Microlensing constraints on clustered primordial black holes

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The discovery of black-hole-binary mergers through their gravitational wave (GW) emission has reopened the exciting possibility that dark matter is made, at least partly, of primordial black holes (PBHs). However, this scenario is challenged by many observational probes that set bounds on the relative PBH abundance across a broad range of viable PBH masses. Among these bounds, the ones coming from microlensing surveys lead to the strongest constraints in the mass range from $\sim 10^{-10}$ to a few $M_\odot$. The upper part of this range precisely corresponds to the mass window inside which the formation of PBHs should be boosted due to the QCD phase transition in the early Universe, which makes the microlensing probes particularly important. However, it has been argued that taking into account the inevitable clustering of PBH on small scales can significantly relax or entirely remove these bounds. While the impact of PBH clustering on the GW event rate has been studied in detail, its impact on the microlensing event rate has not yet been fully assessed. In this paper, we address this issue, and show that clusters arising from isocurvature perturbations, that originating from PBHs formed from Gaussian initial curvature perturbations, do not alter the current microlensing constraints, as they are not sufficiently compact.

I. INTRODUCTION

The advent of gravitational wave (GW) astronomy [1] has opened a new window on the Universe. In particular, the peculiar mass range inferred from the first observations of binary-black-hole (BBH) mergers [1–3] has been interpreted as a hint for a possible primordial-black-hole (PBH) origin of these events [4,5]. This revived the exciting idea that the still mysterious dark matter (DM), which pervades the Universe and is currently the pillar of the theory of structure formation [6–8], could be made, at least partly, of PBHs [4,9,10].

However, this scenario has to confront several observational constraints that currently bound the PBH abundance along an extended range of masses (for recent reviews, see, e.g., [11,12]). Among them, microlensing surveys provide a powerful probe for constraining the abundance of PBHs with masses between asteroid and Solar mass scales [13,14]. The higher end of this mass range is particularly interesting as it corresponds to PBHs that could have been formed during the QCD phase transition, when a drop of pressure decreases the overdensity threshold above which density fluctuations can collapse directly to black holes (BHs) [15]. For approximately scale-invariant preexisting

density perturbations, one expects thus a peak in the mass function of PBHs around one Solar mass [16], which is within the reach of numerous Galactic, but also extragalactic, microlensing surveys [17–21].

Current constraints limit PBHs to represent $f_{\rm pbh} \lesssim 0.1$ of the total DM abundance in the aforementioned mass range. However, these limits have been derived assuming the PBHs are randomly distributed along a given DM density profile. Throughout the literature it has been suggested that the inevitable clustering of PBHs could significantly relax these bounds [22–24]. In this paper, we wish to explore in detail how such clustering affects the microlensing limits. We shall see that for Gaussian inflationary perturbations they are actually unlikely to change.

Where do these clusters come from? On large scales, the behavior of PBHs is indistinguishable from that of any viable cold dark matter (CDM) particle candidate [25]. However, on small scales there are two important differences compared to the particle hypothesis, i.e., the macroscopic PBH masses and their essentially vanishing initial peculiar velocities. For Gaussian perturbations overdense regions exceeding the threshold for PBH formation are randomly distributed in space [26,27]. This subsequently leads to Poisson fluctuations of PBH numbers in any given volume [28,29]. These fluctuations represent the dominant source of isocurvature perturbations on small scales and inevitably seed the formation of PBH clusters already at very early times [30]. However, due to small

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relaxation times, smaller clusters are subject to later evaporation.

The Poisson fluctuations in PBH numbers and their vanishing initial velocities lead to efficient formation of PBH binaries [30–32]. These binaries are highly eccentric due the vanishing initial PBH peculiar velocities, and one expects a large BBH merger rate if PBHs make a significant fraction of CDM. In fact for $f_{pbh} = 1$, it was thought that the merger rate of PBHs should be orders of magnitude larger than the one observed by LIGO/Virgo [33]. However, these estimates did not take into account the presence of dense clusters, and PBH three-body interactions within, which significantly perturb the orbits and thus coalescence times of binaries. Detailed studies found that the merger rate could indeed be consistent with the observed one [34,35], in case that most of the PBH binaries are actually residing in clusters. As of now, it seems that a $f_{pbh} = 1$ with $M \sim M_\odot$ is marginally ruled out due to the coalescence of isolated and highly eccentric binaries [36].

In this work we carefully examine the impact of PBH clustering on the interpretation of microlensing searches for massive compact halo objects (MACHOs). In our analysis, we pay particular attention to the differences in the probability distribution for the expected number of events, as well as the probability for recurring microlensing events, which were typically discarded from the observational samples. For concreteness, we provide quantitative results for the EROS-like microlensing survey [19] of the Large Magellanic Cloud (LMC). However, the implications of our findings can be easily generalized to other similar Galactic surveys, such as those of MACHO [37] and OGLE [20], or even Subaru/HSC observations of the Andromeda galaxy [38].

II. MICROLENSING SEARCHES FOR MACHOS

Microlensing surveys search for temporary brightening of distant sources (e.g., stars) due to the passage of a MACHO near the line-of-sight (l.o.s.). In essence, this phenomenon is a particular case of strong lensing, where the angular separation between multiple images of the source is smaller than the resolution of the telescope (i.e., of order of microarcseconds, hence the name microlensing). The observed effect is the amplification of the source's brightness by a factor [13,39]:

$$A(u) = \frac{u^2 + 2}{u\sqrt{u^2 + 4}}, \tag{1}$$

where $u = R/R_E$ is the displacement of the lens from the l.o.s. axis, expressed in units of Einstein radius,

$$R_E = \sqrt{\frac{2r_S(D_s - D_l)D_l}{D_s}}, \tag{2}$$

where r_S, D_s, and D_l are the Schwarzschild radius of the lens, the distance of the observer to the source, and the distance of the observer to the lens, respectively. The observed light curves are typically fitted using the following parametrization of u [40]:

$$u(t) = \sqrt{u_0^2 + \frac{(t - t_0)^2}{t_E^2}} \tag{3}$$

where u_0 is the impact parameter of the lens, t_0 the time of maximum amplitude, while t_E is the characteristic crossing time, i.e., $t_E = R_E/v_\perp$ with $v_\perp$ being the lens transverse velocity with respect to the l.o.s.

The main quantity in predicting the observational signatures of MACHOs is the differential event rate $d\Gamma/dt_E$. The latter quantifies the rate at which lenses enter the "microlensing tube," defined as the volume along the l.o.s. axis that leads to an amplification above a certain threshold $A_T \equiv A(u_T)$ (for more details see, e.g., [14] and Fig. 1 therein). $d\Gamma/dt_E$ can be used to compute the expected number of microlensing events in a given survey with detection efficiency $\varepsilon(t_E)$ and total exposure E:

$$\bar{N} = E \int dt_E \varepsilon(t_E) \times \frac{d\Gamma}{dt_E}(t_E). \tag{4}$$

In microlensing studies, it is commonly assumed that PBHs follow a smooth and spherically symmetric Galactic halo density profile $\rho(r)$, where r denotes the galactocentric distance, and have an isotropic Maxwellian velocity distribution with a dispersion $\sigma(r)$. In that case, the differential event rate can be computed by performing a single l.o.s. integral [14,17].
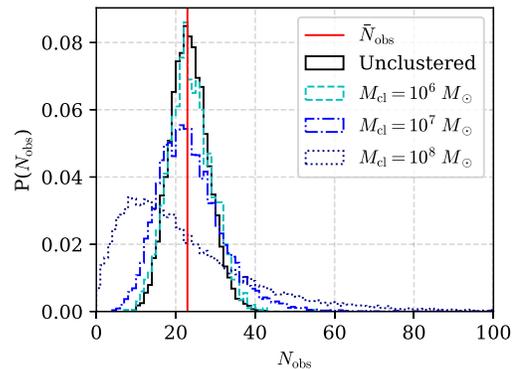

FIG. 1. Probability distribution functions for the number of observed microlensing events $N_{\rm obs}$ in EROS-2-like survey of the LMC. The black curve shows the standard result assuming smoothly distributed PBHs, while color curves represent our results for PBHs bound in clusters of different masses. The vertical red line shows the mean number of events, $\bar{N}_{\rm obs}$, which is independent of PBH clustering.

However, as argued in [22–24,41], these assumptions may no longer be justified if PBHs are bound into compact clusters, but rather should be applied to the clusters themselves and no longer to individual PBHs. This argument was used to evade microlensing constraints on PBH dark matter simply from the fact that (compact) cluster masses fall outside from the microlensing target mass range. It is straightforward and intuitively clear to write down the approximate condition for when a cluster should be treated as a single lens (compact cluster) and when as a compilation of spatially correlated individual lenses (extended cluster). Imagine a perfectly aligned observer-lens-source event where the bending of light rays by the Einstein angle $\Theta_E = R_E(M_{cl})/D_l$ is maximal, where M_{cl} is the cluster mass. These light rays pass approximately $\Theta_E D_l = R_E(M_{cl})$ from the center of the cluster and do not enter it if its radius is $r_{cl} \ll R_E(M_{cl})$. Therefore, a cluster has to be treated as compact from the point of view of microlensing surveys when $r_{cl} \ll R_E(M_{cl})$. We will see that for Gaussian perturbations this is not the case, as the clusters are orders of magnitude larger than the corresponding Einstein radius, i.e., $r_{cl} \gg R_E(M_{cl})$. In such situations, characterized by low optical depth, accounting only for the PBH that is the closest to the l.o.s. is sufficient to model the net lensing effect [42].

III. SOME PROPERTIES OF PBH CLUSTERS

For Gaussian primordial curvature perturbations the probability for the collapse into BHs is uniform in space and, therefore, results in initially randomly distributed PBHs. This unavoidably seeds Poissonian isocurvature perturbations on small scales, which later lead to the formation of PBH clusters. The overdensities associated with scales that on average contain N PBHs are of the size of $\delta(N) \sim \Delta N/N = 1/\sqrt{N}$. In the radiation dominated era, these fluctuations only grow logarithmically, however, after matter-radiation equality, they enter the linear growth regime and collapse. Throughout their subsequent evolution, smaller clusters evaporate, but are themselves aggregated into more massive structures. The typical evaporation time can be estimated as [43]:

$$t_{evap} \sim 140 t_{relax} \sim \frac{14 N_{pbh}}{\log N_{pbh}} t_{cross}, \qquad (5)$$

where t_{relax} is the system relaxation time rewritten in terms of the number of PBHs within the cluster, N_{pbh}, and the typical crossing time $t_{cross} \sim r_{cl}/v_{cl}$ determined by the cluster size, r_{cl}, and its virial velocity, v_{cl}. At present time, one expects to find the largest abundance of PBH clusters with total masses $M_{cl} \sim 10^4 \, M_\odot$, consisting of $\sim 10^3$–10^4 individual PBHs with masses ~ 1–$10 \, M_\odot$, and with typical cluster sizes $r_{cl} \sim 10$ pc [34,35,44]. Larger clusters, which are less dense, are made up of these basic building blocks,

i.e., the of $M_{cl} \sim 10^4 \, M_\odot$ clusters, as these are the smallest structures that have not yet evaporated.

IV. MICROLENSING CONSTRAINTS FOR CLUSTERED PBH

As mentioned above, if PBHs live in compact clusters, one can readily rescale the microlensing limits on the DM fraction in PBHs by considering single lenses of mass M_{cl}. This can completely lift the current microlensing bounds on sub-Solar mass PBHs for $M_{cl} \gg M_\odot$ [22]. However, the condition for PBH clusters to act as individual lenses in surveys of the Magellanic Clouds is that $r_{cl} \ll 10 \, au(M_{cl}/M_\odot)^{1/2}$ (for surveys targeting the Andromeda galaxy this ratio is larger, but only by a factor of ~ 20). This criterion is definitely *not* fulfilled for PBH clusters formed through the collapse of Gaussian primordial adiabatic perturbations [32], whose typical features have been recalled above. Such dense PBH clusters can only form in the early Universe through some other mechanism (e.g., [45–47]), or different initial conditions, such as non-Gaussian perturbations [48] (see also [49]). We postpone a dedicated study of such scenarios to future work, however, we anticipate that substantial fine-tuning of the corresponding PBH production mechanism would be required to significantly relax the microlensing bounds, while still obeying other observational constraints. Therefore, in the following, we focus on the effect of extended PBH clusters on the microlensing limits derived from the surveys of the LMC.

A. Probability distribution for the number of observed events

To carry out the statistical analysis of the event rate, we resort to dedicated Monte-Carlo (MC) simulations of microlensing signals produced by clustered PBHs. While semianalytical estimates are possible for sparsely distributed sources, the crowded stellar fields of nearby galaxies make it necessary to perform a numerical convolution of associated complex stellar surface densities with the probability distribution of PBH cluster positions, rendering it analytically intractable.

1. Monte Carlo simulations

Our MC simulations consist in drawing $N_{cl} \sim \text{Pois}(\lambda_{LMC})$ PBH clusters, whose positions and velocities are distributed according to a chosen global model of the Galactic DM halo. The latter also defines $\lambda_{LMC} = M_{l.o.s.}/M_{cl}$, i.e., the expected number of PBH clusters within the observed volume ($M_{l.o.s.}$ corresponds to the DM mass contained in a cone with the apex at the position of the observer and the base in the approximate plane of the stellar source population, encompassing the observed area in the sky). We additionally assume that all PBH clusters have the same mass M_{cl}, which is justified by the fact that the smallest nonevaporated

clusters will be the most abundant and represent the elementary building blocks of larger clusters [34,35,44]. For each of the clusters, we draw a number of microlensing events from a Poisson distribution with the appropriate mean. The latter is determined by computing the differential event rate, $d\Gamma$, from its standard expression (see, e.g., Eq. (8) in [14]). We further assume that all PBHs have equal masses, $M_{\mathrm{PBH}} = 1\,M_\odot$, and that the velocity dispersion of PBHs inside a cluster is negligible with respect to the cluster velocity in the Galactic halo, such that the velocity distribution of the cluster members can be modeled as a delta function picked at the cluster's peculiar velocity $\vec{v}_{\mathrm{p}}$. Thus, the differential event rate $d\Gamma$ (integrated over angles) can be written as:

$$d\Gamma = 2n_{\mathrm{PBH}}(x)u_{\mathrm{T}}v_\perp R_{\mathrm{E}}(x)dx, \qquad (6)$$

where $n_{\mathrm{PBH}}(x)$ is the PBH number density at position x along the l.o.s. Consequently, the Einstein crossing time of each event associated with this cluster is fixed to $t_{\mathrm{E}} = R_{\mathrm{E}}(x)u_{\mathrm{T}}/v_\perp$. The corresponding differential event rate *inside a cluster* for a l.o.s with impact parameter b can then be expressed as the l.o.s. integral:

$$\frac{d\Gamma}{dt_{\mathrm{E}}}(b, t_{\mathrm{E}}) = \frac{2v_\perp u_{\mathrm{T}} R_{\mathrm{E}}(x_{\mathrm{cl}})}{M_{\mathrm{PBH}}} \delta\left(\frac{R_{\mathrm{E}}(x_{\mathrm{cl}})u_{\mathrm{T}}}{v_\perp} - t_{\mathrm{E}}\right)$$
$$\times \int_{x_{\mathrm{cl}}-\Delta x}^{x_{\mathrm{cl}}+\Delta x} dx\rho_{\mathrm{cl}}\left(\sqrt{(x-x_{\mathrm{cl}})^2 + b^2}\right), \qquad (7)$$

where x_{cl} denotes the position of the PBH cluster along the l.o.s., $\Delta x \equiv \sqrt{r_{\mathrm{cl}}^2 - b^2}$ the half-length of the l.o.s. segment penetrating the cluster, and $\rho_{\mathrm{cl}}(r)$ the radial density profile of the cluster (we assume that the PBH clusters are spherically symmetric). The expected number of microlensing events for the entire cluster is obtained by convoluting the above expression with the surface density of stellar targets, $\Sigma_\star(\mathbf{q})$, over the area of the cluster (neglecting the small angle correction as $r_{\mathrm{cl}} \ll x_{\mathrm{cl}}$):

$$\frac{d\Gamma_{\mathrm{cl}}}{dt_{\mathrm{E}}}(t_{\mathrm{E}}) = \int_0^{r_{\mathrm{cl}}} dbb\frac{d\Gamma}{dt_{\mathrm{E}}}(b, t_{\mathrm{E}})$$
$$\times \int_0^{2\pi} d\varphi\Sigma_\star(\mathbf{q}_{\mathrm{cl}} - \mathbf{q}(b, \varphi)), \qquad (8)$$

where $\mathbf{q}_{\mathrm{cl}}$ denotes the position of the cluster on the celestial sphere and $\mathbf{q}(b, \varphi)$ the displacement corresponding to given values of b and φ. This differential rate translates into the expected number of lensing events induced by the ith cluster:

$$\bar{N}_{\mathrm{e},i} = t_{\mathrm{obs}} \int dt_{\mathrm{E}}\varepsilon(t_{\mathrm{E}}) \times \frac{d\Gamma_{\mathrm{cl},i}}{dt_{\mathrm{E}}}(t_{\mathrm{E}}). \qquad (9)$$

Finally, we draw the number of events due to each cluster, $N_{\mathrm{e},i} \sim \mathrm{Pois}(\bar{N}_{\mathrm{e},i})$, and sum them up to obtain the total number of events in a given simulation run:

$$N_{\mathrm{obs}} = \sum_{i=1}^{N_{\mathrm{cl}}} N_{\mathrm{e},i}. \qquad (10)$$

Note that the stochastic distribution of PBH clusters in the observational cone and the uneven distribution of target stars on the celestial sphere lead to N_{obs} that does not trivially follow a Poisson distribution. This explains the need for performing dedicated MC simulations.

2. Results

For concreteness, here we apply our method to study the distribution of microlensing events in the EROS-2 survey of the LMC [19]. In particular, we adopt the corresponding target star distribution and detection efficiency, while for the Milky Way DM halo we assume the same cored DM halo model—for details see [19].

The MC simulations allow us to determining the probability distribution for the total number of observed events, $P_{\mathrm{obs}}(N_{\mathrm{obs}})$, which we report in Fig. 1 for various M_{cl} while keeping $r_{\mathrm{cl}} = 10$ pc.[1] First thing to note is that the mean number of events, $\bar{N}_{\mathrm{obs}} \approx 23$, is always the same, regardless of the cluster mass, as it only depends on the total number of PBHs within the probed volume, i.e., $M_{\mathrm{l.o.s.}}/M_{\mathrm{PBH}}$. From the same figure it can also be seen that $P_{\mathrm{obs}}(N_{\mathrm{obs}})$ is virtually indistinguishable from the standard result obtained for unclustered PBH if $M_{\mathrm{cl}} \lesssim 10^6\,M_\odot$. This comes from the fact that for a large number of individual clusters, fluctuations about individual l.o.s.'s average out and $P_{\mathrm{obs}}(N_{\mathrm{obs}})$ simply reduces to Poisson distribution (for $M_{\mathrm{cl}} = 10^6\,M_\odot$ one expects $N_{\mathrm{cl}} \sim 1300$ within the field of view). However, for higher cluster masses, $P_{\mathrm{obs}}(N_{\mathrm{obs}})$ becomes increasingly skewed toward low N_{obs}, but with a thin tail reaching much larger values. For the highest PBH cluster mass considered in this work, $M_{\mathrm{cl}} = 10^8\,M_\odot$, $P_{\mathrm{obs}}(N_{\mathrm{obs}})$ peaks at $N_{\mathrm{obs}} = 8$, but has an extremely long tail, stretching to hundreds of events. This would lead to very different exclusion limits on the allowed fraction of DM in the form of PBHs. For example, one could not exclude (at 95% confidence level) DM being fully made of PBH clusters of masses $M_{\mathrm{cl}} = 10^8\,M_\odot$ if more than three microlensing events were detected, while for smoothly distributed PBH the threshold value shifts to 15. This clearly demonstrates the importance of taking into the account correlations along neighboring l.o.s.'s in the case of PBH clusters with M_{cl} approaching $M_{\mathrm{l.o.s.}}$.

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[1]In the scenario where PBHs are formed from Gaussian curvature perturbations, the adopted value of r_{cl} corresponds to clusters with $M_{\mathrm{cl}} \sim 10^4\,M_\odot$, while more massive clusters are expected to be larger and less dense, since $r_{\mathrm{cl}} \propto M_{\mathrm{cl}}^{5/6}$. However, for $M_{\mathrm{cl}} \gtrsim 10^7\,M_\odot$ this would imply extremely large clusters with high chance of spatial overlap, and, therefore, we conservatively keep $r_{\mathrm{cl}} = 10$ pc for all the considered values of M_{cl}. Note that despite this choice even clusters with $M_{\mathrm{cl}} = 10^8\,M_\odot$ still acts as extended clusters.

While the presented results were obtained for an EROS-like survey of the LMC, we wish to emphasize that analogous conclusions are applicable also to other micro-lensing surveys aimed toward the Magellanic Clouds, as well as the Andromeda galaxy. Indeed, extended PBH clusters can significantly affect $P_{\mathrm{obs}}(N_{\mathrm{obs}})$ only if their individual masses represent a large fraction of the expected DM mass within the observational cone. Otherwise, the fluctuations in the expected number of the events along individual l.o.s.'s of each realization average out and the predicted signal becomes very similar to that of smoothly distributed PBH.

B. Recurrence of events

Microlensing events are extremely rare due to the smallness of the volume that has to be crossed by a compact object to produce an observable amplification. Therefore, multiple microlensing events from the same source are conventionally attributed to its intrinsic vari-ability rather than microlensing. This view has been challenged in several works proposing that highly clustered PBHs could lead to recurring amplifications along the same l.o.s. [22,23]. In the following, we critically reexamine this hypothesis, first focusing on the effect of small separation distances between individual PBHs in extremely compact clusters and, subsequently, on the possibility of recurring events due to PBH binaries, which are expected to be efficiently formed in the early Universe.

1. Small PBH separation distances

For dense PBH clusters, the dynamical time is typically orders of magnitude smaller than their ages. Therefore, they should be virialized and the total energy of the cluster can be expressed as $E_{\mathrm{cl}} = -NM_{\mathrm{pbh}}v_{\mathrm{cl}}^2/2 = -G(N)^2/(4r_{\mathrm{cl}})$ [43], where v_{cl} is root-mean-square velocity of an individ-ual PBH of mass M_{pbh}, and N is the number of PBHs within the cluster. This gives $v_{\mathrm{cl}}^2 = GNM_{\mathrm{pbh}}/(2r_{\mathrm{cl}})$. On the other hand, the separation distance between the PBH can be estimated as $\Delta r \approx (V_{\mathrm{cl}}/N)^{1/3} = r_{\mathrm{cl}}(4\pi/(3N))^{1/3}$. From this, the typical time required for a PBH to cross Δr reads:

$$\Delta t \approx \frac{\Delta r}{v_{\mathrm{cl}}} = \left(\frac{4\pi}{3N}\right)^{1/3} \left(\frac{2r_{\mathrm{cl}}^3}{GNM_{\mathrm{pbh}}}\right)^{1/2}. \qquad (11)$$

By adopting a conservative approximation, $r_{\mathrm{cl}} \geq \sqrt{N}R_{\mathrm{E}}(M_{\mathrm{pbh}})$, for which the extended-cluster condition is only marginally fulfilled, we get:

$$\Delta t \gtrsim 9.8 \text{ yr} \left(\frac{x(1-x)}{0.5}\right)^{3/4} \left(\frac{D_s}{55 \text{ kpc}}\right)^{3/4}$$
$$\times \left(\frac{M_{\mathrm{pbh}}}{M_\odot}\right)^{1/4} \times \left(\frac{10^4}{N}\right)^{1/12}, \qquad (12)$$

where[2] $x = D_{\mathrm{l}}/D_{\mathrm{s}}$. While this is comparable to the duration of a typical microlensing survey, it is important to note that most of the events would not produce light curves consistent with Eq. (1) if $r_{\mathrm{cl}} \sim \sqrt{N}R_{\mathrm{E}}(M_{\mathrm{pbh}})$, as the light rays would likely be affected by multiple PBHs. Therefore, in reality, Δt has to be significantly (orders of magnitude) larger than the above value, making recurring events due to small PBH separation distances very unlikely, even for clusters consisting of an overwhelming number of sub-Solar mass PBHs.

2. Binary PBHs

PBH binaries are efficiently formed in the early Universe around matter-radiation equality. A large fraction of these binaries survive three-body interactions in clusters [31,32,34] and could potentially lead to recurring events [22,23,41]. We can derive a lower bound on the time separation of microlensing events under the assumption that one of such pairs happens to lie along a given l.o.s. aligned with its orbital plane. The latter can be simply estimated as one half of the orbital period of a two-body system:

$$\Delta t = \frac{T}{2} = \pi\sqrt{\frac{a^3}{G\mu}}, \qquad (13)$$

where a is the orbital semimajor axis and μ is the reduced mass of the PBH. To minimize Δt, we assume that both PBHs have the same mass, $\mu = M_{\mathrm{pbh}}/2$, while we set $a \geq 2R_{\mathrm{E}}(M_{\mathrm{pbh}})$, so that the single-lens assumption in Eq. (1) is still marginally applicable. This yields:

$$\Delta t \geq 116 \text{ yr} \left(\frac{x(1-x)}{0.5}\right)^{3/4} \left(\frac{D_s}{55 \text{ kpc}}\right)^{3/4} \left(\frac{M_{\mathrm{pbh}}}{M_\odot}\right)^{1/4}. \qquad (14)$$

This is again very conservative, since only a fraction of PBHs should be in the form of binaries with the orbital plane aligned along the l.o.s. and, additionally, one should have $a \gg 2R_E$ to observe light curves that are not distorted by the presence of the second object (such anomalies have already been detected in a binary BH lens [50]). Therefore, we conclude that PBH binaries are also very unlikely sources of recurring events, even for $M_{\mathrm{pbh}} \ll M_\odot$.

V. SUMMARY AND CONCLUSIONS

In this work we have carefully examined the impact of PBH clustering on the interpretation of microlensing searches for MACHOs. Heuristically, it has often been argued that microlensing constraints on DM in the form of PBHs can be significantly relaxed, or even entirely

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[2]Without impacting our conclusions below, for less dense clusters v_{p} should be taken in Eq. (11).

removed, if one accounts for the fact that PBHs are bound into clusters. However, our dedicated MC simulations of EROS-2-like experiment show that microlensing limits can be lifted only if clusters are extremely compact, $r_{cl} \ll R_E(M_{cl})$, or very massive, $M_{cl} \gtrsim 10^8 \ M_\odot$. In the most commonly considered scenarios, where PBHs are formed through the collapse of Gaussian primordial curvature perturbations, both of these conditions are far from being fulfilled. Our conclusions equally apply to other microlensing surveys of the Magellanic Clouds, as well as those of the Andromeda galaxy. Similar results have also been obtained in recent semianalytical study [51], which has appeared during the publication of this work. While it is important to acknowledge that there exist alternative models of PBH formation, such as those with non-Gaussian initial conditions or formation through topological defects, which might produce sufficiently dense PBH clusters to avoid the microlensing limits, further work is needed to establish the survivability of such dense clusters to the present days.

Similarly, it has been argued that PBH clustering can lead to recurring microlensing events which have been spuriously discarded from the observational samples. However, using simple analytical estimates, we have shown that recurring microlensing events due to dense PBH clusters or PBH binaries should occur on significantly longer timescales than the typical duration of microlensing surveys. Therefore, it is unlikely that any of the observed recurring events were in fact of a PBH origin.

[1] LIGO Scientific Collaboration and Virgo Collaboration, Phys. Rev. Lett. **116**, 061102 (2016).

[2] LIGO Scientific Collaboration and Virgo Collaboration, Phys. Rev. X **6**, 041015 (2016).

[3] LIGO Scientific Collaboration and Virgo Collaboration, Astrophys. J. **882**, L24 (2019).

[4] S. Bird, I. Cholis, J. B. Muñoz, Y. Ali-Haïmoud, M. Kamionkowski, E. D. Kovetz, A. Raccanelli, and A. G. Riess, Phys. Rev. Lett. **116**, 201301 (2016).

[5] S. Clesse and J. García-Bellido, Phys. Dark Universe **15**, 142 (2017).

[6] P. J. E. Peebles, Astrophys. J. **263**, L1 (1982).

[7] J. M. Bardeen, J. R. Bond, N. Kaiser, and A. S. Szalay, Astrophys. J. **304**, 15 (1986).

[8] J. Silk and G. A. Mamon, Res. Astron. Astrophys. **12**, 917 (2012).

[9] B. J. Carr and S. W. Hawking, Mon. Not. R. Astron. Soc. **168**, 399 (1974).

[10] G. F. Chapline, Nature (London) **253**, 251 (1975).

[11] B. Carr, K. Kohri, Y. Sendouda, and J. Yokoyama, Rep. Prog. Phys. **84**, 116902 (2021).

[12] A. M. Green and B. J. Kavanagh, J. Phys. G Nucl. Phys. **48**, 043001 (2021).

[13] B. Paczynski, Astrophys. J. **304**, 1 (1986).

[14] K. Griest, Astrophys. J. **366**, 412 (1991).

[15] K. Jedamzik, Phys. Rev. D **55**, R5871 (1997).

[16] C. T. Byrnes, M. Hindmarsh, S. Young, and M. R. S. Hawkins, J. Cosmol. Astropart. Phys. 08 (2018) 041.

[17] The MACHO Collaboration, Astrophys. J. **461**, 84 (1996).

[18] The MACHO Collaboration, Astrophys. J. **550**, L169 (2001).

[19] P. Tisserand, L. Le Guillou *et al.* (The EROS-2 Collaboration), Astron. Astrophys. **469**, 387 (2007).

[20] J. Wyrzykowski, J. Skowron, S. Kozlowski *et al.* (The OGLE Collaboration), Mon. Not. R. Astron. Soc. **416**, 2949 (2011).

[21] M. Zumalacárregui and U. Seljak, Phys. Rev. Lett. **121**, 141101 (2018).

[22] S. Clesse and J. García-Bellido, Phys. Dark Universe **22**, 137 (2018).

[23] J. Calcino, J. García-Bellido, and T. M. Davis, Mon. Not. R. Astron. Soc. **479**, 2889 (2018).

[24] B. Carr, S. Clesse, J. Garcia-Bellido, and F. Kuhnel, Phys. Dark Universe **31**, 100755 (2021).

[25] B. Carr and J. Silk, Mon. Not. R. Astron. Soc. **478**, 3756 (2018).

[26] Y. Ali-Haïmoud, Phys. Rev. Lett. **121**, 081304 (2018).

[27] V. Desjacques and A. Riotto, Phys. Rev. D **98**, 123533 (2018).

[28] P. Meszaros, Astron. Astrophys. **37**, 225 (1974), https://ui.adsabs.harvard.edu/abs/1974A%26A....37..225M/abstract.

[29] P. Meszaros, Astron. Astrophys. **38**, 5 (1975), https://ui.adsabs.harvard.edu/abs/1975A%26A....38....5M.

[30] N. Afshordi, P. McDonald, and D. N. Spergel, Astrophys. J. **594**, L71 (2003).

[31] M. Raidal, C. Spethmann, V. Vaskonen, and H. Veermäe, J. Cosmol. Astropart. Phys. 02 (2019) 018.

[32] D. Inman and Y. Ali-Haïmoud, Phys. Rev. D **100**, 083528 (2019).

[33] M. Sasaki, T. Suyama, T. Tanaka, and S. Yokoyama, Phys. Rev. Lett. **117**, 061101 (2016); **121**, 059901(E) (2018).
[34] K. Jedamzik, J. Cosmol. Astropart. Phys. 09 (2020) 022.
[35] K. Jedamzik, Phys. Rev. Lett. **126**, 051302 (2021).
[36] K. Jedamzik, Proceedings of the "Primordial Black Holes confront GW data", Rome 2021 (unpublished).
[37] The MACHO Collaboration, Astrophys. J. **542**, 281 (2000).
[38] H. Niikura, M. Takada, N. Yasuda, R. H. Lupton, T. Sumi, S. More, T. Kurita, S. Sugiyama, A. More, M. Oguri, and M. Chiba, Nat. Astron. **3**, 524 (2019).
[39] S. Refsdal, Mon. Not. R. Astron. Soc. **128**, 295 (1964).
[40] E. Roulet and S. Mollerach, Phys. Rep. **279**, 67 (1997).
[41] J. García-Bellido and S. Clesse, Phys. Dark Universe **19**, 144 (2018).
[42] P. Fleury and J. García-Bellido, Phys. Dark Universe **29**, 100567 (2020).

[43] J. Binney and S. Tremaine, *Galactic Dynamics*, 2nd ed., Princeton Series in Astrophysics (Princeton University Press, Princeton, NJ, 2008).
[44] G. Hütsi, M. Raidal, V. Vaskonen, and H. Veermäe, J. Cosmol. Astropart. Phys. 03 (2021) 068.
[45] V. A. Berezin, V. A. Kuzmin, and I. I. Tkachev, Phys. Lett. **120B**, 91 (1983).
[46] C. J. Hogan, Phys. Lett. **143B**, 87 (1984).
[47] K. M. Belotsky, V. I. Dokuchaev, Y. N. Eroshenko, E. A. Esipova, M. Y. Khlopov, L. A. Khromykh, A. A. Kirillov, V. V. Nikulin, S. G. Rubin, and I. V. Svadkovsky, Eur. Phys. J. C **79**, 246 (2019).
[48] S. Young and C. T. Byrnes, J. Cosmol. Astropart. Phys. 03 (2020) 004.
[49] B. Carr and F. Kuhnel, arXiv:2110.02821.
[50] S. Dong, A. Udalski, A. Gould *et al.* (The OGLE Collaboration), Astrophys. J. **664**, 862 (2007).
[51] M. Gorton and A. M. Green, arXiv:2203.04209.