# Angular asymmetries in $B \rightarrow \boldsymbol{\Lambda} \bar{p} M$ decays 

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#### Abstract

The forward-backward angular asymmetry $\left(\mathcal{A}_{\mathrm{FB}}\right)$ for $\bar{B}^{0} \rightarrow \Lambda \bar{p} \pi^{+}$measured by Belle has presented an experimental value in the range of $-30 \%$ to $-50 \%$. In our study, we find that $\mathcal{A}_{\mathrm{FB}}\left[\bar{B}^{0} \rightarrow \Lambda \bar{p} \pi^{+}\left(B^{-} \rightarrow\right.\right.$ $\left.\left.\Lambda \bar{p} \pi^{0}\right)\right]$ can be as large as $\left(-14.6_{-1.5}^{+0.9} \pm 6.9\right) \%$. In addition, we present $\mathcal{A}_{\mathrm{FB}}\left[\bar{B}^{0} \rightarrow \Lambda \bar{p} \rho^{+}\left(B^{-} \rightarrow \Lambda \bar{p} \rho^{0}\right)\right]=$ $\left(4.1_{-0.7}^{+2.8} \pm 2.0\right) \%$ as the first prediction involving a vector meson in the charmless $B \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime} M$ decays. While $\mathcal{A}_{\mathrm{FB}}(B \rightarrow \Lambda \bar{p} M)$ indicates an angular correlation caused by the rarely studied baryonic form factors in the timelike region, LHCb and Belle II are capable of performing experimental examinations.


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## I. INTRODUCTION

For the three-body baryonic $B \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime} M_{(c)}$ decays with $M_{(c)}$ denoting a (charmed) meson, the threshold effect has been commonly observed with a rapidly raising peak around the threshold area of $m_{\mathbf{B}^{\prime}} \simeq m_{\mathbf{B}}+m_{\overline{\mathbf{B}}^{\prime}}$ in the dibaryon invariant mass ( $m_{\mathbf{B} \overline{\mathbf{B}}^{\prime}}$ ) spectrum [1]. It is regarded to enhance the branching fraction of $B \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime} M_{(c)}\left[\mathcal{B}\left(B \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime} M_{(c)}\right)\right]$ $[2,3]$, such as $\mathcal{B}\left(B^{-} \rightarrow p \bar{p} M\right) \sim 10^{-6} \quad$ with $\quad M=$ $\left(\pi^{-}, K^{-}, K^{*-}\right)[4-7]$ and $\mathcal{B}\left(\bar{B}^{0} \rightarrow p \bar{p} D^{(*) 0}\right) \sim 10^{-4}[8,9]$. On the other hand, $\mathcal{B}\left(\bar{B}^{0} \rightarrow p \bar{p}\right)$ is as small as $10^{-8}$ [10,11], whose suppression reflects the fact that in the two-body baryonic $B$ decays the $\mathbf{B} \overline{\mathbf{B}}^{\prime}$ formation with $m_{\mathbf{B B}^{\prime}} \sim m_{B}$ is away from the threshold area. Theoretically, the baryonic form factors that parametrize the dibaryon formation have been used to describe the threshold effect [12-18], such that $\mathcal{B}\left(B \rightarrow \boldsymbol{B} \overline{\mathbf{B}}^{\prime} M_{(c)}\right)$ can be explained.

The partial branching fraction can be a function of $\cos \theta_{\mathbf{B}\left(\overline{\mathbf{B}}^{\prime}\right)}$, where $\theta_{\mathbf{B}\left(\overline{\mathbf{B}}^{\prime}\right)}$ is the angle between the (anti) baryon and meson moving directions in the $\mathbf{B} \overline{\mathbf{B}}^{\prime}$ rest frame. It leads to the forward-backward angular asymmetry: $\mathcal{A}_{\mathrm{FB}}\left(B \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime} M_{(c)}\right) \equiv\left(\mathcal{B}_{+}-\mathcal{B}_{-}\right) /\left(\mathcal{B}_{+}+\mathcal{B}_{-}\right)$, where $\mathcal{B}_{+}=\mathcal{B}\left(\cos \theta_{\mathbf{B}\left(\overline{\mathbf{B}}^{\prime}\right)}>0\right)$ and $\mathcal{B}_{-}=\mathcal{B}\left(\cos \theta_{\mathbf{B}\left(\overline{\mathbf{B}}^{\prime}\right)}<0\right)$. The

[^0]forward-backward asymmetries have been found in several decay channels $[5,6,19,20]$, of which the interpretations have caused theoretical difficulties $[18,21,22]$. This indicates that the dibaryon production in $B \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime} M$ has not been fully understood [23].

One has measured the angular asymmetries of $B \rightarrow \Lambda \bar{p} M_{(c)}$ versus $\cos \theta_{\bar{p}}$ in Refs. [19,20], that is,

$$
\begin{align*}
\mathcal{A}_{\mathrm{FB}}\left(\bar{B}^{0} \rightarrow \Lambda \bar{p} D^{+}\right) & =(-8 \pm 10) \%, \\
\mathcal{A}_{\mathrm{FB}}\left(\bar{B}^{0} \rightarrow \Lambda \bar{p} D^{*+}\right) & =(55 \pm 17) \%, \\
\mathcal{A}_{\mathrm{FB}}\left(\bar{B}^{0} \rightarrow \Lambda \bar{p} \pi^{+}\right) & =(-41 \pm 11 \pm 3) \%, \\
\mathcal{A}_{\mathrm{FB}}\left(B^{-} \rightarrow \Lambda \bar{p} \pi^{0}\right) & =(-16 \pm 18 \pm 3) \% . \tag{1}
\end{align*}
$$

According to the calculation [22], $\mathcal{A}_{\mathrm{FB}}\left(\bar{B}^{0} \rightarrow \Lambda \bar{p} D^{+}\right)=$ $(-3.0 \pm 0.2) \%$ is in agreement with the data; $\mathcal{A}_{\mathrm{FB}}\left(\bar{B}^{0} \rightarrow \Lambda \bar{p} D^{*+}\right)=(15.0 \pm 0.0) \%$ presents a sizeable asymmetry despite of two standard deviation compared to the data in Eq. (1). For $B \rightarrow \Lambda \bar{p} \pi$, different angular observables have been studied. One is the angular distribution of the cascade $B \rightarrow \bar{p} \pi(\Lambda \rightarrow) p \pi^{-}$decay versus $\cos \theta$ [24], where $\theta$ denotes the angle between proton and $B$ meson moving directions in the $\Lambda$ rest frame. Subsequently, C. K. Chua and W. S. Hou of Ref. [24] demonstrate that $\Lambda$ is dominantly a left-handed state [25], consistent with the experimental result in Ref. [26]. The other study is from Ref. [21], where S. Y. Tsai presents $\mathcal{A}_{\mathrm{FB}}\left(\bar{B}^{0} \rightarrow \Lambda \bar{p} \pi^{+}\right) \simeq 0$, not verified by the later measurement as in Eq. (1). Clearly, it indicates a sizeable angular correlation to be discovered in the charmless baryonic $B \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime} M$ decays. Moreover, the isospin symmetry should lead to $\mathcal{B}\left(\bar{B}^{0} \rightarrow\right.$ $\left.\Lambda \bar{p} \pi^{+}\right)=2 \mathcal{B}\left(B^{-} \rightarrow \Lambda \bar{p} \pi^{0}\right) \quad$ and $\quad \mathcal{A}_{\mathrm{FB}}\left(\bar{B}^{0} \rightarrow \Lambda \bar{p} \pi^{+}\right)=$ $\mathcal{A}_{\mathrm{FB}}\left(B^{-} \rightarrow \Lambda \bar{p} \pi^{0}\right)$ [24], which seem to disagree with $\mathcal{B}\left(\bar{B}^{0} \rightarrow \Lambda \bar{p} \pi^{+}\right)=(3.14 \pm 0.29) \times 10^{-6}[11,20,26]$,
$\mathcal{B}\left(B^{-} \rightarrow \Lambda \bar{p} \pi^{0}\right)=\left(3.0_{-0.6}^{+0.7}\right) \times 10^{-6} \quad[11,20]$, and angular asymmetries in Eq. (1) [20]. This suggests a possible isospin symmetry violation to be tested.

Compared to the charmful $\bar{B}^{0} \rightarrow \Lambda \bar{p} D^{(*)+}$ decay channels, where the $\Lambda \bar{p}$ formation is from the (axial)vector current, the penguin-dominant $B \rightarrow \Lambda \bar{p} \pi$ decay has an additional contribution from the (pseudo)scalar current. Consequently, there might exist an interference between the (axial)vector and (pseudo)scalar currents, which can cause a possible angular asymmetry. Therefore, we propose to investigate $B \rightarrow \Lambda \bar{p} \pi$, along with the rarely studied
baryonic form factors in the timelike region. We will also study $\mathcal{A}_{\mathrm{FB}}(B \rightarrow \Lambda \bar{p} \rho)$, which can be the first prediction involving a vector meson in the charmless $B \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime} M$ decays. The isospin relations will be discussed.

## II. FORMALISM

According to Fig. 1, where the decay process is drawn with the $B$ meson transition to a meson, along with the dibaryon production, the amplitude of $B \rightarrow \Lambda \bar{p} M$ can be factorized as [21,24,27-29]

$$
\begin{equation*}
\mathcal{M}(B \rightarrow \Lambda \bar{p} M)=\frac{G_{F}}{\sqrt{2}}\left[\alpha_{1}\langle\Lambda \bar{p}|(\bar{s} u)_{V-A}|0\rangle\langle M|(\bar{u} b)_{V-A}|B(b \bar{q})\rangle+\alpha_{6}\langle\Lambda \bar{p}|(\bar{s} u)_{S+P}|0\rangle\langle M|(\bar{u} b)_{S-P}|B(b \bar{q})\rangle\right], \tag{2}
\end{equation*}
$$

with $q=d$ and $u$ for $\bar{B}^{0} \rightarrow \Lambda \bar{p} \pi^{+}\left(\rho^{+}\right)$and $B^{-} \rightarrow$ $\Lambda \bar{p} \pi^{0}\left(\rho^{0}\right)$, respectively, where $G_{F}$ is the Fermi constant, $\left(\bar{q}_{i} q_{j}\right)_{V-A}=\bar{q}_{i} \gamma_{\mu}\left(1-\gamma_{5}\right) q_{j}, \quad\left(\bar{q}_{i} q_{j}\right)_{S \pm P}=\bar{q}_{i}\left(1 \pm \gamma_{5}\right) q_{j}$, and $|0\rangle$ in $\langle\Lambda \bar{p}|(\bar{s} u)|0\rangle$ denotes the vacuum state. We define $\alpha_{1}=V_{u b} V_{u s}^{*} a_{1}-V_{t b} V_{t s}^{*} a_{4}$ and $\alpha_{6}=V_{t b} V_{t s}^{*} 2 a_{6}$, where $V_{q_{i} q_{j}}$ are the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, and the parameters $a_{1,4,6}$ consist of the effective Wilson coefficients $c_{i}^{\text {eff }}$ [30], given by
$a_{1}=c_{1}^{\mathrm{eff}}+\frac{1}{N_{c}} c_{2}^{\mathrm{eff}}, \quad a_{4}=c_{4}^{\mathrm{eff}}+\frac{1}{N_{c}} c_{3}^{\mathrm{eff}}, \quad a_{6}=c_{6}^{\mathrm{eff}}+\frac{1}{N_{c}} c_{5}^{\mathrm{eff}}$,
with $N_{c}$ the color number.
In Eq. (2), the matrix elements of the $B$ to $\pi(\rho)$ transition can be written as [31]
$\langle\pi|(\bar{u} b)_{V-A}|B\rangle=p^{\mu} F_{\pi 1}+\frac{m_{B}^{2}-m_{\pi}^{2}}{t} q^{\mu}\left(F_{\pi 0}-F_{\pi 1}\right)$,
$\langle\rho|(\bar{u} b)_{V-A}|B\rangle=\epsilon_{\mu \nu \alpha \beta} \varepsilon^{* \nu} p_{B}^{\alpha} p_{\rho}^{\beta} \frac{2 V_{1}}{m_{+}}-i\left\{\left[\varepsilon_{\mu}^{*}-\frac{\varepsilon^{*} \cdot q}{t} q_{\mu}\right]\left(m_{+}\right) A_{1}+\frac{\varepsilon^{*} \cdot q}{t} q_{\mu}\left(2 m_{\rho}\right) A_{0}-\left[p_{\mu}-\frac{m_{B}^{2}-m_{\rho}^{2}}{t} q_{\mu}\right]\left(\varepsilon^{*} \cdot q\right) \frac{A_{2}}{m_{+}}\right\}$,
with $p^{\mu}=\left(p_{B}+p_{M}\right)^{\mu}, q^{\mu}=\left(p_{B}-p_{M}\right)^{\mu}, m_{+}=m_{B}+m_{M}$, $t \equiv q^{2}$, and $\varepsilon_{\mu}^{*}$ defined as the polarization four-vector of the $\rho$ meson, where $F_{A}=\left(F_{\pi 1}, V_{1}, A_{0}\right)$ and $F_{B}=\left(F_{\pi 0}, A_{1,2}\right)$ are the mesonic form factors. Using the equation of motion, we obtain $\langle\pi|(\bar{u} b)_{S-P}|B\rangle=\left(p \cdot q / m_{b}\right) F_{0}^{\pi}$ and $\quad\langle\rho|(\bar{u} b)_{S-P}|B\rangle=2 i\left(m_{\rho} / m_{b}\right) A_{0} \varepsilon^{*} \cdot q$. The form factor $F_{A(B)}$ can be given in the three-parameter representation [31]:

$$
\begin{align*}
F_{A}(t) & =\frac{F_{A}(0)}{\left(1-\frac{t}{M_{A}^{2}}\right)\left(1-\frac{\sigma_{1} t}{M_{A}^{2}}+\frac{\sigma_{2} t^{2}}{M_{A}^{4}}\right)}, \\
F_{B}(t) & =\frac{F_{B}(0)}{1-\frac{\sigma_{1} t}{M_{B}^{2}}+\frac{\sigma_{2} t^{2}}{M_{B}^{4}}}, \tag{5}
\end{align*}
$$

where $F_{A(B)}(0)$ is the form factor at the zero momentum transfer squared $(t=0), \sigma_{1,2}$ the parameters, and $M_{A(B)}$ the


FIG. 1. Feynman diagrams for $B \rightarrow \Lambda \bar{p} M$ that depict (a) tree and (b) penguin-level processes, respectively.
pole mass. One has determined $F_{A(B)}(0), \sigma_{1,2}$ and $M_{A(B)}$ with the results of the model calculation, such that the momentum transfer squared dependences for $F_{A(B)}$ can be described.

For the baryon-pair production, the matrix elements are in the forms of [17,24,27,32,33]

$$
\begin{align*}
\left\langle\mathbf{B} \overline{\mathbf{B}}^{\prime}\right| \bar{q} \gamma_{\mu} q^{\prime}|0\rangle & =\bar{u}\left[F_{1} \gamma_{\mu}+\frac{F_{2}}{m_{\mathbf{B}}+m_{\overline{\mathbf{B}}^{\prime}}} i \sigma_{\mu \nu} q_{\mu}\right] v, \\
\left\langle\mathbf{B} \overline{\mathbf{B}}^{\prime}\right| \bar{q} \gamma_{\mu} \gamma_{5} q^{\prime}|0\rangle & =\bar{u}\left[g_{A} \gamma_{\mu}+\frac{h_{A}}{m_{\mathbf{B}}+m_{\overline{\mathbf{B}}^{\prime}}} q_{\mu}\right] \gamma_{5} v, \\
\left\langle\mathbf{B} \overline{\mathbf{B}}^{\prime}\right| \bar{q} q^{\prime}|0\rangle & =f_{S} \bar{u} v, \\
\left\langle\mathbf{B} \overline{\mathbf{B}}^{\prime}\right| \bar{q} \gamma_{5} q^{\prime}|0\rangle & =g_{P} \bar{u} \gamma_{5} v, \tag{6}
\end{align*}
$$

where the spinor $u(v)$ represents the spin-1/2 (anti-)baryon state, $F_{\mathbf{B E}^{\prime}}=\left(F_{1,2}, g_{A}, h_{A}\right)$ and $\left(f_{S}, g_{P}\right)$ are the baryonic form factors in the timelike region. In the approach of pQCD counting rules, one derives that $F_{\mathbf{B}_{\overline{\mathbf{B}}^{\prime}}} \propto\left(\alpha_{s} / t\right)^{n}$ [12,12-15], where $n$ is to account for the gluon propagators that attach the valence quarks in $\mathbf{B} \overline{\mathbf{B}}^{\prime}$. Besides, $\alpha_{s}(t)=\left(4 \pi / \beta_{0}\right)\left[\ln \left(t / \Lambda_{0}^{2}\right)\right]^{-1}$ is the running coupling constant in the strong interaction [14], with the parameter $\beta_{0} \equiv 11-2 n_{f} / 3$, the flavor number $n_{f}=3$, and the scale factor $\Lambda_{0}=0.3 \mathrm{GeV}$. Subsequently, $\left(F_{1}, g_{A}, f_{S}, g_{P}\right)$ correspond to $n=2$; however, $F_{2}$ and $h_{A}$ need an additional gluon to flip the chirality, such that $n=3$. Explicitly, we present $F_{\mathbf{B}_{\overline{\mathbf{B}}^{\prime}}}$ as $[12,15,17,18]$

$$
\begin{align*}
& \left(F_{1}, g_{A}\right)=\frac{\left(C_{F_{1}}, C_{g_{A}}\right)}{t^{2}} \ln \left(\frac{t}{\Lambda_{0}^{2}}\right)^{-\gamma} \\
& \left(f_{S}, g_{P}\right)=\frac{\left(C_{f_{S}}, C_{g_{P}}\right)}{t^{2}} \ln \left(\frac{t}{\Lambda_{0}^{2}}\right)^{-\gamma} \\
& \left(F_{2}, h_{A}\right)=\frac{\left(C_{F_{2}}, C_{h_{A}}\right)}{t^{3}} \ln \left(\frac{t}{\Lambda_{0}^{2}}\right)^{-\gamma^{\prime}} \tag{7}
\end{align*}
$$

with $\gamma^{(\prime)}=2.148(3.148)$.

Using the $S U(2)$ helicity $\left[S U(2)_{h}\right]$ symmetry, $F_{1}$ and $g_{A}$ can be related. To this end, we parametrize $\left\langle\mathbf{B}_{R+L}\right| J_{\mu}^{R}\left|\mathbf{B}_{R+L}^{\prime}\right\rangle$ in the spacelike region as $[13,32]$

$$
\begin{equation*}
\left\langle\mathbf{B}_{R+L}\right| J_{\mu}^{R}\left|\mathbf{B}_{R+L}^{\prime}\right\rangle=\bar{u}\left[\gamma_{\mu} \frac{1+\gamma_{5}}{2} F_{R}+\gamma_{\mu} \frac{1-\gamma_{5}}{2} F_{L}\right] u, \tag{8}
\end{equation*}
$$

where $J_{\mu}^{R}=\left(V_{\mu}+A_{\mu}\right) / 2$ is a right-handed chiral current, $\left|\mathbf{B}_{R+L}^{(\prime)}\right\rangle \equiv\left|\mathbf{B}_{R}^{(\prime)}\right\rangle+\left|\mathbf{B}_{L}^{(\prime)}\right\rangle$, and $F_{R, L}$ the chiral form factors. Furthermore, we define $Q \equiv J_{0}^{R}$ as the chiral charge to act on the valence quark $q_{i}$ in $\mathbf{B}^{\prime}\left(q_{1} q_{2} q_{3}\right)$, such that one transforms $\mathbf{B}^{\prime}$ into $\mathbf{B}$. With the chirality that is regarded as the helicity at $t \rightarrow \infty$, the helicity of $q_{i}$ can be (anti)parallel $[\|(\overline{\|})]$ to the helicity of $\mathbf{B}^{\prime}$, such that we denote the chiral charge for $q_{i}$ by $Q_{\|(\|))}(i)(i=1,2,3)$. Thus, we obtain [13,32]

$$
\begin{align*}
\left(F_{R}, F_{L}\right) & =\left(e_{\|}^{R} F_{\|}+e_{\overline{\|}}^{R} F_{\overline{\|}}, e_{\|}^{L} F_{\|}+e_{\overline{\|}}^{L} F_{\bar{\Pi}}\right), \\
e_{\|(\overline{\Pi l})}^{R} & =\Sigma_{i}\left\langle\mathbf{B}_{R}\right| Q_{\|(\overline{\|})}(i)\left|\mathbf{B}_{R}^{\prime}\right\rangle, \\
e_{\|(\overline{\|})}^{L} & =\Sigma_{i}\left\langle\mathbf{B}_{L}\right| Q_{\|(\overline{\|})}(i)\left|\mathbf{B}_{L}^{\prime}\right\rangle \tag{9}
\end{align*}
$$

where $\quad F_{\|(\overline{\|})} \equiv C_{\|(\overline{(1)})} / t^{2}\left[\ln \left(t / \Lambda^{2}\right)\right]^{-\gamma}$. This results in $C_{F_{1}}\left(C_{g_{A}}\right)=\left(e_{\|}^{R} \pm e_{\|}^{L}\right) C_{\|}+\left(e_{\|}^{R} \pm e_{\|}^{L}\right) C_{\bar{\Pi}}$. In the crossing symmetry, since the spacelike form factors can be seen to behave as the timelike ones, the derivation can be applied to those in Eqs. (6) and (7). Similarly, we relate $f_{S}$ and $g_{P}$. It is hence obtained that

$$
\begin{align*}
& \left(C_{F_{1}}, C_{g_{A}}\right)=\sqrt{\frac{3}{2}}\left(C_{\|}, C_{\|}^{*}\right),\left(C_{f_{S}}, C_{g_{P}}\right)=-\sqrt{\frac{3}{2}}\left(\bar{C}_{\|}, \bar{C}_{\|}^{*}\right), \quad \text { for }\langle\Lambda \bar{p}|(\bar{s} u)_{V, A, S, P}|0\rangle \\
& \left(C_{F_{1}}, C_{g_{A}}\right)=\frac{1}{3}\left(4 C_{\|}-C_{\overline{\|}}, 4 C_{\|}^{*}+C_{\Pi}^{*}\right), \quad \text { for }\langle n \bar{p}|(\bar{d} u)_{V, A}|0\rangle \tag{10}
\end{align*}
$$

with $C_{\|(\overline{\|})}^{*} \equiv C_{\|(\overline{\|})}+\delta C_{\|(\overline{\|})}$ and $\bar{C}_{\|}^{*} \equiv \bar{C}_{\|}+\delta \bar{C}_{\|}$, where the second line for $\langle n \bar{p}|(\bar{d} u)_{V, A}|0\rangle$ is to include more data in the numerical analysis. Note that our derivation depends on the $S U(2)$ helicity $\left[S U(2)_{h}\right]$ symmetry at large $t$ $(t \rightarrow \infty)$, where quarks can be seen as massless particles. In the baryonic $B$ decay processes, since $t$ is ranging from
$\left(m_{\mathbf{B}}+m_{\overline{\mathbf{B}}^{\prime}}\right)^{2}$ to $\left(m_{B}-m_{M}\right)^{2}$, instead of $t \rightarrow \infty$, the fact that the quarks are no longer massless can induce the $S U(2)_{h}$ symmetry breaking. Hence, $\delta C_{\|(\overline{\|})}$ and $\delta \bar{C}_{\|}$are added in Eq. (10) to estimate the possible broken symmetry effect. One has derived $F_{2}=F_{1} /\left(t \ln \left[t / \Lambda_{0}^{2}\right]\right)$ in the pQCD model [34], which verifies the parametrization in Eq. (7).

By contrast, the model calculation for $h_{A}$ has not been available yet. In the $S U(3)$ flavor $\left[S U(3)_{f}\right]$ symmetry, $C_{h_{A}}$ can be related as [32]

$$
\begin{equation*}
C_{h_{A}}=-\frac{1}{\sqrt{6}}\left(C_{D}+3 C_{F}\right), \quad C_{h_{A}}=C_{D}+C_{F} \tag{11}
\end{equation*}
$$

for $\langle\Lambda \bar{p}|(\bar{s} u)_{A}|0\rangle$ and $\langle n \bar{p}|(\bar{d} u)_{A}|0\rangle$, respectively.
To integrate over the phase space in the three-body $B \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime} M$ decays, we adopt the equation as $[18,22,35]$

$$
\begin{equation*}
\Gamma=\int_{-1}^{+1} \int_{\left(m_{\mathbf{B}}+m_{\overline{\mathbf{B}}^{\prime}}\right)^{2}}^{\left(m_{B}-m_{M}\right)^{2}} \frac{\beta_{t}^{1 / 2} \lambda_{t}^{1 / 2}}{\left(8 \pi m_{B}\right)^{3}}|\overline{\mathcal{M}}|^{2} d t d \cos \theta \tag{12}
\end{equation*}
$$

where $\quad \beta_{t}=\left[1-\left(m_{\mathbf{B}}+m_{\overline{\mathbf{B}}^{\prime}}\right)^{2} / t\right]\left[1-\left(m_{\mathbf{B}}-m_{\overline{\mathbf{B}}^{\prime}}\right)^{2} / t\right]$, $\lambda_{t}=\left[\left(m_{B}+m_{M}\right)^{2}-t\right]\left[\left(m_{B}-m_{M}\right)^{2}-t\right]$, and $\Gamma$ represents the decay width. Moreover, $|\overline{\mathcal{M}}|^{2}$ denotes the squared amplitude summed over the baryon spins. We choose $\theta$ as the angle between $\overline{\mathbf{B}}^{\prime}$ and $M$ moving directions in the $\mathbf{B} \overline{\mathbf{B}}^{\prime}$ rest frame. Accordingly, the (anti)baryon energy can be a function of $\cos \theta$, given by

$$
\begin{align*}
E_{\mathbf{B}} & =\frac{t+m_{B}^{2}-m_{\pi}^{2}+\beta_{t}^{1 / 2} \lambda_{t}^{1 / 2} \cos \theta}{4 m_{B}} \\
E_{\overline{\mathbf{B}}^{\prime}} & =\frac{t+m_{B}^{2}-m_{\pi}^{2}-\beta_{t}^{1 / 2} \lambda_{t}^{1 / 2} \cos \theta}{4 m_{B}} \tag{13}
\end{align*}
$$

We reduce $|\bar{M}|^{2}(B \rightarrow \Lambda \bar{p} \pi)$ as

$$
\begin{align*}
|\bar{M}|^{2}(B \rightarrow \Lambda \bar{p} \pi) \simeq & a+b \cos \theta+c \cos ^{2} \theta \\
a= & 2\left|\alpha_{1}\right|^{2} F_{\pi 1}^{2}\left\{F_{1}^{2}\left(m_{B}^{2}-t\right)^{2}+g_{A}^{2}\left[4 m_{p}^{2}\left(2 m_{B}^{2}-t\right)+\left(m_{B}^{2}-t\right)^{2}\right]\right\} \\
& +2\left|\alpha_{6}\right|^{2}\left(m_{B}^{2} / m_{b}\right)^{2} F_{\pi 0}^{2}\left[f_{S}^{2}\left(t-4 m_{p}^{2}\right)+g_{P}^{2} t\right]-8 \Re\left(\alpha_{1} \alpha_{6}^{*}\right) F_{\pi 0} F_{\pi 1} g_{A} g_{P} m_{p}\left(m_{B}^{4} / m_{b}\right), \\
b= & -8 \Re\left(\alpha_{1} \alpha_{6}^{*}\right) F_{\pi 0} F_{\pi 1} F_{1} f_{S} m_{p}\left(m_{B}^{2} / m_{b}\right)\left[\left(t-4 m_{p}^{2}\right)\left(m_{B}^{2}-t\right)^{2} / t\right]^{1 / 2}, \\
c= & 2\left|\alpha_{1}\right|^{2}\left[\left(t-4 m_{p}^{2}\right)\left(m_{B}^{2}-t\right)^{2} / t\right] F_{\pi 1}^{2}\left(F_{1}^{2}+g_{A}^{2}\right), \tag{14}
\end{align*}
$$

with $m_{\Lambda}-m_{p} \simeq 0, m_{\pi} / m_{B} \simeq 0$ and $F_{\pi 0}-F_{\pi 1} \simeq 0$. Note that $|\bar{M}|^{2}(B \rightarrow \Lambda \bar{p} \pi)$ in the reduced form is for a simple presentation; however, no approximation is made in the real calculation. Similarly, we obtain

$$
\begin{align*}
|\bar{M}|^{2}(B \rightarrow \Lambda \bar{p} \rho) \simeq & a^{*}+b^{*} \cos \theta+c^{*} \cos ^{2} \theta \\
a^{*}= & \left|\alpha_{1}\right|^{2}\left(m_{B}^{2}-t\right)^{2} /\left(2 m_{\rho}^{2} m_{B}^{2} t\right)\left[A_{1} m_{B}^{2}-A_{2}\left(m_{B}^{2}-t\right)\right]^{2}\left[F_{1}^{2} t+g_{A}^{2}\left(t-4 m_{p}^{2}\right)\right] \\
& +2\left|\alpha_{6}\right|^{2} / m_{b}^{2}\left[A_{0}\left(m_{B}^{2}-t\right)\right]^{2}\left[f_{S}^{2}\left(t-4 m_{p}^{2}\right)+g_{P}^{2} t\right] \\
b^{*}= & 4 \Re\left(\alpha_{1} \alpha_{6}^{*}\right) m_{p} /\left(m_{\rho} m_{b} m_{B}\right)\left[\left(t-4 m_{p}^{2}\right)\left(m_{B}^{2}-t\right)^{2} / t\right]^{1 / 2}\left(m_{B}^{2}-t\right) A_{0}\left[A_{1} m_{B}^{2}-A_{2}\left(m_{B}^{2}-t\right)\right] F_{1} f_{S}, \\
c^{*}= & \left|\alpha_{1}\right|^{2} /\left(2 m_{\rho}^{2} m_{B}^{2}\right)\left[\left(t-4 m_{p}^{2}\right)\left(m_{B}^{2}-t\right)^{2} / t\right]\left[A_{1} m_{B}^{2}-A_{2}\left(m_{B}^{2}-t\right)\right]^{2}\left(F_{1}^{2}+g_{A}^{2}\right) . \tag{15}
\end{align*}
$$

For the angular asymmetry, we define

$$
\begin{equation*}
\mathcal{A}_{\mathrm{FB}} \equiv \frac{\int_{0}^{+1} \frac{d \Gamma}{d \cos \theta} d \cos \theta-\int_{-1}^{0} \frac{d \Gamma}{d \cos \theta} d \cos \theta}{\int_{0}^{+1} \frac{d \Gamma}{d \cos \theta} d \cos \theta+\int_{-1}^{0} \frac{d \Gamma}{d \cos \theta} d \cos \theta} \tag{16}
\end{equation*}
$$

where $d \Gamma / d \cos \theta$ is the angular distribution.

## III. NUMERICAL ANALYSIS

In the numerical analysis, we take the CKM matrix elements as [11]

$$
\begin{align*}
\left(V_{u d}, V_{u s}, V_{u b}\right) & =\left(1-\lambda^{2} / 2, \lambda, A \lambda^{3}(\rho-i \eta)\right) \\
\left(V_{t d}, V_{t s}, V_{t b}\right) & =\left(A \lambda^{3}(1-\rho-i \eta),-A \lambda^{2}, 1\right) \tag{17}
\end{align*}
$$

with $(\lambda, A, \rho, \eta)=(0.2265,0.790,0.145 \pm 0.017,0.366 \pm$ 0.011 ) in the Wolfenstein parametrization. We use the mesonic form factors from Ref. [31], which can be found in Table I. From Ref. [30], $c_{i}^{\text {eff }}(i=1,2, \ldots, 6)$ can lead to

$$
\begin{align*}
& \alpha_{1}=(-13.7-10.7 i,-15.2-11.4 i,-18.3-12.7 i), \\
& \alpha_{6}=(47.5+6.4 i, 49.6+6.9 i, 53.7+7.7 i) \tag{18}
\end{align*}
$$

with $N_{c}=(2,3, \infty)$, where $N_{c}$ is taken as a floating number, in order that the nonfactorizable QCD corrections can be estimated in the generalized edition of the factorization [30]. In Eqs. (10) and (11), there are totally eight constants that correspond to the baryonic form factors:

TABLE I. The form factors of $B \rightarrow \pi, \rho$ are adopted from Ref. [31], with $M_{A(B)}=5.32 \mathrm{GeV}$ and $\left(M_{A}, M_{B}\right)=$ $(5.27,5.32) \mathrm{GeV}$ for $\pi$ and $\rho$, respectively. In the second row, $F_{A}(0)$ and $F_{B}(0)$ correspond to $\left[F_{\pi 1}(t), V_{1}(t), A_{0}(t)\right]$ and [ $\left.F_{\pi 0}(t), A_{1,2}(t)\right]$, respectively, with the zero value of the momentum transfer squared $(t=0)$.

| $B \rightarrow \pi, \rho$ | $F_{\pi 0}$ | $F_{\pi 1}$ | $V_{1}$ | $A_{0}$ | $A_{1}$ | $A_{2}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $F_{A, B}(0)$ | 0.29 | 0.29 | 0.31 | 0.30 | 0.26 | 0.24 |
| $\sigma_{1}$ | 0.76 | 0.48 | 0.59 | 0.54 | 0.73 | 1.40 |
| $\sigma_{2}$ | 0.28 | $\cdots$ | $\cdots$ | $\cdots$ | 0.10 | 0.50 |

TABLE II. Fit results of the constants $\left(C_{i}\right)$ derived from the baryonic form factors, along with the $\chi^{2}$ value; n.d.f denotes the number of degrees of freedom.

| $\left(\chi^{2}\right.$, n.d.f, $\left.C_{i}\right)$ | Fit values |
| :--- | :---: |
| $\chi^{2}$ | 24.4 |
| n.d.f | 9 |
| $C_{\\|}$ | $(150.8 \pm 5.7) \mathrm{GeV}^{4}$ |
| $\delta C_{\\|}$ | $(31.9 \pm 7.1) \mathrm{GeV}^{4}$ |
| $C_{\overline{\\|}}$ | $(27.4 \pm 27.3) \mathrm{GeV}^{4}$ |
| $\delta C_{\bar{\Pi}}$ | $(-735.0 \pm 293.0) \mathrm{GeV}^{4}$ |
| $\bar{C}_{\\|}$ | $(511.2 \pm 74.4) \mathrm{GeV}^{4}$ |
| $\delta \bar{C}_{\\|}$ | $(-317.8 \pm 169.1) \mathrm{GeV}^{4}$ |
| $C_{D}$ | $(-761.1 \pm 128.0) \mathrm{GeV}^{6}$ |
| $C_{F}$ | $(905.7 \pm 119.8) \mathrm{GeV}^{6}$ |

$C_{\|}, \quad \delta C_{\|}, \quad C_{\bar{\Pi}}, \quad \delta C_{\bar{\Pi}}, \quad \bar{C}_{\|}, \quad \delta \bar{C}_{\|}, \quad C_{D}, \quad C_{F}$.
We perform the minimum $\chi^{2}$-fit to extract the constants, which includes the experimental inputs from $\mathcal{B}\left(\bar{B}^{0} \rightarrow\right.$ $\left.\Lambda \bar{p} \pi^{+}\right), \quad \mathcal{B}\left(B^{-} \rightarrow \Lambda \bar{p} \pi^{0}\left(\rho^{0}\right)\right), \quad \mathcal{A}_{\mathrm{FB}}\left(\bar{B}^{0} \rightarrow \Lambda \bar{p} \pi^{+}\right), \quad$ and $\mathcal{A}_{\mathrm{FB}}\left(B^{-} \rightarrow \Lambda \bar{p} \pi^{0}\right)$ in Table III, and the angular distribution of $\bar{B}^{0} \rightarrow \Lambda \bar{p} \pi^{+}$in Fig. 2; the branching fractions of

TABLE III. Branching fractions and angular asymmetries of the baryonic decay channels, where the first error of our results estimates the nonfactorizable effects, while the second one combines the uncertainties from CKM matrix elements and the hadronic parameters.

| Decay modes | Our results | Experimental data |
| :--- | :---: | :---: |
| $10^{6} \mathcal{B}\left(\bar{B}^{0} \rightarrow \Lambda \bar{p} \pi^{+}\right)$ | $3.2_{-0.3-1.1}^{+0.6+2.5}$ | $3.1 \pm 0.3[11,20]$ |
| $10^{6} \mathcal{B}\left(B^{-} \rightarrow \Lambda \bar{p} \pi^{0}\right)$ | $1.8_{-0.2-0.4}^{+0.3+1.6}$ | $3.0 \pm 0.7[11,20]$ |
| $10^{6} \mathcal{B}\left(\bar{B}^{0} \rightarrow \Lambda \bar{p} \rho^{+}\right)$ | $9.2_{-1.9-3.5}^{+0.9+5}$ |  |
| $10^{6} \mathcal{B}\left(B^{-} \rightarrow \Lambda \bar{p} \rho^{0}\right)$ | $5.0_{-1.0-1.9}^{+0.5+3.1}$ | $4.8 \pm 0.9[11,40]$ |
| $\mathcal{A}_{\mathrm{FB}}\left(\bar{B}^{0} \rightarrow \Lambda \bar{p} \pi^{+}\right)$ | $\left(-14.6_{-1.5}^{+0.9} \pm 6.9\right) \%$ | $(-41 \pm 11 \pm 3) \%[20]$ |
| $\mathcal{A}_{\mathrm{FB}}\left(B^{-} \rightarrow \Lambda \bar{p} \pi^{0}\right)$ | $\left(-14.6_{-1.5}^{+0.9} \pm 6.9\right) \%$ | $(-16 \pm 18 \pm 3) \%[20]$ |
| $\mathcal{A}_{\mathrm{FB}}\left(\bar{B}^{0} \rightarrow \Lambda \bar{p} \rho^{+}\right)$ | $\left(4.1_{-0.7}^{+2.8} \pm 2.0\right) \%$ |  |
| $\mathcal{A}_{\mathrm{FB}}\left(B^{-} \rightarrow \Lambda \bar{p} \rho^{0}\right)$ | $\left(4.1_{-0.7}^{+2.8} \pm 2.0\right) \%$ |  |



FIG. 2. The angular distribution of $\bar{B}^{0} \rightarrow \Lambda \bar{p} \pi^{+}$with the solid (dotted) line for the central value (error), where the data points are adopted from Ref. [20].
$\bar{B}^{0} \rightarrow n \bar{p} D^{*+}$ [36], $\bar{B}^{0} \rightarrow \Lambda \bar{p} D^{(*)+}$ [19], $B^{-} \rightarrow \Lambda \bar{p}$ [37], $D_{s}^{-} \rightarrow n \bar{p}[38,39]$, and $\mathcal{A}_{\mathrm{FB}}\left(\bar{B}^{0} \rightarrow \Lambda \bar{p} D^{(*)+}\right)$ [19] are also included. We thus present the results of the global fit in Table II.

With the extracted constants, we calculate the branching fractions and angular asymmetries of $B \rightarrow \Lambda \bar{p} M$, and draw the angular distribution of $\bar{B}^{0} \rightarrow \Lambda \bar{p} \pi^{+}$with $m_{\Lambda \bar{p}}<2.8 \mathrm{GeV}$, which are given in Table III and Fig. 2, respectively.

## IV. DISCUSSIONS AND CONCLUSIONS

We study the penguin-dominant $B \rightarrow \Lambda \bar{p} \pi$ decay with the branching fraction and angular asymmetry. With $\chi^{2} /$ n.d.f $\simeq 2.7$ calculated from Table II, it demonstrates that the theoretical study can accommodate the experimental data. Particularly, we find that $\Delta \chi^{2}=14.3$ that comes from $\mathcal{A}_{\mathrm{FB}}(B \rightarrow \Lambda \bar{p} \pi)$ and five data points of $d \mathcal{B}\left(\bar{B}^{0} \rightarrow\right.$ $\left.\Lambda \bar{p} \pi^{+}\right) / d \cos \theta$ gives sizeable contribution to the total $\chi^{2}$ value, indicating that more accurate measurements of the angular distribution (asymmetry) can improve the fit.

In Eq. (14), $|\bar{M}|^{2}(B \rightarrow \Lambda \bar{p} \pi) \simeq a+b \cos \theta+c \cos ^{2} \theta$ can be useful for our investigation. It is found that $2\left|\alpha_{6}\right|^{2}\left(m_{B}^{2} / m_{b}\right)^{2} F_{\pi 0}^{2}\left[f_{S}^{2}\left(t-4 m_{p}^{2}\right)+g_{P}^{2} t\right]$ in the $a$ term gives the main contribution to the total branching fraction. In the $b$ term, $-8 \Re\left(\alpha_{1} \alpha_{6}^{*}\right) F_{\pi 0} F_{\pi 1} F_{1} f_{S} m_{p}\left(m_{B}^{2} / m_{b}\right)$ $\left[\left(t-4 m_{p}^{2}\right)\left(m_{B}^{2}-t\right)^{2} / t\right]^{1 / 2}$ is responsible for the angular asymmetry. However, the $c$ term with $\left|\alpha_{1}\right|^{2}$ is insignificant. As a consequence, we obtain $\mathcal{B}\left(\bar{B}^{0} \rightarrow \Lambda \bar{p} \pi^{+}\right)=$ $\left(3.2_{-0.3-1.1}^{+0.6+2.5}\right) \times 10^{-6}$. Besides, we obtain $\mathcal{A}_{\mathrm{FB}}\left(\bar{B}^{0} \rightarrow \Lambda \bar{p} \pi^{+}\right)=$ $\left(-14.6_{-1.5}^{+0.9} \pm 6.9\right) \%$ that has 2 standard deviation departure from the experimental value of $(-41 \pm 11 \pm 3) \%$.

Since we get $C_{g_{P}}=(0.38 \pm 0.37) C_{f_{S}}$ different from $C_{g_{P}}=C_{f_{S}}$ in the $S U(2)$ helicity symmetry at $t \rightarrow \infty$, it clearly indicates a broken symmetry effect with $\delta \bar{C}_{\|}$.

Currently, $\delta \bar{C}_{\|}$is determined with a large uncertainty, which reflects the fact that $\mathcal{A}_{\mathrm{FB}}(B \rightarrow \Lambda \bar{p} \pi)$ and the angular distribution in Fig. 2 have not been precisely measured. Without a model calculation, we obtain $C_{h_{A}}$ from the fit. It is found that $C_{h_{A}}=-798.6 \mathrm{GeV}^{6}$ for $\langle\Lambda \bar{p}|(\bar{s} u)_{A}|0\rangle$ gives $2.8 \%$ of $\mathcal{B}\left(\bar{B}^{0} \rightarrow \Lambda \bar{p} \pi^{+}\right)$and $3.4 \%$ of $\mathcal{A}_{\mathrm{FB}}\left(\bar{B}^{0} \rightarrow \Lambda \bar{p} \pi^{+}\right)$.

The angular asymmetry of $\bar{B}^{0} \rightarrow \Lambda \bar{p} \pi^{+}$decay was once studied in Ref. [21], where $\mathcal{A}_{\mathrm{FB}}\left(\bar{B}^{0} \rightarrow \Lambda \bar{p} \pi^{+}\right) \simeq 0$ is not verified by the observation. The cause is that $g_{P}=f_{S}$ as a strong relation has been used, such that $g_{P}$ with $2\left|\alpha_{6}\right|^{2}\left[f_{S}^{2}\left(t-4 m_{p}^{2}\right)+g_{P}^{2} t\right]$ in the $a$ term becomes the dominant form factor in the branching fraction. By contrast, $f_{S}$ turns out to be a less important form factor both in the $a$ and $b$ terms, leading to $\mathcal{A}_{\mathrm{FB}}\left(\bar{B}^{0} \rightarrow \Lambda \bar{p} \pi^{+}\right) \simeq 0$.

For the first time, we study the angular asymmetry of the charmless $B \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime} M$ decay with $M$ as a vector meson. We predict $\mathcal{A}_{\mathrm{FB}}\left(B^{-} \rightarrow \Lambda \bar{p} \rho^{0}\right)=\left(4.1_{-0.7}^{+2.8} \pm 2.0\right) \%$. It is interesting to note that $\mathcal{A}_{\mathrm{FB}}(B \rightarrow \Lambda \bar{p} \rho)$ is not as large as $\mathcal{A}_{\mathrm{FB}}(B \rightarrow \Lambda \bar{p} \pi)$. This is due to $A_{1} \simeq A_{2}$, which suppresses $4 \Re\left(\alpha_{1} \alpha_{6}^{*}\right) m_{p} /\left(m_{\rho} m_{b} m_{B}\right)\left[\left(t-4 m_{p}^{2}\right)\left(m_{B}^{2}-t\right)^{2} / t\right]^{1 / 2}\left(m_{B}^{2}-\right.$ t) $A_{0}\left[A_{1} m_{B}^{2}-A_{2}\left(m_{B}^{2}-t\right)\right] F_{1} f_{S}$ in the $b^{*}$ term, resulting in a suppressed angular asymmetry.

We find no source to violate the isospin relation. Since $\pi^{0}\left(\rho^{0}\right)=(u \bar{u}-d \bar{d}) / \sqrt{2}$ and $\pi^{+}\left(\rho^{+}\right)=u \bar{d}$, it results in $\sqrt{2}\left\langle\pi^{0}\left(\rho^{0}\right)\right|(\bar{u} b)\left|B^{-}\right\rangle=\left\langle\pi^{+}\left(\rho^{+}\right)\right|(\bar{u} b)\left|\bar{B}^{0}\right\rangle \quad[24,41]$. We hence obtain

$$
\begin{align*}
\mathcal{B}\left(\bar{B}^{0} \rightarrow \Lambda \bar{p} \pi^{+}\left(\rho^{+}\right)\right) & \simeq 2 \mathcal{B}\left(B^{-} \rightarrow \Lambda \bar{p} \pi^{0}\left(\rho^{0}\right)\right), \\
\mathcal{A}_{\mathrm{FB}}\left(\bar{B}^{0} \rightarrow \Lambda \bar{p} \pi^{+}\left(\rho^{+}\right)\right) & =\mathcal{A}_{\mathrm{FB}}\left(B^{-} \rightarrow \Lambda \bar{p} \pi^{0}\left(\rho^{0}\right)\right), \tag{20}
\end{align*}
$$

which suggests $\mathcal{B}\left(\bar{B}^{0} \rightarrow \Lambda \bar{p} \rho^{+}\right) \simeq 10^{-5}$ that has not been measured yet.

In summary, we have investigated the angular asymmetry of $B \rightarrow \Lambda \bar{p} M$. In particular, we have obtained $\mathcal{A}_{\mathrm{FB}}\left(\bar{B}^{0} \rightarrow\right.$ $\left.\Lambda \bar{p} \pi^{+}\right)=\left(-14.6_{-1.5}^{+0.9} \pm 6.9\right) \%$ with 2 standard deviation departure from the experimental value of $(-41 \pm 11 \pm 3) \%$. We have hence reduced the deviation caused by $\mathcal{A}_{\mathrm{FB}}\left(\bar{B}^{0} \rightarrow\right.$ $\left.\Lambda \bar{p} \pi^{+}\right) \simeq 0$ previously studied in the literature. We have calculated $\mathcal{A}_{\mathrm{FB}}\left(B^{-} \rightarrow \Lambda \bar{p} \rho^{0}\right)=\left(4.1_{-0.7}^{+2.8} \pm 2.0\right) \%$, which can be the first prediction for the charmless $B \rightarrow \mathbf{B} \overline{\mathbf{B}}^{\prime} M$ decay with $M$ as a vector meson. According to the isospin relation, it has been calculated that $\mathcal{B}\left(\bar{B}^{0} \rightarrow \Lambda \bar{p} \rho^{+}\right)=\left(9.2_{-1.9-3.5}^{+0.9+5.7}\right) \times 10^{-6}$, promising to be measured by the LHCb and Belle II experiments.

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