Erratum: $\pi\pi \to \pi\gamma^*$ amplitude and the resonant $\rho \to \pi\gamma^*$ transition from lattice QCD [Phys. Rev. D 93, 114508 (2016)]

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Since the publication of our paper [Phys. Rev. D **93**, 114508 (2016)], we have found a series of typographical mistakes in the equations shown. In particular, expressions written for the Lellouch-Lüscher factor and the transition amplitudes were those relevant for a basis of definite $\pi\pi$ isospin, while the numerical results presented were those for a basis of definite pion charge states. In this erratum, we explain and correct the typographical errors, which do not alter the numerical results presented.

Reference [1] determined the amplitude for the process $\pi^+\gamma \to \pi^+\pi^0$ where definite charge states of the pions are specified. As discussed in the text, this "charge-basis" amplitude differs from the amplitude where the final state is one of definite isospin $\pi^+\gamma \to (\pi\pi)_{I=1}$ by a Clebsch-Gordan factor of $1/\sqrt{2}$. Although this factor was explained correctly and highlighted in the paper, the expressions presented were systematically in the isospin basis, while the numerical results were those in the charge basis. This results in a factor of $1/\sqrt{2}$ discrepancy in amplitudes and a $1/2$ discrepancy in cross section between the expressions in the paper and the numerical results. Here we explain and correct the key expressions that were affected by this typographical mistake.

First, the generalized Lellouch-Lüscher factor, as derived in Ref. [2], depends on the basis choice. This factor labeled $\mathcal{R}$ was given in Ref. [1] in Eqs. (18) and (19). These equations should be replaced with

$$\frac{|\mathcal{H}^\mu_{\pi\pi,\pi}|^2}{|L^3\langle\pi,\Lambda_\pi;L|\mathcal{J}^\mu(0)|\pi\pi,\Lambda_{\pi\pi};L\rangle|^2} = \frac{2E_\pi}{\mathcal{R}},$$
$$= 2E_\pi \frac{8\pi}{q^\star_{\pi\pi}} E_{\pi\pi} \frac{\partial(\delta_1 + \phi)}{\partial E^\star_{\pi\pi}}, \tag{1}$$

where the prefactor in the second line was previously 32, but should instead be 16. The factor of r present in the corresponding equations in Ref. [1] has been set equal to unity.[1]

The second quantity impacted by the basis mismatch is the dynamical part of the transition amplitude, namely, $\mathcal{A}_{\pi\pi,\pi\gamma^\star}$. An on-shell representation of this was given in Eq. (25) in Ref. [1] in terms of the phase shift for P-wave $\pi\pi$ scattering, the energy-dependent decay width, and the energy-dependent transition form factor. This amplitude also depends on the basis chosen for the $\pi\pi$ state, and the expression given in Ref. [1] should be replaced with

$$\mathcal{A}_{\pi\pi,\pi\gamma^\star}(E^\star_{\pi\pi}, Q^2) = F(E^\star_{\pi\pi}, Q^2)\left(\frac{1}{\cot\delta_1(E^\star_{\pi\pi}) - i}\right)\sqrt{\frac{8\pi}{q^\star_{\pi\pi}\Gamma(E^\star_{\pi\pi})}}, \tag{2}$$

where the difference is an overall $1/\sqrt{2}$ factor.

The final physical quantity considered that is affected by the basis mismatch is the $\pi^+\gamma \to \pi^+\pi^0$ cross section. The expression written in Eq. (E4) of Ref. [1] requires no correction, but Eq. (E5) should be replaced with

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[1]The factor of r was proposed in Ref. [1] in order to introduce a systematic error associated with any mismatch between the lattice calculated energy levels and the finite-volume energy levels given by the scattering amplitude. As described in Ref. [3], this procedure can be shown to be ambiguous and not useful, and we instead set $r = 1$.

$$\sigma(\pi^+\gamma \to \pi^+\pi^0) = 8\pi\alpha \frac{q_i^\star}{m_\pi^2}|F(E_{\pi\pi}^\star,0)|^2 \frac{\sin^2\delta_1(E_{\pi\pi}^\star)}{\Gamma(E_{\pi\pi}^\star)}. \tag{3}$$

All numerical results presented in the paper correspond to the expressions for $\mathcal{R}$, $\mathcal{A}_{\pi\pi,\pi\gamma^\star}$, and $\sigma(\pi^+\gamma \to \pi^+\pi^0)$ given in this erratum and not those in Ref. [1].

An unrelated typographical error is also present in the manuscript in the expression for the radiative decay width $\Gamma(\rho^+ \to \pi^+\gamma)$ given in Eq. (E6) in Ref. [1]. This expression should be replaced with

$$\Gamma(\rho^+ \to \pi^+\gamma) = \alpha \frac{4}{3} \frac{q_\gamma^{\star 3}}{m_\pi^2}|F(m_\rho,0)|^2, \tag{4}$$

where the correction is to provide the correct power of $q_\gamma^\star$, the momentum of the photon in the ρ rest frame. The correct expression (given in this erratum) was used to generate the experimental photocoupling presented in Fig. 13 in Ref. [1], from the radiative decay width data in Ref. [4].

[1] R. A. Briceño, J. J. Dudek, R. G. Edwards, C. J. Shultz, C. E. Thomas, and D. J. Wilson, Phys. Rev. D **93**, 114508 (2016).
[2] R. A. Briceno, M. T. Hansen, and A. Walker-Loud, Phys. Rev. D **91**, 034501 (2015).
[3] R. A. Briceño, J. J. Dudek, and L. Leskovec, Phys. Rev. D **104**, 054509 (2021).
[4] L. Capraro, P. Levy, M. Querrou, B. Van Hecke, M. Verbeken *et al.*, Nucl. Phys. **B288**, 659 (1987).