

Meson masses and decay constants in holographic QCD consistent with ChPT and HQET

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We focus on the chiral and heavy quark mass expansion of meson masses and decay constants. We propose a light-front QCD formalism for the evaluation of these quantities, consistent with chiral perturbation theory and heavy quark effective theory.

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I. INTRODUCTION

In Refs. [1,2] we studied the chiral properties and heavy quark mass behavior of masses and decay constants of mesons and tetraquarks. We proposed longitudinal light-front wave functions (LFWFs) of mesons, which helps to provide a systematic chiral expansion of masses and decay constants of light pseudoscalar mesons $P = \pi, K, \eta$ and heavy hadrons/tetraquarks containing u, d, s quarks. In particular, the longitudinal part of the LFWF was constructed in terms of current quark masses, which helped to introduce the mechanism of explicit chiral symmetry breaking. The idea of the current quark mass dependence of the longitudinal LFWF was originally proposed in two-dimensional large N_c QCD [3], later was used in the context of the two-dimensional massive Schwinger model [4–6], and then was applied in holographic QCD [1,2,7,8]. In addition, in Refs. [1,2], for the case of LFWFs of heavy hadrons and tetraquarks, we implemented the constraints of heavy quark effective theory (HQET) and potential models for heavy quarkonia. In particular, in the heavy quark limit $m_Q \rightarrow \infty$, we reproduced the mass splitting and scaling of leptonic decay constants of heavy-light mesons and heavy quarkonia.

Recently, in Refs. [9–13], our ideas were further developed by the construction of the so-called longitudinal potential, which produces longitudinal wave functions of hadrons. We feel that we can improve the construction of

such potentials, by requiring a more exact correspondence with chiral perturbation theory (ChPT) [14,15], which is the low-energy limit of quantum chromodynamics (QCD). In particular, such correspondence requires that the chiral Lagrangian/Hamiltonian must vanish in the limit of vanishing current quark masses of light u, d, s quarks. Notice that this is not a case in the formalisms proposed in Refs. [9–13]. There are a few conditions that should be imposed. First of all, the quark condensate B is a Lorentz invariant quantity [the vacuum expectation of scalar quark operator $B = |\langle 0 | \bar{q}q | 0 \rangle| / (2F_\pi^2)$, where F_π is the pion leptonic decay constant] with no preference of transverse or longitudinal direction, i.e., it obeys rotational invariance. Second, the quark condensate in QCD and ChPT is defined as the partial derivative of the generating functional (or Lagrangian/Hamiltonian) with respect to current quark mass. This means that the quark condensate must be included into the holographic Hamiltonian in such a way that the derivative of the Hamiltonian with respect to the current quark mass gives the condensate. The solution is clear. One should add the chiral mass term $H_\chi = \mathcal{M}B$ into the holographic Hamiltonian, where $\mathcal{M} = \text{diag}\{m_u, m_d, m_s\}$ is the mass matrix of light (u, d, s) quarks, which are the constituents of the respective pseudoscalar meson $P = \pi, K, \eta$. Such modification of the holographic Hamiltonian guarantees that its partial derivative with respect to the current quark masses leads to the condensate. Another point, which requires one to reconsider the formalisms developed in Refs. [9–13], is the condition that the dependence on quark condensate in a Lagrangian/Hamiltonian should vanish in the chiral limit (i.e., when current quark masses vanish, $m_i \rightarrow 0$).

The main objective of the present paper is to extend our ideas proposed and developed in Refs. [1,2] and derive the

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Hamiltonians and equations of motion (EOMs) producing masses and leptonic decay constants of light mesons and mesons containing heavy quarks, consistent with ChPT and HQET.

The paper is organized as follows. In Sec. II, we present our formalism. We consistently study the chiral expansion of light meson masses and leptonic decay constants. Then, we extend our analysis on mesons containing heavy c or b quarks and derive a heavy quark mass expansion of their masses and lepton decay constants. Finally, Sec. III contains our conclusions.

II. FRAMEWORK

As we stressed in the Introduction, the task of deriving the longitudinal Hamiltonian (potential) in the context of light-front QCD should necessary take into account

$$\begin{aligned} |\pi^+\rangle &= |P(u_1, \bar{d}_2)\rangle, & |\pi^-\rangle &= |P(d_1, \bar{u}_2)\rangle, & |\pi^0\rangle &= \frac{1}{\sqrt{2}}[|P(u_1, \bar{u}_2)\rangle - |P(d_1, \bar{d}_2)\rangle], \\ |K^+\rangle &= |P(u_1, \bar{s}_2)\rangle, & |K^-\rangle &= |P(s_1, \bar{u}_2)\rangle, & |K^0\rangle &= |P(d_1, \bar{s}_2)\rangle, & |\bar{K}^0\rangle &= |P(s_1, \bar{d}_2)\rangle, \\ |\eta\rangle &= \frac{1}{\sqrt{6}}[|P(u_1, \bar{u}_2)\rangle + |P(d_1, \bar{d}_2)\rangle - 2|P(s_1, \bar{s}_2)\rangle]. \end{aligned} \quad (1)$$

Next, we define the Hamiltonian of pseudoscalar mesons, which produces their masses, as

$$\hat{H}_P = \sum_{k=1}^2 \hat{H}_P^{(k)}, \quad (2)$$

where k is the index numbering quarks in the pseudoscalar mesons, and \hat{H}_P will be specified below. Such Hamiltonian obeys the light-front Schrödinger type EOM:

$$\hat{H}_P |P\rangle = M_P^2 |P\rangle, \quad (3)$$

where M_P^2 is the mass of pseudoscalar meson squared.

The master formula for the mass spectrum of pseudoscalar mesons reads:

$$M_P^2 = \langle P | \hat{H}_P | P \rangle = \int_0^1 dz \int_0^1 dx \psi_P(z, x) H_P(z, x) \psi_P(z, x), \quad (4)$$

where z is the holographic variable, corresponding to the scale—fifth dimension in the anti de-Sitter (AdS) space, x is the light-cone variable, $\psi_P(z, x)$ is the holographic wave function of the pseudoscalar meson, and $H_P(z, x)$ is the representation of the Hamiltonian \hat{H}_P in the (z, x) space. Now we specify $H_P(z, x)$

symmetry breaking term. In particular, it is not correct that these symmetry breaking terms be generated by the longitudinal part of Hamiltonian (potential) as was proposed in Refs. [9–13]. Another important point is that the full Hamiltonian cannot be fully universal and must be specific for each type of meson. We proceed step by step, starting from light pseudoscalar mesons.

A. Light mesons

First, we define the Fock states describing the light pseudoscalar mesons—quark-antiquark state $|P(q_i^i \bar{q}_2^j)\rangle = |q_1^i\rangle |\bar{q}_2^j\rangle$ with spin-parity $J^P = 0^-$, where i and j are the $SU(3)$ flavor indices. The Fock states of light pseudoscalar mesons $|P\rangle$, based on the $SU(3)$ classification, are defined in terms of $|P(q_i^i \bar{q}_2^j)\rangle$ as

$$H_P(z, x) = \sum_{k=1}^2 [H_{\text{kin}}^{(k)}(z, x) + H_{\text{CF}}^{(k)}(z, x) + H_{\chi}^{(k)} + H_I^{(k)}(z, x)]. \quad (5)$$

Here

$$H_{\text{kin}}^{(1)}(z, x) = -\frac{d^2}{2dz^2} + \frac{m_1^2}{x}, \quad H_{\text{kin}}^{(2)}(z, x) = -\frac{d^2}{2dz^2} + \frac{m_2^2}{1-x} \quad (6)$$

are the kinetic parts of the Hamiltonian acting on quark q_1^i and antiquark \bar{q}_2^j , respectively, m_1 and m_2 are the masses of the corresponding current quarks,

$$H_{\text{CF}}^{(1)}(z) = H_{\text{CF}}^{(2)}(z) = \frac{4L^2 - 1}{8z^2} \quad (7)$$

are centrifugal parts, where L is the angular orbital momentum,

$$H_{\chi}^{(1)} = m_1 B, \quad H_{\chi}^{(2)} = m_2 B, \quad (8)$$

and

$$H_I^{(k)}(z, x) = H_{I:T}^{(k)}(z) + H_{I:L}^{(k)}(x) \quad (9)$$

is the interaction term. The latter conventionally splits into a transversal part [16]

$$H_{I;T}^{(1)}(z) = H_{I;T}^{(2)}(z) = \frac{U_0(z)}{2}, \quad U_0(z) = \kappa^4 z^2 - 2\kappa^2 \quad (10)$$

and a longitudinal part $H_{I;L}^{(k)}(z, x)$. The latter was discussed in the series of papers [9–13]. In particular, it was proposed that the longitudinal interaction potential generates explicitly breaking of chiral symmetry. As we stressed before, this is not correct since it contradicts ChPT. On the other hand, in the constructions of Refs. [9–13] this longitudinal interaction Lagrangian is universal for all mesons, which again contradicts ChPT and HQET. We found that the Hamiltonian $H_{I;L}^{(k)}(x)$, in the case of light pseudoscalar mesons P , must have the form

$$\begin{aligned} H_{I;L}^{(1)}(x) &= H_{I;L}^{(2)}(x) \\ &= -\frac{\kappa^2}{2} [\partial_x(x(1-x)\partial_x) + (\alpha_1 + \alpha_2)(1 + \alpha_1 + \alpha_2)], \end{aligned} \quad (11)$$

where $\alpha_i = m_i/\kappa$ are the parameters specifying the longitudinal part of the meson wave function and κ is the dilaton scale parameter in the soft-wall AdS/QCD approach [17]. We remind the reader that the total mesonic LFWF function is defined as a product of transversal $\varphi_T(z)$, longitudinal $f_L(x)$, and flavor χ_P parts (see details in Ref. [1]):

$$\psi_P(z, x) = \varphi_T(z) f_L(x) \chi_P. \quad (12)$$

Transverse wave functions for mesons with arbitrary spin, angular orbital momentum, and radial quantum number can be found in Ref. [16]. As it was shown by t'Hooft in Ref. [3], and confirmed in Refs. [4–6], the longitudinal function reads

$$f_L(x) = N x^{\alpha_1} (1-x)^{\alpha_2}, \quad (13)$$

where N is the normalization constant fixed from the condition

$$1 = \int_0^1 dx [f_L(x)]^2, \quad (14)$$

and the α_i parameters are proportional to current quark masses.

The resulting masses of the mesons get contributions from the transverse part $M_T^2 = 4\kappa^2[n + (J + L)/2]$ [16], the longitudinal part M_L^2 , and additional term encoding symmetry breaking. For example, in the case of pseudoscalar mesons one gets: (i) $M_L^2 = 0$ due to the fact that the contribution of the longitudinal potential is fully compensated by the contribution of the mass term in kinetic term; (ii) term $H_{\chi}^{(k)}$ produces the leading order chiral corrections consistent with ChPT by construction.

After straightforward calculations we reproduce, for the masses of pseudoscalar mesons, both the Gell-Mann-Oakes-Renner and the Gell-Mann-Okubo relations:

$$\begin{aligned} M_{\pi}^2 &= M_{\pi^{\pm}}^2 = M_{\pi^0}^2 = 2B\hat{m}, & \hat{m} &= \frac{m_u + m_d}{2}, \\ M_{K^+}^2 &\equiv M_{K^{\pm}}^2 = B(m_u + m_s), \\ M_{K^0}^2 &\equiv M_{K^0/\bar{K}^0}^2 = B(m_d + m_s), \\ M_{\eta}^2 &= \frac{B}{3}(2\hat{m} + 4m_s), \\ 4M_K^2 &= M_{\pi}^2 + 3M_{\eta}^2, \end{aligned} \quad (15)$$

where

$$M_K^2 = \frac{M_{K^+}^2 + M_{K^0}^2}{2} \quad (16)$$

is the average kaon mass squared.

In the case of vector mesons the chiral symmetry breaking corrections were consistently studied in Refs. [18–23]. In particular, it was shown [23] that in this case there appears the same term which explicitly breaks chiral symmetry for the pseudoscalar mesons, but here it shows up with an arbitrary coupling a :

$$H_{\chi}^{V;a(k)} = a H_{\chi}^{(k)}. \quad (17)$$

In addition, for singlet states ω and ϕ there is an additional term, which distinguishes them from members of the ρ mesons triplet (ρ^+, ρ^-, ρ^0) and the two K^* doublets (K^{*+}, K^{*0}) and (K^{*-}, \bar{K}^{*0}). This second term is produced by the Hamiltonian construction, using H_{χ} multiplied with an additional and independent coupling b ,

$$H_{\chi}^{V;b(k)} = b H_{\chi}^{(k)}, \quad (18)$$

which is projected between matrices V^S of singlet states:

$$V^S = \text{diag} \left\{ \frac{\omega}{\sqrt{2}}, \frac{\omega}{\sqrt{2}}, -\phi \right\}. \quad (19)$$

The second term gives additional corrections, in the case of ω and ϕ states:

$$\delta M_{\omega}^2 = 2bB\hat{m}, \quad \delta M_{\phi}^2 = bBm_s, \quad (20)$$

Combining together the contributions of the two terms responsible for explicit chiral symmetry breaking, one gets for vector meson masses:

$$\begin{aligned}
M_\rho^2 &= M_{\rho^\pm}^2 = M_{\rho^0}^2 = 2aB\hat{m} = aM_\phi^2, \\
M_\omega^2 &= 2(a+b)B\hat{m} = (a+b)M_\pi^2, \\
M_{K^{*\pm}}^2 &= aB(m_u + m_s) = aM_{K^{\pm}}^2, \\
M_{K^{*0}/\bar{K}^{*0}}^2 &= aB(m_d + m_s) = aM_{K^0/\bar{K}^0}^2, \\
M_\phi^2 &= (2a+b)Bm_s = \left(a + \frac{b}{2}\right)(M_{K^\pm}^2 + M_{K^0/\bar{K}^0}^2 - M_\pi^2).
\end{aligned} \tag{21}$$

Other important quantities of light pseudoscalar mesons are leptonic decay constants. In this respect, pion leptonic decay constant was calculated for the first time in soft-wall AdS/QCD in Refs. [24–26]. In addition, in Ref. [25] leptonic decay constants of other pseudoscalar and vector mesons composed of both light and heavy quarks were also calculated. Later on, in Ref. [1], the effects of current quark masses in leptonic decay constants of light and heavy-light mesons, and heavy quarkonia, have been investigated, where full consistency with ChPT and HQET was achieved. In particular, the expression for the leptonic decay constant of pseudoscalar and vector mesons, in terms of the $\alpha_i = m_i/\kappa$ parameters, are given by the expression

$$\begin{aligned}
f_M(\alpha_1, \alpha_2) &= \kappa \frac{\sqrt{6}\Gamma(3/2 + \alpha_1)\Gamma(3/2 + \alpha_2)}{\pi \Gamma(3 + \alpha_1 + \alpha_2)} \\
&\times \sqrt{\frac{\Gamma(2 + 2\alpha_1 + 2\alpha_2)}{\Gamma(1 + 2\alpha_1)\Gamma(1 + 2\alpha_2)}}.
\end{aligned} \tag{22}$$

At leading order of chiral expansion, the leptonic decay constant is given by [1,24–26]

$$f_M^{(0)} = \frac{\kappa\sqrt{6}}{8}. \tag{23}$$

One can see that, in agreement with ChPT [15,21], the leading chiral symmetry breaking correction starts with the term linear in current quark mass:

$$f_M = f_M^{(0)} \left[1 + \frac{m_1 + m_2}{\kappa} \zeta + \mathcal{O}(m_1^2, m_2^2, m_1 m_2) \right], \tag{24}$$

where $\zeta = \frac{3}{2} - \log 4$. In particular, for the physical states of light pseudoscalar and vector mesons, one gets the following expressions for decay constant, including leading result and first-order chiral symmetry breaking correction

$$\begin{aligned}
f_{\pi^\pm} &= \frac{\kappa\sqrt{6}}{8} \left[1 + \frac{m_u + m_d}{\kappa} \zeta \right], \\
f_{K^\pm} &= \frac{\kappa\sqrt{6}}{8} \left[1 + \frac{m_u + m_s}{\kappa} \zeta \right]
\end{aligned} \tag{25}$$

and

$$\begin{aligned}
f_{\rho^\pm} &= \frac{\kappa\sqrt{6}}{8} \left[1 + \frac{m_u + m_d}{\kappa} \zeta \right], \\
f_{\rho^0} &= \frac{\kappa\sqrt{3}}{8} \left[1 + \frac{2m_u + m_d}{3\kappa} \zeta \right], \\
f_\omega &= \frac{\kappa\sqrt{3}}{8} \left[\frac{1}{3} + \frac{2m_u - m_d}{3\kappa} \zeta \right], \\
f_\phi &= \frac{\kappa\sqrt{6}}{8} \left[\frac{1}{3} + \frac{2m_s}{3\kappa} \zeta \right], \\
f_{K^{*\pm}} &= \frac{\kappa\sqrt{6}}{8} \left[1 + \frac{m_u + m_s}{\kappa} \zeta \right], \\
f_{K^{*0}/\bar{K}^{*0}} &= \frac{\kappa\sqrt{6}}{8} \left[1 + \frac{m_d + m_s}{\kappa} \zeta \right].
\end{aligned} \tag{26}$$

B. Heavy-light mesons

Next we discuss the mass spectrum and leptonic decay constants of heavy-light mesons. In this case we consider an expansion in inverse powers of the heavy quark mass and prove that we have full correspondence with HQET. In the following we define by q and Q the light and heavy quark, respectively.

Here, the longitudinal potential is similar to the case of light mesons and it reads

$$\begin{aligned}
H_{i;L}^{(1);q\bar{Q}}(x) &= H_{i;L}^{(2);q\bar{Q}}(x) = -\frac{\kappa^2}{2} [\partial_x(x(1-x)\partial_x) \\
&+ (\alpha_q + \alpha_Q)(1 + \alpha_q + \alpha_Q)],
\end{aligned} \tag{27}$$

where the α parameters are fixed as [1]

$$\alpha_Q = \frac{1}{2}, \quad \alpha_q = \frac{\bar{\Lambda}}{m_Q} \left[1 + \frac{m_q^2 + \bar{\Lambda}^2}{2m_Q\bar{\Lambda}} \right] - \frac{1}{2}, \tag{28}$$

where m_q and m_Q are the masses of light and heavy quark, $\bar{\Lambda}$ is the leading (of order Λ_{QCD}) and flavor independent correction to the heavy quark mass in the expansion of the mass of heavy-light meson $M_{q\bar{Q}}$ in HQET [27]:

$$M_{q\bar{Q}} = m_Q + \bar{\Lambda} + \mathcal{O}(1/m_Q) \tag{29}$$

Due to our choice of the α_q and α_Q parameters, we exactly reproduce the expansion for the mass of heavy-light mesons, i.e., Eq. (29). Note that this expansion is governed by the longitudinal potential (27).

Now we turn to discussion of the results for leptonic decay constants f_{qQ} of heavy-light mesons. Taking Eq. (22) for the leptonic decay constant of a meson with arbitrary quarks and substituting $\alpha_1 = \alpha_q$ and $\alpha_2 = \alpha_Q = 1/2$ we get

$$f_{qQ} = \frac{\kappa\sqrt{6}\Gamma(3/2 + \alpha_q)}{\pi \Gamma(7/2 + \alpha_q)} \sqrt{\frac{\Gamma(3 + 2\alpha_q)}{\Gamma(1 + 2\alpha_q)}} \\ = \frac{\kappa\sqrt{6}}{\pi} \frac{\sqrt{r(1+r)}}{(1+r/2)(2+r/2)}, \quad (30)$$

where

$$r = 1 + 2\alpha_q = \frac{2\bar{\Lambda}}{m_Q} \left[1 + \frac{m_q^2 + \bar{\Lambda}^2}{2m_Q\bar{\Lambda}} \right] \quad (31)$$

is the small parameter of order $\mathcal{O}(1/m_Q)$ in which powers we can expand. We get

$$f_{q\bar{Q}} = \frac{\kappa\sqrt{6r}}{2\pi} \left[1 - \frac{r}{4} + \mathcal{O}(r^2) \right] \sim \sqrt{\frac{\bar{\Lambda}}{m_Q}}. \quad (32)$$

One can see that at leading order of the heavy quark mass expansion, the decay constant $f_{q\bar{Q}}$ scales as $\sqrt{1/m_Q}$, in full agreement with HQET [27]. Another interesting result is that the chiral corrections appear at order m_q^2 and are suppressed in comparison with the linear m_q correction, which could be induced by the chiral Hamiltonian H_χ explicitly breaking chiral symmetry.

C. Heavy quarkonia

Finally, we consider the heavy quark mass expansion of masses and decay constants of heavy quarkonia. We start by specifying the longitudinal potential for heavy quarkonia

$$H_{i:L}^{(1);Q_1\bar{Q}_2}(x) = H_{i:L}^{(2);Q_1\bar{Q}_2}(x) \\ = -\frac{8E^4}{(m_{Q_1} + m_{Q_2})^2} [\partial_x(x(1-x)\partial_x) \\ + (\alpha_{Q_1} + \alpha_{Q_2})(1 + \alpha_{Q_1} + \alpha_{Q_2})], \quad (33)$$

and dilaton parameter as

$$\kappa_{Q_1\bar{Q}_2} = \kappa \left(\frac{\mu_{Q_1\bar{Q}_2}}{E} \right)^{3/4}, \quad (34)$$

where

$$\mu_{Q_1\bar{Q}_2} = \frac{m_{Q_1}m_{Q_2}}{m_{Q_1} + m_{Q_2}} \quad (35)$$

is the reduced mass of the bound state composed of heavy quarks Q_1 and Q_2 , and κ is the parameter of dimension of mass, which is of order $\mathcal{O}(1)$, i.e., independent on heavy flavor. E is the binding energy, which is defined as the leading correction to the heavy quark masses m_{Q_1} and m_{Q_2}

in the heavy quark mass expansion of the mass of heavy quarkonia $M_{Q_1\bar{Q}_2}$:

$$M_{Q_1\bar{Q}_2} = m_{Q_1} + m_{Q_2} + E + \mathcal{O}(1/m_{Q_1}, 1/m_{Q_2}). \quad (36)$$

In order to get consistency with HQET we fix the α_{Q_i} parameters as [1]

$$\alpha_{Q_i} = \frac{m_{Q_i}}{4E} \left[1 - \frac{E}{2m_{Q_i}} \right]. \quad (37)$$

Now we look at the leptonic decay constants of heavy quarkonia. First, we consider the leptonic decay constant of heavy quarkonia composed of quark and antiquark of the same flavor $Q_1 = Q_2 = Q$. Using Eq. (22) and substituting there $\alpha_{Q_1} = \alpha_{Q_2} = \alpha_Q$ one gets for leading term in the heavy quark mass expansion:

$$f_{Q\bar{Q}} = \frac{\kappa_{Q\bar{Q}}\sqrt{6}}{\pi^{3/4}} \frac{1}{(2\alpha)^{1/4}} \\ = \frac{\kappa\sqrt{3}}{\pi^{3/4}} \sqrt{\frac{m_Q}{E}} \sim \sqrt{m_Q} \quad (38)$$

in full agreement with HQET.

Another interesting case is the leptonic decay constant of the $B_c^+(c\bar{b})$ meson. Here, we apply the condition that the mass of charm quark is much smaller than the mass of the bottom quark $m_c \ll m_b$. In this limit one gets

$$f_{c\bar{b}} = \frac{2\kappa\sqrt{6}}{\pi^{3/4}} \frac{m_c}{\sqrt{m_b}E} \sim \frac{m_c}{\sqrt{m_b}}. \quad (39)$$

III. CONCLUSION

In this paper we continue our study of the consistency of light-front QCD motivated by soft-wall AdS/QCD, with ChPT and HQET. In particular, in Refs. [1,2] we preliminary studied chiral properties and heavy quark mass behavior of masses and decay constants of mesons and tetraquarks. We proposed longitudinal LFWFs of mesons and tetraquarks providing systematic and consistent chiral expansion of masses of mesons and tetraquarks. In Refs. [1,2] we did not specify the longitudinal potential, which should accompany the corresponding LFWFs. In recent papers [9–13] our ideas were further developed by derivation of the longitudinal potential, which produces masses of mesons and leading-order chiral corrections. As we stressed in the Introduction, Refs. [9–13] actually failed in the construction of this longitudinal potential. We claim that the source of the explicit breaking of chiral symmetry should be introduced following ChPT and it is a Lorenz invariant quantity and cannot be related to the longitudinal

dynamics of the bound state in LF QCD. We showed how to construct the longitudinal potential in order to get consistency with ChPT and also with HQET. Analytical results for leading correction for meson masses and leptonic decay constants were derived. Also we demonstrated how to proceed in the case of heavy-light mesons and heavy quarkonia, to get correspondence with HQET for the expansion of masses of these states and the power scaling of their leptonic decay constants at infinitely large values of heavy quark masses.

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- [1] T. Gutsche, V.E. Lyubovitskij, I. Schmidt, and A. Vega, *Phys. Rev. D* **87**, 056001 (2013).
 - [2] T. Gutsche, V.E. Lyubovitskij, and I. Schmidt, *Phys. Rev. D* **96**, 034030 (2017).
 - [3] G. 't Hooft, *Nucl. Phys.* **B75**, 461 (1974).
 - [4] H. Bergknoff, *Nucl. Phys.* **B122**, 215 (1977).
 - [5] Y. Ma and J.R. Hiller, *J. Comput. Phys.* **82**, 229 (1989).
 - [6] Y.-Z. Mo and R. J. Perry, *J. Comput. Phys.* **108**, 159 (1993).
 - [7] S. S. Chabysheva and J. R. Hiller, *Ann. Phys. (Amsterdam)* **337**, 143 (2013).
 - [8] J.R. Forshaw and R. Sandapen, *Phys. Rev. Lett.* **109**, 081601 (2012).
 - [9] Y. Li and J. P. Vary, *Phys. Lett. B* **825**, 136860 (2022).
 - [10] G.F. de Teramond and S. J. Brodsky, *Phys. Rev. D* **104**, 116009 (2021).
 - [11] M. Ahmady, H. Dahiya, S. Kaur, C. Mondal, R. Sandapen, and N. Sharma, *Phys. Lett. B* **823**, 136754 (2021).
 - [12] M. Ahmady, S. Kaur, S. L. MacKay, C. Mondal, and R. Sandapen, *Phys. Rev. D* **104**, 074013 (2021).
 - [13] Y. Li and J. P. Vary, *arXiv:2202.05581*.
 - [14] S. Weinberg, *Physica (Amsterdam)* **96A**, 327 (1979).
 - [15] J. Gasser and H. Leutwyler, *Ann. Phys. (N.Y.)* **158**, 142 (1984).
 - [16] T. Gutsche, V.E. Lyubovitskij, I. Schmidt, and A. Vega, *Phys. Rev. D* **85**, 076003 (2012); T. Branz, T. Gutsche, V. E. Lyubovitskij, I. Schmidt, and A. Vega, *Phys. Rev. D* **82**, 074022 (2010).
 - [17] A. Karch, E. Katz, D. T. Son, and M. A. Stephanov, *Phys. Rev. D* **74**, 015005 (2006).
 - [18] E. E. Jenkins, A. V. Manohar, and M. B. Wise, *Phys. Rev. Lett.* **75**, 2272 (1995).
 - [19] J. Bijnens and P. Gosdzinsky, *Phys. Lett. B* **388**, 203 (1996).
 - [20] J. Bijnens, P. Gosdzinsky, and P. Talavera, *Nucl. Phys.* **B501**, 495 (1997).
 - [21] J. Bijnens, P. Gosdzinsky, and P. Talavera, *Phys. Lett. B* **429**, 111 (1998).
 - [22] P. C. Bruns and U. G. Meißner, *Eur. Phys. J. C* **40**, 97 (2005).
 - [23] M. Dax, T. Isken, and B. Kubis, *Eur. Phys. J. C* **78**, 859 (2018).
 - [24] A. Vega, I. Schmidt, T. Branz, T. Gutsche, and V. E. Lyubovitskij, *Phys. Rev. D* **80**, 055014 (2009).
 - [25] T. Branz, T. Gutsche, V. E. Lyubovitskij, I. Schmidt, and A. Vega, *Phys. Rev. D* **82**, 074022 (2010).
 - [26] S. J. Brodsky and G. F. de Teramond, *Phys. Rev. D* **77**, 056007 (2008).
 - [27] M. Neubert, *Phys. Rep.* **245**, 259 (1994).