Consistency of the dual formulation of axion solutions to the strong *CP* problem

Otari Sakhelashvili
*Max-Planck-Institut für Physik, Föhringer Ring 6, 80805 München, Germany
and Arnold Sommerfeld Center, Ludwig-Maximilians-Universität,
Theresienstraße 37, 80333 München, Germany*

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An exact duality between an axion with arbitrary potential and an antisymmetric form field has been derived some time ago. Using this duality, the axion solution to a strong *CP* problem has been formulated as a gauge invariant theory of forms. In this description, the QCD axion is represented by a Kalb-Ramond field that is eaten up by the Chern-Simons 3-form of QCD, thereby making it massive. This ensures the *CP* invariance of the vacuum. Although viewed as an effective low energy theory, this formulation accomplishes the same goal as an ordinary Peccei-Quinn mechanism, due to its gauge invariance, it is protected against unwanted UV corrections. In a previous work it has been shown that dual formulation is insensitive to UV physics in the sense that the corrections to *CP*-conserving vacuum from arbitrary massive sources are strictly zero. By going carefully through duality transformations and source resolution, we reproduce this curious result and give some further consistency checks. We apply a similar analysis to other approaches to naturalness problems based on form fields and axions, such as a cosmological relaxation of the standard model Higgs boson mass via the attractor mechanism.

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I. INTRODUCTION

The dualities involving form fields play an important role in descriptions of physical systems. In [1] it was shown that there is an exact duality between a massive pseudoscalar a (axion) with arbitrary potential $V(a)$ and a gauge-invariant theory of coupled antisymmetric 2- and 3-forms. Using this connection, in the above work, the strong-*CP* puzzle and its axion solution were reformulated as the gauge theory of forms. The 3-form gauge field C is an organic part of QCD in form of the Chern-Simons 3-form. In the absence of axions (or chiral massless quarks[1]), the 3-form is massless. Such a field has no propagating degrees of freedom. However, its "electric" field strength, $F \equiv dC$, can assume an arbitrary constant value in the vacuum. In the absence of axion, this electric field ${}^*E = F$ cannot be changed neither classically nor quantum mechanically. The different values of E form different superselection sectors. Vacua with $E \neq 0$ are *CP* nonconserving. Therefore, the vacua with different values of E represent an alternative description of familiar θ vacua [2] of QCD. In this description, E plays the same role in parametrizing *CP* breaking as the vacuum angle θ plays in the conventional picture.

In the formulation of [1], the QCD axion is introduced in the theory in the form of a Kalb-Ramond antisymmetric 2-form B. The ordinary Peccei-Quinn (PQ) global symmetry under the shift of axion is replaced by the gauge symmetry of B and C forms. The 2-form axion B becomes a longitudinal (Stückelberg) component of the 3-form C, forming a massive 3-form field. The massive 3-form gauge field can no longer sustain a nonzero constant electric field E in the vacuum, which becomes screened. Thus, in the presence of a B axion, we end up with an unique *CP*-conserving vacuum $E = 0$. In ordinary language, this is equivalent to relaxing $\theta = 0$.

At the level of low energy effective theory, the dual formulation of [1] accomplishes exactly the same goal as the conventional axion [3] of a Peccei-Quinn scenario [4]. However, the sensitivities of the two formulations with respect to UV physics can be drastically different. In the language of original Peccei-Quinn formulation [4] the UV corrections are essentially uncontrollable, due to the fact that an explicit breaking of global Peccei-Quinn symmetry is not forbidden by any fundamental principle. For example, one can argue that in gravity such a breaking is triggered by high-dimensional operators suppressed by the Planck scale [5].

In contrast, the gauge symmetry of the dual formulation must be respected by arbitrary UV completion of the theory [1]. This puts the potentially dangerous corrections

[1]In a chiral limit of a massless quark, QCD contains a built-in axion in form of a η' meson [1].

under much stricter control as compared to ordinary formulation.

As shown in [1], due to its gauge invariance, the dual formulation is fully insensitive to UV physics. Namely, it was proven that, provided new physics contains no long-range 3-form correlators (i.e., no massless 3-form fields), the CP invariance of the vacuum ($E = 0$), is not affected at all. That is, CP-violating corrections to the vacuum, coming from arbitrary massive modes, are exactly zero. The proof is based on effective field theory methods relying on gauge invariance and the analysis of the pole structure of the propagators brought by new physics.

From the first look, the above sounds like a rather strong statement. However, the formulation in terms of 3-form gauge theory makes the story sufficiently transparent. In the present paper we shall reproduce this result by explicitly resolving heavy sources and shall further elaborate on it.

As discussed in [1], the insensitivity of gauge formulation to massive physics gives a useful tool for parametrizing and avoiding unwanted corrections to an axion solution of strong CP from the sources such as gravity. Namely, the UV-insensitivity criterion tells us that, in dual formulation, the unwanted corrections can only appear if the theory contains additional massless 3-forms that can also mix with the QCD axion. If such massless forms exist, they can “disrupt” an axion from fully screening the QCD Chern-Simons 3-form field. For example, in gravity such a potential danger to an axion can come uniquely from a gravitational Chern-Simons 3-form, provided the latter can give a long-range correlator.

As explained in [1], even if gravity contains such unwanted corrections, the dual gauge-invariant formulation offers a way out. A disruption from gravitational Chern-Simons can be easily avoided if the theory contains an additional chiral symmetry that is anomalous with respect to gravity. In fact, the role of a “protector” for an axion can be played by a chiral symmetry of a light fermion, such as a neutrino [1,6].[2]

Another application of 3-form/axion system is the Dvali-Vilenkin “attractor” mechanism [8,9]. Originally this mechanism was used for the cosmological relaxation of the mass of the standard model Higgs boson. This scenario is motivated by the hierarchy problem, an inexplicable smallness of the Higgs mass relative to Planck scale, an ultimate cutoff of the theory. The conventional approaches, such as low energy supersymmetry, or low scale quantum gravity [10,11], predict the existence of new physics not far from the weak scale. The mechanism of [8,9] intended to offer a potential way to relax the Higgs mass (and the vacuum expectation value (VEV)) to acceptably low values, without the need of low energy stabilizing physics. The idea is to use a 3-form field as a control parameter for the Higgs mass. The vacua with different values of the Higgs mass are then actualized due to the change of the electric field E by its sources. Their role can be played by branes or by solitons (domain walls) of heavy axionlike fields. The key ingredient of the mechanism is that the step of the variation of E diminishes towards a certain critical value E_*. This results into a divergent density of vacuum states with values of E arbitrarily close to E_*. Due to this, the vacuum $E = E_*$ acts as an attractor point of the cosmological evolution.

The 3-form attractor mechanism has also been applied to the strong-CP problem [12]. In this setup, the role of C is played by the QCD Chern-Simons and the attractor point is at the vanishing electric field $E_* = 0$, corresponding to CP-conserving vacuum. The idea is that the Universe can be driven to it by cosmic evolution without the need of the axion field.

In order to represent a legitimate solution of the naturalness problems, it is important for the attractor mechanism not to be UV sensitive. In particular, the attractor point should not be sensitive to the details of short distance physics such as the inner structure of the brane. For this, it is important to understand under what circumstances the brane structure becomes important. In the present paper we shall also address such issues.

The paper is organized as follows. After carefully reviewing the duality between 3-forms and axions and a dual formulation of an axion given in [1], we study resolutions of the heavy sources. We observe that the screening of E (equivalently θ) by an axion in QCD is not sensitive to the presence of the heavy sources and/or massive states. We thus confirm the findings of [1].

We then turn to the situations when no light axion is present in low energy theory and C is sourced exclusively by massive branes. In such a case C remains massless and can produce a long-range electric field E. This situation does not correspond to the case of QCD axion, but is relevant for the attractor setups [8,9,12]. We compute the back reaction from the electric field E to the brane in the leading order and establish the timescale after which the inner structure of the brane is affected. Applying this to the attractor mechanism, we shall find out that the physics near the attractor point is insensitive to this resolution. As another sort of back reaction, the motion of branes leads to the particle creation. In case of [8,9] the radiated quanta are Higgs bosons. This is due to the change of the Higgs boson mass triggered by the electric field E. Again, the effect vanishes near the attractor point but can be significant when the system is far from it.

II. DUALITY

The dualities between the form fields have long history. In particular, it has been known for a long time that a theory of a free massless 2-form Kalb-Ramond field $B_{\mu\nu}$, described by the Lagrangian

$$\mathcal{L} = \frac{1}{12}(dB)^2, \tag{1}$$

[2]Possible phenomenological implications of this effect for neutrino masses were explored in [7].

is dual to a free massless pseudoscalar a,

$$\mathcal{L} = \frac{1}{2}(\partial a)^2, \quad (2)$$

where d denotes an exterior derivative. The proof of duality at the level of free theory is straightforward. However, for some time, the dualities at the level of massive interacting theories were a source of controversy. It has even been suggested [13] that duality ceases to exist at the level of an interacting theory and that interaction and mass terms break duality.

This issue was clarified in [1] where it was proven that duality continues to hold for a massive and interacting axion field with arbitrary scalar potential $V(a)$. The caveat is that the nonderivative interaction terms of the axion dualize to a nontrivial kinetic function of a massive 3-form field $C_{\mu\nu\alpha}$ in which the Kalb-Ramond $B_{\mu\nu}$ enters as the longitudinal (Stückelberg) degree of freedom. This duality has smooth limits both for the zero coupling and the zero mass. The number of degrees of freedom remains intact in both limits. We shall start by reviewing this duality closely following [1].

Let us consider a theory of a massive pseudoscalar axion with arbitrary potential

$$\mathcal{L} = \frac{1}{2}(\partial a)^2 - V(a). \quad (3)$$

We wish to show that this theory is dual to the following theory of an interacting massive 3-form C with the field strength $F_{\mu\alpha\beta\gamma} = \partial_{[\mu}C_{\alpha\beta\gamma]} = \epsilon_{\mu\alpha\beta\gamma}E$,

$$\mathcal{L} = \Lambda^4\mathcal{K}\left(\frac{E}{\Lambda^2}\right) + \frac{1}{2}m^2C^2, \quad (4)$$

where Λ and m are parameters of mass dimensionality. The quantity E shall be referred to as the "electric" field. The $\mathcal{K}(\frac{E}{\Lambda^2})$ is a nonderivative function of its argument E, which satisfies

$$V(a) = \frac{1}{\sqrt{6}}m\Lambda^2\int da\,\mathbf{inv}\,\mathcal{K}'\left(\frac{ma}{\sqrt{6}\Lambda^2}\right), \quad (5)$$

where prime denotes a derivative with respect to the argument and **inv** stands for an inverse function. For a simplest choice $\mathcal{K}(x) = \frac{1}{2}x^2$, we get quadratic and canonically normalized Lagrangian.

In order to prove the above, let us first decompose the massive field C in its transverse and longitudinal modes,

$$C = C^T - dB. \quad (6)$$

The longitudinal mode, which is the only propagating mode of the massive 3-form field, is a Kalb-Ramond field. It also serves as the Stückelberg field for explicitly maintaining the following gauge redundancy,

$$C^T \to C^T + d\Omega, \qquad B \to B + \Omega. \quad (7)$$

Here Ω is a gauge shift parameter, which represents an arbitrary 2-form function of spacetime coordinates.

Notice that (6) is just a decomposition and does not amount to any modification of the theory. The original theory (4), as well as its decomposed version,

$$\mathcal{L} = \Lambda^4\mathcal{K}\left(\frac{E}{\Lambda^2}\right) + \frac{1}{2}m^2(C^T - dB)^2, \quad (8)$$

are physically equivalent for arbitrary values of the mass, including the limit $m \to 0$. This is obvious from the fact that they give identical physical observables, such as the interactions mediated between arbitrary external sources.

Following [1], we now perform dualization. As the first step, we treat $dB \equiv X$ as a fundamental 3-form, simultaneously imposing the Bianchi identity ($dX = 0$) as a constraint through a Lagrange multiplier a,

$$\mathcal{L} = \Lambda^4\mathcal{K}\left(\frac{E}{\Lambda^2}\right) + \frac{1}{2}m^2(C^T - X)^2 + \frac{1}{\sqrt{6}}ma\epsilon_{\mu\alpha\beta\gamma}\partial_\mu X_{\alpha\beta\gamma}. \quad (9)$$

Integrating out X through the equations of motion ($X_{\alpha\beta\gamma} = C^T_{\alpha\beta\gamma} - \frac{1}{\sqrt{6}m}\partial_\mu a\epsilon_{\mu\alpha\beta\gamma}$) and using the definition of the electric field (4), we get the following effective theory:

$$\mathcal{L} = \Lambda^4\mathcal{K}\left(\frac{E}{\Lambda^2}\right) + \frac{1}{2}(\partial a)^2 - \frac{1}{\sqrt{6}}maE. \quad (10)$$

Notice that the gauge symmetry (7) of the Kalb-Ramond field is replaced by the global shift symmetry of the axion field by an arbitrary constant,

$$a \to a + \text{const}. \quad (11)$$

The equation of motion for C gives

$$\partial_\mu\left(\mathcal{K}'\left(\frac{E}{\Lambda^2}\right) - \frac{m}{\sqrt{6}\Lambda^2}a\right) = 0, \quad (12)$$

whereas the one of axion is

$$\Box a = \frac{1}{\sqrt{6}}mE. \quad (13)$$

Solving for E as a function of a from (12) and taking into account (13) makes it obvious that

$$E(a) = \frac{\sqrt{6}}{m}\frac{dV(a)}{da}. \quad (14)$$

This is the source of the relation (5). Thus, integrating out the E field, we finally arrive to the theory (3) with the potential $V(a)$ determined by the function $\mathcal{K}'$ through the relation (5). Thus, we reproduce the result of [1] that theory of an axion field a (3) with an arbitrary potential has an exact dual in the form of a gauge theory (8) of C^T and B, where the axion potential is translated into the kinetic function $\mathcal{K}$ of the 3-form through the relation (5).

Next, using the above duality, in [1] the solution of the strong-CP problem was formulated as the Higgs-like phase of the gauge theory of forms. This formulation contains no reference to global chiral symmetry and only relies on gauge redundancy. This dual formulation is described by the Lagrangian (8), where C^T represents a Chern-Simons 3-form of QCD and B is a Kalb-Ramond dual of the axion a. The theory (8) eliminates the strong-CP violation in a gauge redundant formulation. In this formulation, the global Peccei-Quinn symmetry, which acts on an axion (11) is replaced by the gauge symmetry (7). Viewed as low energy theories, the two formulations accomplish exact same goal but exhibit different sensitivities towards UV physics. In particular, in formulation (8) the vacuum of the theory exhibits zero UV sensitivity.

An example of the $\mathcal{K}(x)$ function, $\mathcal{K}(x) \propto (x \arcsin x + \sqrt{1-x^2})$, provided in [1], gives a simplest prototype potential $V(a) \propto \cos(\frac{\mathrm{a}}{\mathrm{f_a}})$ (here f_a is an axion decay constant) commonly used in axion literature. Conventionally, this potential is obtained by instanton calculation in dilute gas approximation [14]. The 3-form language shows that the vanishing of CP violation is insensitive to a precise form of $V(a)$.

In QCD, the function $\mathcal{K}$ is not known exactly, but the important thing is that none of the key aspects of the axion physics, such as the generation of the axion mass gap and elimination of θ, are sensitive to its exact form. In particular, from (14) it is clear that $E = 0$ at every extremum of the axion potential, regardless the form of the function $\mathcal{K}$. This constitutes an alternative support of Vafa-Witten theorem [15] and its generalization to all extrema of $V(a)$.

Thus, the above duality allows us to understand the generation of axion mass in QCD as the 3-form Higgs effect. Notice that this description is exact [1], as the entire information about the axion mass and its potential is contained in a nonderivative function $\mathcal{K}$. None of the high derivative contributions affect either the mass or the potential, since such contributions vanish in a zero momentum limit. Thus, as long as no additional massless 3-form fields are added to QCD, the generation of the axion mass with simultaneous elimination of the θ vacua, can be understood in the language of the Lagrangian (8) [equivalently (4)] or its dual (3).

This language also makes it transparent why the existence of an additional potential $\tilde{V}(a)$ (not generated via a QCD 3-form) un-Higgses the 3-form and regenerates θ vacua. Indeed, in a theory with an additional potential

$$\mathcal{L} = \Lambda^4 \mathcal{K}\left(\frac{E}{\Lambda^2}\right) + \frac{1}{2}(\partial a)^2 - \frac{1}{\sqrt{6}} maE - \tilde{V}(a), \qquad (15)$$

the 3-form remains massless. The way to understand this in the 3-form Higgs language is to notice that the additional potential can be traded for an additional 3-form [1]. It then becomes clear that only one superposition of the two 3-forms can become massive by eating up a single scalar field. This is the reason why any external explicit breaking of the axion shift symmetry (PQ symmetry) in QCD brings back the observable CP violation.

III. UV IN SENSITIVITY OF A DUAL AXION MECHANISM

We now wish to discuss the point of [1] that the axion solution, when formulated as 3-form gauge theory (8), is insensitive towards UV physics. That is, the generation of the mass gap cannot be affected by the presence of the heavy fields at some mass scale M_f. In particular, coupling the QCD 3-form to arbitrary massive branes cannot regenerate a nonzero CP violation. A curious thing about this statement is that it is meant to be exact.

Naïvely, one would think that the introduction of massive states at some high scale M_f can affect the low energy physical observables by a small but nonzero amount. The corrections, while possibly exponentially small or suppressed by powers of the scale M_f, are usually nonzero. In the case of the gauge formulation (4) of the QCD axion, this would imply that coupling the QCD Chern-Simons term to heavy banes would perturb the observable θ parameter by a small amount and trigger the CP violation.

However, this is not what is happening. The CP-violating order parameter E remains exactly zero, as long as new physics does not come with new massless poles in 3-form correlators. The proof, which can be found in [1], is based on integration out of the heavy physics and writing down the effective propagator for the 3-form C. We shall not repeat it here. However, we shall do a cross-check in the example below.

We can test the above claim by using the explicit resolution of the brane, analogous to the one in [9]. For this, let us assume the C form in the Lagrangian (10) to stand for a QCD Chern-Simons field. Let us now add its couplings to a heavy brane coming from some UV physics. This heavy brane can be resolved in the form of a soliton of a heavy axion b with the effective potential $V(b)$. The theory now becomes

$$\mathcal{L} = -\frac{1}{48}F^2 - \frac{1}{2\pi}\partial_\mu(qa + q_b b)\epsilon_{\mu\alpha\beta\gamma}C_{\alpha\beta\gamma} + \frac{1}{2}f_a^2(\partial a)^2 + \frac{1}{2}f_b^2(\partial b)^2 - V_b(b). \qquad (16)$$

Here f_a and f_b are decay constants of a and b axions that are not canonically normalized. $\mathcal{K}(x) = \frac{1}{2}x^2$ and we also parametrize the mixing between 3-forms and axions with constants q and q_b with mass dimension of 2. The connection between the parameters of (16) and (10) is straightforward.

Solving the equations of motion shows that, despite the presence of the heavy field b, in the vacuum state the electric field E of C remains exactly zero. This is immediately apparent from the equation,

$$f_a^2 \Box a = \frac{q}{2\pi} E, \tag{17}$$

which shows that no vacuum can be reached for any nonzero value of E. This zero sensitivity of the $E = 0$ vacuum towards heavy physics (in this case the heavy axion b) is again very transparent in the Language of 3-form Higgs effect. Using the exact duality, we can rewrite the potential for axion b as a Higgs effect with respect to a second 3-form C^b. The Lagrangian then becomes

$$\mathcal{L} = -\frac{1}{48}F^2 - \frac{1}{2\pi}\partial_\mu(qa + q_b b)\epsilon_{\mu\alpha\beta\gamma}C_{\alpha\beta\gamma} + \frac{1}{2}f_a^2(\partial a)^2 + \frac{1}{2}f_b^2(\partial b)^2 - \frac{q_{bb}}{2\pi}\partial_\mu b \epsilon_{\mu\alpha\beta\gamma}C^b_{\alpha\beta\gamma} + \Lambda^4 \mathcal{K}_b\left(\frac{E^b}{\Lambda^2}\right), \tag{18}$$

where $\mathcal{K}_b$ satisfies the same condition (5) with respect to $V_b(b)$. This theory describes a Higgs effect with two gauge 3-forms (C and C_b) and two axions (a and b). As a result, no massless degree of freedom is remaining in the theory and there exist no electric fields in the vacuum with respect to any 3-form. Thus, all CP-violating order parameters are exactly zero.

The situation would change drastically if the new physics, coupled to an ordinary axion, would come in form of a 3-form without its own axion partner,

$$\mathcal{L} = -\frac{1}{48}F^2 - \frac{1}{2\pi}\partial_\mu a \epsilon_{\mu\alpha\beta\gamma}(qC_{\alpha\beta\gamma} + q_b C^b_{\alpha\beta\gamma}) - \frac{1}{48}F_b^2 + \frac{1}{2}f_a^2(\partial a)^2. \tag{19}$$

This is an example considered in [1] as violating the criterion of absence of massless 3-forms. In such a case, the new physics will be represented solely by C_b, which has no axion partner. Correspondingly, there is a massless pole that new physics brings in. Integrating out C_b and a, we recover (up to notations) Eq. (35) of [1],

$$\mathcal{L} = -\frac{1}{48}F^2 - \frac{1}{48}\frac{q^2}{q_b^2}F_{\mu\alpha\beta\gamma}\frac{m^2}{\Box + m^2}F_{\mu\alpha\beta\gamma}, \tag{20}$$

where $m^2 = \frac{q_b^2}{4\pi^2 f_a^2}$. Since in zero momentum limit ($\Box \to 0$) the only effect of the second term is that it corrects kinetic term normalization, the theory contains a long-range correlator in the form of a massless 3-form field. Correspondingly, there exists a vacuum solution with a constant E, which violates CP symmetry.

At the same time, if we perform analogous integration in (18), assuming $\mathcal{K}_b(x) = \frac{1}{2}x^2$, then the effective theory for C will be

$$\mathcal{L} = \frac{1}{2}C_{\alpha\beta\gamma}\left(\Box + m_a^2 + \left(\frac{q_b}{q_{bb}}\right)^2 \frac{m_b^2 \Box}{\Box + m_b^2}\right)\Pi_{\alpha\mu}C_{\mu\beta\gamma}, \tag{21}$$

where $\Pi_{\alpha\mu} = \eta_{\alpha\mu} - \frac{\partial_\alpha \partial_\mu}{\Box}$ is a transverse projector, $m_a^2 = \frac{q^2}{4\pi^2 f_a^2}$, and $m_b^2 = \frac{q_{bb}^2}{4\pi^2 f_b^2}$. This theory describes a theory with mass gap in which the longitudinal mode has been integrated out. There is no constant electric field in the vacuum and correspondingly no CP violation. This confirms the statement of [1] about the insensitivity of the dual formulation of axion with respect to heavy physics.

IV. MORE ON RESOLVING SOURCES

The explicit resolution of sources in the form of a concrete UV physics gives the possibility for accounting for backreaction. This is important for understanding UV sensitivity of mechanisms that involve brane sources of 3-forms. The implications for the attractor mechanism will be discussed in Sec. V.

Our next steps are the following. First we consider a 3-form sourced by a brane in the zero width approximation. We then resolve the brane in the form of a soliton as this was done in [9]. As shown there, we obtain that the topological charge of the soliton in low energy theory acts as an electric (Noether) charge for the 3-form field. We then take into account the backreaction that branes experience from the 3-form dynamics and we ask how sensitive the resolution is to this backreaction.

For resolving the brane as a soliton of heavy axionlike field a, we choose the Lagrangian in the following form [9]:

$$\mathcal{L} = -\frac{1}{48}F^2 - \frac{q}{2\pi}\partial_\mu a \epsilon_{\mu\alpha\beta\gamma}C_{\alpha\beta\gamma} + \frac{1}{2}f_a^2(\partial a)^2 - V(a), \tag{22}$$

where the field a is a noncanonically normalized axion, with decay constant f_a and a periodic potential $V(a)$, which we choose in the form

$$V(a) = V_0[1 - \cos(a)], \tag{23}$$

where V_0 is constant with the dimension of the energy density. The parameter q is a coupling between the 3-form and the axion. The connection between the parameters of the above and (15) is straightforward. This is a typical sine-Gordon potential, which we choose as a simple prototype, but our conclusion holds for an arbitrary periodic potential.

Equation of motions for C field will give

$$-\partial_\mu F_{\mu\alpha\beta\gamma} = J_{\alpha\beta\gamma}, \tag{24}$$

where J is an axionic current

$$J_{\alpha\beta\gamma} = -\frac{q}{2\pi}\epsilon_{\mu\alpha\beta\gamma}\partial_\mu a. \tag{25}$$

This current is trivially conserved due to its topological nature.

For the full treatment we must solve the coupled system of equations of motion. We shall however proceed iteratively by first ignoring the backreaction from F to a. The field a will be replaced by the sine-Gordon soliton,

$$a(z) = 4\tan^{-1} e^{\frac{\sqrt{V_0}(z-z_0)}{f_a}} + 2\pi N, \tag{26}$$

which depends on a single coordinate z. This soliton describes a topologically nontrivial configuration in which the axion field a interpolates between the two nearest neighboring minima of the periodic potential,

$$\begin{aligned} a &= 2\pi N, \qquad z = -\infty, \\ a &= 2\pi(N+1), \qquad z = \infty, \end{aligned} \tag{27}$$

where N is an integer. The parameter z_0 is arbitrary and represents a collective coordinate of the soliton. The configuration (26) solves the equation motion for a in the limit $q = 0$, when the backreaction from the 3-form can be ignored.

Substituting the solution (26) into the topological current we get an external source for C. This approximation amounts to a limit in which we take $\frac{\sqrt{2V_0}}{f_a} \to \infty$ and keeping q small. In this approximation, the soliton field acts as an external source for the 3-form, while experiencing no backreaction from it. In fact, in this limit, we have

$$a'(z) = 2\pi\delta(z - z_0), \tag{28}$$

where $'$ denotes a derivative with respect to z and the sine-Gordon soliton becomes effectively a delta-function source.

The equation of motion (24) and are gauge redundant and require gauge fixing. We fix the Coulomb gauge. Then using the substitution of $C_{\alpha\beta\gamma} = \epsilon_{\mu\alpha\beta\gamma}C_\mu$ and taking into account (28), the above equation reduces to

$$\epsilon_{z\alpha\beta\gamma}C_z'' = J_{\alpha\beta\gamma} = -q\delta(z - z_0)\epsilon_{z\alpha\beta\gamma}. \tag{29}$$

This equation is identical to an equation of an electrostatic field produced by a static point charge in one space dimension with coordinate z.

The solution for C_z is

$$C_z = -\frac{1}{2}q|z - z_0| - 4E_0(z - z_0) + c_2, \tag{30}$$

where E_0 and c_2 are integration constants. The electric field corresponding to this solution has the form

$$E = \frac{1}{8}q\, sign(z - z_0) + E_0. \tag{31}$$

Thus, the topological charge of the soliton effectively acts as an electric Noether charge for the 3-form gauge field.

A. Backreaction

We now wish to take into account the leading back reaction in q on the soliton from the electric field. In particular, the backreaction is expected to take place because of the difference between the electric fields on the two sides of the kink. This creates an energy difference which shall accelerate the kink. In other words, the kink carries a charge under the 3-form electric field and is pushed towards infinity.

This effect can be taken into account by considering an effective theory for a time-dependent fluctuation of the collective coordinate,

$$z_0 \to z_0 + \delta z_0(t). \tag{32}$$

Inserting this ansatz back into the Lagrangian together with the 3-form solution, we get an effective Lagrangian for the fluctuation

$$\mathcal{L} = -\frac{1}{48}F^2 - \frac{q}{2\pi}aE + \frac{1}{2}f_a^2 a'^2 \delta\dot{z_0}^2 - \frac{1}{2}f_a^2 a'^2 - V(a). \tag{33}$$

In the above Lagrangian, first term is constant and the last two terms give a total derivative, since the fluctuation of Kink does not change its topological properties. We are therefore left with a simpler Lagrangian density,

$$L = \int dz\mathcal{L} = \int dz\left(\frac{1}{2}f_a^2 a'^2 \delta\dot{z_0}^2 - \frac{q}{8\pi}a'C_z\right). \tag{34}$$

Next we integrate over the z coordinate. In order to resolve the square of the delta function in the first term, we explicitly take into account the profile (26) of the sine-Gordon soliton and integrate over it. In the second term, we can perform the integration within delta function approximation (28). Choosing proper normalization we get the following Lagrangian,

$$L = \frac{1}{2}M\delta\dot{z_0}^2 + \frac{1}{f_a^2}\left(\frac{1}{8}q^2|\delta z_0| + qE_0\delta z_0\right), \tag{35}$$

this system can be understood as a point charge with mass $M = 8\frac{\sqrt{V_0}}{f_a}$ and coordinate δz_0 in an one-dimensional external electric field [16]. For $\delta z_0 > 0$ the above system has solutions in which the soliton (domain wall) is moving accelerated

$$\delta z_0(t) = \frac{1}{2Mf_a^2}\left(\frac{1}{8}q^2 + qE_0\right)t^2 + vt + \delta z_0(0), \tag{36}$$

where v is the initial velocity. Without a loss of generality, let us assume that the initial coordinate and initial velocity are zero.

Let us try to identify the validity timescale of the above approximation. This is the timescale after which the effective theory in which we treat a soliton without changing the internal structure breaks down.

Most conservatively, this can be estimated by demanding that the variation of the collective coordinate should not exceed the width of the soliton. This gives a condition

$$\frac{f_a}{\sqrt{V_0}} \gtrsim |\delta z_0|, \qquad (37)$$

which, when applied to the solution (36), implies

$$\frac{f_a}{\sqrt{V_0}} \gtrsim \frac{1}{16\sqrt{V_0} f_a} \left| \frac{1}{8} q^2 + qE_0 \right| t^2. \qquad (38)$$

This leads us to the following upper bound on the validity timescale,

$$t \lesssim \frac{4 f_a}{|q| \sqrt{|\frac{1}{8} + \frac{E_0}{q}|}} \equiv t_*. \qquad (39)$$

After this time, the internal structure of the soliton is affected by the action of the electric field. Of course, for a long distance observer, the description of the soliton as a point particle accelerated by a constant electric field is still valid. The full relativistic solution for such a particle is well known [16].

V. IMPLICATIONS FOR COSMIC ATTRACTOR

We shall now apply the above results to the attractor mechanism [8,9]. The idea of the attractor is that the charge of the brane q with respect to a massless 3-form C, through a certain chain of influences, effectively depends on the electric field $q(E)$. An important point is that, for a certain critical value E_*, it vanishes. That is $q(E_*) = 0$. Since the change of the field E among the neighboring vacua is given by the charge

$$\Delta E \propto q(E), \qquad (40)$$

and the step vanishes when we approach E_*. That is, the number of distinct vacua required to traverse through for reaching E_* is infinite. In other words, there exist infinitely many vacua with values of the electric field that are arbitrarily close to the attractor value E_*.

One of the requirements to the attractor mechanism is that it should not be sensitive to UV physics. That is, the singularity in number of vacua at the attractor point E_* should be trusted without knowing the internal structure of the brane. We can now explicitly verify this by evaluating the scaling of the critical time t_* near the attractor point. Since at the attractor point the electric field is finite E_*, whereas the brane change $q(E_*)$ vanishes, it is clear from (39) that $t_* \to \infty$. That is, the closer we are to the attractor vacuum, the longer it takes for the thin-brain approximation to break down.

A. Effect of particle creation

An important ingredient of the attractor solution to the hierarchy problem [8,9] is that the value of the Higgs mass and the VEV is determined by the 3-form electric field. In this way, the Higgs mass changes from vacuum to vacuum.

Thus, in the brane background, the Higgs mass is effectively dependent on the space-time coordinates. The part of the Lagrangian in which this information is encoded has the following form:

$$\mathcal{L} = \frac{1}{2} (\partial h)^2 - \frac{1}{2} h^2 \left(m_h^2 + \frac{F^2}{48 M_f^2} \right), \qquad (41)$$

where h is a Higgs field, m_h is its “bare” mass and M_f, is some fundamental scale, which will be assumed to be large.

Taking in the account solution for electric field (31), we get effectively a mass term for Higgs which depends on a position on the z axis. Basically, since the electric field changes across the brane, so does the Higgs mass. Correspondingly, we have two different masses M_- and M_+,

$$M_\pm^2 = m_h^2 - \frac{1}{128} \frac{q^2}{M_f^2} - \frac{E_0^2}{2M_f^2} \pm \frac{1}{8} \frac{E_0 q}{M_f^2}, \qquad (42)$$

on the left and right sides of the brane, respectively.

Without loss of generality, we assume that the constant part of the electric field is positive. For the opposite case we could just change the labeling. Let us assume that, for some initial time $t = 0$, on both sides of the wall the Higgs field is in corresponding vacuum states. That is, no Higgs particles are excited.

Now, since the wall is moving accelerated (36), the electric field changes in time and correspondingly changes the vacuum of the Higgs field. This leads to a particle creation.

In order to compute the rate of particle creation, let us start with a region in the vacuum that corresponds to M_+. After the brane passes by this region, the mass decreases to M_-.

In this case the number operator of M_--mass particles is defined in the following way

$$N = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2\omega_-} a_-^\pm(\vec{p}) a_-(\vec{p}), \qquad (43)$$

where $\omega_\pm^2 = p^2 + M_\pm^2$, and the subscript on the ladder operators has the same meaning. Performing a Bogoliubov transformation (A6) and averaging in the vacuum $|0_+\rangle$ (which is the vacuum in this case), we get following expression:

$$N = SL\frac{1}{4}\int d^3p\frac{\omega_+}{\omega_-}\left(\frac{\omega_-}{\omega_+}-1\right)^2, \quad (44)$$

where S is the surface area of the brane and L is the length traveled by it in the z direction. Taking into account that

$$\omega_+^2 - \omega_-^2 = \frac{1}{16}\frac{E_0 q}{M_f^2}, \quad (45)$$

for $M_f^2 \gg \frac{E_0 q}{M_-^2}$, we get

$$\omega_+ \approx \omega_- + \frac{1}{32}\frac{E_0 q}{M_f^2 \omega_-}. \quad (46)$$

At the lowest order, the produced particle number per unit surface is given by

$$\frac{N}{S} = L\frac{1}{4096}\frac{E_0^2 q^2}{M_f^4}\int d^3p\frac{1}{\omega_-^4}. \quad (47)$$

After integration we obtain

$$\frac{N}{S} = L\frac{\pi^2}{4096}\frac{E_0^2 q^2}{M_f^4\, M_-}. \quad (48)$$

Substituting the distance traveled by the domain wall in time t, we get the final result

$$\frac{N}{S} = \left(\frac{1}{16\sqrt{V_0}f_a}\left(\frac{1}{8}q^2 + qE_0\right)t^2 + vt\right)\frac{\pi^2}{4096}\frac{E_0^2 q^2}{M_f^4\, M_-}. \quad (49)$$

The above expression gives us the amount of Higgs particles created by a passing-by domain wall.

However, we should remember that we can trust the above analysis until the time (39). During this time, we get the following amount of particles created per unit surface

$$\frac{N}{S} = \frac{f_a}{\sqrt{V_0}}\frac{\pi^2}{4096}\frac{E_0^2 q^2}{M_f^4\, M_-}. \quad (50)$$

Far from the attractor, where q is large, the effect of particle creation can be significant and affect the dynamics of the system. However, not surprisingly, since the effect is proportional to q^2, it is vanishingly small near the attractor point. Again, the attractor behavior is largely unaffected by the high energy effects.

VI. CONCLUSIONS

The dual formulation of the axion solution of the strong-*CP* problem in the form of a 3-form gauge theory [1], by the power of gauge invariance, gives a possibility of controlling the unwanted UV corrections. In particular, it has been shown that corrections from arbitrary massive physics to the *CP* invariance of the vacuum are exactly zero. In the present paper we reproduced this result and performed consistency checks on some explicit examples of heavy physics. In accordance with the proof given in [1], we find that the only possibility of destabilizing the *CP*-invariant vacuum is through the appearance of additional massless 3-form fields coupled to axions. In their absence, the QCD vacuum remains exactly *CP* conserving.[3]

We also gave a resolution of brane sources coupled to massless 3-form fields and estimated a back reaction from the three electric field to leading order. Such a setup has been applied to the solutions of the hierarchy [8,9] and strong-*CP* [12] problems via the attractor mechanism. We observe that the existence of the attractor point is not sensitive to the brane resolution or to backreaction. We also estimated particle creation in the background of time dependent 3-form electric field.

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APPENDIX: BOGOLYUBOV TRANSFORMATIONS

In the above section, we have a scalar field the mass of which effectively depends on time. This means that this field does not have a fixed vacuum. Using Bogolyubov transformations, we can properly account for the change of the vacuum. Let us consider a free scalar field ϕ, in the Schrödinger picture

$$\phi(\vec{x}) = \int\frac{d^3p}{(2\pi)^3}\frac{1}{2\omega(p,t)}(a_t(\vec{p})e^{i\vec{p}\vec{x}} + a_t^+(-\vec{p})e^{i\vec{p}\vec{x}}), \quad (A1)$$

where $\omega(\vec{p},t)$ is the time dependent frequency. Therefore, ladder operators depend on time as well (despite being described in the Schrödinger picture). The corresponding canonical momenta have the following form

[3] We note that, perhaps, an alternative physical way of understanding this UV insensitivity is that the generation of an axion mass from a 3-form Higgs effect can be understood in purely topological terms, as discussed in [17].

$$\pi(\vec{x}) = \int \frac{d^3p}{(2\pi)^3} \frac{-i}{2} (a_t(\vec{p}) e^{i\vec{p}\,\vec{x}} - a_t^+(-\vec{p}) e^{i\vec{p}\,\vec{x}}). \quad \text{(A2)}$$

The important point here is that the field and the canonical momenta do not depend on time explicitly. Thus, for any given time we can write

$$\frac{1}{2\omega(p,t)}(a(\vec{p})_t + a^+(-\vec{p})_t) = \int d^3x e^{-i\vec{p}\,\vec{x}} \phi(\vec{x}),$$
$$\frac{-i}{2}(a(\vec{p})_t - a^+(-\vec{p})_t) = \int d^3x e^{-i\vec{p}\,\vec{x}} \pi(\vec{x}). \quad \text{(A3)}$$

From this equation immediately follows the following transformation:

$$a(\vec{p}) = \frac{1}{2}\left(\frac{\omega}{\omega_0}+1\right) a_0(\vec{p}) + \frac{1}{2}\left(\frac{\omega}{\omega_0}-1\right) a_0^+(-\vec{p}),$$
$$a(-\vec{p})^+ = \frac{1}{2}\left(\frac{\omega}{\omega_0}+1\right) a_0(-\vec{p})^+ + \frac{1}{2}\left(\frac{\omega}{\omega_0}-1\right) a_0(\vec{p}), \quad \text{(A4)}$$

where subscript 0 means that the quantities are evaluated at $t_0 = 0$. The quantities without a subscript are evaluated at time t and all frequencies are evaluated for momentum p. We can compute different quantities after we get the connection between these operators. One example is the number operator, which at the time t has following form:

$$N = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2\omega} a^+(\vec{p}) a(\vec{p}). \quad \text{(A5)}$$

Rewriting it in the terms of operators in time $t_0 = 0$, we get

$$N = \frac{1}{4}\int \frac{d^3p}{(2\pi)^3} \frac{1}{2\omega} \left[\left(\frac{\omega}{\omega_0}+1\right) a_0(\vec{p})^+ + \left(\frac{\omega}{\omega_0}-1\right) a_0(-\vec{p})\right]$$
$$\times \left[\left(\frac{\omega}{\omega_0}-1\right) a_0(-\vec{p})^+ + \left(\frac{\omega}{\omega_0}+1\right) a_0(\vec{p})\right], \quad \text{(A6)}$$

which shows that, in vacuum, corresponding $t_0 = 0$ after certain time particles will be created.

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