Transverse positron polarization in the polarized μ^+ decay related with the muonium-to-antimuonium transition

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The constructions of the new high-intensity muon beamlines are progressing in facilities around the world, and new physics searches related to the muons are expected. The facilities can observe the transverse positron polarization of the polarized μ^+ decay to test the standard model. The transition of muonium into antimuonium (Mu-to-$\overline{\text{Mu}}$ transition), which is one of the interesting possibilities in the models beyond the standard model, can be also tested. An observation of the transition in the near future would have a great impact since it would indicate that there is an approximate discrete symmetry in the lepton sector. If the Mu-to-$\overline{\text{Mu}}$ transition operator is generated, a new muon decay operator can exist, and it may interfere with the standard model muon decay operator to induce the corrections to the transverse positron polarization in the μ^+ decay. We examine the possibility that the Mu-to-$\overline{\text{Mu}}$ transition and the correction to the transverse positron polarization are related, and we show that the two are related in the model of a neutral flavor gauge boson. We also investigate the models to generate the Mu-to-$\overline{\text{Mu}}$ transition, such as an inert $SU(2)_L$ doublet, an $SU(2)_L$ triplet for the type-II seesaw model, a dilepton gauge boson, and a left-right model. The nonzero value of the transverse polarization for one of the two directions, P_{T_2}, violates the time-reversal invariance, and the experimental constraint of the electron electric dipole moment can provide a severe constraint on P_{T_2}, depending on the model.

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I. INTRODUCTION

The operating high-intensity muon beamlines at Japan Proton Accelerator Research Complex (J-PARC) are being upgraded [1], and muon fundamental properties, such as the anomalous magnetic moment $(g-2)$ and the electric dipole moment (EDM), will be accurately examined [2]. The high-intensity muon beamlines are being planned [3]. The facilities can produce muonium (a bound state of μ^+e^-), and they will examine the muonium-to-antimuonium (Mu-to-$\overline{\text{Mu}}$) transition [4,5], which is an interesting phenomenological possibility with lepton flavor violation (LFV) [6–9]. Using the beamline, the transverse positron polarization in the polarized μ^+ decay [10–13] will be also measured to find clues about new physics [14].

LFV is one of the keys to new physics in the lepton sector since it directly indicates that there is a new particle and interaction beyond the standard model (SM) at the TeV scale. LFV processes such as $\mu \to e\gamma$ and $\mu \to 3e$ decays and $\mu - e$ conversion in nuclei have not yet been observed, and nonobservation only gives severe bounds to the model parameters at present [15–17]. We remark that the absence of such ΔL_e, $\Delta L_\mu = \pm 1$ processes does not necessarily mean that there is no new physics at the TeV scale. Even if these processes are absent, there is still plenty of room left for new physics in the lepton sector. Unlike the quark sector, the lepton sector may have a high affinity with discrete symmetry; e.g., the atmospheric neutrino mixing is nearly maximal. Although it is surely important to search for ΔL_e, $\Delta L_\mu = \pm 1$ processes, we need a close examination of the models beyond the SM so as not to have a preconception about those processes being dominant in new physics with LFV. Indeed, if there is an approximate discrete flavor symmetry, the Mu-to-$\overline{\text{Mu}}$ transition as a ΔL_e, $\Delta L_\mu = \pm 2$ process can be important in finding new physics in the lepton sector while the ΔL_e, $\Delta L_\mu = \pm 1$ processes are suppressed. Since the constraints from the

ΔL_e, $\Delta L_\mu = \pm 1$ processes to obtain the Mu-to-$\overline{\text{Mu}}$ transition were intensively investigated in Ref. [18], we assume that the ΔL_e, $\Delta L_\mu = \pm 1$ processes are absent due to a discrete symmetry for simplicity in this paper.

Let us suppose that the Mu-to-$\overline{\text{Mu}}$ transition rate is just below the current experimental bound. We take notice of the existence of a new muon decay operator, if at least one of the two muons and one of the two electrons are left-handed in the transition operator. The coupling strength of the four-fermion operator is less than $O(10^{-3})$ in the unit of the Fermi coupling constant of the $(V-A)\times(V-A)$ muon decay in the SM, from the experimental result at the Paul Scherrer Institute (PSI) [19]. If the new effective operator for the muon decay is of the type $(S-P)\times(S+P)$, the interference of the decay amplitudes can contribute to the transverse polarization of the $e^\pm$ from the polarized $\mu^\pm$ decay in the primary order of the new physics. Although the new coupling is bounded by the result of the Mu-to-$\overline{\text{Mu}}$ transition, near-future experiments have the potential to observe the contributions from the new physics in the transverse positron polarization. Actually, the high-intensity muon beam facility can examine both the Mu-to-$\overline{\text{Mu}}$ transition and the transverse positron polarization in the polarized μ^+ decay.

There are two independent transverse directions, and the positron polarizations are named P_{T_1} and P_{T_2}. Let $\boldsymbol{k}_e$ be the momentum of the positron, and let $\boldsymbol{P}_\mu$ be the polarization vector (which specifies the degree and the direction of the polarization) of μ^+ at rest. The direction of P_{T_2} is defined to be that of $\boldsymbol{k}_e \times \boldsymbol{P}_\mu$. A nonzero value of P_{T_2} violates the time-reversal invariance (namely, CP invariance), and P_{T_2} is extremely small in the SM. On the other hand, P_{T_1} is nonzero even in the SM, and its size is $\sim m_e/m_\mu$ and becomes smaller for larger positron energy. Observing the transverse polarizations serves as a useful test of the fundamental interaction in the lepton sector.

In this paper, we examine the relation between the Mu-to-$\overline{\text{Mu}}$ transition and the transverse positron polarization in the polarized μ^+ decay. The four-lepton operators (without right-handed neutrinos) are generated by the tree-level exchange of the inert $SU(2)_L$ doublet, $SU(2)_L$ triplet, dilepton gauge boson, and neutral flavor gauge boson. We show that they are related in the case of the neutral flavor gauge boson, and the model could be tested in a near-future experiment. The CP phases in the models are severely bounded by the experimental constraint of the electron EDM (eEDM). We discuss whether the eEDM can allow a nonzero P_{T_2} in the models. We also describe the transverse positron polarization in the left-right model.

This paper is organized as follows: In Sec. II, we review the formulation of the muon decay and give the formula for the transverse polarization of the decayed $e^\pm$ in the polarized $\mu^\pm$ decay. In Sec. III, we review the expressions of the Mu-to-$\overline{\text{Mu}}$ transition. In Sec. IV, we describe the model of the neutral flavor gauge boson and show the relation between the Mu-to-$\overline{\text{Mu}}$ transition and the transverse positron polarization in the μ^+ decay. In Sec. V, we consider other models which can generate the transverse positron polarization through the tree-level exchange of mediators. Section VI is the conclusion. In Appendix A, we will give expressions of the Fierz transformation of the muon decay operators for Majorana neutrinos. In Appendix B, we comment on the physical background of the model discussed in Sec. IV. In Appendix C, we describe the constraints on the heavy-light neutrino mixing to evaluate the transverse positron polarization from the muon decay operators with right-handed neutrinos.

II. TRANSVERSE POLARIZATION OF THE DECAYED $e^\pm$ IN THE $\mu^\pm$ DECAY

In this section, we review the formalism for the transverse polarizations of the decayed $e^\pm$ in the polarized $\mu^\pm$ decay [10,11,13]. We follow the convention given in the review by the Particle Data Group [20]. In general, we can write the four-fermion interaction for the $\mu \to e\nu_\mu\bar{\nu}_e$ decay in the Lagrangian [21] as

$$-\mathcal{L}_{\mu\to e\nu_\mu\bar{\nu}_e} = \frac{4G_F}{\sqrt{2}} \sum_{\gamma=S,V,T} \sum_{\epsilon,m=R,L} g^\gamma_{\epsilon m} (\bar{e}_\epsilon \Gamma^\gamma \nu_e)(\bar{\nu}_\mu \Gamma_\gamma \mu_m) + \text{H.c.}, \tag{2.1}$$

where G_F is the Fermi constant, $\Gamma_S = \mathbf{1}$, $\Gamma_V = \gamma_\mu$, and $\Gamma_T = \sigma_{\mu\nu}/\sqrt{2}$. For simplicity, we refer to the operators using the dimensionless couplings as $g^\gamma_{\epsilon m}$. Since the two operators of g^T_{LL} and g^T_{RR} are identically zero, there are ten independent couplings. The standard model corresponds to $g^V_{LL} = 1$, with the other couplings being zero. We note that the flavor violating "wrong" muon decay $\mu \to e\nu_e\bar{\nu}_\mu$ can interfere with the $\mu \to e\nu_\mu\bar{\nu}_e$ decay if the neutrinos are Majorana fermions. We list the Fierz transformation of the operators for the Majorana neutrinos in Appendix A.

When we neglect the radiative correction, the differential decay rate of $\mu^\pm$ is given by

$$\frac{d^2\Gamma}{dx\, d\cos\theta} = \frac{\bar{G}_F^2}{4\pi^3} m_\mu W^4_{e\mu} \sqrt{x^2 - x_0^2} \{F_{\mathrm{IS}}(x) \pm P_\mu \cos\theta F_{\mathrm{AS}}(x)\}\{1 + \hat{\zeta}\cdot \boldsymbol{P}_e(x,\theta)\} \tag{2.2}$$

for the emitted $e^\pm$ with its spin parallel to the arbitrary direction $\hat{\zeta}$. Here P_μ is the magnitude of the $\mu^\pm$ polarization vector $\boldsymbol{P}_\mu$ and θ is the angle between $\boldsymbol{P}_\mu$ and the $e^\pm$ momentum $\boldsymbol{k}_e$. Defining the maximal $e^\pm$ energy as

$$W_{e\mu} = \frac{m_\mu^2 + m_e^2}{2m_\mu}, \tag{2.3}$$

we use the dimensionless variables

$$x = \frac{E_e}{W_{e\mu}}, \qquad x_0 = \frac{m_e}{W_{e\mu}} \tag{2.4}$$

instead of the energy E_e and the rest energy $E_0 = m_e$. The allowed value of x is between x_0 and 1. We note that $\bar{G}_F$ in Eq. (2.2) includes the new physics effect $\bar{G}_F^2 = G_F^2 A/16$. See below for the parameter $A/16 \simeq 1$. The muon decay constant determined by the muon lifetime is $\bar{G}_F$. As we will note in Sec. III, the quadratic corrections from the new muon decay couplings are bounded from the precision data relating to the universality of the weak couplings.

The polarization vector $\boldsymbol{P}_e$ of $e^\pm$ is defined using the differential decay rate in Eq. (2.2), and the transverse components of $\boldsymbol{P}_e$ are defined as

$$P_{\mathrm{T}_1} \equiv \hat{x}_1 \cdot \boldsymbol{P}_e, \qquad P_{\mathrm{T}_1} \equiv \hat{x}_2 \cdot \boldsymbol{P}_e, \tag{2.5}$$

where the following three unit vectors are defined using the two specific directions $\boldsymbol{k}_e$ and $\boldsymbol{P}_\mu$:

$$\hat{x}_3 = \frac{\boldsymbol{k}_e}{|\boldsymbol{k}_e|}, \qquad \hat{x}_2 = \frac{\boldsymbol{k}_e \times \boldsymbol{P}_\mu}{|\boldsymbol{k}_e \times \boldsymbol{P}_\mu|}, \qquad \hat{x}_1 = \hat{x}_2 \times \hat{x}_3. \tag{2.6}$$

The transverse components can be written as

$$P_{\mathrm{T}_1}(x,\theta) = \frac{P_\mu \sin\theta F_{\mathrm{T}_1}(x)}{F_{\mathrm{IS}}(x) \pm P_\mu \cos\theta F_{\mathrm{AS}}(x)}, \tag{2.7}$$

$$P_{\mathrm{T}_2}(x,\theta) = \frac{P_\mu \sin\theta F_{\mathrm{T}_2}(x)}{F_{\mathrm{IS}}(x) \pm P_\mu \cos\theta F_{\mathrm{AS}}(x)}, \tag{2.8}$$

where the functions F_{IS}, F_{AS}, F_{T_1}, and F_{T_2} depend on the coefficients $g^\gamma_{\epsilon m}$. Instead of using the coefficients directly, it is practical to define the muon decay parameters for the spectrum and the transverse polarization [10,22–24]. For example, the function F_{T_2} is written as

$$F_{\mathrm{T}_2}(x) = \frac{1}{3}\sqrt{x^2 - x_0^2}\left[3\frac{\alpha'}{A}(1-x) + 2\frac{\beta'}{A}\sqrt{1-x_0^2}\right], \tag{2.9}$$

where

$$\alpha' = 8\mathrm{Im}[g^V_{LR}(g^{S*}_{RL} + 6g^{T*}_{RL}) - g^V_{RL}(g^{S*}_{LR} + 6g^{T*}_{LR})], \tag{2.10}$$

$$\beta' = 4\mathrm{Im}[g^V_{RR} g^{S*}_{LL} - g^V_{LL} g^{S*}_{RR}]. \tag{2.11}$$

It is important to note that the nonzero value of P_{T_2} indicates the CP violation in the interaction. See Refs. [11,13,20] for the definition of the other CP conserving parameters ($a, a', b, b', c, c', \alpha, \beta$, or their recombination $\rho, \delta, \eta, \eta', \xi, \xi', \xi'', A$) and the functions F_{IS}, F_{AS}, and F_{T_1}. Here we write about the case when there are only the following relevant operators:

$$\begin{aligned} -\mathcal{L}_{\mu\to e\nu_\mu\bar{\nu}_e} = \frac{4G_F}{\sqrt{2}}[&g^V_{LL}(\bar{e}\gamma^\alpha P_L \nu_e)(\bar{\nu}_\mu \gamma_\alpha P_L \mu) \\ &+ g^S_{RR}(\bar{e}P_L\nu_e)(\bar{\nu}_\mu P_R \mu) \\ &+ g^V_{RR}(\bar{e}\gamma^\alpha P_R \nu_e)(\bar{\nu}_\mu(x)\gamma_\alpha P_R \mu) \\ &+ g^S_{LL}(\bar{e}P_R\nu_e)(\bar{\nu}_\mu P_L \mu)] + \mathrm{H.c.} \end{aligned} \tag{2.12}$$

We obtain

$$a = a' = \alpha = \alpha' = c = c' = 0, \tag{2.13}$$

$$b = 4(|g^V_{LL}|^2 + |g^V_{RR}|^2) + |g^S_{RR}|^2 + |g^S_{LL}|^2, \tag{2.14}$$

$$b' = -4(|g^V_{LL}|^2 - |g^V_{RR}|^2) + |g^S_{RR}|^2 - |g^S_{LL}|^2, \tag{2.15}$$

$$\beta = 4\mathrm{Re}[-g^V_{RR} g^{S*}_{LL} - g^V_{LL} g^{S*}_{RR}], \tag{2.16}$$

$$\beta' = 4\mathrm{Im}[g^V_{RR} g^{S*}_{LL} - g^V_{LL} g^{S*}_{RR}], \tag{2.17}$$

and the parameters for the muon decay spectrum (at the tree level) are given by

$$\rho = \delta = \frac{3}{4}, \qquad \eta = \eta' = 0, \qquad \xi'' = 1, \tag{2.18}$$

$$\xi = \xi' = -\frac{4b'}{A}, \tag{2.19}$$

$$A \equiv a + 4b + 6c = 4b. \tag{2.20}$$

We obtain the following functions for the spectrum and the transverse polarization of the emitted $e^\pm$:

$$F_{\mathrm{IS}}(x) = \frac{1}{6}(-2x^2 + 3x - x_0^2) - \frac{2\beta}{A}(1-x)x_0, \tag{2.21}$$

$$F_{\mathrm{AS}}(x) = \frac{\xi}{6}\sqrt{x^2 - x_0^2}\left(2x - 2 + \sqrt{1-x_0^2}\right), \tag{2.22}$$

$$F_{\mathrm{T}_1}(x) = -\frac{1}{6}(1-x)x_0 + \frac{2\beta}{3A}(x - x_0^2), \tag{2.23}$$

$$F_{\mathrm{T}_2}(x) = \frac{2\beta'}{3A}\sqrt{(1-x_0^2)(x^2 - x_0^2)}. \tag{2.24}$$

As can be seen in Eq. (2.16), the g^S_{RR} operator can directly interfere with the g^V_{LL} operator in the SM, and thus it can provide the primary contribution from the new physics beyond the SM. The other contributions are all quadratic (including all the other operators). Since x_0 in F_{IS} is small ($x_0 = 9.67 \times 10^{-3}$) due to $m_e \ll m_\mu$, it is important to observe P_{T_1} and P_{T_2} to extract the primary effect in the muon decay. An analysis of the current experimental results at PSI shows that [12,13,25]

$$\frac{\beta}{A} = (1.1 \pm 3.5(\text{statistical}) \pm 0.5(\text{systematic})) \times 10^{-3}, \tag{2.25}$$

$$\frac{\beta'}{A} = (-1.3 \pm 3.5(\text{statistical}) \pm 0.6(\text{systematic})) \times 10^{-3} \tag{2.26}$$

if g^S_{RR} is the only source of the new physics. The experimental accuracy has not yet reached the size of the SM contribution in P_{T_1}. Although the current experimental bounds for β, β' are still loose, we expect the hundred-times-intense new muon beamlines in the world to develop the observation of the transverse positron polarization for $\beta/A, \beta'/A \sim 10^{-3}$. The sensitivity of the order of 10^{-3} of those values in the near-future experiment at J-PARC is also expected from the simulation in Ref. [14].

III. MU-TO-$\overline{\mathrm{Mu}}$ TRANSITION

The effective Mu-to-$\overline{\mathrm{Mu}}$ transition operators in the Lagrangian are given as [26]

$$\begin{aligned} -\mathcal{L}_{\text{Mu-}\overline{\text{Mu}}} = \frac{4G_F}{\sqrt{2}} &[g_1(\bar{\mu}\gamma^\alpha P_L e)(\bar{\mu}\gamma_\alpha P_L e) \\ &+ g_2(\bar{\mu}\gamma^\alpha P_R e)(\bar{\mu}\gamma_\alpha P_R e) \\ &+ g_3(\bar{\mu}\gamma^\alpha P_L e)(\bar{\mu}\gamma_\alpha P_R e) + g_4(\bar{\mu}P_L e)(\bar{\mu}P_L e) \\ &+ g_5(\bar{\mu}P_R e)(\bar{\mu}P_R e)] + \text{H.c.} \end{aligned} \tag{3.1}$$

There are four states $(F, m) = (0,0)$, (1,0), and $(1, \pm 1)$ in the $1S$ orbital of Mu. The transition amplitudes for the (F, m) states are

$$\mathcal{M}_{0,0} = -\frac{8(m_{\text{red}}\alpha_{\text{em}})^3}{\sqrt{2}\pi}\left(g_1 + g_2 - \frac{3}{2}g_3 - \frac{1}{4}g_4 - \frac{1}{4}g_5\right)G_F, \tag{3.2}$$

$$\mathcal{M}_{1,m} = -\frac{8(m_{\text{red}}\alpha_{\text{em}})^3}{\sqrt{2}\pi}\left(g_1 + g_2 + \frac{1}{2}g_3 - \frac{1}{4}g_4 - \frac{1}{4}g_5\right)G_F, \tag{3.3}$$

where $m_{\text{red}} = m_e m_\mu/(m_e + m_\mu) \simeq m_e$ is the reduced mass between the muon and the electron and α_{em} is the fine structure constant.

The external magnetic field mixes the (0,0) and (1,0) states, and the amplitudes in the magnetic flux density B are given as [27,28]

$$\mathcal{M}^B_{0,0} = \frac{1}{2}\left(\mathcal{M}_{0,0} - \mathcal{M}_{1,0} + \frac{\mathcal{M}_{0,0} + \mathcal{M}_{1,0}}{\sqrt{1+X^2}}\right), \tag{3.4}$$

$$\mathcal{M}^B_{1,0} = \frac{1}{2}\left(-\mathcal{M}_{0,0} + \mathcal{M}_{1,0} + \frac{\mathcal{M}_{0,0} + \mathcal{M}_{1,0}}{\sqrt{1+X^2}}\right), \tag{3.5}$$

where $X = 6.31 \times B/\text{T}$. On the other hand, the magnetic field splits the $(1, \pm 1)$ states, and the oscillations of the $(1, \pm 1)$ states are dropped in the magnetic field for $B \gtrsim 0.01$ T.

The time-integrated transition probability at the PSI experiment is given as

$$P = 2\tau^2(|c_{0,0}|^2|\mathcal{M}^B_{0,0}|^2 + |c_{1,0}|^2|\mathcal{M}^B_{1,0}|^2), \tag{3.6}$$

where $|c_{F,m}|^2$ gives the population of the Mu states and τ is the Mu lifetime. The result from the PSI experiment at the magnetic flux density $B = 0.1$ T is [19]

$$P < 8.3 \times 10^{-11}. \tag{3.7}$$

If $g_3 = 0$, we obtain

$$P = \frac{64 m^6_{\text{red}}\alpha^6\tau^2 G^2_F}{\pi^2}\left|g_1 + g_2 - \frac{1}{4}g_4 - \frac{1}{4}g_5\right|^2 \frac{|c_{0,0}|^2 + |c_{1,0}|^2}{1+X^2}. \tag{3.8}$$

The PSI experimental result is decoded as

$$\left|g_1 + g_2 - \frac{1}{4}g_4 - \frac{1}{4}g_5\right| < 3.0 \times 10^{-3}. \tag{3.9}$$

If $g_3 \neq 0$ and the others are 0, we find that

$$|g_3| < 2.1 \times 10^{-3}. \tag{3.10}$$

We use the population of Mu states $|c_{0,0}|^2 = 0.32$, $|c_{1,0}|^2 = 0.18$.

The Mu-to-$\overline{\mathrm{Mu}}$ transition operators are generated at the tree level using the following mediators [18]:

(1) Neutral flavor gauge boson ($\to g_1, g_2, g_3$).
(2) Neutral scalar in an inert $SU(2)_L$ doublet ($\to g_3, g_4, g_5$).
(3) Doubly charged scalar in the $SU(2)_{L,R}$ triplet ($\to g_1, g_2$).
(4) Dilepton gauge boson in an $SU(3)_l \times U(1)_X$ extension of electroweak symmetry ($\to g_3$).

If the Mu-to-$\overline{\mathrm{Mu}}$ transition is generated, new muon decay operators are also induced, especially for the g_1 and g_3 terms. For the g_2, g_4, and g_5 terms, new muon decays are induced if the right-handed neutrinos are lighter than muon. Through the mixings of the left- and right-handed neutrinos, muons can also decay to active neutrinos plus an electron.

We note that the four-fermion operators induced by the mediators can modify the decay constant $\bar{G}_F$ in Eq. (2.2), and therefore the universality of the decay constants can constrain the masses of the mediators and their flavor-dependent couplings. However, the Mu-to-$\overline{\mathrm{Mu}}$ transition (if induced) can provide stronger experimental constraints on the couplings than the electroweak precision and high energy Bhabha scattering. Corrections to the muon decay operators to induce the transverse positron polarization in the μ^+ decay for our target are about 10^{-3}, and they can be a deeper probe than the current experiments. Therefore, we do not describe the current experimental constraints from the electroweak precision in this paper. See Refs. [29–33] for the experimental constraints. We also note that the contributions to muon and electron $g-2$ are too small to explain their anomalies as a consequence of the 10^{-3} size of the induced coupling in our context.

In Sec. IV, we consider the model of the neutral flavor gauge boson, and we show that the Mu-to-$\overline{\mathrm{Mu}}$ transition and the correction to P_{T_1} in the polarized μ^+ decay are related. In Sec. V, we study the other models.

IV. NEUTRAL FLAVOR GAUGE BOSON

The interactions to the neutral gauge boson X to generate the Mu-to-$\overline{\mathrm{Mu}}$ transition are written as

$$\begin{aligned}\mathcal{L} = g_X(\bar{\ell}_\mu\gamma_\alpha\ell_e + \bar{\ell}_e\gamma_\alpha\ell_\mu)X^\alpha \\ + a g_X(e^{-i\varphi}\overline{\mu_R}\gamma_\alpha e_R + e^{i\varphi}\overline{e_R}\gamma_\alpha\mu_R)X^\alpha, \end{aligned} \tag{4.1}$$

where ℓ_e and ℓ_μ are the left-handed lepton doublets,

$$\ell_e = \begin{pmatrix}\nu_{eL}\\ e_L\end{pmatrix}, \qquad \ell_\mu = \begin{pmatrix}\nu_{\mu L}\\ \mu_L\end{pmatrix}, \tag{4.2}$$

and a is a $U(1)'$ charge for the right-handed charged leptons. In Appendix B, we give the construction of this model. Here we mention only that the interactions have a discrete lepton flavor symmetry, and they do not induce the ΔL_e, $\Delta L_\mu = \pm 1$ processes, such as $\mu \to e\gamma$, $\mu \to 3e$. See Ref. [18] for the bound of the ΔL_e, $\Delta L_\mu = \pm 1$ processes in the case where the discrete symmetry is not exact. There can be a physical phase parameter φ in the coupling in general. The phase in the left-handed lepton couplings can be rotated away without loss of generality.

The Mu-to-$\overline{\mathrm{Mu}}$ transition operators can be generated by the exchange of the neutral gauge boson, and we obtain

$$\begin{aligned} g_1 = \frac{g_X^2}{4\sqrt{2}M_X^2 G_F}, \qquad g_2 = a^2 e^{-2i\varphi}\frac{g_X^2}{4\sqrt{2}M_X^2 G_F}, \\ g_3 = 2a e^{-i\varphi}\frac{g_X^2}{4\sqrt{2}M_X^2 G_F}, \end{aligned} \tag{4.3}$$

where M_X is the mass of the neutral gauge boson.

The interaction can also generate the following eEDM via a loop diagram with the muon mass insertion in the internal line:

$$\frac{d_e}{e} = m_\mu \frac{2a g_X^2 \sin\varphi}{64\pi^2 M_X^2} G\left(\frac{m_\mu^2}{M_X^2}\right) = \frac{m_\mu G_F \mathrm{Im} g_3^*}{8\sqrt{2}\pi^2} G\left(\frac{m_\mu^2}{M_X^2}\right), \tag{4.4}$$

where G is a loop function,

$$G(x) = \frac{4 - 3x - x^3 + 6x\ln x}{(1-x)^3}. \tag{4.5}$$

The experimental bound of the eEDM is [34]

$$|d_e| < 1.1 \times 10^{-29}\ e\,\mathrm{cm}. \tag{4.6}$$

One needs $|\varphi| \lesssim 10^{-5}$ if we consider the $|g_3| \sim 2 \times 10^{-3}$ region which is near the current bound from the Mu-to-$\overline{\mathrm{Mu}}$ transition experiment in Eq. (3.10). Therefore, we suppose that there is no CP phase in the interaction ($\varphi = 0$).

The exchange of the neutral gauge boson can also induce the muon decay operators. Using the Fierz transformation

$$(\overline{\nu_{\mu L}}\gamma\nu_{eL})(\overline{e_R}\gamma\mu_R) = -2(\overline{e_R}\nu_e)(\overline{\nu_\mu}\mu_R), \tag{4.7}$$

we find that

$$g_{RR}^S = -2g_3. \tag{4.8}$$

Similarly, the exchange can induce the correction of g_{LL}^V,

$$\Delta g_{LL}^V = g_1. \tag{4.9}$$

If the neutrinos are Majorana fermions, we also find that

$$g_{RR}^V = -g_3, \qquad g_{LL}^S = 2g_1. \tag{4.10}$$

From the definitions of the muon decay parameters β in Eq. (2.16), we obtain

$$\beta = 4(+2g_1g_3 + 2g_3(1+g_1)) \simeq 8g_3. \tag{4.11}$$

The model parameters are constrained by the time-integrated transition probability (with $B = 0.1$ T) given in Eq. (3.6). Since there are no ΔL_e, $\Delta L_\mu = \pm 1$ processes in the model in Eq. (4.1) and the Mu-to-$\overline{\mathrm{Mu}}$ transition gives the strongest constraints, the experimental constraints for the model parameters g_X, M_X, and a are governed by the bound of the transition probability in Eq. (3.7). We assume that the neutral gauge boson is heavier than the muon. In the future, Belle II (the ILC) can directly search for the boson with $M_X \lesssim 10$ GeV ($\mathcal{O}(100)$ GeV) if its integrated luminosity accumulates enough. The transition probability is proportional to

$$|c_{0,0}|^2|-1.68g_3 + g_1 + g_2|^2 + |c_{1,0}|^2|0.68g_3 + g_1 + g_2|^2. \tag{4.12}$$

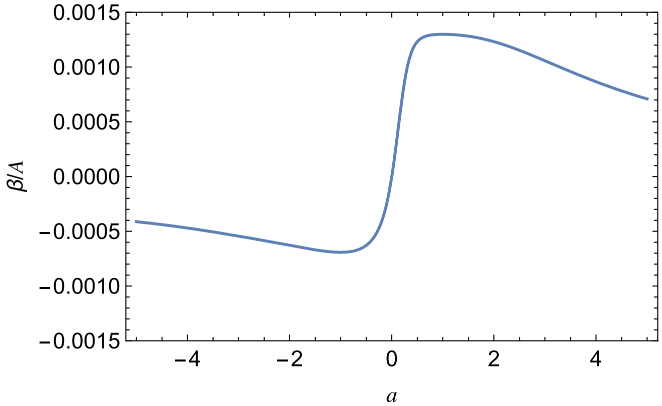

FIG. 1. Muon decay parameter β/A plotted as a function of the model parameter a assuming that the probability of the Mu-to-$\overline{\mathrm{Mu}}$ transition is at the current experimental bound. The transition experiment allows a larger magnitude of β/A for positive values than for negative values.

Notice that g_1 and g_2 are positive, and that g_3 can be either positive and negative. One can find that the magnitude of g_3 allowed by the transition experiment depends on the sign of g_3. Indeed, the larger magnitude is allowed for $g_3 > 0$. In Fig. 1, we show a plot of β/A as a function of a assuming that the transition probability is the upper bound from the PSI experiment.

In Fig. 2, we plot the P_{T_1} given in Eq. (2.7). We choose $\beta/A = 0.0012$ and $\beta/A = -0.0007$, which are allowed by the experimental result of the Mu-to-$\overline{\mathrm{Mu}}$ transition. The electron mass can induce P_{T_1} in the SM case, $\beta/A = 0$. Around the maximal energy of the positron, P_{T_1} tends to zero. As can be found in Eq. (2.23), P_{T_1} changes its sign for $\beta > 0$ for larger energy. We note that the differential decay width given in Eq. (2.2) is larger for the larger positron energy, and near-future experiments may observe the change of the sign.

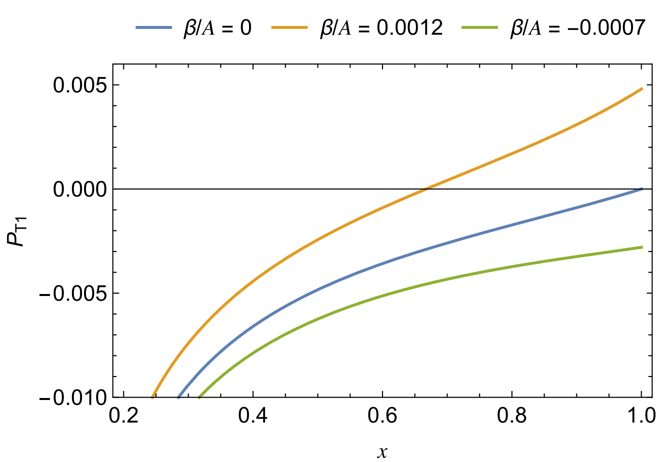

FIG. 2. $P_{\mathrm{T}_1}(\theta = \pi/2)$ plotted as a function of the reduced positron energy $x = E_e/W_{e\mu}$ for various β/A. The SM case corresponds to $\beta/A = 0$.

V. OTHER MODELS

As examined in Sec. IV, the flavor neutral gauge boson can generate both the Mu-to-$\overline{\mathrm{Mu}}$ transition and the transverse positron polarization P_{T_1} in the muon decay, and the two are related. In this section, we study other models to generate the new muon decay operators and see if the transverse polarizations are related to the Mu-to-$\overline{\mathrm{Mu}}$ transition.

We first enumerate the interactions to generate the following new muon decay operators at the tree level:

$$\text{(a)} \quad h_{ee}\overline{e_R}\ell_e\Phi + h_{\mu\mu}\overline{\mu_R}\ell_\mu\Phi + \mathrm{H.c.} \tag{5.1}$$

$$\text{(b)} \quad h_{e\mu}\overline{e_R}\ell_\mu\Phi + h_{\mu e}\overline{\mu_R}\ell_e\Phi + \mathrm{H.c.} \tag{5.2}$$

$$\text{(c)} \quad \kappa_{ee}\overline{\ell^c_e}\ell_e\Delta_L + \kappa_{\mu\mu}\bar{\ell}^c_\mu\ell_\mu\Delta_L + \mathrm{H.c.} \tag{5.3}$$

$$\text{(d)} \quad \kappa_{e\mu}(\bar{\ell}^c_e\ell_\mu\Delta_L + \bar{\ell}^c_\mu\ell_e\Delta_L) + \mathrm{H.c.} \tag{5.4}$$

$$\text{(e)} \quad f(\bar{\ell}^c_e\ell_\mu S^+ - \bar{\ell}^c_\mu\ell_e S^+) + \mathrm{H.c.} \tag{5.5}$$

$$\text{(f)} \quad g_{3l}(\overline{(e_R)^c}\gamma_\alpha\ell_e Y^\alpha + \overline{(\mu_R)^c}\gamma_\alpha\ell_\mu Y^\alpha) + \mathrm{H.c.} \tag{5.6}$$

$$\text{(g)} \quad g_{3l}(\overline{(e_R)^c}\gamma_\alpha\ell_\mu Y^\alpha + \overline{(\mu_R)^c}\gamma_\alpha\ell_e Y^\alpha) + \mathrm{H.c.} \tag{5.7}$$

$$\text{(h)} \quad g_X(\bar{\ell}_e\gamma_\alpha\ell_\mu X^\alpha + a\overline{e_R}\gamma_\alpha\mu_R X^\alpha) + \mathrm{H.c.} \tag{5.8}$$

Here Φ is an $SU(2)_L$ inert doublet which does not have a vacuum expectation value (VEV), Δ_L is an $SU(2)_L$ triplet whose VEV can generate the type-II neutrino masses [35–38], and S^+ is an $SU(2)_L$ singlet with the hypercharge $Y = 1$. The couplings to Δ_L and S^+ are written in terms of the components as

$$\overline{\ell^c_a}\ell_b\Delta_L = \overline{(\nu_{aL})^c}\nu_{bL}\Delta_L - \frac{1}{\sqrt{2}}(\overline{(\nu_{aL})^c}e_{bL} + \overline{(e_{aL})^c}\nu_{bL})\Delta_L^+ + \overline{(e_{aL})^c}e_{bL}\Delta_L^{++}, \tag{5.9}$$

$$\bar{\ell}^c_a\ell_b S^+ = (\overline{(\nu_{aL})^c}e_{bL} - \overline{(e_{aL})^c}\nu_{bL})S^+. \tag{5.10}$$

The vector field $Y_\alpha = (Y_\alpha^{++}, Y_\alpha^+)$ denotes a multiplet of the dilepton gauge boson in a model with gauge extension, and X_α is a flavor neutral gauge boson, which we studied in Sec. IV. The coexistence of cases (a) and (b) suffers from the LFV decay constraints if the couplings are sizable. The same is true for the coexistence of cases (c) and (d), and that of cases (f) and (g). A discrete flavor symmetry can forbid their coexistence. For example, we assign the discrete charges c_i to the lepton fields and the SM Higgs H as

$$\ell_e, e_R : c_1, \qquad \ell_\mu, \mu_R : c_2, \qquad \ell_\tau, \tau_R : c_3, \qquad H : 0, \tag{5.11}$$

TABLE I. List of which muon decay operators are induced from the respective interactions and mediators given in Eqs. (5.1)–(5.8). We assume that the neutrinos are Majorana to make it interfere with the SM decay operator if the induced operator is for $\mu \to \bar{\nu}_\mu \nu_e e$ (see Appendix A). In the fourth column, we put "Im" if the phase of the coefficient is allowed. The $\sharp$ mark is attached if the phase is constrained from the existence of the $(\Phi\tilde{H})^2$ term in the scalar potential (see explanations in the text). If the eEDM constrains the phase, we put "Re ($\because$ eEDM)." In the fifth column, we put the transition operators if they are induced.

Interaction(s)	Mediator	Operator	Phase	Mu-to-$\overline{\text{Mu}}$
(a)	Φ^+	g^S_{RR}	Im	
(b)	Φ^+	g^V_{RR}	Im ($\sharp$)	g_3
(c)	Δ^+_L	g^S_{LL}	Im	g_1
(d) or (e)	Δ^+_L or S^+	g^V_{LL}	Im	
(a) + (c)	Φ^+–Δ^+_L	$g^S_{LR,RL}$	Im	g_1
(b) + [(d) or (e)]	Φ^+–$(\Delta^+_L$ or $S^+)$	$g^{S,T}_{LR,RL}$	Im	g_3
(b) + (d)	Φ^0–Δ^0_L	$g^{S,T}_{LR,RL}$	Im	g_3
(b) + (d)	Φ^{0*}–Δ^0_L	$g^V_{LR,RL}$	Im	g_3
(f)	Y^+	g^V_{RR}	Im	g_3
(g)	Y^+	g^S_{RR}	Re ($\because$ eEDM)	
(h)	X	$g^{S,V}_{RR}$, $g^{S,V}_{LL}$	Re ($\because$ eEDM)	g_1,g_2,g_3

and the c_i's are all different. If the charge of Φ is 0, case (a) is obtained. If we assign the discrete charge n to Φ under Z_{2n} symmetry and $c_1 - c_2 \equiv n$, we obtain case (b). In fact, Φ and H should not mix in case (b) to suppress the LFV, which can be also controlled by the discrete symmetry. For case (e), it also suffers from the LFV if S^+ also couples to τ. These are all the lepton bilinear couplings (without right-handed neutrinos) that cause muon decays at the tree level. The muon decay operators from the couplings with the right-handed neutrinos are suppressed by the heavy-light neutrino mixings, which will be studied in the context of the left-right model later.

In Table I, we list the muon decay operators which can be induced by the interactions in cases (a)–(h). If the induced muon decay process is $\mu \to e\nu_e\overline{\nu_\mu}$, which does not interfere with $\mu \to e\overline{\nu_e}\nu_\mu$, the Fierz-transformed operator given in Appendix A is shown by assuming that the neutrinos are Majorana. One can see that g^S_{RR} can be induced in cases (a), (g), and (h), and P_{T_1} can be modified from the SM since β/A can be $\sim 10^{-3}$. Owing to the constraint of the eEDM, only case (a) can generate $\beta'/A \sim 10^{-3}$, and P_{T_2} can be observed in the near-future experiments. Only case (h) can relate the transverse polarization and the Mu-to-$\overline{\text{Mu}}$ transition discussed in Sec. IV.

A. Inert Higgs doublet Φ

Interaction (a) can directly induce g^S_{RR} via Φ^+ exchange,

$$g^S_{RR} \propto h_{ee} h^*_{\mu\mu}. \tag{5.12}$$

The EDMs for electrons and muons are obtained with a Φ^0-loop diagram, and

$$d_a \propto m_a \mathrm{Im} h^2_{aa} (f(m_a, M^2_{\mathrm{Re}}) - f(m_a, M^2_{\mathrm{Im}}))(a = e, \mu), \tag{5.13}$$

where M_{Re} and M_{Im} are the masses of the real and imaginary parts of Φ^0 and f is a loop function. One can find that g^S_{RR} can be complex without contradicting EDMs for electrons and muons if h^2_{ee} is real or $M_{\mathrm{Re}} = M_{\mathrm{Im}}$. The magnitude of the muon EDM (μEDM) for $P_{T_2} \sim O(10^{-3})$ in the case of imaginary $h_{\mu\mu}$ and $M_{\mathrm{Re}} \neq M_{\mathrm{Im}}$ is estimated to be $O(10^{-24})$ e cm, which is far below the current experimental bound [39,40]. The absence of a $(\Phi\tilde{H})(\Phi\tilde{H})$ term (H is a Higgs doublet which acquires a VEV) can make $M_{\mathrm{Re}} = M_{\mathrm{Im}}$, though discrete symmetries cannot realize it in nonsupersymmetric models. We note that the size of the $(\Phi\tilde{H})(\Phi\tilde{H})$ term is related with the radiative neutrino mass with the inert doublet [41]. In any event, g^S_{RR} can be complex, and therefore both P_{T_1} and P_{T_2} could be observed in a near-future experiment in this case. The Mu-to-$\overline{\text{Mu}}$ transition is not induced.

In case (b), the operator which is induced by the charge scalar Φ^+ exchange is

$$(\overline{e_R}\nu_\mu)(\overline{\nu_e}\mu_R), \tag{5.14}$$

which is not g^S_{RR}. If the neutrinos are Majorana, the induced operator can become g^V_{RR} (instead of g^S_{RR}). We find that the relation of the Mu-to-$\overline{\text{Mu}}$ transition is

$$g^V_{RR} \simeq -g^*_3 \propto h_{e\mu} h^*_{\mu e}. \tag{5.15}$$

The eEDM is

$$d_e \propto m_\mu \mathrm{Im}(h_{\mu e} h_{e\mu})(f(m_\mu, M_{\mathrm{Re}}^2) - f(m_\mu, M_{\mathrm{Im}}^2)). \tag{5.16}$$

The coupling of the $(\Phi\tilde{H})(\Phi\tilde{H})$ term and only one of $h_{\mu e}$ and $h_{e\mu}$ can be made real by the redefinition of the lepton fields and Φ. Therefore, to bring $\mathrm{Im} g_{RR}^V \neq 0$ into agreement with the eEDM, $M_{\mathrm{Re}} = M_{\mathrm{Im}}$ is needed in this case. We note that the eEDM diagram hits the muon mass at the internal line, while the electron mass is hit for the μEDM, and the μEDM thus becomes much smaller than the eEDM.

If we take into account the light-heavy neutrino mixings, g_{RR}^S can be induced even in case (b). The current neutrino state can be written using the mass eigenstates as

$$\nu_a = U_{ai}\nu_i + X_{aI}N_I. \tag{5.17}$$

See Appendix C for the neutrino mixing matrix. We find that

$$\{\beta, \beta'\} \sim \{\mathrm{Re} g_3, \mathrm{Im} g_3\} \sum_{i,j} U_{ei} U_{\mu i}^* U_{ej}^* U_{\mu j}. \tag{5.18}$$

We suppose that N_I's are heavier than muon. Using the unitarity relation, we find that

$$\sum_i U_{ei} U_{\mu i}^* = -\sum_I X_{eI} X_{\mu I}^*. \tag{5.19}$$

This magnitude is constrained by the $\mu \to e\gamma$ decay process, and the transverse positron polarizations are minuscule in case (b).

B. Type-II seesaw

Interactions (c) and (d) are available for type-II seesaw neutrino masses when the $SU(2)_L$ triplet Δ_L acquires a VEV.

In case (c), the g_{LL}^S muon decay operator is generated using a Δ_L^+ exchange if the neutrinos are Majorana. The Mu-to-$\overline{\mathrm{Mu}}$ transition operator (g_1) is also generated by a Δ_L^{++} exchange:

$$g_{LL}^S = 2g_1^*. \tag{5.20}$$

In case (d), the g_{LL}^V contribution of the muon decay is generated, while it does not induce the Mu-to-$\overline{\mathrm{Mu}}$ transition. The type-II seesaw interactions do not generate the EDMs, and thus the induced coefficients can be imaginary.

C. Type-II seesaw + inert doublet Φ

The β, β' parameters for the transverse polarizations in Eqs. (2.16) and (2.17) are not generated from the type-II seesaw terms alone. If we add the inert doublet Φ and there are multiple contributions (b) + (c), β, β' can be generated and they relate to the Mu-to-$\overline{\mathrm{Mu}}$ transitions as follows:

$$\{\beta, \beta'\} = \{\mathrm{Re}, \mathrm{Im}\}(8g_1^* g_3). \tag{5.21}$$

These magnitudes are less than $O(10^{-5})$ from the PSI bound of the Mu-to-$\overline{\mathrm{Mu}}$ transition.

The scalar trilinear term $\Phi H \Delta_L$ is allowed, and it can induce a $\Phi - \Delta_L$ mixing since the SM Higgs doublet H acquire a VEV. $g_{LR,RL}^{S,V,T}$ operators can then be generated (see Table I). Among them, $g_{LR,RL}^V$ can be generated by the neutral scalar exchange with $(\Phi\tilde{H})(\Phi\tilde{H})$ insertion in the interaction with (b) + (d). The generated operators are

$$\begin{aligned} g_{LR}^{S,T} &\propto \kappa_{e\mu}^* h_{\mu e}^*, \qquad g_{RL}^{S,T} \propto \kappa_{e\mu} h_{e\mu}, \\ g_{LR}^V &\propto \kappa_{e\mu} h_{\mu e}^*, \qquad g_{RL}^V \propto \kappa_{e\mu}^* h_{e\mu}. \end{aligned} \tag{5.22}$$

We note that the coupling of $\Phi H \Delta_L$ can be made real by the phase redefinition of Δ_L. We obtain the CP violating parameter α' in Eq. (2.10) for P_{T_2} as

$$\alpha' \propto |\kappa_{e\mu}|^2 \mathrm{Im}(h_{\mu e} h_{e\mu}). \tag{5.23}$$

As explained, the existence of the $(\Phi\tilde{H})(\Phi\tilde{H})$ term can conflict with the eEDM, and $h_{\mu e}$ and $h_{e\mu}$ should be real. As a consequence, the muon decay parameter α' is severely constrained by the eEDM in this model.

D. Dilepton gauge boson

In the dilepton gauge model [42–47] whose gauge symmetry is $SU(3)_c \times SU(3)_l \times U(1)_X$, the leptons are unified in one multiplet, $\mathbf{3}^*$ representation of $SU(3)_l$, $L_a = (l_a, -\nu_a, l_a^c)$. The gauge interaction (in two-component spinor notation) of the dilepton gauge boson is given as

$$\mathcal{L} = g_{3l}(\nu_a \sigma^\mu \overline{l_a^c} Y_\mu^+ - l_a \sigma^\mu \overline{l_a^c} Y_\mu^{++}) + \mathrm{H.c.} \tag{5.24}$$

The Higgs boson to generate the charged-lepton masses are $\mathbf{3}^*$ and $\mathbf{6}$ under $SU(3)_l$. Remember that the coupling matrices with $\mathbf{3}^*$ and $\mathbf{6}$ are antisymmetric and symmetric, respectively, under the generation index a. If the Yukawa couplings with the $\mathbf{3}^*$ Higgs boson are absent, the mass matrix is symmetric and the gauge interaction of the mass eigenstates is given by case (f). By adopting a discrete flavor symmetry, e.g., L_1:1, L_2:2, L_3:0, $\mathbf{3}^*$:0, and 6:0 under Z_3, the allowed Yukawa couplings can be

$$L_1 L_2 \mathbf{3}^* - L_2 L_1 \mathbf{3}^* + L_1 L_2 \mathbf{6} + L_2 L_1 \mathbf{6} + L_3 L_3 \mathbf{6}. \tag{5.25}$$

In this case, the gauge interaction is given by case (g) because the multiplets are $L_a = (e, -\nu_e, \mu^c)$, $(\mu, -\nu_\mu, e^c)$, and $(\tau, -\nu_\tau, \tau^c)$. To make $m_e \ll m_\mu$, one needs a fine-tuning.

In case (f), the Mu-to-$\overline{\mathrm{Mu}}$ transition operator g_3 is generated by the Y^{++} exchange. The g_{RR}^V muon decay operator can be generated by the Y^+ exchange,

$$g^V_{RR} = -g_3^*, \tag{5.26}$$

assuming that the neutrinos are Majorana.

In case (g), the g^S_{RR} muon decay operator is generated by the Y^+ exchange. Therefore, the modification of P_{T_1} from the SM can be sizable enough to detect in the muon decay experiments. In general, the couplings with Y^+ are complex in the basis where the charged-lepton masses are real. However, the complex couplings can induce the eEDM via the Y^{++}-loop diagram. Owing to the eEDM bound, the phase of g^S_{RR} has to be minuscule, and P_{T_2} will not be observed in near-future experiments.

E. Left-right model

The involvement of the right-handed neutrinos also provides muon decay operators which can interfere with the SM decay amplitude. Although their contributions are small due to the heavy-light neutrino mixings, as we mentioned, we describe the contributions to g^S_{RR} in the left-right model, $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge theory as a pedagogical guide.

We introduce an $SU(2)_R$ triplet Δ_R and consider the following interaction:

$$\overline{\ell^c_{eR}}\ell_{eR}\Delta_R + \bar{\ell^c_{\mu R}}\ell_{\mu R}\Delta_R + \text{H.c.}, \tag{5.27}$$

where ℓ_R is an $SU(2)_R$ doublet, e.g., $\ell_{eR} = (N_{eR}, e_R)^T$. The VEV of the $SU(2)_R$ triplet breaks $SU(2)_R \times U(1)_{B-L}$ down to $U(1)_Y$, and also generates the Majorana masses of the right-handed neutrinos. The Δ_R^{++} exchange generates the g_2 operator of the Mu-to-$\overline{\text{Mu}}$ transition. The Δ_R^+ exchange generates

$$(\overline{e_R}(N^c_e)_L)(\overline{(N^c_\mu)_L}\mu_R). \tag{5.28}$$

We use the notation of the neutrino mixing matrix given in Appendix C, and the current neutrino state can be written using the mass eigenstates as

$$(N^c_\alpha)_L = V_{\alpha i}\nu_i + Y_{\alpha I}N_I. \tag{5.29}$$

We obtain

$$\{\beta, \beta'\} = \{\text{Re}, \text{Im}\}\left(-8g_2^*\sum_{i,j}U_{ei}V^*_{ei}U^*_{\mu j}V_{\mu j}\right). \tag{5.30}$$

We remark that the magnitude of $U_{ei}V^*_{ei}$ is directly constrained by the neutrinoless double beta decay ($0\nu 2\beta$). The unitarity and the decay universality restrict $U_{\mu i}V^*_{\mu i} = -X_{\mu I}Y^*_{\mu I}$ (see Appendix C). The induced size of $|\beta^{(\prime)}|$ is estimated to be less than $O(10^{-7})$.

The tree-level W_R gauge boson exchange can generate the g^S_{RR} operator if the neutrinos are Majorana [48]. Using

$$(\overline{e_R}\gamma N_{eR})(\overline{N_{\mu R}}\gamma\mu_R) = 2(\overline{e_R}(N^c_\mu)_L)(\overline{(N^c_e)_L}\mu_R), \tag{5.31}$$

we obtain the muon decay parameter from the W_R exchange as

$$\{\beta, \beta'\} = \{\text{Re}, \text{Im}\}\left(-8\frac{g_R^2}{g_L^2}\frac{M^2_{W_L}}{M^2_{W_R}}\sum_{i,j}U_{ei}V^*_{\mu i}U^*_{\mu j}V_{ej}\right), \tag{5.32}$$

which is also minuscule due to the W_R mass bound from the LHC [49–52] and $0\nu 2\beta$. For a native estimation of the quantity $|\sum_{i,j}U_{ei}V^*_{\mu i}U^*_{\mu j}V_{ej}|$ (see Appendix C), we determine that $|\beta^{(\prime)}|$ is less than $O(10^{-8})$. Even for a more conservative estimation of the quantity, $|\beta^{(\prime)}|$ is less than $O(10^{-6})$.

VI. CONCLUSION

The new leptonic interactions with a discrete flavor symmetry can induce the Mu-to-$\overline{\text{Mu}}$ transition and the transverse polarization of $e^\pm$ in the polarized $\mu^\pm$ decay, which can be of a size that will be observable at the facilities with high-intensity muon beamlines. We have studied whether the transition rate and the transverse polarization can be related.

There will be three candidates of mediators to induce the testable muon decay parameter β for the transverse positron polarization in the near future:

(a) Neutral flavor gauge boson.
(b) Inert doublet.
(c) Dilepton gauge boson.

Among these candidates, in the model of the neutral flavor gauge boson, the Mu-to-$\overline{\text{Mu}}$ transition and the β parameter (the correction of the transverse positron polarization P_{T_1}) are indeed related. A larger contribution is allowed by the Mu-to-$\overline{\text{Mu}}$ transition experiment for the positive value of β than for the negative value (P_{T_1} changes its sign depending on the positron energy for positive β). The other direction of the transverse polarization, P_{T_2}, is constrained by the nonobservation of the eEDM.

In the model with an inert scalar doublet (which does not acquire a VEV), either the Mu-to-$\overline{\text{Mu}}$ transition or the correction to the transverse positron polarization can be observed. The nonzero value of P_{T_2} does not conflict with the eEDM in this model. In the dilepton gauge boson, either the Mu-to-$\overline{\text{Mu}}$ transition or the correction to the transverse positron polarization can be observed. The nonobservation of the eEDM restricts P_{T_2}. Although the observable size of the Mu-to-$\overline{\text{Mu}}$ transition can be induced in the model with $SU(2)_L$ and $SU(2)_R$ triplet scalars, the correction to the transverse polarization is smaller than the three above.

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APPENDIX A: FIERZ TRANSFORMATIONS OF THE MUON DECAY OPERATORS FOR MAJORANA NEUTRINOS

The following identical equations hold for the four-component fermions ψ and χ:

$$\overline{\psi}\chi = \overline{\chi^c}\psi^c, \qquad \overline{\psi}\gamma_\mu\chi = -\overline{\chi^c}\gamma_\mu\psi^c, \qquad \overline{\psi}\sigma_{\mu\nu}\chi = -\overline{\chi^c}\sigma_{\mu\nu}\psi^c, \tag{A1}$$

where $\psi^c = C\overline{\psi}^T$ and C is a charge conjugation matrix. Using these, one finds for Majorana neutrinos $\nu = \nu^c$

$$\begin{aligned}(\overline{e_R}\nu_{aL})(\overline{\nu_{bL}}\mu_R) &= -\frac{1}{2}(\overline{e_R}\gamma\mu_R)(\overline{\nu_{bL}}\gamma\nu_{aL}) \\ &= \frac{1}{2}(\overline{e_R}\gamma\mu_R)(\overline{\nu_{aR}}\gamma\nu_{bR}) \\ &= \frac{1}{2}(\overline{e_R}\gamma\nu_{bR})(\overline{\nu_{aR}}\gamma\mu_R).\end{aligned} \tag{A2}$$

Similarly,

$$(\overline{e_L}\nu_{aR})(\overline{\nu_{bR}}\mu_L) = \frac{1}{2}(\overline{e_L}\gamma\nu_{bL})(\overline{\nu_{aL}}\gamma\mu_L). \tag{A3}$$

Here we omit the obvious Lorentz indices of γ_μ for their contraction. For example, although $(\overline{e_R}\nu_{\mu L})(\overline{\nu_{eL}}\mu_R)$ does not interfere with the usual $\mu \to e\overline{\nu_e}\nu_\mu$ decay amplitude for Dirac neutrinos, it can be made an operator to interfere with it for Majorana neutrinos using the above equation.

One can also obtain, for $\nu = \nu^c$,

$$(\overline{e_L}\gamma\nu_{aL})(\overline{\nu_{bR}}\gamma\mu_R) = (\overline{e_L}\gamma\nu_{bL})(\overline{\nu_{aR}}\gamma\mu_R), \tag{A4}$$

$$\begin{aligned}&(\overline{e_L}\nu_{aR})(\overline{\nu_{bL}}\mu_R) \\ &\quad = -\frac{1}{2}(\overline{e_L}\nu_{bR})(\overline{\nu_{aL}}\mu_R) + \frac{1}{8}(\overline{e_L}\sigma\nu_{bR})(\overline{\nu_{aL}}\sigma\mu_R),\end{aligned} \tag{A5}$$

$$\begin{aligned}&(\overline{e_L}\sigma\nu_{aR})(\overline{\nu_{bL}}\sigma\mu_R) \\ &\quad = 6(\overline{e_L}\nu_{bR})(\overline{\nu_{aL}}\mu_R) + \frac{1}{2}(\overline{e_L}\sigma\nu_{bR})(\overline{\nu_{aL}}\sigma\mu_R),\end{aligned} \tag{A6}$$

and the same holds for the exchange of $L \leftrightarrow R$.

APPENDIX B: THE MODEL WITH THE NEUTRAL FLAVOR GAUGE BOSON

We describe the construction of the model with the neutral flavor gauge boson discussed in Sec. IV.

TABLE II. $U(1)_1 \times U(1)_2$ charge assignments of the left-handed lepton doublets ℓ_i, the right-handed charged leptons e_{iR}, and the SM singlet scalar fields ϕ, ϕ_1, and ϕ_2. The scalar field ϕ breaks $U(1)_1 \times U(1)_2$ down to $U(1)'$. The scalar fields ϕ_1 and ϕ_2 break the remaining $U(1)'$ symmetry and can generate the Yukawa interaction of the first and second generations of the charged leptons. The $U(1)$ charges of the quark fields are all zero.

Fields	ℓ_1	ℓ_2	ℓ_3	e_{1R}	e_{2R}	e_{3R}	ϕ	ϕ_1	ϕ_2
$U(1)_1$ charge	$+1$	-1	0	$+1$	-1	0	a_1	1	1
$U(1)_2$ charge	$+1$	-1	0	-1	$+1$	0	a_2	1	-1
$U(1)'$ charge	$+1$	-1	0	a	$-a$	0	0	1	a

Table II shows extra $U(1)$ charge assignments of the lepton fields. The extra $U(1)$ symmetries do not cause gauge anomalies: $[SU(3)_c]^2U(1)_n$, $[SU(2)_L]^2U(1)_n$, $[U(1)_Y]^2U(1)_n$, $[U(1)_n]^2U(1)_Y$, $[U(1)_n]^3$, $[U(1)_1]^2U(1)_2$, $[U(1)_2]^2U(1)_1$, and $[\text{gravity}]^2U(1)_n$ $(n = 1, 2)$.

The ℓ_3 and e_{3R} fields are identified to the third generation, and the Yukawa interaction to generate the mass of the tau lepton can be directly written. The Yukawa interaction to generate the electron and muon masses can be obtained by introducing the vectorlike fermions L and E as in the usual flavor models:

$$\begin{aligned}-\mathcal{L}_Y = {}& y_1\phi_1^*\overline{L_R}\ell_1 + y_2\phi_1\overline{L_R}\ell_2 + y_1'\phi_2^*\overline{e_{1R}}E_L \\ &+ y_2'\phi_2\overline{e_{2R}}E_L + y\bar{E}LH + M_L\bar{L}L + M_E\bar{E}E.\end{aligned} \tag{B1}$$

By integrating out the vectorlike fermions, one obtains

$$-\mathcal{L}_Y = (Y_\ell)_{ij}\overline{e_{iR}}\ell_i H \tag{B2}$$

and

$$Y_\ell = -\frac{y}{M_L M_E}\begin{pmatrix} y_1y_1'\phi_1^*\phi_2^* & y_1y_2'\phi_1^*\phi_2 \\ y_2y_1'\phi_1\phi_2^* & y_2y_2'\phi_1\phi_2 \end{pmatrix}. \tag{B3}$$

We note that the electron is massless (at the tree level) if only one set of vectorlike fermions is introduced, as given in Eq. (B1). Introducing one more set of vectorlike fermions, one obtains a tree-level electron mass, though we do not write it explicitly to avoid complicating the expression.

We suppose that the $U(1)_1 \times U(1)_2$ symmetry is broken down to $U(1)'$ by a VEV of a scalar ϕ whose charges are given in Table II. By redefining the normalization of the $U(1)'$ charge, we find that the $U(1)'$ charge for the right-handed charged lepton is

$$a = \frac{a_2 + a_1}{a_2 - a_1}. \tag{B4}$$

We note that $U(1)_2$ $(U(1)_1)$ is broken if $a_1 = 0$ $(a_2 = 0)$, and one obtains $a = 1$ $(a = -1)$ trivially, which returns us to the special charge assignments given in Ref. [53]. We assume that the VEV of ϕ is much larger than the VEVs of ϕ_1 and ϕ_2 which break $U(1)'$, and we ignore the contribution from the exchange of the heavier extra gauge boson in Sec. IV.

If the Lagrangian in Eq. (B1) has an exchange symmetry under $\ell_1 \leftrightarrow \ell_2$, $e_{1R} \leftrightarrow e_{2R}$, (namely, $y_1 = y_2$, $y_1' = y_2'$; their phases can be different in more general exchange symmetry), the fields ℓ_i and e_{iR} can be written in terms the mass eigenstates ℓ_e, ℓ_μ, e_R, and μ_R as

$$\ell_1 = \frac{\ell_e + e^{i\varphi_L}\ell_\mu}{\sqrt{2}}, \qquad \ell_2 = \frac{\ell_e - e^{i\varphi_L}\ell_\mu}{\sqrt{2}},$$
$$e_{1R} = \frac{e_R + e^{i\varphi_R}\mu_R}{\sqrt{2}}, \qquad e_{2R} = \frac{e_R - e^{i\varphi_R}\mu_R}{\sqrt{2}}. \tag{B5}$$

We remark that the Yukawa couplings can have phases in general and there can be phases in the linear combinations in Eq. (B5) in the basis where the electron and muon mass is real. The gauge interaction is calculated as

$$\begin{aligned}\mathcal{L} &= g_X(\bar{\ell}_1\gamma_\alpha\ell_1 - \bar{\ell}_2\gamma_\alpha\ell_2)X^\alpha \\ &\quad + a g_X(\overline{e_{1R}}\gamma_\alpha e_{1R} - \overline{e_{2R}}\gamma_\alpha e_{2R})X^\alpha \\ &= g_X(e^{-i\varphi_L}\overline{\ell_\mu}\gamma_\alpha\ell_e + e^{i\varphi_L}\overline{\ell_e}\gamma_\alpha\ell_\mu)X^\alpha \\ &\quad + a g_X(e^{-i\varphi_R}\overline{\mu_R}\gamma_\alpha e_R + e^{i\varphi_R}\overline{e_R}\gamma_\alpha\mu_R)X^\alpha. \end{aligned}\tag{B6}$$

Using a phase redefinition, $\ell_\mu \to e^{-i\varphi_L}\ell_\mu$ and $\mu_R \to e^{-i\varphi_L}\mu_R$, which does not change the phase of the muon mass, we obtain Eq. (4.1) with one physical phase $\varphi = \varphi_R - \varphi_L$.

APPENDIX C: HEAVY-LIGHT NEUTRINO MIXINGS

To evaluate the muon decay operators which contain right-handed neutrinos, we need to know the size of the heavy-light neutrino mixings. Here we list information about it.

Before we study the constraints on the mixings, we define the neutrino mixing matrix. We work on the basis where the charged-lepton mass matrix is diagonal. The neutrino mass term is given as

$$-\mathcal{L}_m = \frac{1}{2}\left(\overline{(\nu^c)_R} \quad \overline{N_R}\right)\mathcal{M}\begin{pmatrix}\nu_L \\ (N^c)_L\end{pmatrix} + \text{H.c.}, \tag{C1}$$

where ν and N are current-basis left- and right-handed neutrinos, and the 6×6 neutrino mass matrix $\mathcal{M}$ is written as

$$\mathcal{M} = \begin{pmatrix} 0 & m_D \\ m_D^T & M_N \end{pmatrix}. \tag{C2}$$

The mass eigenstates ν' and N' are given as

$$\begin{pmatrix}\nu_L \\ (N^c)_L\end{pmatrix} = \mathcal{U}\begin{pmatrix}\nu_L' \\ N_L'\end{pmatrix} \tag{C3}$$

and

$$\mathcal{U}^T\mathcal{M}\mathcal{U} = \mathrm{diag}(M_\mathcal{I}) = \mathrm{diag}(m_i, M_I). \tag{C4}$$

We choose phases in $\mathcal{U}$ so that the $M_\mathcal{I}$'s are real. We use index i for the light neutrino mass eigenstates, index I for the "heavy" neutrino mass eigenstates, and index $\mathcal{I}$ for both states. For the generation index in the current basis, we use a, b. For convenience, we define

$$\mathcal{U} = \begin{pmatrix} U & X \\ V & Y \end{pmatrix}. \tag{C5}$$

Namely,

$$\nu_{aL} = U_{ai}\nu_{iL}' + X_{aI}N_{IL}', \tag{C6}$$

$$N_{aL}^c = V_{ai}\nu_{iL}' + Y_{aI}N_{IL}'. \tag{C7}$$

In the following, the mass eigenstates ν_i and N_I are defined as Majorana fermions, namely, $\nu_i \equiv \nu_{iL}' + (\nu_{iL}')^c$ and $N_I \equiv N_{IL}' + (N_{IL}')^c$.

Our concern is the constraints of the size of V_{ai}, i.e., the mass eigenstate of the active neutrino in the current basis of the right-handed neutrino N_{aL}^c. Because of the unitarity of the mixing matrix $\mathcal{U}$, we obtain

$$U_{ai}V_{bi}^* + X_{aI}Y_{bI}^* = 0. \tag{C8}$$

Therefore, let us first enumerate the constraints on X_{aI} [54,55].

(1) The mixings are bounded by electroweak precision data

$$\sum_I |X_{eI}|^2, \sum_I |X_{\mu I}|^2 \lesssim 0.003 \tag{C9}$$

individually. This obeys the unitarity $\sum_i |U_{ai}|^2 = 1 - \sum_I |X_{aI}|^2$ and the universality of the four-fermion decays. If the new muon decay operators are added, the bound can be modified, but the contribution to the muon decay parameters from the new operators will then be dominant. If N_I is lighter than the Z boson, the new decay modes constrain the mixing more severely depending on their channels.

(2) The product of $|X_{eI}X_{\mu I}|$ is bounded by the $\mu \to e\gamma$ decay process as follows:

$$\left|\sum_I X_{\mu I}^* X_{eI} F\left(\frac{M_I^2}{M_W^2}\right)\right| \lesssim 4\times10^{-5}, \tag{C10}$$

where

$$F(x) = \frac{x(1-6x+3x^2+2x^3-6x^2\ln x)}{(1-x)^4}. \quad \text{(C11)}$$

(3) For one generation (2×2 neutrino mass matrix), the light neutrino mass in type-I seesaw is $m_\nu = m_D^2/M_N$, and the mixing is $(X_{\alpha I})^2 = m_D^2/M_N^2 = m_\nu/M_N$. Therefore, the Mu-to-$\overline{\text{Mu}}$ transition is minuscule. For a three-generation case, there are degrees of freedom to enlarge the mixings, X_{eI} and $X_{\mu I}$, while keeping the tree-level active neutrino masses minuscule.

(4) If the light-heavy neutrino mixing is enlarged, a sizable active neutrino mass can be generated by the Z boson loop diagram [56],

$$(M_\nu)_{ab}^{\text{1-loop}} \simeq \frac{\alpha_2}{4\pi\cos^2\theta_W}\sum_I X_{aI}X_{bI}\frac{M_I^3}{M_I^2-M_Z^2}\ln\frac{M_Z^2}{M_I^2}. \quad \text{(C12)}$$

The loop-induced neutrino mass can be canceled if the heavy neutrino masses are degenerate ($M_1 = -M_2$, $X_{a1} = X_{a2}$). If the heavy neutrino masses are not degenerate and one wants to avoid unnatural cancellation between the tree-level and one-loop neutrino masses, we need $X_{aI} \lesssim O(10^{-5})$ for $M_I \sim 1$ TeV. Therefore, we usually suppose that there is a mass degeneracy in the heavy neutrino sector to obtain a size of the mixing X_{aI}.

(5) The neutrinoless double beta decay ($0\nu 2\beta$) process via the heavy neutrinos X_{eI}^2/M_I, which can be canceled for the degenerate heavy neutrino masses. If it is not canceled, the current half life gives the bound

$$|X_{eI}|^2 \lesssim 10^{-5}\times\frac{M_I}{1\ \text{TeV}}. \quad \text{(C13)}$$

Next, let us see the direct constraints on V_{ai} [57,58]. In the left-right model, the $0\nu 2\beta$ process can be induced via $W_L - W_R$ mixing and the W_R coupling to the right-handed electron, and $U_{ei}V_{ei}^*$ is bounded as

$$|U_{ei}V_{ei}^*| \lesssim O(10^{-4})\times\frac{g_L}{g_R}\frac{10^{-5}}{\xi_{LR}}, \quad \text{(C14)}$$

where ξ_{LR} is a $W_L - W_R$ mixing and g_L and g_R are the W_L and W_R coupling constants.

One often considers the so-called inverse seesaw by adding singlet fermions N_S. The 9×9 mass matrix for $\mathcal{N} = (\nu_L, (N^c)_L, N_S)^T$ is

$$\mathcal{M} = \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & \mu_N & M_S \\ 0 & M_S^T & \mu_S \end{pmatrix}. \quad \text{(C15)}$$

We denote the 9×9 unitary matrix to diagonalize the mass matrix $\mathcal{M}$ as

$$\mathcal{U} = \begin{pmatrix} U & X \\ V & Y \\ W & Z \end{pmatrix}, \quad \text{(C16)}$$

where U, V, W are 3×3 matrices and X, Y, Z are 3×6 matrices. The light neutrino mass matrix is

$$M_\nu^{\text{light}} \simeq m_D(M_S^T)^{-1}\mu_S M_S^{-1} m_D^T \quad \text{(C17)}$$

for the small Majorana mass μ_S. In the left-right model, the Dirac mass m_D is the naively similar size of the charged-lepton masses, which is not good for obtaining sub-eV neutrino masses in the TeV-scale model. In the inverse seesaw, the minuscule neutrino masses can be explained by the smallness of μ_S. The size of X is naively m_D/M_S, which can be sizable. However, the size of V is minuscule, $\sim m_D\mu_S/M_S^2 \simeq M_\nu/m_D$, while W can be as large as X. Therefore, the inverse seesaw is not suitable if one wants a sizable V_{ai} mixing in the left-right model.

If one wants a sizable V_{ai} mixing avoiding unnatural cancellation of the tree-level and one-loop active neutrino mass, one needs a special structure of the 6×6 neutrino mass matrix in Eq. (C2) (or a more complicated setup) with the mass degeneracy in the 3×3 Majorana neutrino mass matrix, although describing this in detail is beyond the purpose of this Appendix. The structure restricts the estimation of the quantity $J = \sum_{i,j} U_{ei}V_{\mu i}^* U_{\mu j}^* V_{ej} = \sum_{I,J} X_{eI}Y_{\mu I}^* X_{\mu J}^* Y_{eJ}$, which affects the discussion in Sec. V E. If the sizable mixings are $X_{a1} = X_{a2}$ for $M_1 = -M_2$, the $\mu\to e\gamma$ process bounds $X_{e1}X_{\mu 1}^*$ in Eq. (C10), and therefore the magnitude of the quantity J is restricted to less than $O(10^{-5})$. Even if one can somehow evade the $\mu\to e\gamma$ constraint and also make $|\sum U_{\mu i}V_{ei}^*| \ll |\sum U_{ei}V_{ei}^*|$ avoid the $0\nu 2\beta$ constraint in Eq. (C14), the bound in Eq. (C9) will restrict $|J|$ to less than $O(10^{-3})$.

[1] N. Kawamura *et al.*, New concept for a large-acceptance general-purpose muon beamline, Prog. Theor. Exp. Phys. (2018) 113G01.
[2] M. Abe *et al.*, A new approach for measuring the muon anomalous magnetic moment and electric dipole moment, Prog. Theor. Exp. Phys. (2019) 053C02.
[3] M. Aiba *et al.*, Science case for the new high-intensity muon beams HIMB at PSI, arXiv:2111.05788.
[4] N. Kawamura, R. Kitamura, H. Yasuda, M. Otani, Y. Nakazawa, H. Iinuma, and T. Mibe, A new approach for Mu-$\overline{\text{Mu}}$ conversion search, J. Phys. Soc. Jpn. Conf. Proc. **33**, 011120 (2021).
[5] C. Han, D. Huang, J. Tang, and Y. Zhang, Probing the doubly charged Higgs with muonium to antimuonium conversion experiment, Phys. Rev. D **103**, 055023 (2021).
[6] B. Pontecorvo, Mesonium and anti-mesonium, Sov. Phys. JETP **6**, 429 (1957).
[7] G. Feinberg and S. Weinberg, Conversion of muonium into antimuonium, Phys. Rev. **123**, 1439 (1961).
[8] B. W. Lee and R. E. Shrock, Natural suppression of symmetry violation in gauge theories: Muon- and electron-lepton-number nonconservation, Phys. Rev. D **16**, 1444 (1977); B. W. Lee, S. Pakvasa, R. E. Shrock, and H. Sugawara, Muon and Electron Number Nonconservation in a $V-A$ Gauge Model, Phys. Rev. Lett. **38**, 937 (1977); **38**, 1230(E) (1977).
[9] A. Halprin, Neutrinoless Double-Beta Decay and Muonium-Antimuonium Transitions, Phys. Rev. Lett. **48**, 1313 (1982).
[10] T. Kinoshita and A. Sirlin, Polarization of electrons in muon decay with general parity-nonconserving interactions, Phys. Rev. **108**, 844 (1957).
[11] F. Scheck, Muon physics, Phys. Rep. **44**, 187 (1978).
[12] N. Danneberg *et al.*, Muon Decay: Measurement of the Transverse Polarization of the Decay Positrons and its Implications for the Fermi Coupling Constant and Time Reversal Invariance, Phys. Rev. Lett. **94**, 021802 (2005).
[13] W. Fetscher, Muon decay, SciPost Phys. Proc. **5**, 006 (2021).
[14] T. Fukuyama, S. Kanda, D. Nomura, and K. Shimomura, Hunting for T-violation and Majoranality of neutrinos in muon decays, arXiv:1908.01630.
[15] J. Adam *et al.* (MEG Collaboration), New Constraint on the Existence of the $\mu^+ \to e^+\gamma$ Decay, Phys. Rev. Lett. **110**, 201801 (2013); A. M. Baldini *et al.* (MEG Collaboration), Search for the lepton flavour violating decay $\mu^+ \to \mathrm{e}^+\gamma$ with the full dataset of the MEG experiment, Eur. Phys. J. C **76**, 434 (2016).
[16] U. Bellgardt *et al.* (SINDRUM Collaboration), Search for the decay $\mu^+ \to e^+e^+e^-$, Nucl. Phys. **B299**, 1 (1988).
[17] W. H. Bertl *et al.* (SINDRUM II Collaboration), A search for muon to electron conversion in muonic gold, Eur. Phys. J. C **47**, 337 (2006).
[18] T. Fukuyama, Y. Mimura, and Y. Uesaka, Models of muonium to antimuonium transition, Phys. Rev. D **105**, 015026 (2022).
[19] L. Willmann *et al.*, New Bounds from a Search for Muonium to Antimuonium Conversion, Phys. Rev. Lett. **82**, 49 (1999).
[20] P. A. Zyla *et al.* (Particle Data Group), Review of particle physics, Prog. Theor. Exp. Phys. (2020) 083C01.
[21] W. Fetscher, H. J. Gerber, and K. F. Johnson, Muon decay: Complete determination of the interaction and comparison with the standard model, Phys. Lett. B **173**, 102 (1986).
[22] L. Michel, Interaction between four half-spin particles and the decay of the μ-meson, Proc. Phys. Soc. London Sect. A **63**, 514 (1950).
[23] C. Bouchiat and L. Michel, Theory of μ-meson decay with the hypothesis of nonconservation of parity, Phys. Rev. **106**, 170 (1957).
[24] T. Kinoshita and A. Sirlin, Muon decay with parity nonconserving interactions and radiative corrections in the two-component theory, Phys. Rev. **107**, 593 (1957).
[25] C. A. Gagliardi, R. E. Tribble, and N. J. Williams, Global analysis of muon decay measurements, Phys. Rev. D **72**, 073002 (2005).
[26] R. Conlin and A. A. Petrov, Muonium-antimuonium oscillations in effective field theory, Phys. Rev. D **102**, 095001 (2020).
[27] K. Horikawa and K. Sasaki, Muonium-antimuonium conversion in models with dilepton gauge bosons, Phys. Rev. D **53**, 560 (1996).
[28] W. S. Hou and G. G. Wong, Magnetic field dependence of muonium-antimuonium conversion, Phys. Lett. B **357**, 145 (1995).
[29] F. Cuypers and S. Davidson, Bileptons: Present limits and future prospects, Eur. Phys. J. C **2**, 503 (1998).
[30] A. Falkowski and K. Mimouni, Model independent constraints on four-lepton operators, J. High Energy Phys. 02 (2016) 086.
[31] T. Li and M. A. Schmidt, Sensitivity of future lepton colliders and low-energy experiments to charged lepton flavor violation from bileptons, Phys. Rev. D **100**, 115007 (2019).
[32] A. Crivellin, F. Kirk, C. A. Manzari, and L. Panizzi, Searching for lepton flavor universality violation and collider signals from a singly charged scalar singlet, Phys. Rev. D **103**, 073002 (2021).
[33] A. Crivellin, M. Hoferichter, and C. A. Manzari, Fermi Constant from Muon Decay versus Electroweak Fits and Cabibbo-Kobayashi-Maskawa Unitarity, Phys. Rev. Lett. **127**, 071801 (2021).
[34] V. Andreev *et al.* (ACME Collaboration), Improved limit on the electric dipole moment of the electron, Nature (London) **562**, 355 (2018).
[35] J. Schechter and J. W. F. Valle, Neutrino masses in $\mathrm{SU}(2) \times \mathrm{U}(1)$ theories, Phys. Rev. D **22**, 2227 (1980).
[36] T. P. Cheng and L. F. Li, Neutrino masses, mixings and oscillations in $\mathrm{SU}(2) \times \mathrm{U}(1)$ models of electroweak interactions, Phys. Rev. D **22**, 2860 (1980).
[37] G. Lazarides, Q. Shafi, and C. Wetterich, Proton lifetime and fermion masses in an SO(10) model, Nucl. Phys. **B181**, 287 (1981).
[38] R. N. Mohapatra and G. Senjanovic, Neutrino masses and mixings in gauge models with spontaneous parity violation, Phys. Rev. D **23**, 165 (1981).
[39] G. W. Bennett *et al.* (Muon $g-2$ Collaboration), An improved limit on the muon electric dipole moment, Phys. Rev. D **80**, 052008 (2009).

[40] Y. Ema, T. Gao, and M. Pospelov, Improved Indirect Limits on Muon EDM, Phys. Rev. Lett. **128**, 131803 (2022).

[41] E. Ma, Verifiable radiative seesaw mechanism of neutrino mass and dark matter, Phys. Rev. D **73**, 077301 (2006).

[42] P. H. Frampton and B. H. Lee, SU(15) Grand Unification, Phys. Rev. Lett. **64**, 619 (1990).

[43] P. H. Frampton, Chiral Dilepton Model and the Flavor Question, Phys. Rev. Lett. **69**, 2889 (1992).

[44] P. H. Frampton and D. Ng, Dileptons: Present status and future prospects, Phys. Rev. D **45**, 4240 (1992).

[45] P. H. Frampton and M. Harada, Constraints from precision electroweak data on leptoquarks and bileptons, Phys. Rev. D **58**, 095013 (1998).

[46] H. Fujii, S. Nakamua, and K. Sasaki, Constraints on dilepton mass from low-energy muon experiments, Phys. Lett. B **299**, 342 (1993).

[47] H. Fujii, Y. Mimura, K. Sasaki, and T. Sasaki, Muonium hyperfine structure and the decay $\mu^+ \to e^+ + \overline{\nu}_e + \nu_\mu$ in models with dilepton gauge baryons, Phys. Rev. D **49**, 559 (1994).

[48] M. Doi, T. Kotani, H. Nishiura, K. Okuda, and E. Takasugi, Majorana neutrinos and μ decay, Prog. Theor. Phys. **67**, 281 (1982).

[49] M. Nemevšek, F. Nesti, and G. Popara, Keung-Senjanović process at the LHC: From lepton number violation to displaced vertices to invisible decays, Phys. Rev. D **97**, 115018 (2018).

[50] A. M. Sirunyan *et al.* (CMS Collaboration), Search for a heavy right-handed W boson and a heavy neutrino in events with two same-flavor leptons and two jets at $\sqrt{s} = 13$ TeV, J. High Energy Phys. 05 (2018) 148.

[51] A. Tumasyan *et al.* (CMS Collaboration), Search for a right-handed W boson and a heavy neutrino in proton-proton collisions at $\sqrt{s} = 13$ TeV, arXiv:2112.03949.

[52] M. Aaboud *et al.* (ATLAS Collaboration), Search for a right-handed gauge boson decaying into a high-momentum heavy neutrino and a charged lepton in pp collisions with the ATLAS detector at $\sqrt{s} = 13$ TeV, Phys. Lett. B **798**, 134942 (2019).

[53] R. Foot, X. G. He, H. Lew, and R. R. Volkas, Model for a light Z-prime boson, Phys. Rev. D **50**, 4571 (1994).

[54] A. Atre, T. Han, S. Pascoli, and B. Zhang, The search for heavy Majorana neutrinos, J. High Energy Phys. 05 (2009) 030.

[55] F. F. Deppisch, P. S. Bhupal Dev, and A. Pilaftsis, Neutrinos and collider physics, New J. Phys. **17**, 075019 (2015).

[56] A. Pilaftsis, Radiatively induced neutrino masses and large Higgs neutrino couplings in the standard model with Majorana fields, Z. Phys. C **55**, 275 (1992).

[57] J. Barry and W. Rodejohann, Lepton number and flavour violation in TeV-scale left-right symmetric theories with large left-right mixing, J. High Energy Phys. 09 (2013) 153.

[58] P. S. Bhupal Dev, S. Goswami, and M. Mitra, TeV-scale left-right symmetry and large mixing effects in neutrinoless double beta decay, Phys. Rev. D **91**, 113004 (2015).