

Scanning the landscape of axion dark matter detectors: Applying gradient descent to experimental design

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 (Received 28 September 2021; accepted 23 March 2022; published 19 April 2022)

The hunt for dark matter remains one of the principal objectives of modern physics and cosmology. Searches for dark matter in the form of axions are proposed or underway across a range of experimental collaborations. As we look to the next generation of detectors, a natural question to ask is whether there are new experimental designs waiting to be discovered and how we might find them. Here we take a new approach to the experimental design procedure by using gradient descent techniques to search for optimal detector designs. We provide a proof of principle for this technique by searching 1D detectors varying the bulk properties of the detector until the optimal detector design is obtained. Remarkably, we find the detector is capable of outperforming a human designed experiment on which the search was initiated. This opens the door to further gradient descent searches of more complex 2D and 3D designs across a wider variety of materials and boundary geometries of the detector. There is also an opportunity to use more sophisticated gradient descent algorithms to complete a more exhaustive scan of the landscape of designs.

DOI: [10.1103/PhysRevD.105.083010](https://doi.org/10.1103/PhysRevD.105.083010)

I. INTRODUCTION

The quest for an understanding of dark matter remains one of the foremost challenges of modern particle physics. Axions [1–5] represent one of the leading candidates for dark matter [6–8] and are predicted by a range of theories beyond the standard model [9,10]. One possibility to detect axions is via their astrophysical signatures, for instance via galactic halo axions decaying into two photons [11–22] or through observations of neutron stars [14,23–32]. It is also of vital importance to complement these efforts through laboratory searches and there are a number of current and proposed experiments aiming to detect axion dark matter here on Earth. Notable examples are HAYSTAC [33], ADMX [34], MADMAX [35], ABRACADABRA [36], and ORGAN [37] to name a few.

In recent years there has also been a new proposal [38] which seeks to push dark matter searches in new directions by exploiting novel material structures. This gives rise to detectors with new and unusual properties. In that work, the authors explored metamaterials capable of mimicking the properties of plasma at radio/microwave frequencies, providing an entirely new way to probe axion dark matter.

Furthermore, in light of the wonderful array of exotic and tunable metamaterials [39–42], the door is now open to detectors with a wider and more complex variety of structures with a greater flexibility in demanding materials with prespecified properties.

In this spirit, the purpose of the present paper is to perform the following thought experiment. If one is free to choose the properties of the detector (subject to reasonable constraints, e.g., total volume, fixed magnetic field strength, and some unavoidable losses in the materials etc.), what is the best design and how can we find it? Given the increasingly large variety of experimental designs, a selection of which is listed above, answering this question is of paramount importance for the following reasons. Any experimentalist wants to be able to answer the question as to whether a better design awaits discovery, or whether their design is in fact already effectively optimal and can only be improved via iterative adjustments such as better refrigeration, more sensitive photodetectors and lower loss materials, and so on.

At present, answering this question is a trial-and-error process whereby anyone wishing to propose a new experiment must generate a new design and hope that the idea they conceive can outperform its current competitors. The proposal of the present paper seeks instead to take the first steps towards automating the experimental design procedure by using a gradient descent search to locate optimal detector designs. Though this should not be considered an exhaustive tool for the experimental design procedure, it is nonetheless fascinating to see what the landscape of

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potential designs might look like and ultimately to ask the question: can a computer design a better experiment than a human being, which has far-reaching consequences beyond the field of axion physics.

This paper is *not* any kind of white paper for a specific experimental proposal but *is* an attempt to take the first steps towards asking whether machine intelligence may be able to assist in the design of new experiments whose objective can be phrased in terms of a simple optimizable function. This is especially timely in light of the many other recent advances in artificial intelligence and machine learning which are increasingly demonstrating their ability to match or outperform humans across various disciplines.

The structure of the remainder of this paper is organised as follows. In Sec. II we present a 1D axion haloscope model consisting of metamaterial components, providing a simple setup on which to demonstrate the gradient descent approach to experimental design. Here we also introduce gradient descent and explain how the detector design can be reformulated in terms of an objective function (the haloscope stored energy) to be maximized. In Sec. III we present detector designs resulting from our search with fiducial axion mass values at $150 \mu\text{eV}$ and compare the performance of these optimized designs to a uniform plasma haloscope considered in [38]. Finally in sec. IV we offer our conclusions and discuss several directions for future work.

II. SETUP

Our starting point is the equations for axion-electrodynamics given by augmenting Maxwell's equations with an additional term $\mathcal{L} = \frac{g_{a\gamma\gamma}}{4} a F_{\mu\nu} \tilde{F}^{\mu\nu}$, where a is the axion field and $g_{a\gamma\gamma}$ a coupling constant. These read

$$\nabla \cdot \mathbf{D} = -g_{a\gamma\gamma} \mathbf{B} \cdot \nabla a, \quad (1)$$

$$\nabla \times \mathbf{B} - \dot{\mathbf{D}} = g_{a\gamma\gamma} \dot{a} \mathbf{B} - g_{a\gamma\gamma} \mathbf{E} \times \nabla a, \quad (2)$$

$$\nabla \cdot \mathbf{B} = 0, \quad (3)$$

$$\dot{\mathbf{B}} + \nabla \times \mathbf{E} = 0, \quad (4)$$

where \mathbf{E} and \mathbf{B} are the electric and magnetic fields. We shall assume a simple constitutive relation $\mathbf{D} = \boldsymbol{\epsilon} \cdot \mathbf{E}$ where $\boldsymbol{\epsilon}$ is the permittivity tensor of the medium. Furthermore, we shall neglect spatial gradients of the axion field. This is valid when the detector size is much less than axion wavelengths, as is the case for nonrelativistic axions and the masses and detector sizes we shall consider. With these assumptions, one obtains the following equation for the electric field:

$$-\nabla^2 \mathbf{E} + \nabla(\nabla \cdot \mathbf{E}) + \boldsymbol{\epsilon} \cdot \ddot{\mathbf{E}} = g_{a\gamma\gamma} \ddot{a} \mathbf{B}. \quad (5)$$

Inspired by the results of [38] we consider an idealized detector consisting of N metamaterial layers with permittivities $\epsilon^{(i)}$. We assume the detector contains a magnetic field sufficiently strong that the permittivities have a single scalar component in the direction of \mathbf{B} which can be described by a Drude-like model via

$$\epsilon^{(i)} = 1 - \frac{\omega_p^{(i)}}{\omega^2 + i\Gamma^{(i)}\omega}, \quad 1 \leq i \leq N, \quad (6)$$

where $\Gamma^{(i)}$ is a loss rate and ω the frequency of the electric field. This provides a useful relation between the plasma frequencies and losses in a physically well-motivated way, though of course the true relation will depend on the precise properties of the (meta)material in question. We shall consider a 1D prototypical detector design. In future work we will include 2D and 3D effects, but the computational overhead associated with solving axion-electrodynamics in higher dimensions would require more careful thought in light of the many iterations which must be performed during a gradient descent search. We therefore consider a simple 1D setup in which the electric field is sourced only via its component E_{\parallel} parallel to a constant magnetic field magnitude B_0 with the equations expressible in terms of a single integration parameter x perpendicular to the electric field which vanishes at the endpoints of the detector $x = 0$ and $x = L$, where L is the length of the detector. This leads to the following equation:

$$-\frac{d^2 E_{\parallel}}{dx^2} + \omega^2 \epsilon E_{\parallel} = g_{a\gamma\gamma} \omega^2 a_0 B_0, \quad (7)$$

where ϵ is a discontinuous function taking the piecewise values (6) in each of the layers and a_0 is the amplitude of the axion field. This setup is sufficient to illustrate the gradient descent approach, and also provides a good proxy for higher dimensional configurations as shown in e.g., MADMAX [35] and plasma haloscopes [38].

Since the purpose of the present work is to achieve the best design through an iterative optimization procedure, we must first decide what function to maximize. In the future, one may want to minimize the total scan time over a desired frequency range, or minimize the amount of tuning required by e.g., finding a broadband configuration. For now we take the simplest approach and seek to maximize the stored energy U for a given axion mass. This works since for a critically coupled haloscope, the power entering probe is proportional to U which for a plasmlike material is defined by [43]

$$U[\{\omega_p^{(i)}\}] = \int dV [\partial_{\omega}(\omega \text{Re}[e]) \mathbf{E}^2 + \mathbf{B}^2]. \quad (8)$$

The aim of the remainder of the present work is to find those plasma values and permittivities which maximize U

and therefore the sensitivity to axion dark matter. To this end, we identify an *objective function* to be minimized as

$$F[X] \equiv -U[\{\omega_p^{(i)}\}], \quad X = \{\omega_p^{(1)}, \dots, \omega_p^{(N)}\}. \quad (9)$$

Note the minus sign in (9) appears owing to the convention that one minimizes rather than maximizes the objective function. We emphasize that in future work, the objective function arguments could be extended so that X includes e.g., the shape of the boundary in 2D and 3D searches¹ in a way reminiscent of novel cavity shapes [44,45]. One might also consider varying the magnetic permeability in various layers or adjusting the nature of boundary conditions and so on. Similarly, if there are practical limitations e.g., maximum number of layers, minimum size of components, these can be incorporated as constraints in the optimization procedure. This allows one to find an optimal design subject to whatever material limitations one wishes to impose.

We then perform a gradient descent as follows. First some initial search point X_0 is chosen, successive points X_n (the “designs”) are then computed according to

$$X_{n+1} = X_n - \gamma_n \nabla_X F(X_n), \quad (10)$$

where ∇F is the gradient of the cost function and γ is the learning rate, which decreases with successive iterations as the minimum is approached and makes use of rmijo backtracking along the direction of the gradient to ensure the objective function decreases when moving in the direction of the negative gradient.

We terminate the iteration scheme when convergence in $F(X)$ and each coordinate of X has reached 0.1%. Note we also decrease the learning rate γ_0 until this tolerance condition is exceeded by the first step, which avoids the algorithm terminating artificially on the first iteration from trivially small initial value of γ . We illustrate the gradient descent approach for a toy two-layer haloscope in Fig. 1 searching for the minimizing plasma values $(\omega_p^{(1)}, \omega_p^{(2)})$. We mention in passing that optimization procedures have also been used to explore the best disc spacings in MADMAX [46], though this work used random sampling techniques rather than gradient descent. We also emphasize that the rationale behind the present paper is to provide a much more general and far-reaching approach to experimental design beyond simply optimizing one aspect of a particular experiment.

¹This could be achieved by discretizing the boundary of the detector into a mesh of nodes $\mathbf{x}_i = (x_i, y_i, z_i)$ connected by smooth surfaces and finding those combinations which maximize the axion signal, treating the \mathbf{x}_i as free parameters in X . In other words, moulding the exterior geometry iteratively until the optimum shape is achieved.

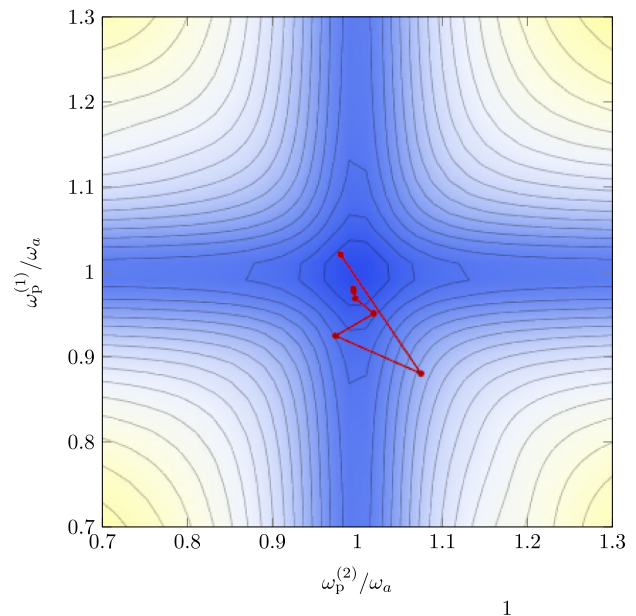


FIG. 1. Gradient descent search of detector design. For a toy two-layer haloscope one can visualize the gradient descent search superimposed on a contour plot. The purple lines display the gradient descent search points $X = (\omega_p^{(1)}, \omega_p^{(2)})$ on top of the contours of the objective function $F = -U$ given by the haloscope stored energy.

III. RESULTS

We now scan detector configurations by varying the number and value of layers within the experimental design. We choose a detector length $L = 2.7$ m, with vanishing boundary conditions at $x = 0$ and $x = L$ and fix the damping rate $\Gamma = 0.1\omega_p$ to be uniform across each layer. This is the same conservative fiducial value for losses chosen in [38]. We performed a gradient descent starting from initial values $\omega_p^{(i)} \simeq \omega_a$ close to the axion frequency ω_a and keep the size $L = 2.7$ m fixed. At lower masses there may be some advantage in adjusting the geometry of the detector to achieve resonances through its spatial configuration. This would be especially interesting in 2D and 3D studies in future work. However, in the present setup, for the larger axion masses considered here, we found that if L is treated as a free parameter, the gradient descent simply increases L without limit to maximize the volume (and hence total stored energy) of the detector, which is a trivial form of optimization, and so we keep L fixed. In Fig. 2 we display the permittivities corresponding to the optimal gradient descent search for different numbers of layers and Fig. 3 shows how the power of the optimal design scales with the number of layers.

Unsurprisingly, one finds that periodic structure can enhance the signal relative to uniform plasma haloscopes. One can offer some words of explanation as to what is occurring physically. On the one hand the permittivity has the effect of altering the photons dispersion relation, so that

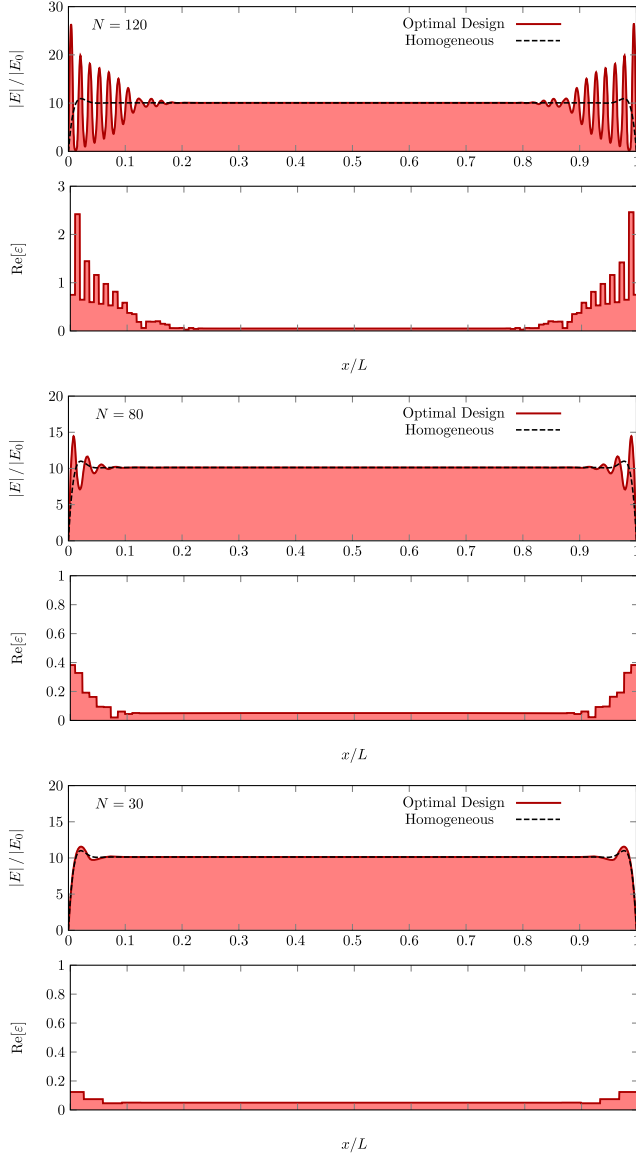


FIG. 2. Gradient descent designs. Optimal axion dark matter detector designs from applying a gradient descent search of designs. As an example we illustrate designs for $m_a = 150 \mu\text{eV}$ for differing number N of layers with $L = 60 \text{ cm}$ we display results for $N = 30, 80, 120$. We took $\Gamma = 0.1\omega_p$ and defined $E_0 = g_{a\gamma\gamma} a_0 B_0$. For each value of N we display the permittivities and the and the electric field of the optimal design (pink) and for the initial design (black dashed) on which the search was initiated with $\omega_p \simeq \omega_a$. For comparison, we also show the electric field (gray solid) of a homogeneous detector with $\omega_p = \omega_a$ considered in Ref. [38].

by matching the photon and axion masses a resonance is created. This is the effect exploited in [38] and would occur in an infinite medium without edge effects. On the other hand, the permittivity also plays the role of an effective potential for the photon. In this instance, we know that potential wells or periodic structure can be used to create additional resonances and constructive interferences [35].

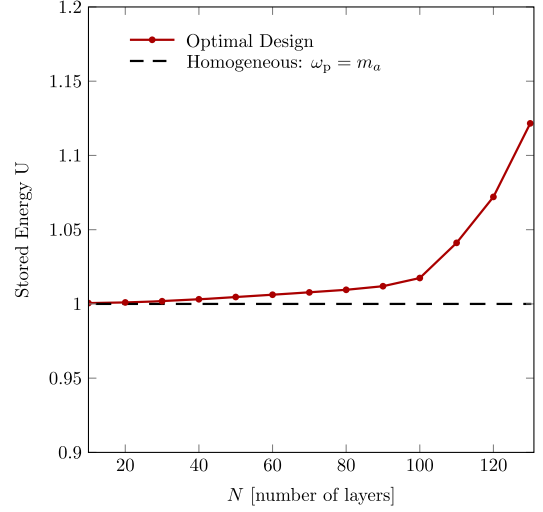


FIG. 3. Parametric dependence on N . The optimized detector stored energy U (red) given by performing a gradient descent search on the detector design for different values of N for fixed $m_a = 150 \mu\text{eV}$. We display U normalized to the homogeneous case with $\omega_p = m_a$ (black dashed) as considered in [38].

It seems that the optimization routine is harnessing the power of the hard reflective boundary at the endpoints of the detector and using periodic permittivity structures to create additional enhancements in the stored energy. Of course the optimal design must strike some balance between these two effects. The fact the edge effects and periodic structure do not persist into the bulk of the detector may be related to the finite propagation lengths associated to losses in the medium.

Of course widening the class of free parameters in future work could lead to an even greater variety of detector designs giving rise to more exotic detectors than those produced here. Quite remarkably, we also see that when there is sufficient substructure in the detector for larger N , the power can be slightly improved over that of a homogeneous plasma haloscope, motivating further study for a larger family of free parameters in the detector design. It is also worth emphasizing that gradient descent can be sensitive to the choice of initial conditions. Here we took as our starting point designs with plasma values close to a homogeneous case and evolved the optimization procedure from there. Gradient descent will of course seek a minimum close to the initial search point. A more exhaustive search of initial conditions has the possibility to reveal an even greater variety of optimal designs.

IV. CONCLUSIONS

The aim of this work was to open a discussion towards a new way of designing experiments in particle physics and address the question of what is the optimal detector design according to a gradient descent search of haloscope properties. In this work, we have illustrated a new gradient

descent approach to axion dark matter detector design using a simple setup. We have shown how the power of a detector can be maximized by choosing an optimal configuration of detector components and that the precise specifications are given straightforwardly by an iterative search. We achieved this by considering simple 1D haloscopes consisting of N layers of metamaterial components forming an effective plasma frequency $\omega_p^{(i)}$ for $i = 1, \dots, N$ and performed a gradient descent search over the $\omega_p^{(i)}$ to maximize the stored energy. We find that by beginning the search from near a uniform tuned haloscope with all plasma masses set to $\omega_p^{(i)} \simeq \omega_a$, an improvement in power can be achieved through periodic structures within the detector in a similar vein to the MADMAX design [35].

While the seed of any idea for a new experiment inevitably must overcome future practical challenges, the present study nonetheless opens up many more avenues for future work, including a more detailed study of higher dimensional 2D and 3D detector designs as alluded to in the footnote above, as well as a wider class of material properties including the magnetic permeability μ . In practice, antenna (coupling) and photon detectors have another big impact on scan speed. Explicitly including the readout antenna and optimizing its coupling to obtain maximal scan speed assuming a quantum limited amplifier would also be an interesting and important extension of the present work.

The existing example, and indeed these more complicated considerations may also benefit from more

sophisticated gradient descent searches than those considered here. This would facilitate a more exhaustive search of the detector design landscape and perhaps reveal new designs hidden in local minima whose location requires state-of-the-art machine learning techniques.

Although more work is needed to confront issues of tunability and incorporate practical limitations into the design procedure as constraints in the optimization, the scheme presented here may help to guide us in searching the landscape of detectors when planning our designs for the next generation of experiments. This is vital in order that we do not miss any opportunities for new designs hiding just around the corner.

ACKNOWLEDGMENTS

I am grateful for support from the F. R. S.-FNRS under the Excellence of Science (EOS) Project No. 30820817 (be.h). I also wish to thank Alex Millar and Stefan Knirck for discussions and their comments on the draft and Béla Majorovits for his time in answering various questions concerning haloscopes as well as John Pendry for correspondence on metamaterials and artificial wire plasmas. I also thank Michael Spannowsky for exchanges on gradient descent. I am indebted to Richard Battye for encouraging me to prepare this manuscript and to Shankar Srinivasan for his collaboration during the early stages of this project. I also thank the referee for insightful comments on the manuscript.

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