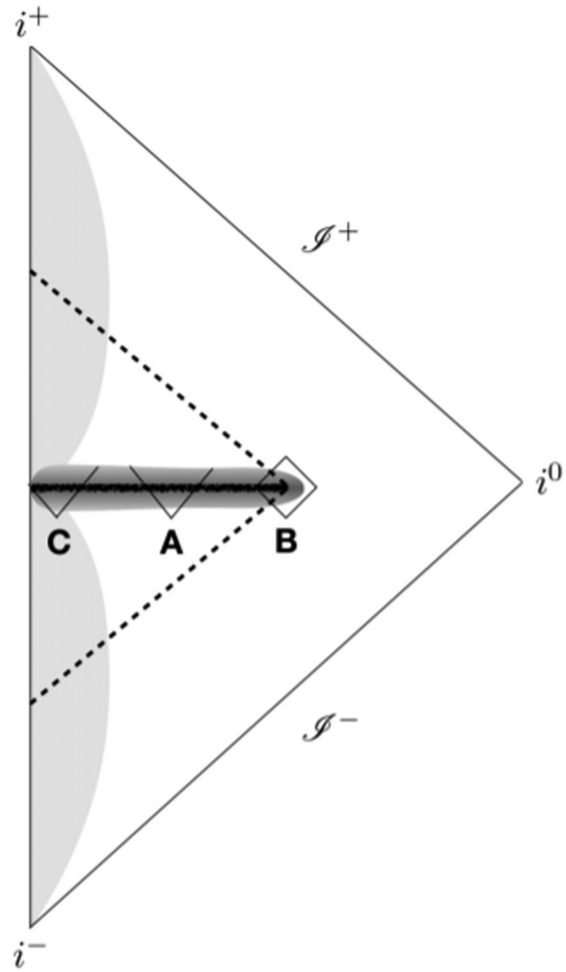

FIG. 2. The Carter-Penrose diagram of the black to white transition. The dark gray region is the quantum gravity region. The black hole (trapped region) is below the quantum gravity region, while the white hole is above. The trapping horizons are the dashes lines.

III. THE MAXIMALLY EXTENDED REISSNER-NORDSTRÖM METRIC

The Reissner-Nordström metric describing a massive charged star (or black hole) is

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\Omega^2 \tag{4}$$

with

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}, \tag{5}$$

where M is the mass of the star and Q its charge. Together with the Maxwell potential

$$A_a = \left(\frac{Q}{r}, 0, 0, 0\right), \tag{6}$$

Eqs. (4) and (5) solve the Maxwell-Einstein equations.

Notice that the $Q \to 0$ limit that reduces Reissner-Nordström metric to Schwarzschild metric is subtle at small radius, as

$$\lim_{Q\to 0}\lim_{r\to 0} f(r) = \infty, \quad \text{while} \quad \lim_{r\to 0}\lim_{Q\to 0} f(r) = -\infty, \tag{7}$$

which shows that even a small charge changes the inner geometry radically.

As the Schwarzschild metric, the Reissner-Nordström metric is static outside the horizon and spherically symmetric. The main difference with the Schwarzschild metric is that the equation $f(r) = 0$ that gives the position of the trapping horizons has two solutions rather than one:

$$r_\pm = M \pm \sqrt{M^2 - Q^2}. \tag{8}$$

The two solutions r_+ and r_- define the outer and inner horizon, respectively. They separate trapped, nontrapped, and antitrapped regions: The surfaces of the constant r coordinate are timelike for $r > r_+$, become spacelike for $r_- < r < r_+$, and are timelike again for $r < r_-$. The difference with the Schwarzschild metric is therefore the presence of the inner horizon r_-, a feature in common with the Kerr metric. As for the Schwarzschild metric, it has four Killing fields, the three associated with the rotations, and the Killing field $\frac{\partial}{\partial t} = \xi$ associated with the invariance with respect to the t coordinate. Notice that ξ is timelike for $r > r_+$ and $r < r_-$, null at the two horizons, and spacelike for $r_- < r < r_+$.

The Penrose diagram of the maximally extended Reissner-Nordström spacetime is given in Fig. 3. The interior region continues into an antitrapped region, namely, a white hole region, that in turns exits via another outer horizon into a different asymptotic region.

Consider a neutral particle with mass m and energy momentum $p_\alpha = m u_\alpha$ falling into a Reissner-Nordström black hole. The quantity $E = -g_{\alpha\beta} p^\alpha \xi^\beta$ is a constant of motion. A straightforward calculation gives

$$\dot{r}^2 + f(r) = E^2, \tag{9}$$

where $\dot{r} = u^r$. Notice that $\dot{r}$ vanishes at the radius r_b determined by

$$f(r_b) = E^2. \tag{10}$$

If the initial radial velocity of the particle vanishes at large r, then $E = 1$ and

$$r_b = \frac{Q^2}{2M} < r_-. \tag{11}$$

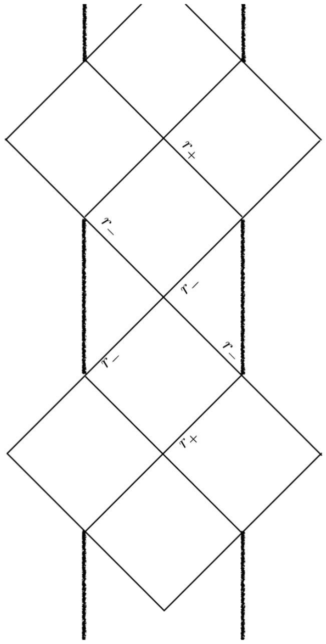

FIG. 3. Maximally extended Reissner-Nordström metric.

Hence, the particle enters the black hole, crosses the outer horizon at r_+, then crosses the inner horizon at r_-, and reaches $r = r_b$ where its velocity goes to zero. The particle bounces and starts moving outward. This is permitted because the interior region is not trapped. By time inversion symmetry, its geodesics exit then into the next asymptotic region through the antitrapped region; see Fig. 4.

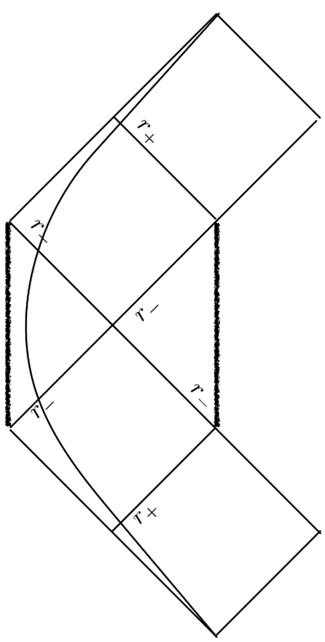

FIG. 4. The portion of the maximally extended Reissner-Nordstrom spacetime relative for the black to white bounce. The gray line is a geodesic that enters the black hole from a asymptotic region and exits in a different asymptotic region.

The conclusion is also valid for a charged particle. Because of (6), the electromagnetic effect on the energy of a charged particle is simply to shift it to $p_0 = mu_0 + qA_0$, where $A_0 = \frac{Q}{r}$ (we use units where $\frac{1}{4\pi\epsilon_0} = 1$). The conserved quantity is $E = -g^{\alpha\beta} p_\alpha \xi_\beta = -p_\alpha \xi^\alpha$, giving

$$\left(E - \frac{Q^2}{Mr}\right)^2 = f(r) + \dot{r}^2. \tag{12}$$

Charged particles do not follow geodesics, but (12) is similar to (9) up to a shift in the energy. If the initial radial velocity vanishes, it implies from (12) that $E \simeq 1$ by assuming that the initial radius is sufficiently large. Hence, we have to solve Eq. (12) with the turning point condition ($\dot{r}^2 = 0$), which gives

$$\left(1 - \frac{Q^2}{Mr}\right)^2 = f(r). \tag{13}$$

Interestingly, the solution of this equation is still (9) where we did not take the electrostatic repulsion into account.

Thus, a collapsing massive spherical charged mass that enters its own outer horizon can cross the inner horizon and bounce out at r_b.

In the Schwarzschild case, the bounce of the star was a hypothesis based on quantum gravity. In the charged case, the bounce of the star is predicted by classical general relativity. The physical problem of region C defined above is solved without the need of quantum mechanics. In other words, the presence of a charge opens up a classical throat for the passage of the star and its surrounding spacetime from the black to the white hole region.

If the charge is small, this passage is narrow. Its size can be estimated from the spacelike distance of the boundary of the star to the outer boundary of the classical region on a constant t surface. This is

$$\begin{aligned} d &= \int_{r_b}^{r_-} \frac{dr}{\sqrt{f(r)}} = \int_{r_b}^{r_-} \frac{r}{\sqrt{(r - r_+)(r - r_-)}} dr \\ &\leq \int_0^{r_-} \frac{r_-}{\sqrt{(r_- - r_+)(r - r_-)}} dr \\ &= \sqrt{2} M \frac{(1 - \sqrt{1 - \eta^2})^{\frac{3}{2}}}{(1 - \eta^2)^{\frac{1}{4}}}, \end{aligned} \tag{14}$$

where $\eta = \frac{Q}{M}$. If η is small, the throat is small. Only matter falling just after the star collapses reaches it, rather than the Cauchy horizon, But if η is close to 1,

$$ds = \frac{r}{\sqrt{(r - r_+)(r - r_-)}} dr \simeq \frac{r}{r - r_-} dr, \tag{15}$$

which is not integrable in r_-. Hence, the throat can be arbitrarily large.

The result suggests that the Reissner-Nordström black to white transition may be more natural than the Schwarzschild one: Already in the classical theory, the white hole is in the future of the black hole.

IV. THE REISSNER-NORDSTRÖM BLACK TO WHITE TRANSITION

Here we are not interested in a hypothetical emergence in a different universe. We are interested in the compatibility of classical general relativity with a bounce within the same universe, generated by a quantum tunneling in a small compact spacetime region.

To show that this is possible, we can construct a solution of the Maxwell-Einstein equations, following [1], by cutting and gluing relevant portions of the maximally extended Reissner-Nordström metrics. The way this can be done is sketched in Fig. 5.

To construct the spacetime we are interested in, we proceeded as follows: (i) We cut away from the maximally extended Reissner-Nordström spacetime all the regions on the left of the gray line in Fig. 5 and replace it with the interior of a classical bouncing star. (ii) We cut away all the region to the right of the blue line in Fig. 5. (iii) We glue the two dotted portions of the blue line. These are both constant-Schwarzschild-time surfaces, and therefore, the gluing gives a smooth junction where the Einstein equations are satisfied.

More precisely, to glue the spacetime smoothly, both metric and extrinsic curvature must match. The metric of the two surfaces of constant time coordinate t is clearly the same. By construction, the extrinsic curvature k changes sign (because of the time reversal symmetry exploited in cutting the spacetime). But the curvature vanishes on these surfaces, as the normal to the spacelike surfaces of constant t is the vector $N_\alpha = \partial_\alpha t = \xi$ where ξ is the Killing vector associated with the t-coordinate invariance and

$$k = \frac{1}{2}\mathcal{L}_N q = \frac{1}{2}\mathcal{L}_\xi q = 0, \tag{16}$$

where q is the induced metric on the spacelike hypersurface.

The resulting spacetime is depicted in Fig. 6, and in Fig. 7 by including the star. The central gray area represents the quantum tunneling region, where the classical evolution is violated. Let us do so more explicitly.

Let us break the spacetime of Fig. 5 into two overlapping regions, as depicted in Fig. 8, and introduce distinct coordinates in the two regions. The left panel includes the exterior $r > r_+$, the black hole $r_- < r < r_+$, and the interior $r < r_-$. A set of coordinates covering this entire left region is given by r, and the advanced time null coordinate v defined by

$$v = t + r^*(r), \tag{17}$$

where

$$r^*(r) = \int \frac{dr}{f(r)}. \tag{18}$$

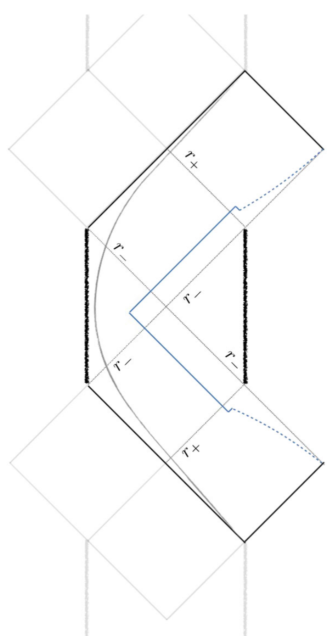

FIG. 5. The surface of the star (gray line), the boundary of the quantum region (continuous blue line) and the constant time surface that can be identified.

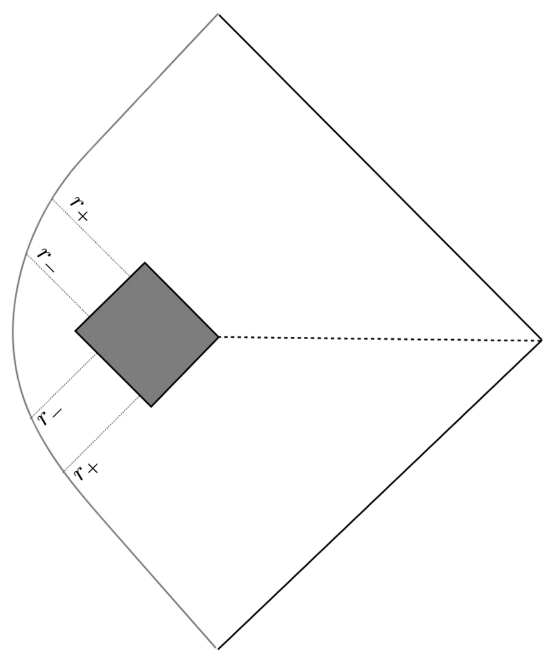

FIG. 6. The spacetime of the black to white charged transition outside the star.

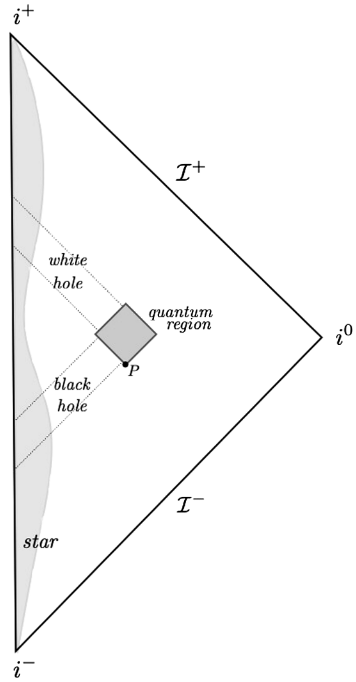

FIG. 7. The full black to white transition for a charged black hole.

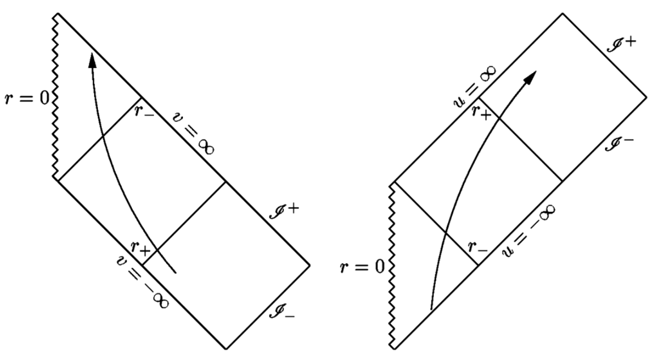

FIG. 8. The part of the maximally extended Reissner-Nordström spacetime surrounding the black hole region on the left that can be labeled by (r, v) coordinates, and the part corresponding to the white hole region on the right by the (r, u) patch. Picture taken from [31].

The integration must be done separately in the regions separated by r_+ and r_-, because the integral is ill behaved at the horizons. (The t coordinates in the three regions are unrelated to one another, each going from $-\infty$ to $+\infty$ in its own region. Properly speaking, they should have different names. For instance, we can call t_{out} the coordinate outside the horizon, t_{in} the innermost one, and so on.) In the interior region, it is convenient to choose the integration constant (or the lowest boundary of the integral) so that the t coordinate of the bounce point b is zero. In the external region, it is convenient to choose it so that the t coordinate of the outermost point of the quantum region is zero.

In these coordinates, the metric reads

$$ds^2 = -f(r)dv^2 + 2drdv + r^2 d\Omega^2. \tag{19}$$

The right panel of Fig. 8 includes the exterior, the white hole, and again the interior region separated by r_+ and r_-. A set of coordinates covering the left region is given by r and the retarded time

$$u = t - r^*(r). \tag{20}$$

In these coordinates, the metric reads

$$ds^2 = -f(r)du^2 - 2drdu + r^2 d\Omega^2. \tag{21}$$

The two coordinate patches overlap in the interior ($r < r_-$) and exterior ($r > r_+$) regions, where their relation is easily deduced by equating the t coordinate of the two:

$$u + r^*(r) = v - r^*(r). \tag{22}$$

Notice that identifying the coordinates in the two overlapping regions (exterior and interior) adds a parameter to the definition of the spacetime (irrespective of the detailed location of the quantum region). This can be seen as follows. Consider two points p_1 and p_2 both on $t = 0$ (hence, $u_1 = -v_1$ and $u_2 = -v_2$), and let r_1 and r_2 be their radius. If the two points are both in the exterior or both in the interior region, the difference of their retarded (or advanced) time is

$$u_2 - u_1 = r^*(r_2) - r^*(r_1) = \int_{r_1}^{r_2} \frac{dr}{f(r)}. \tag{23}$$

But if p_2 is in the interior and p_1 in the exterior, the above integral must be broken into two and therefore depends on an arbitrary integration constant, not determined by the radius of the points. This additional parameter is a global topological parameter which distinguished spacetimes with the same M and Q obtained in this manner.

To specify the metric entirely (including the location of the quantum region), it is convenient to focus on three special events: the event b where the radius of the star reaches its minimal value r_b given in (11), the event P where the quantum tunnelling starts, and the point P' where it ends. We take the quantum region to be the causal diamond defined by P and P', namely, the intersection between the causal future of P and the causal past of P'; see Fig. 9.

Let the coordinates of the points b, P, and P' be $(v_b, u_b), (v_P, u_P)$, and $(v_{P'}, u_{P'})$, respectively. The two points of the quantum region min and max with the minimal and maximal radius have coordinates $(v_P, u_{P'})$ and $(v_{P'}, u_P)$, respectively.

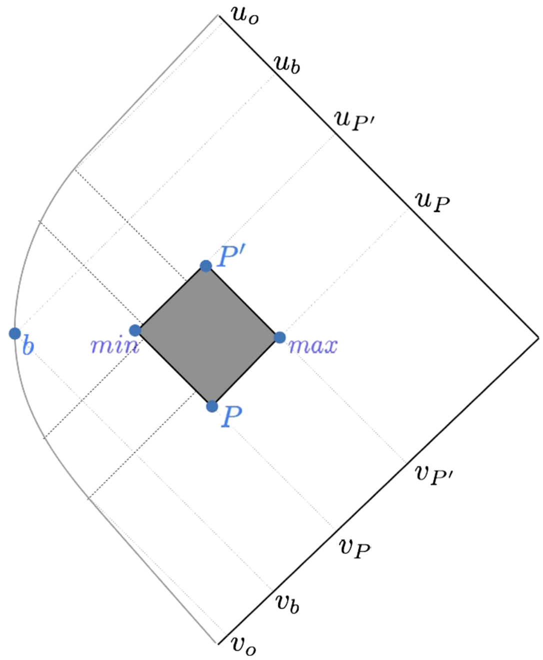

FIG. 9. The parameters characterizing the bouncing geometry.

As a first step, consider a time reversal symmetric bounce geometry. In this case, we can assume $u_b = -u_v$, $u_{P'} = -v_P$, and $u_P = -v_{P'}$ (see Fig. 9). The maximal radius reached by the quantum region is $r_{\rm max} = (v_{P'} - u_P)/2 > r_+$ at max. The minimal radius reached by the quantum region is $r_{\rm min} = v_P - u_{P'} < r_-$ at min. We can choose the zero of the time coordinate $t_{\rm in}$ so that b and min are on $t_{\rm in} = 0$ and the zero of the Schwarzschild-like time coordinate $t_{\rm out}$ so that max is on $t_{\rm out} = 0$.

Let us discuss the interpretation of these quantities. The quantities v_0 and u_0 are the advanced time of the collapse of the star into the black hole and the retarded time of exit from the white hole. The difference $v_b - v_o$ is the (short) time determined by the details of the dynamics of the collapsing star. For simplicity, we can take the limit of a star collapsing at very high relativistic speed so that $v_b - v_o \sim 0$ and neglect their difference.

The advanced time $\tau_t = v_{P'} - v_P$ is the time of the tunneling transition (likely to be short). The crucial physical parameter of the process is the difference $\tau_{\rm BH} = v_P - v_b$. Notice that we must have $r_{\rm min} < r_-$, but as $r_{\rm min}$ approaches r_-, the advanced time v_P goes to infinity. Hence, $\tau_{\rm BH}$ can be arbitrarily long.

Disregarding the time $v_b - v_o$, the symmetric bounce is then fully characterized by five parameters: $M, Q, r_P, \tau_{\rm BH}$, and τ_t. A theory of quantum gravity must give the transition probability $W(M, Q, r_P, \tau_{\rm BH}, \tau_t)$ between the black hole state and the white hole state. For the formulation of this computation in loop quantum gravity, see Refs. [5,6,10,28].

A non-time-reversal-symmetric bounce can be obtained by having b and $(v_P, u_{P'})$ on different constant t surfaces and P and P' at different radius. This gives $u_{P'} \neq -v_P$ and $\tau_{\rm WH} = u_b - u_{P'}$ different from $\tau_{\rm BH}$. The two long times $\tau_{\rm BH}$ and $\tau_{\rm WH}$ can be interpreted as the lifetimes of the black hole and the white hole. The general case is therefore characterized by seven parameters: $M, Q, r_P, r_{P'}, \tau_{\rm BH}$, and $\tau_{\rm WH}$, plus the tunneling time τ_t.

V. THE POINT P AND THE ONSET OF THE TUNNELING

The spacetime described above is characterized by the region $(u_P < u < u_{P'}, v_P < v < v_{P'})$ where the Einstein equations are not satisfied. Following [10], we call this region the B region. As mentioned in the Introduction, this is the region where we assume quantum mechanics alters the classical spacetime dynamics. Why is quantum theory relevant here? There are three answers to this question.

(i) Quantum mechanics allows a tunneling transition with a given finite probability. A rough estimate of the tunneling probability may include a factor e^{-S}, where S is the action of the process. Since Q is of little relevance near the outer horizon, we may expect this factor to be proportional to e^{-M^2}. In the classical theory, the lifetime of a black hole is infinite, and during an infinite time, an event may happen even if its probability per unit of time is very small. This would give an exponentially large factor e^{M^2} in the black hole lifetime, but still a finite lifetime.

(ii) However, we expect an isolated black hole to evaporate by Hawking radiation. During the evaporation, the mass of the hole decreases, reaching a Planckian value in a time of the order M_0^3, where M_0 is the initial mass of the hole. When M approaches the Planck mass, the suppressing factor e^{-M^2} approaches one, and the tunneling becomes likely to happen. Hence, we may expect a transition to happen shortly before the end of the Hawking evaporation, hence, in a time of order M_0^3 after the collapse. It is important to recall again that when M is near the Planck mass, the curvature becomes Planckian outside the horizon. We can view the point P as a point where the curvature is close enough to a Planckian curvature to trigger the quantum tunneling.

If the transition happens near the very end of the evaporation, the forming white hole will be small, and the transition may not change much of the Hawking radiation. But the small white hole can be stabilized by quantum gravity [18], be long-living, and have astrophysical or cosmological relevance [19,22].

(iii) Finally, we also recall the hypothesis considered in [1]: Quantum corrections to the Einstein equations can be small but are never exactly zero, so they can pile up in proper time τ. Assuming a linear deviation to first order in $\hbar$, a non-negligible quantum phenomenon may start when $q = K\tau$ is Planckian, where K is a curvature scalar such as the Kretchman invariant

$$K^2 = C_{\alpha\beta\gamma\delta}C^{\alpha\beta\gamma\delta} = 48\frac{(M - \frac{Q^2}{r})^2}{r^6}, \qquad (24)$$

and τ is the proper time along a stationary timelike geodesic. Since the curvature near the horizon is of order M^{-2}, this has suggested the (speculative) possibility of a transition triggered already after a black hole lifetime of order M_0^2. This consideration also suggests what the possible radius of P is. The parameter q is maximized at a finite distance from the horizon, of the order of M. This is because the curvature increases with smaller radius, but the proper time at a stationary point is redshifted near the outer horizon as $\tau = \sqrt{f(r)}\tau_{\rm BH}$, so that

$$q = K\tau \sim \frac{M - \frac{Q^2}{r}}{r^3}\sqrt{1 - \frac{2M}{r} + \frac{Q^2}{r^2}}\tau_{\rm BH}, \qquad (25)$$

which has a minimum in r just outside the outer horizon. The minimum sets the radius of P, while $q \sim 1$ at this minimum sets $\tau_{\rm BH}$ [1].

One way or the other, at some point P outside the horizon the dynamics of the gravitational field enters the quantum tunneling region. By causality, this region must then be in the causal future of P. In the future of this region, we can assume spacetime to be described again by a solution of the Einstein equations. Depending on the interpretation of quantum theory one prefers, this can be seen as a decohered many world branch, or the result of a ("measuring") interaction with other degrees of freedom (the "observer") after the quantum process.

Notice that the timelike singularity of the Reissner-Nordström spacetime situated outside the star (on the right in the diagram) is replaced by the quantum region, and so is the entire full entire triangular region bounded by this singularity and r_-.

This completes the construction of the Reissner-Nordström black to white hole transition spacetime.

Before concluding this section, we observe that the above result can also be taken as a possible form of the effective metric in the C and A regions in the case of vanishing electrical charge. (Another simple guess for the effective metric in the A region can be obtained by replacing r with $\sqrt{r^2 + L^2}$ in the Schwarzschild metric [32]; see also Ref. [33] for the same idea). Loop quantum cosmology [34] suggests that the dominant quantum effect at high curvature is a repulsive force, precisely as in the Reissner-Nordström geometry (where repulsion does not act on charged matter only). This is the short-scale quantum pressure which is also responsible for the quantum bounce of a Planck star [2,14]. Hence, the Reissner-Nordström geometry can also be taken as a guess for the quantum corrections to the metric at short radius for an effective value of a "charge" $Q^2_{\rm eff}$ determined by M (and the Planck mass $M_{\rm Planck}$). This can be estimated assuming that the corrections become relevant when the curvature is Planckian, which gives

$$Q_{\rm eff} \sim M^{\frac{2}{3}}. \qquad (26)$$

Hence, restoring physical units

$$\eta_{\rm eff} \sim \left(\frac{M}{M_{\rm Planck}}\right)^{-\frac{1}{3}}. \qquad (27)$$

For a macroscopic mass M, $\eta_{\rm eff} \ll 1$ and so the resulting metric is very close to Schwarzschild metric, but since, as pointed out above, the $Q \to 0$ and $r \to 0$ limits do not commute, the global structure of the metric is radically changed nevertheless. If this is correct, the actual effect of quantum gravity makes the Reissner-Nordström geometry studied here more realistic than the Schwarzschild geometry.

VI. CAUCHY HORIZON INSTABILITIES

There are three important physical phenomena that the model defined in this paper disregards: rotation, instabilities, and the Hawking radiation. In this last part of the paper, we briefly comment on their possible effect.

To account for rotation, we must study the Kerr-Newman metric. As already noticed, the Carter-Penrose maximal extension of the Kerr-Newman metric is similar to the one of the maximal extension of the Reissner-Nordström metric. It is therefore reasonable to expect that some qualitative aspects of the black to white bounce extend to the rotating case. This will be done elsewhere.

Instabilities are expected before the Cauchy horizon (the boundary of the future of a singularity) [35,36]. There is a simple argument that illustrates why. Consider a sequence of pulses emitted radially at regular time intervals from past infinity. They all reach an observer moving toward the Cauchy horizon in a finite proper time. Thus, if perturbations enter the black hole for an arbitrary long external time, they all pile up in the finite time of the observer before the Cauchy horizon. In turn, this is likely to cause a concentration of energy near the Cauchy horizon. This is going to generate a strong curvature, which at some point may become Planckian.

Poisson and Israel [26,27] have in fact shown that small perturbations of the metric outside the black hole during or after the star collapsing become unstable once they penetrate inside the black hole and lead the Cauchy horizon to become a null curvature singularity. This phenomenon is known as mass inflation and is confirmed by numerical investigations [37–42]. Furthermore, physical and numerical indications point to the expectation that the perturbation grows into a spacelike singularity in the classical theory.

Before diverging, the curvature must become Planckian. Hence, the region along the Cauchy horizon is situated inside a quantum gravity region. This result strengthens the hypothesis that the spacetime dynamics enters a quantum region before the development of the Cauchy horizon, as in the hypothesis of this paper, and no Cauchy horizon develops in reality. Quantum gravity should correct not only the singularities but also the lack of global hyperbolicity of classical general relativity (see also Refs. [41,42]).

A quick estimate of where the instability drives the spacetime dynamics into the quantum region can be obtained as follows. Near the inner horizon, the Reissner-Nordström metric can be written as (see Ref. [31])

$$ds^2 \simeq -2e^{-\kappa_- v} e^{\kappa_- u} du dv + r^2 d\Omega^2, \tag{28}$$

where $\kappa_- = \frac{1}{2} f'(r_-)$ is the surface gravity on the inner horizon. The proper time interval $d\tau$ between the reception of signals by an observer approaching the Cauchy horizon along a (timelike) constant $u + v = 2t$ trajectory inside the black hole is related to the proper time interval dv of emission of these signals from infinity by

$$d\tau \simeq \sqrt{2} e^{\kappa_- t} e^{-\kappa_- v} dv \tag{29}$$

approaching the Cauchy horizon ($v \to \infty$), giving a blueshift growing exponentially as

$$\frac{d\tau}{dv} \sim e^{-\kappa_- v}. \tag{30}$$

This blueshift is expected to hold whatever is the observer motion across the inner horizon [38]. According to [37,38], the perturbation generated by this instability gives rise, for $v \to \infty$, to a curvature of the order of

$$K^2 = g(u) \frac{e^{2\kappa_- v}}{v^{2q}}, \tag{31}$$

where $g(u)$ is a function that vanishes at the outer horizon and increases monotonically, and q is a positive integer depending on the characteristics of the infalling perturbations. Since v is large, the term v^{2q} can be neglected and we have

$$v \simeq -\frac{\log g(u)}{2\kappa_-}, \tag{32}$$

so we may expect that the spacetime enters the quantum region just because of the instabilities. Since this formula is only reliable in the large v limit, is not clear to us whether this can happen before the onset of the quantum regime studied above. But notice that in any case, these instabilities happen only in the interior of the black hole as $g(u)$ vanishes on the outer boundary. The black to white transition requires that the region affected by quantum gravity leaks outside the horizon. Without this, the apparent horizon becomes an event horizon, and the black hole remains such forever, if observed from the exterior.

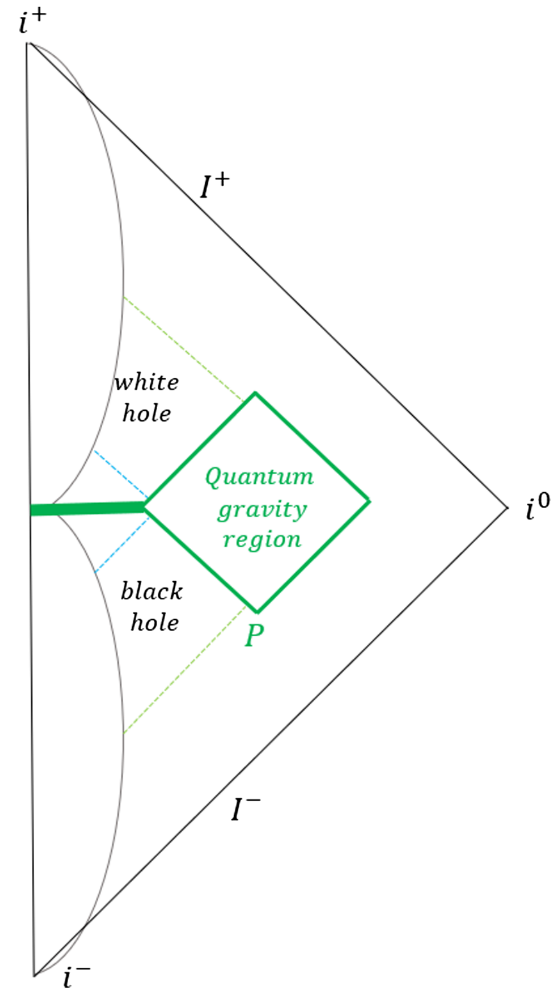

FIG. 10. The Carter-Penrose diagram for the black to white transition of the charged black hole when the quantum gravity region (in green) extends up to the entire internal region. The difference with Fig. 7 is that here the star encounters a quantum gravity region, and the white hole region is not classically connected to the black hole region anymore.

As Reissner-Nordström and Kerr share the same causal diagram, the existence of the infinite blueshift and the consequences discussed above, i.e., the instability the (Kerr) metric, the null weak singularity along the Cauchy horizon, and its shrinking up to a spacelike singularity at $r = 0$, are also present for the rotating black hole and the charged and rotating Kerr-Newman black hole.

Notice that there is also the possibility that the quantum gravity region generated by the instabilities extends all the way to the trajectory of the bouncing star, in which case, the physics would be more similar to the one studied in [1]; see Fig. 10.

VII. HAWKING EVAPORATION

In the Schwarzschild case, the mass of the hole decreases because of the Hawking radiation (HR) falling into the hole. What about its charge?

One may speculate that the HR is dominated by massless particles that can escape to infinity more easily. Massless particles in the standard model are not charged, so they may not carry charge away and diminish the black hole charge Q. The same is not true for the rotating case, because photons can carry angular momentum, although the HR might still be dominated by s waves. According to [43], charged black holes do not necessarily evolve toward the Schwarzschild limit, contrary to Kerr black holes. If they are massive enough and if their charge is large enough ($\frac{3}{4} < \eta^2 < 1$ according to [43]), they evolve toward the extremal limit ($\eta = 1$). If so, the ratio $\eta = Q/M$ increases at the end of the Hawking evaporation, making the Reissner-Nordsrtöm model more realistic than the Schwarzschild model for the transition.

In the absence of Hawking evaporation, the black to white transition might take exponentially long times to happen. The Hawking evaporation shrinks the mass, making it increasingly more probable; hence, it is likely to happen within a time M_0^3. But part of the HR falls into the black hole, affecting the internal metric. Some consequences of the backreaction of the HR on the bounce have been explored in [8]. In the rest of this section, we present some sketchy considerations on the possible effect of this backreaction in the charged case. These are speculative, because not much is solidly known about the energy-momentum tensor of the HR falling inside the hole and its backreaction. For simplicity, we assume here that the ratio $\eta = \frac{Q}{M}$ remains constant. If so, both the outer $r_+ = M(1 + \sqrt{1-\eta^2})$ and inner $r_- = M(1 - \sqrt{1-\eta^2})$ horizons decrease.

There are two simple models of the HR falling inside the hole. The first is that it is made by negative energy quanta moving along ingoing null geodesics. The second is that it is made by negative energy quanta moving along outgoing null geodesics (the radius of which is still decreasing, since it is a trapped region). The reality is probably between the two [8,44,45].

Consider a Hawking quantum of mass $-\delta m$ and charge $-\delta q$ infalling along a constant v null line. A simple estimate of its backreaction is to modify the geometry by shifting the Reissner-Nordström mass and charges by these amounts in its future. Notice that this shifts the two horizons inward. Let r'_- be the radius of the new inner horizon; see Fig. 11.

Call c the point where the infalling quantum meets the bouncing star and r_c the radius of this point. There are two possibilities. If $r'_- > r_c$, the negative energy quanta crosses the new inner horizon r'_- before reaching the star. This does not change the overall picture much. However, it might also happen that $r'_- < r_c$; namely, the negative energy infalling quanta meets the bouncing star before entering the new inner horizon. If so, c is in a trapped region; hence, the star

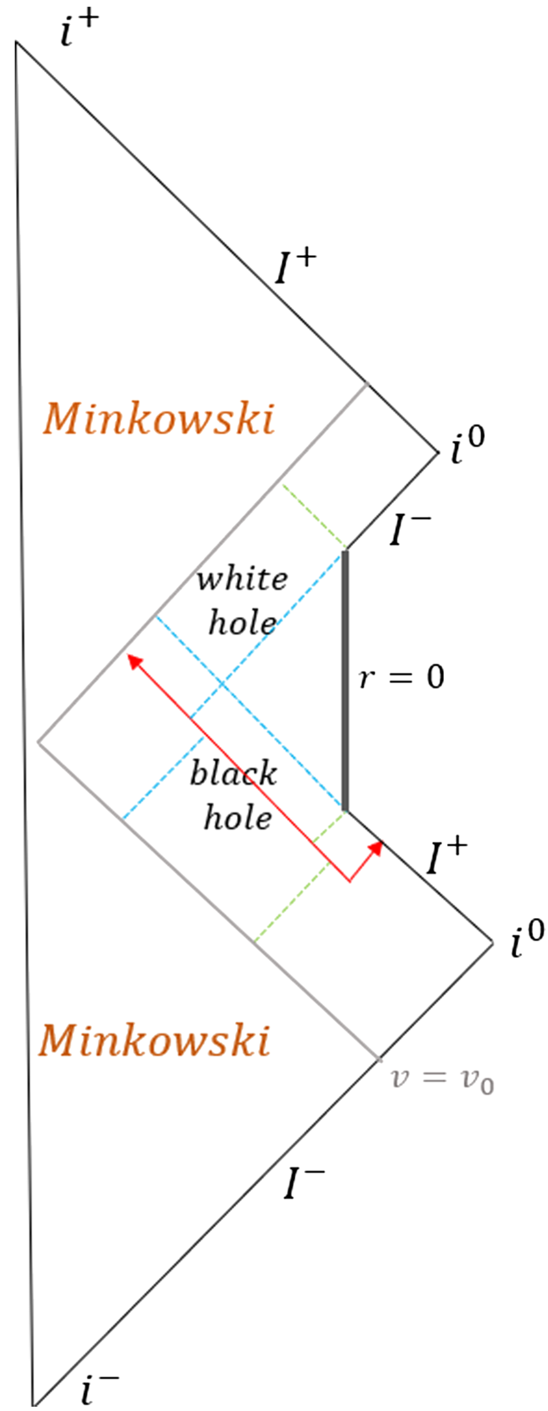

FIG. 11. A pair of particles is created just outside the horizon (red lines). One, with mass $+\delta m$ and charge $+\delta q$, escapes to null infinity. The other, with mass $-\delta m$ and charge $-\delta q$, falls into the black hole along a constant v null line. The infalling particle modifies the metric inside the black hole in its future, producing a Reissner-Nordström metric of a black hole of mass $M - \delta m$ and charge $Q - \delta q$. Both inner and outer horizons shrink. (Here and in the next figure depicted without the gluing of the asymptotic regions).

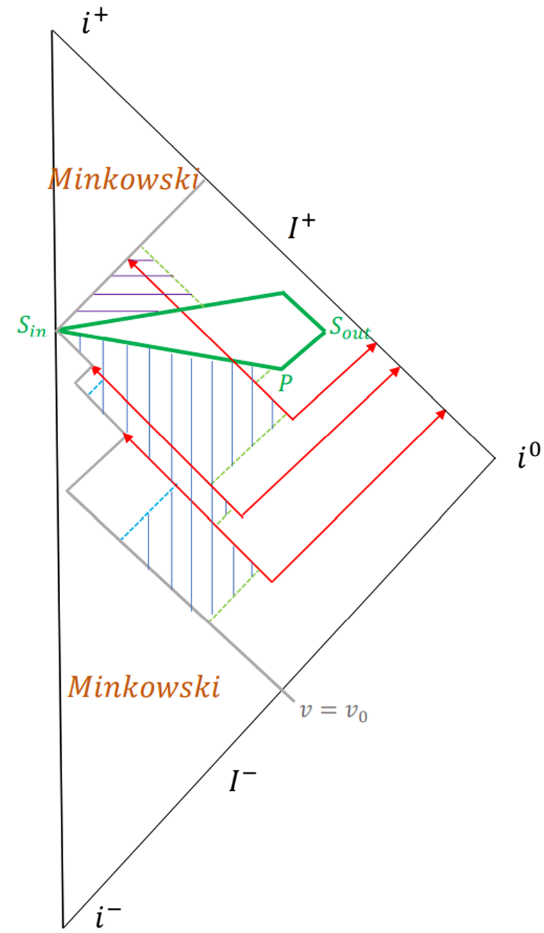

FIG. 12. The Hawking radiation shifts the inner horizon for each quanta. A shell (gray line) falls again and bounces at increasingly smaller radius. The vertically hatched area is trapped; the horizontally hatched area is antitrapped. Radiation continues to be emitted outside until the point P outside the trapped region becomes Planckian.

has to fall again until it crosses new inner horizon and be allowed to bounce at a point b' with radius $r_{b'} < r_b$. This process might disrupt the classical throat. In particular, notice that if r_P is near Planckian, so must be the outer and inner horizons at v_P, and hence, so must be the corresponding r_c, which implies that the star itself must have entered a quantum regime; see Fig. 12. While in the noncharged case the infalling HR always encounters the star in the antitrapped region, in the charged case, it is the HR that may make its charge and mass decrease to Planckian values before the final bounce. Our control of the energy momentum tensor of the HR and its backreaction is still insufficient to judge if this is truly the case.

An argument can be given, suggesting that the onset of the quantum gravity region may be spacelike, especially for small η (on this, see also Ref. [46]). As ingoing HR falls into the hole, it carries negative energy and therefore decreases the local mass of the geometry.

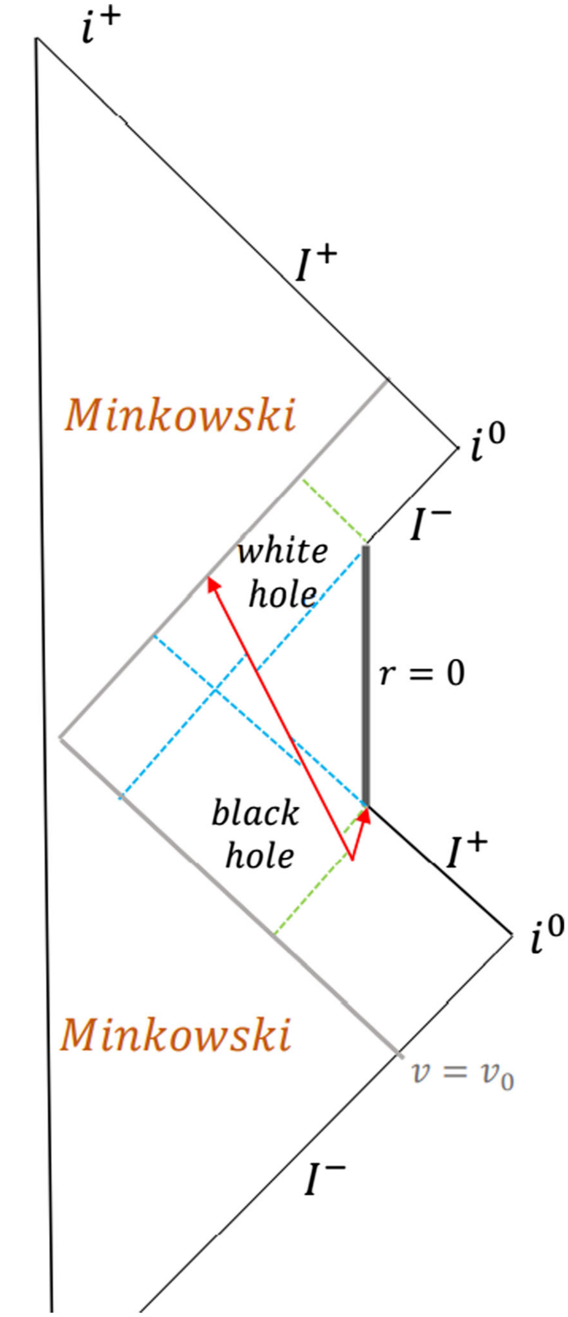

FIG. 13. Massive Hawking radiation quanta enter the quantum region and make the inner (and outer) horizon shrink. (Depicted without the gluing of the two asymptotic regions).

We may expect the metric to be a Reissner-Nordström metric locally but with a decreasing mass function $M(v)$. On the inner horizon, the curvature of a given slice [from Eq. (24)] is

$$K^2 = 48\frac{\left(1-\frac{\eta^2}{1-\sqrt{1-\eta^2}}\right)^2}{M(v)^4(1-\sqrt{1-\eta^2})^6} \simeq \frac{3072}{M(v)^4\eta^{12}}, \quad (33)$$

where the last approximation holds if η is small. This indicates when the inner horizon enters a region of Planckian curvature. Let $S_{\rm in}$ be the point where the curvature becomes Planckian on the inner horizon and $v_{\rm in}$ its advanced time. This is the innermost point included in the quantum region. The curvature at the outer horizon $r_+ = M(v)(1+\sqrt{1-\eta^2})$ is

$$K^2 = 48\frac{\left(1-\frac{\eta^2}{1+\sqrt{1-\eta^2}}\right)^2}{M(v)^4(1+\sqrt{1-\eta^2})^6} \simeq \frac{3}{4}\frac{1}{M(v)^4}, \quad (34)$$

where again the last approximation holds if η is small. Comparing (33) and (34) for $K^2 \sim 1$,

$$\frac{M(v_{\rm in})}{M(v_P)} = \frac{8}{\eta^3} > 1. \quad (35)$$

Hence, $v_{\rm in} < v_P$ as M is a decreasing function of v, and so the line joining the points P and $S_{\rm in}$ is spacelike. The line joining P and $S_{\rm in}$ can be taken as the boundary of the quantum region. This picture has similarities to the Schwarzschild black to white transition.

Finally, outgoing HR quanta inside the hole, on the other hand, fall into the quantum region. The negative energy they carry can emerge into the white hole and decrease the mass and charge of the star inside the white hole and the classical throat is not destroyed; see Refs. [8,47]. The same may happen if the Hawking quanta are massive (see Fig. 13).

VIII. CONCLUSION

The presence of charge renders the black to white transition more interesting than the noncharged case. The bounce of the star and the region immediately surrounding it evolve into a white hole simply by following the classical dynamics. Only the horizon region tunnels. The white hole is in the same location of the same asymptotic region as the black hole that originates it.

The extension of the spacetime region surrounding the star that evolves into a white hole classically depends on the charge: In the limit of vanishing charge, the situation is similar to the Schwarzschild tunneling.

We have also tentatively explored the effects of the classical instabilities and the backreaction of the HR on the process. These may alter the picture of the interior, creating a spacelike onset of the quantum region (as in the Schwarzschild case) and decreasing the range of the classical region, but they do not seem to alter the basic possibility of black to white hole quantum tunneling.

The qualitative similarities of the Reissner-Nordström and Kerr-Newman metrics suggest that the entire black to white hole quantum tunneling may be a general possibility, and therefore represent a likely scenario for the fate of all real black holes.

ACKNOWLEDGMENTS

This work was supported by the ID# 61466 grant from the John Templeton Foundation, as part of the QISS (The Quantum Information Structure of Spacetime) project.

[1] H. M. Haggard and C. Rovelli, Quantum-gravity effects outside the horizon spark black to white hole tunneling, Phys. Rev. D **92**, 104020 (2015).
[2] C. Rovelli and F. Vidotto, Planck stars, Int. J. Mod. Phys. D **23**, 1442026 (2014).
[3] M. Christodoulou, C. Rovelli, S. Speziale, and I. Vilensky, Realistic observable in background-free quantum gravity: The Planck-star tunnelling-time, Phys. Rev. D **94**, 084035 (2016).
[4] C. Rovelli, Black hole evolution traced out with loop quantum gravity, Physics **11**, 127 (2018).
[5] E. Bianchi, M. Christodoulou, F. D’Ambrosio, H. M. Haggard, and C. Rovelli, White holes as remnants: A surprising scenario for the end of a black hole, Classical Quantum Gravity **35**, 225003 (2018).
[6] M. Christodoulou and F. D’Ambrosio, Characteristic time scales for the geometry transition of a black hole to a white hole from spinfoams, arXiv:1801.03027.
[7] P. Martin-Dussaud and C. Rovelli, Interior metric and ray-tracing map in the firework black-to-white hole transition, Classical Quantum Gravity **35**, 147002 (2018).
[8] P. Martin-Dussaud and C. Rovelli, Evaporating black-to-white hole, Classical Quantum Gravity **36**, 245002 (2019).
[9] F. D’Ambrosio, M. Christodoulou, P. Martin-Dussaud, C. Rovelli, and F. Soltani, The end of a black hole’s evaporation: Part I, Phys. Rev. D **103**, 106014 (2021).
[10] F. Soltani, C. Rovelli, and P. Martin-Dussaud, The end of a black hole’s evaporation: Part II, Phys. Rev. D **104**, 066015 (2021).
[11] A. Barrau, C. Rovelli, and F. Vidotto, Fast radio bursts and white hole signals, Phys. Rev. D **90**, 127503 (2014).
[12] A. Barrau and C. Rovelli, Planck star phenomenology, Phys. Lett. B **739**, 405 (2014).
[13] F. Vidotto, A. Barrau, B. Bolliet, M. Shutten, and C. Weimer, Quantum-gravity phenomenology with primordial black holes, Springer Proc. Phys. **208**, 157 (2018).
[14] C. Rovelli, Planck stars: New sources in radio and gamma astronomy?, Nat. Astron. **1**, 0065 (2017).
[15] A. Barrau, B. Bolliet, M. Schutten, and F. Vidotto, Bouncing black holes in quantum gravity and the Fermi gamma-ray excess, Phys. Lett. B **772**, 58 (2017).

[16] A. Raccanelli, F. Vidotto, and L. Verde, Effects of primordial black holes quantum gravity decay on galaxy clustering, J. Cosmol. Astropart. Phys. 08 (2018) 003.
[17] F. Vidotto, Quantum insights on primordial black holes as dark matter, Proc. Sci. EDSU2018 (**2018**) 046 [arXiv:1811.08007].
[18] C. Rovelli and F. Vidotto, Small black/white hole stability and dark matter, Universe **4**, 127 (2018).
[19] C. Rovelli and F. Vidotto, Pre-big-bang black-hole remnants and past low entropy, Universe **4**, 129 (2018).
[20] F. Vidotto, Measuring the last burst of non-singular black holes, Found. Phys. **48**, 1380 (2018).
[21] E. Barausse *et al.*, Prospects for fundamental physics with LISA, Gen. Relativ. Gravit. **52**, 81 (2020).
[22] A. Barrau, L. Ferdinand, K. Martineau, and C. Renevey, Closer look at white hole remnants, Phys. Rev. D **103**, 043532 (2021).
[23] C. Rovelli, Black holes have more states than those giving the Bekenstein-Hawking entropy: A simple argument, arXiv:1710.00218.
[24] C. Rovelli, The subtle unphysical hypothesis of the firewall theorem, Entropy **21**, 839 (2019).
[25] R. Penrose, The question of cosmic censorship, J. Astrophys. Astron. **20**, 233 (1999).
[26] E. Poisson and W. Israel, Inner-Horizon Instability and Mass Inflation in Black Holes, Phys. Rev. Lett. **63**, 1663 (1898).
[27] E. Poisson and W. Israel, Internal structure of black holes, Phys. Rev. D **41**, 1796 (1990).
[28] M. Christodoulou and F. D'Ambrosio, Characteristic time scales for the geometry transition of a black hole to a white hole from spinfoams, arXiv:1801.03027.
[29] H. M. Haggard and C. Rovelli, Quantum gravity effects around Sagittarius A*, Int. J. Mod. Phys. D **25**, 1644021 (2016).
[30] R. Carballo-Rubio, F. D. Filippo, S. Liberati, and M. Visser, Geodesically complete black holes, Phys. Rev. D **101**, 084047 (2020).
[31] E. Poisson, An advanced course in general relativity, lecture notes at University of Guelph, (2002).
[32] F. D'Ambrosio and C. Rovelli, How information crosses Schwarzschild's central singularity, Classical Quantum Gravity **35**, 215010 (2018).
[33] E. Franzin, S. Liberati, J. Mazza, A. Simpson, and M. Visser, Charged black-bounce spacetimes, J. Cosmol. Astropart. Phys. 07 (2021) 036.
[34] I. Agullo and A. Corichi, Loop quantum cosmology, arXiv:1302.3833.
[35] M. Simpson and R. Penrose, Internal instability in a Reissner-Nordström black hole, Int. J. Theor. Phys. **7**, 183 (1973).
[36] M. Dafermos, Stability and instability of the Cauchy horizon for the spherically symmetric Einstein-Maxwell-scalar field equations, Ann. Math. **158**, 875 (2003).
[37] P. R. Brady and J. D. Smith, Black Hole Singularities: A Numerical Approach, Phys. Rev. Lett. **75**, 1256 (1995).
[38] P. R. Brady, The internal structure of black holes, Prog. Theor. Phys. Suppl. **136**, 29 (1999).
[39] L. M. Burko, Structure of the Black Hole's Cauchy-Horizon Singularity, Phys. Rev. Lett. **79**, 4958 (1997).
[40] L. M. Burko and A. Ori, Analytic study of the null singularity inside spherical charged black holes, Phys. Rev. D **57**, R7084 (1998).
[41] R. Carballo-Rubio, F. Di Filippo, S. Liberati, C. Pacilio, and M. Visser, Inner horizon instability and the unstable cores of regular black holes, J. High Energy Phys. 05 (2021) 132.
[42] R. Carballo-Rubio, F. Di Filippo, S. Liberati, C. Pacilio, and M. Visser, On the viability of regular black holes, J. High Energy Phys. 07 (2018) 023.
[43] W. A. Hiscock and L. D. Weems, Evolution of charged evaporating black holes, Phys. Rev. D **41**, 1142 (1990).
[44] W. Hiscock, Models of evaporating black holes. I, Phys. Rev. D **23**, 2813 (1981).
[45] M. K. Parikh and F. Wilczek, Hawking Radiation as Tunneling, Phys. Rev. Lett. **85**, 5042 (2000).
[46] E. Bianchi and H. M. Haggard, Spin fluctuations and black hole singularities: The onset of quantum gravity is spacelike, New J. Phys. **20**, 103028 (2018).
[47] P. Martin-Dussaud and C. Rovelli, Evaporating black-to-white hole, Classical Quantum Gravity **36**, 245002 (2019).