Final-state interaction in the process $e^+e^- \to \Lambda_c\bar{\Lambda}_c$

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(Received 21 February 2022; accepted 24 March 2022; published 7 April 2022)

We show that the final-state interaction explains the nontrivial near-threshold energy dependence of the cross section of the process $e^+e^- \to \Lambda_c\bar{\Lambda}_c$ observed by the Belle and BESIII collaborations. This energy dependence is the result of the mixture of S-wave and D-wave components of the $\Lambda_c\bar{\Lambda}_c$ wave function due to a tensor interaction. The Coulomb potential is important only in the narrow energy region about a few MeV above the threshold of the process. It is shown that the widely used assumption that the impact of the Coulomb interaction on the cross sections of hadron production is reduced to the Sommerfeld-Gamow-Sakharov factor is not correct.

DOI: 10.1103/PhysRevD.105.074002

I. INTRODUCTION

During the last few decades, a lot of processes with hadron production have been investigated in detail in the corresponding near-threshold energy regions. In these energy regions, a strong energy dependence of the cross sections was observed. For instance, this phenomenon is manifested in the processes $e^+e^- \to p\bar{p}$ [1–6], $e^+e^- \to n\bar{n}$ [7], $e^+e^- \to B\bar{B}$ [8], $J/\psi(\psi') \to p\bar{p}\pi^0(\eta)$ [9–11], $J/\psi(\psi') \to p\bar{p}\omega(\gamma)$ [11–15], and $e^+e^- \to \phi\Lambda\bar{\Lambda}$ [16]. Note that there is no conventional view on the origin of such strong energy dependence. One of the most natural explanations is the effect of the final-state interaction of produced hadrons. Indeed, a small relative velocity of hadrons in the near-threshold energy region results in a strong effect due to a large effective time of interaction. In a set of publications [17–26], it was shown that the account for the final-state interaction allows one to explain the available experimental data with good accuracy. At present, it is impossible to describe the interaction of hadrons at small relative velocities using QCD. As a result, it is necessary to employ various phenomenological models. Comparison of the theoretical predictions with the available experimental data allows one to fix the parameters of the models.

Much attention of researchers was attracted to the process $e^+e^- \to \Lambda_c\bar{\Lambda}_c$. The corresponding cross section demonstrates a very nontrivial energy dependence in the vicinity of the threshold. The Belle Collaboration observed a peak in the cross section at the energy of the $\Lambda_c\bar{\Lambda}_c$ pair about 80 MeV above the threshold [27]. Later, the data obtained by the BESIII Collaboration demonstrated a plateau in the cross section in the energy region from 1.5 to 30 MeV above the threshold [28]. At first glance, these two sets of data seem to be inconsistent with each other, and it is not clear if all these data can be explained by the final-state interaction of Λ_c baryons [29]. Note that the authors of Ref. [29] do not see any disagreement between BESIII and Belle data. Recently, there was an attempt to describe the behavior of the cross section of this process using the modified Sommerfeld-Gamow-Sakharov factor [30]. However, this factor alone cannot describe the peak in the cross section at an energy of about 80 MeV. It is necessary to emphasize that the effective model of $\Lambda_c\bar{\Lambda}_c$ interaction should describe not only the energy dependence of the cross section, but also the energy dependence of the ratio $|G_E/G_M|$ of electromagnetic form factors of the Λ_c baryon. This ratio was also measured by the BESIII Collaboration [28]. In the present work, we propose a simple model of $\Lambda_c\bar{\Lambda}_c$ interaction which reproduces all features of the cross section of the process $e^+e^- \to \Lambda_c\bar{\Lambda}_c$ together with the energy dependence of the ratio $|G_E/G_M|$.

II. THEORETICAL APPROACH

The method to account for the effects of a baryon-antibaryon final-state interaction was developed in Refs. [17–20] for the case of a nucleon-antinucleon system. This method is based on the assumption that the process of production of nonrelativistic hadrons can be separated into two stages. In the first stage, virtual hadrons are produced at small distances, and the amplitude of their production is a

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smooth function of the energy of the system. In the second stage, the interaction takes place at large distances, where the hadrons become real, but not virtual. Therefore, any sharp behavior of the cross section of the process is the result of the interaction of hadrons at large distances. This interaction can be described by some effective optical potentials. The imaginary part of the optical potentials takes into account the annihilation of hadrons into mesons. In the case of nucleon-antinucleon pair production [17–20], it is necessary to account for the components of the wave function with the isospins $I = 0$ and $I = 1$. A mixture of these components arises, firstly, due to the electromagnetic interaction, and secondly, due to the difference of proton and neutron masses. Consideration of the process $e^+e^- \to \Lambda_c\bar{\Lambda}_c$ is essentially simpler than that of the processes of nucleon-antinucleon pair production. The effective potential of the $\Lambda_c\bar{\Lambda}_c$ interaction is real, because it is not necessary to account for the $\Lambda_c\bar{\Lambda}_c$ annihilation into mesons. Besides this, the isospin of the $\Lambda_c\bar{\Lambda}_c$ pair is zero.

The process of e^+e^- annihilation into a $\Lambda_c\bar{\Lambda}_c$ pair goes through a virtual photon. Hence, the quantum numbers of the pair are $J^{PC} = 1^{--}$, so that the angular momentum of a pair is $l = 0, 2$ and the total spin is $S = 1$. The S-wave and D-wave components of the wave function are mixed by the tensor forces. The effective potential of $\Lambda_c\bar{\Lambda}_c$ interaction contains several parts and has the form ($\hbar = c = 1$)

$$\mathcal{V}(r) = -\frac{\alpha}{r} + V_S(r)\delta_{l0} + \left(\frac{6}{Mr^2} + V_D(r)\right)\delta_{l2} + V_T(r)S_{12}. \tag{1}$$

Here, α is the fine-structure constant; V_S, V_D, and V_T are the S-wave, the D-wave, and the tensor contributions to the potential, respectively; $S_{12} = 6(\boldsymbol{S}\cdot\boldsymbol{n})^2 - 4$ is the tensor operator; $\boldsymbol{S}$ is the spin operator of the $\Lambda_c\bar{\Lambda}_c$ pair; and $\boldsymbol{n} = \boldsymbol{r}/r$. The corresponding coupled-channels radial Schrödinger equation can be written in the form

$$\left[\frac{p_r^2}{M} + \mathcal{V}(r) - E\right]\Psi(r) = 0, \tag{2}$$

where M is the mass of the Λ_c baryon, E is the energy of the pair counted from the threshold, and $(-p_r^2)$ is the radial part of the Laplace operator. The wave function $\Psi(r)$ of the Schrödinger equation [Eq. (2)] has two components: namely, $\Psi^T(r) = (u(r), w(r))$, where the first component corresponds to the S-wave and the second one to the D-wave. In this basis, the potential $\mathcal{V}(r)$ can be written in a matrix form:

$$\mathcal{V}(r) = \begin{pmatrix} -\frac{\alpha}{r} + V_S & -2\sqrt{2}V_T \\ -2\sqrt{2}V_T & -\frac{\alpha}{r} + \frac{6}{Mr^2} + V_D - 2V_T \end{pmatrix}. \tag{3}$$

Two regular independent solutions of the Schrödinger equation [Eq. (2)] have the following asymptotics at $r \to \infty$ (see Refs. [17–20]):

$$\begin{aligned}
\Psi_1^T(r) &= \frac{1}{2i}(S_{11}\chi_0^+ - \chi_0^-, S_{12}\chi_2^+), \\
\Psi_2^T(r) &= \frac{1}{2i}(S_{21}\chi_0^+, S_{22}\chi_2^+ - \chi_2^-), \\
\chi_l^\pm &= \frac{1}{kr}\exp[\pm i(kr - l\pi/2 + \eta\ln(2kr) + \sigma_l)], \\
\sigma_l &= \frac{i}{2}\ln\frac{\Gamma(1+l+i\eta)}{\Gamma(1+l-i\eta)}, \quad \eta = \frac{M\alpha}{2k}, \quad k = \sqrt{ME},
\end{aligned} \tag{4}$$

where $\Gamma(x)$ is the Euler gamma function and S_{ij} are some functions of energy. In the nonrelativistic approximation, the electric G_E and the magnetic G_M form factors of the Λ_c baryon in the time-like region are expressed in terms of $u_1(0)$ and $u_2(0)$, which are the S-wave components of two independent solutions at $r = 0$:

$$\begin{aligned}
G_E &= \mathcal{G}(u_1(0) - \sqrt{2}u_2(0)), \\
G_M &= \mathcal{G}\left(u_1(0) + \frac{1}{\sqrt{2}}u_2(0)\right).
\end{aligned} \tag{5}$$

Here, $\mathcal{G}$ is the energy-independent amplitude of $\Lambda_c\bar{\Lambda}_c$ pair production at small distances. Note that $u_2(0)$ is nonzero only in order to account for the tensor forces and the D-wave component of the wave function. The energy dependence of the ratio G_E/G_M is determined by the energy dependence of the ratio $f = u_2(0)/u_1(0)$:

$$\frac{G_E}{G_M} = \frac{1 - \sqrt{2}f}{1 + \frac{1}{\sqrt{2}}f}. \tag{6}$$

The integrated cross section of $\Lambda_c\bar{\Lambda}_c$ pair production has the form

$$\sigma = \frac{\pi k\alpha^2}{2M^3}|\mathcal{G}|^2(|u_1(0)|^2 + |u_2(0)|^2). \tag{7}$$

Therefore, near the threshold, both the cross section and the ratio of electromagnetic form factors depend on the energy via the functions $u_1(0)$ and $u_2(0)$. In the present paper, we calculate numerically these functions using some effective potential. The parameters of this potential are fixed by fitting the available experimental data.

III. RESULTS AND DISCUSSION

The exact potential of $\Lambda_c\bar{\Lambda}_c$ interaction is unknown, so a phenomenological potential model has to be proposed. Our previous works [21,26] devoted to the final-state interaction in various hadronic systems showed that the enhancement of the cross section of hadronic pair production is usually

TABLE I. The parameters of the potential of $\Lambda_c\bar{\Lambda}_c$ interaction.

	V_S	V_D	V_T
U (MeV)	$-447^{+5.1}_{-4.1}$	363^{+42}_{-33}	$22.1^{+1.1}_{-1.2}$
a (fm)	$1.425^{+0.006}_{-0.007}$	$2.66^{+0.1}_{-0.09}$	$2.66^{+0.1}_{-0.09}$

associated with the existence of a near-threshold resonant state. For all these cases, the lifetimes of produced hadronic systems before their decays are much higher than a typical hadronic time of $1/\Lambda_{\mathrm{QCD}} \sim 10^{-24}$ s. The shape of the invariant-mass spectrum of hadronic pair production is determined mainly by the parameters of this resonance, and the specific parametrization of the potential is not so important. Therefore, we consider the S-wave, D-wave, and tensor parts of the potential as rectangular potential wells:

$$V_n(r) = U_n\theta(a_n - r), \qquad n = S, D, T, \tag{8}$$

where $\theta(x)$ is the Heaviside function, and U_n and a_n are some fitting parameters. In addition, for the convenience of numerical calculations, the tensor potential is regularized at small distances by the factor

$$F(r) = \frac{(br)^2}{1 + (br)^2} \tag{9}$$

with $b = 10\ \mathrm{fm}^{-1}$. In fact, the results are almost independent of the specific value of the parameter b. The parameters of the potential, as well as the coefficient $\mathcal{G}$, are determined by fitting the experimental data and minimizing χ^2. The experimental data include measurements of the cross section of the process $e^+e^- \to \Lambda_c\bar{\Lambda}_c$ collected by the Belle [27] and BESIII [28] collaborations, as well as two measurements for the ratio of electric and magnetic form factors of the Λ_c baryon obtained by BESIII [28].

The parameters of the potential corresponding to the best fit are listed in Table I. Note that the radii of the D-wave and tensor parts of the potential turned out to be close to each other, and we set them to be equal. For the parameters of the potential obtained within our approach, the value of χ^2 is 7.5, so that $\chi^2/N_{df} = 0.75$, where N_{df} is the number of degrees of freedom. The results of this fit are shown in Fig. 1 by the solid curves.

Let us discuss the effect of various contributions to the potential on the shape of the cross section and the ratio $|G_E/G_M|$. If we set the tensor potential to be zero, then $|G_E/G_M|$ will be unity, and the plateau in the cross section

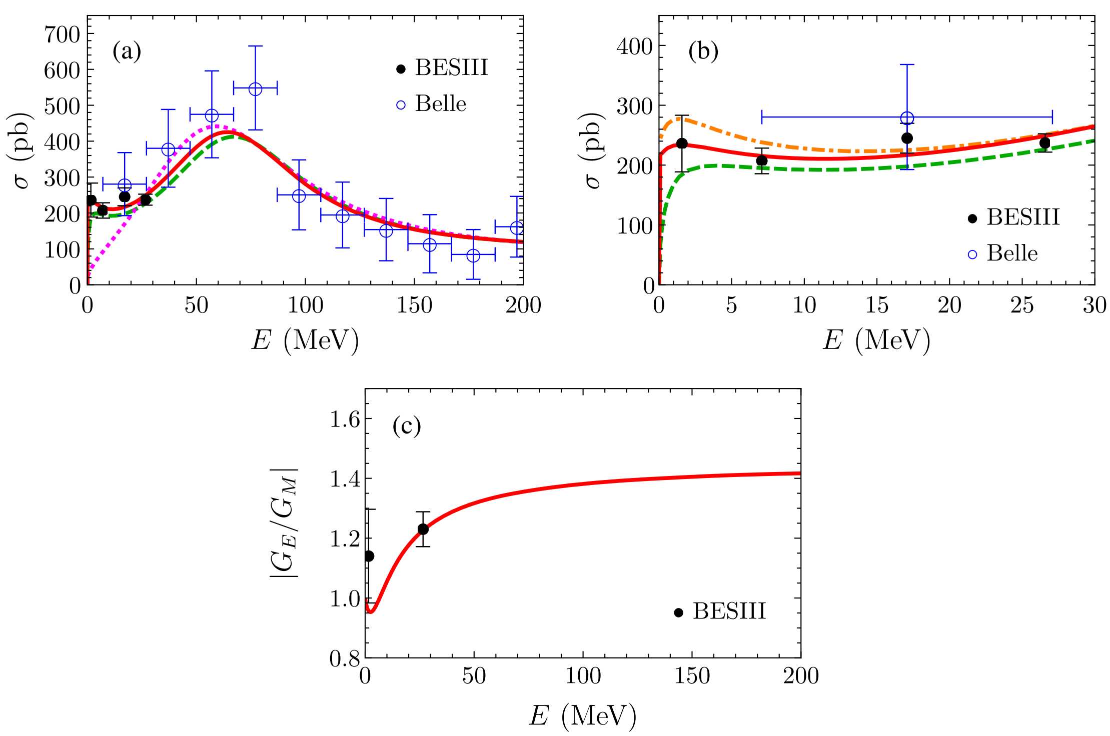

FIG. 1. The cross section of the process $e^+e^- \to \Lambda_c\bar{\Lambda}_c$ (a) in the energy region from the threshold to 200 MeV, and (b) in the narrow energy region. (c) The ratio of electric and magnetic form factors of the Λ_c baryon. The solid curves correspond to our predictions obtained with all contributions taken into account. The dashed curves are obtained without accounting for the Coulomb potential. The dotted curve corresponds to the prediction for zero value of the tensor potential. The dash-dotted curve shows the result for zero Coulomb potential multiplied by the Sommerfeld-Gamow-Sakharov factor. The experimental data are from Refs. [27,28].

at energies below 30 MeV will disappear. However, the peak at energies around 80 MeV and its shape are well reproduced. The corresponding results are shown in Fig. 1(a) by the dotted curve. Only nonzero values of V_S, V_D, and V_T together allow us to reproduce the entire set of experimental data.

It is interesting to investigate the effect of the Coulomb potential. The cross section calculated without the Coulomb potential is shown in Fig. 1 by the dashed curves. It is seen that the Coulomb potential is important only in the energy region very close to the threshold. For energies of $\Lambda_c\bar{\Lambda}_c$ pairs above a few MeV, the impact of the Coulomb interaction is not very important. It is generally accepted that the cross section calculated with the Coulomb interaction taken into account can be represented as the cross section calculated without the Coulomb potential multiplied by the so-called Sommerfeld-Gamow-Sakharov factor C,

$$C = \frac{2\pi\eta}{1 - e^{-2\pi\eta}}, \qquad \eta = \frac{M\alpha}{2k}. \tag{10}$$

The cross section calculated using this approach is shown in Fig. 1(b) by the dash-dotted curve. Obviously, this result is quite different from the result of the exact calculations (the solid curve). Despite the fact that the Sommerfeld-Gamow-Sakharov factor provides a nonzero cross section at the threshold, the factorization of the cross section does not work well enough. The same conclusion was previously made in Ref. [20] when analyzing the cross section of the process $e^+e^- \to p\bar{p}$. Therefore, the absence of factorization is not a specific feature of some process, but a general statement.

IV. CONCLUSION

It is shown that near the threshold, the nontrivial energy dependence of the cross section of the process $e^+e^- \to \Lambda_c\bar{\Lambda}_c$ and the ratio $|G_E/G_M|$ can be well described by the final-state interaction. We used a simple potential of $\Lambda_c\bar{\Lambda}_c$ interaction containing S-wave, D-wave, and tensor parts. Each part of the potential was parametrized by a rectangular potential well. A peak in the spectrum of the process corresponds to the near-threshold resonant state of the $\Lambda_c\bar{\Lambda}_c$ pair. The plateau in the energy region below 30 MeV is due to the tensor and D-wave parts of the potential. These parts of the potential are responsible also for the deviation of the ratio $|G_E/G_M|$ from unity.

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