Black hole remnants from dynamical dimensional reduction?

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A intriguing feature shared by many quantum gravity programs is the dynamical decrease of the spectral dimension from $D_s = 4$ at macroscopic to $D_s \approx 2$ at microscopic scales. In this note, we study the impact of this transition on the energy loss of static, spherically symmetric black holes due to Hawking radiation. We demonstrate that the decrease in the spectral dimension renders the luminosity of a black hole finite. While this slightly increases the lifetime of light black holes, we find that this mechanism is insufficient to generate long-lived black hole remnants. We briefly comment on the relation of our findings to previous work on this topic.

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I. INTRODUCTION

Black holes provide a powerful laboratory for testing our ideas about space and time at both the theoretical as well as the observational frontier. A striking theoretical prediction based on quantum field theory in curved spacetime is that black holes are not entirely black: their event horizon emits black body radiation with a temperature inversely proportional to the mass of the black hole [1–4]. Owed to this so-called Hawking effect, the black hole loses mass and eventually evaporates completely within a finite time span. The robustness of this scenario has been corroborated in a number of different ways comprising perturbative computations in a fixed background spacetime [1], the detector approach [5–9], as well as by analogy to the Unruh effect [10].

The Hawking effect gives rise to a series of theoretical puzzles though. First, one encounters the black hole information paradox reviewed in [11]. The picture above suggests that the physical information about how the black hole was formed could permanently disappear, allowing many physical states to evolve into the same state. Basically, there are three viewpoints on this problem [12,13]: (1) information is indeed lost after the black holes has evaporated completely, (2) evaporation stops and information is preserved inside a stable remnant, and (3) information may be returned outside via Hawking radiation. In particular, with regard to option (2) it is important to understand the final configuration emanating from the black hole evaporation process.

The second puzzle comes from the increase of the horizon temperature as the black hole becomes lighter and lighter. For instance, a black hole with a mass of the order of the Planck mass would have a temperature $T_h \approx 10^{31}$ K. Theoretically, it is then predicted that the final stage of the black hole evaporation process generates a so-called thunderbolt singularity. From the observational perspective, this feature leads to the prediction that a black hole reaching a mass range between 10^9–10^{13} g should create powerful short-lived gamma-ray bursts with energy of a few hundred MeV [14]. So far, all attempts to detect such high energy bursts have failed and resulted only in upper bounds on black hole evaporation rate in the vicinity of the Earth [15,16].

Third, a cold phase in a black hole's life gives an interesting perspective on dark matter [17–19]. If the evaporation process is halted as some mass (potentially set by the Planck mass), then one may end up with a stable configuration which, except for its gravitational interaction, has no or extremely small interaction with ordinary matter and hence fits perfectly into the definition of a weakly interacting massive particle. This has led many authors to claim that in fact such tiny black holes could explain the mystery of dark matter in our Universe, see, e.g., [17–19] for selected references. In Ref. [20] it is shown that no major constraint can be cast upon the properties of Planck-size remnants if they play the role of dark matter at a cosmological scale; nonetheless, the way these remnants can be produced and their stability could be potential weak spots of such scenarios [18].

These puzzles, clearly ask for a better understanding of the black hole evaporation process beyond the quantum

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field theory in curved spacetime analysis. It is conceivable that the ultimate answer lies in the realm of a theory of quantum gravity. Since there are currently many different routes at various stages of development, we take a different angle on the problem: quite strikingly many quantum gravity programs, including string theory [21–23], loop quantum gravity and spin foams [24–27], asymptotically safe gravity [28–35], causal dynamical triangulations [36,37], and Hořava-Lifshitz gravity [38–40] predict a dynamical dimensional reduction of the theories momentum space [41,42] (also see [43] for an early account of this idea). Specifically, the spectral dimension D_s, probing the effective dimension experienced by a random walk, drops from $D_s = 4$ at macroscopic scales to $D_s \approx 2$ at short distances. For instance, the analysis of geometries obtained from the causal dynamical triangulations program reported a scale-dependent spectral dimension [44],

$$D_s(\sigma) = a - \frac{b}{c + \sigma}, \qquad (1)$$

where σ denotes the diffusion time and the free parameters a, b, and c have been fitted to $a = 4.02$, $b = 119$, and $c = 54$. A similar analysis within Euclidean dynamical triangulations [45] obtained $D_s(\sigma) = 3.94 \pm 0.16$ and $D_s(\sigma) = 1.44 \pm 0.19$ in the limit of long and short diffusion times when extrapolating to the continuum and infinite volume limits. Generalizing [46], the scale-dependent spectral dimension along a fixed asymptotically safe renormalization group trajectory has been computed in [47], leading to a three plateau structure with $D_s(\sigma) = 4$, $D_s(\sigma) = 4/3$, and $D_s(\sigma) = 2$ at large, intermediate, and short distances, respectively (also see [48] for a review).[1]

In [51] the scale dependence of the spectral dimension has been linked to the dimensionality of the theories momentum space: a drop of the spectral dimension indicates that there are less degrees of freedom at high energy as compared to the expectation based on the dimension of spacetime observed at macroscopic scales. Thus the dynamical dimensional reduction provides a powerful mechanism for eliminating divergences occurring at high energies. Within the realm of multiscale models [52–55], the phenomenological consequences of this mechanism have been explored, e.g., in the context of quantum field theory [56], cosmology [57,58], and also for the Unruh effect [59], see [60] for an up-to-date review.[2]

In [63] the authors suggested an intricate connection between dynamical dimensional reduction and the formation of cold remnants formed at the end of the black hole evaporation process based on a two-dimensional dilaton-gravity model. The goal of our work is to complement this analysis by implementing the effect of a drop in the spectral dimension in the thermodynamic properties of a four-dimensional Schwarzschild black hole. As our main result, we demonstrate that the mechanism of dynamical dimensional reduction removes the thunderbolt singularity appearing in the last stages of the black hole evaporation process. It does not lead to the formation of long-lived black hole remnants though. The latter requires additional ingredients, with a change in the topology of the black hole solution being the most probable one. Our analysis uses the framework of quantum field theory in a curved background, incorporating the “quantum gravity effect” of a dynamical dimensional reduction at the level of the field forming the Hawking radiation.

The rest of our work is organized as follows. Sections II and III provide a brief introduction to black hole thermodynamics and the concept of generalized dimensions, respectively. Our analysis is presented in Sec. IV and we conclude with a brief discussion and outlook in Sec. V.

II. BLACK HOLE THERMODYNAMICS IN A NUTSHELL

We start by reviewing the basics of black hole thermodynamics, referring to [64–66] for more detailed, pedagogical accounts. For simplicity, we consider spherically symmetric black holes described by the Schwarzschild solution. In natural units where $G = c = \hbar = k_b = 1$, the resulting line element is

$$ds^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \left(1 - \frac{2M}{r}\right)^{-1} dr^2 - r^2 d\Omega^2. \qquad (2)$$

Here $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$ is the line element on the unit two-sphere and M is the mass of the black hole. The geometry (2) possesses an event horizon at

$$r_h = 2M. \qquad (3)$$

Based on (2) one readily deduces that the area of this horizon is

$$A_h = 4\pi r_h^2 = 16\pi M^2. \qquad (4)$$

Any object or photon crossing this horizon inevitably has to move inward, eventually ending at the curvature singularity at $r = 0$. Classically, signals emitted at $r \leq r_h$ cannot reach an observer stationed at $r > r_h$. Hence the terminology “black hole.”

The analysis within the framework of quantum field theory in curved spacetime [1] shows, however, that the event horizon emits black body radiation (Hawking radiation) with a temperature proportional to the surface gravity at the horizon. For the Schwarzschild black hole (2), this results in

[1]For related works in the context of string theory focusing on the dynamical dimensional reduction of the spacetime dimension at short distances, see [49,50].

[2]Along different lines, fractal aspects of black holes have been considered within the “ungravity program” [61,62].

$$T_h = \frac{1}{8\pi M}. \quad (5)$$

The resulting luminosity L is then given by

$$L = A_h I, \quad (6)$$

where I is the integrated black body spectrum. For bosonic fields as the scalar field considered in this work

$$I = \int \frac{d^3 p}{(2\pi)^3} \frac{E}{e^{E/T_h} - 1}. \quad (7)$$

For a massless photon, $E = |\vec{p}| = \omega$, and one recovers the standard result [66][3]:

$$L_{\rm massless} = \frac{8M^2}{\pi} \int_0^\infty d\omega \frac{\omega^3}{e^{8\pi M \omega} - 1}, \\ = \frac{1}{7680\pi M^2}. \quad (8)$$

Here we have performed the integral over frequencies and expressed the result in terms of the black hole mass M by substituting (5). $L_{\rm massless}$ as a function of M is then illustrated as the blue straight line in Fig. 1. Equation (8) exhibits the curious feature that black holes become more and more luminous the lighter they become. In particular L diverges as $M \to 0$. The presence of this so-called thunderbolt singularity suggests that the semiclassical analysis breaks down when describing the final stage of black hole evaporation [67–70].

The life-time of a black hole with initial mass M_0 can then be obtained by integrating the mass-loss formula

$$\frac{dM}{dt} = -L. \quad (9)$$

Substituting (8) gives the black hole evaporation time

$$t_{\rm evap} = 2560\pi M_0^3. \quad (10)$$

Thus the emission of Hawking radiation renders the life-time of the black hole finite.

[3]In general, the power contained in the Hawking radiation associated with a massless scalar field has the form $P = \sum_l \int_0^\infty d\omega P_l(\omega)$ with the lth multipole contributing with $P_l(\omega) = \frac{A_h}{8\pi^2} T_l(\omega)\omega^3 (e^{\omega/T_h} - 1)^{-1}$. Our analysis focuses on the $l = 0$ sector and neglects the gray-body corrections $T_l(\omega)$. Since the latter encode the transmission probability of Hawking radiation reaching future infinity without being backscattered by the gravitational barrier surrounding the black hole, one expects that these will lead to an additional suppression of the massive contributions as compared to the massless ones. Based on Eq. (23) one then expects that the inclusion of the $T_l(\omega)$ will further inhibit the formation of remnants while leaving the leading order analysis unaffected.

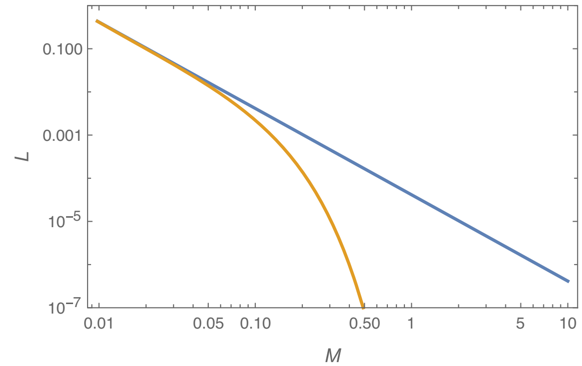

FIG. 1. Luminosity of a Schwarzschild black hole as a function of the mass M. The case of a massless and a massive scalar field with $m^2 = 1$ are illustrated by the straight blue and orange lines, respectively.

The luminosity formula (6) is readily generalized to the case of a scalar field with mass m. In this case the energy appearing in (7) is replaced by the relativistic dispersion relation $E^2 = \vec{p}^2 + m^2$. In general, I does not admit a simple analytic expression. Nevertheless, it is instructive to study the following limits: if $m/T_h \ll 1$, then the particle is relativistic and one essentially recovers the massless case. For temperatures $m/T_h \gg 1$ the particle is nonrelativistic and the Bose-Einstein distribution in (7) may be approximated by the Boltzmann distribution. This shows that the contribution of a massive mode is actually exponentially suppressed at temperatures $T_h \lesssim m$. The full expression for $L_{\rm massive}$ as a function of M is obtained by numerical integration. The result is the orange line shown in Fig. 1.

III. THE SPECTRAL DIMENSION IN A NUTSHELL

An intuitive picture about quantum gravity is that spacetime at short distances will develop nonmanifoldlike features. A first step towards characterizing the resulting structures is to generalize the notion of "dimension" borrowing concepts from fractal geometry [71]. In this way one naturally distinguishes between the Hausdorff dimension (based on covering a set of points with balls of decreasing radius), the spectral dimension (measuring the dimension "felt" by a diffusing particle), or the walk dimension (related to the expectation value of the distance traveled by a random walk as function of the diffusion time) of an Euclidean space. While all of these dimensions agree when working on manifolds, they characterize distinct properties of fractal spaces. In the present work, the key role is played by the spectral dimension D_s, which may be interpreted as the dimension of the theories momentum space [51]. A decrease of the spectral dimension at high energy may then be interpreted as "the theory possessing less degrees of freedom than its analog defined on a background manifold."

Formally, the spectral dimension d_s and its scale-dependent generalization $D_s(\sigma)$ is introduced by studying the diffusion of a test particle on a d-dimensional Euclidean spacetime with metric $g_{\mu\nu}$ with respect to the fiducial diffusion time σ. Denoting the Laplacian constructed from $g_{\mu\nu}$ by $\Delta \equiv -g^{\mu\nu} D_\mu D_\nu$, and introducing $F(\Delta) \equiv G(\Delta)^{-1}$ with $G(\Delta)$, being the position-space representation of the particles propagator, the motion of the test particle is captured by the generalized heat equation

$$\frac{\partial}{\partial\sigma} K_g(\xi, \xi_0; \sigma) = -F(\Delta) K_g(\xi, \xi_0; \sigma) \tag{11}$$

subject to the boundary condition

$$K_g(\xi, \xi_0; \sigma)|_{\sigma=0} = \delta^d(\xi - \xi_0). \tag{12}$$

Here $K_g(\xi, \xi_0; \sigma)$ is the heat-kernel associated with $F(\Delta)$. It describes the probability of the particle defusing from the initial point ξ_0 to ξ during the time-interval σ. In particular, one recovers the standard heat-equation for $F(\Delta) = \Delta$. The return probability $P_g(T)$ is then defined by the particle returning to its initial point after time σ

$$P_g(\sigma) \equiv V^{-1} \int d^d\xi \sqrt{g} K_g(\xi, \xi; \sigma). \tag{13}$$

Here $V \equiv \int d^d\xi \sqrt{g}$ is the volume of the space. Based on (13) the spectral dimension d_s is then defined as

$$d_s \equiv -2 \lim_{\sigma\to 0} \frac{d \ln P_g(\sigma)}{d \ln \sigma}. \tag{14}$$

For the standard heat-equation on a smooth manifold $d_s = d$ agrees with the topological dimension of the manifold. At this stage, it is convenient to generalize (14), allowing for a scale-dependent spectral dimension

$$D_s(\sigma) \equiv -2 \frac{d \ln P_g(\sigma)}{d \ln \sigma}. \tag{15}$$

$D_s(\sigma)$ takes into account the possibility that long random walks may experience a different spectral dimension than the infinitesimal ones entering in the definition (14).

On a flat Euclidean space $\mathbb{R}^d$ with metric $\delta_{\mu\nu}$ the generalized heat-equation (11) can be solved using Fourier-techniques

$$K_\delta(\xi, \xi_0; \sigma) = \int \frac{d^d p}{(2\pi)^d} e^{ip(\xi-\xi_0)} e^{-\sigma F(p^2)}. \tag{16}$$

The resulting return probability is

$$P_\delta(\sigma) = \int \frac{d^d p}{(2\pi)^d} e^{-\sigma F(p^2)}. \tag{17}$$

For $F(p^2) = p^2$, the return probability evaluates to

$$P_\delta(\sigma) = (4\pi\sigma)^{-d/2}. \tag{18}$$

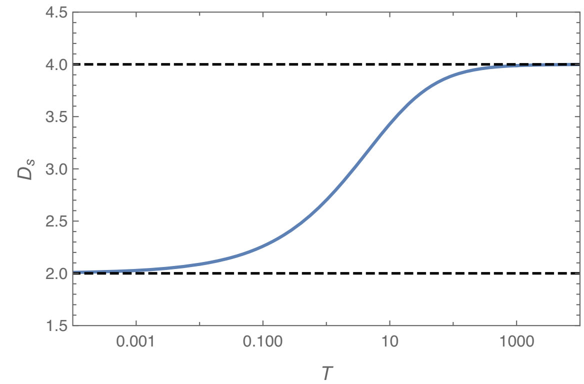

FIG. 2. Illustration of the scale-dependent spectral dimension $D_s(\sigma)$ obtained from the two-scale model (20) with $m^2 = 1$. $D_s(\sigma)$ interpolates smoothly between $D_s = 4$ for $\sigma/m \gg 1$ and $D_s = 2$ for $\sigma/m \ll 1$.

Substituting this result into (15) shows that $D_s(T) = d$ is independent of σ and agrees with the dimension of the momentum space integral entering (17).

The computation is readily generalized to the case where the function $F(p^2)$ has a fixed scaling behavior $F(p^2) = p^{2+\delta}$.[4] Rewriting the integral in (17) in terms of the dimensionless variable $x = p^{2+\delta}\sigma$, one readily finds that [47]

$$D_s(\sigma) = \frac{2d}{2+\delta}, \tag{19}$$

which is again independent of the diffusion time T.

Based on (19) it is then straightforward to construct a simple multiscale model that interpolates between $D_s(\sigma) = 2$ at microscopic and $D_s(\sigma) = 4$ at macroscopic scales [59]. Starting from the momentum-space propagator

$$G(p^2) = \frac{1}{p^2} - \frac{1}{p^2 + m^2}, \tag{20}$$

one obtains $F(p^2) = \frac{1}{m^2} p^2(p^2 + m^2)$. Thus $F(p^2)$ interpolates between $F(p^2) \propto p^2$ for $p^2 \ll m^2$ and $F(p^2) \propto p^4$ for $p^2 \gg m^2$. Evaluating (19) in these scaling regimes suggests that one recovers the desired behavior of the spectral dimension at microscopic and macroscopic scales. The integrals determining $P_\delta(\sigma)$ can be performed analytically and can be expressed in terms of error functions. The resulting spectral dimension is shown in Fig. 2. This confirms

[4]Generically, any function $F(p^2)$ for which the integral (16) is not the Fourier transform of a Gaussian will result in diffusion kernels $K_g(\xi, \xi_0; \sigma)$, which are not positive semidefinite. The occurrence of negative probabilities can be cured by going to fractional calculus [54]. Since this is not relevant in the present analysis, we do not dwell on this technical feature at this point.

that the model indeed interpolates between $D_s = 4$ for $\sigma/m \gg 1$ and $D_s = 2$ for $\sigma/m \ll 1$. Equation (20) then defines a generic toy model realizing the dynamical dimensional reduction encountered in the full-fledged quantum gravity analysis. The latter may also fix the crossover scale m based on microscopic considerations.

At this stage, the following clarifications concerning our toy-model are useful. The massive contribution in Eq. (20) has the structure of a massive ghost field, coming with a wrong sign of the kinetic term. We stress that this contribution should not be interpreted as a particle akin to the ghost degree of freedom found, e.g., in quadratic gravity [72].[5] It merely serves the purpose of implementing the decimation of states in the phase space of the theory at energies above m^2 in a rather minimalistic way. A realistic quantum theory will provide a more refined way for generating this effect, see, e.g., [75] for a specific model achieving this without the introduction of ghost fields.

Moreover, the modified dispersion relation (20) retains local Lorentz invariance. This entails that typical problems associated with Lorentz-symmetry breaking effects, such as, e.g., the ambiguity in defining the event horizon of the black hole and its corresponding temperature [76], are avoided. For both components in (20) the event horizon appears in the same location and there is no ambiguity in defining the horizon temperature. Finally, the detector approach [5–9] derives the black body nature of the Hawking radiation based on the two-point function of the field. This derivation is irrespective of its overall sign of the propagator. Thus the energy loss owed to the massive term can be obtained from a thermal emission spectrum.

IV. BLACK HOLE THERMODYNAMICS INCLUDING A NONTRIVIAL SPECTRAL DIMENSION

At this stage we are in a position to combine our discussions on the thermodynamical properties of black holes and dynamical dimensional reduction. Throughout this section we will assume that the radiation emitted by the event horizon remains thermal also for very light black holes, see [77–79] for a detailed analysis supporting this assumption. The goal of this section is then to go beyond the semiclassical analysis utilizing the concepts of generalized dimensions and dynamical dimensional reduction. Our discussion takes place in the framework of quantum field theory in a nonfluctuating, curved spacetime with the "quantum gravity input" being captured by a change of dimension in the momentum space of the radiated scalar particle.

[5]Note that the presence of the ghost in this context does not automatically entail the violation of unitarity, see [73,74] for recent discussions.

A. Single-scale analysis

From the perspective of generalized dimensions the luminosity formula (6) contains two distinguished elements. First, the black body factor I contains an integral over the theories momentum space. This suggests that the dimension appearing in this term is the spectral dimension

$$I_{d_s} = \int \frac{d^{d_s-1}p}{(2\pi)^{d_s-1}} \frac{E}{e^{E/T_h} - 1}. \tag{21}$$

Second, the horizon area is related to position space properties. This suggests that this term is sensitive to the Hausdorff dimension of the (quantum) spacetime. Owing to the lack of a concrete model that would allow us to describe such an effect for the event horizon, we refrain from including such an effect. If a concrete model is available, then this feature may be included rather straightforwardly in the present setting by considering an effective dimension build from a linear combination of the spectral and Hausdorff dimension.

The generalization (21) then allows us to determine the threshold on the spectral dimension d_s required for creating a long-lived black hole remnant from dynamical dimensional reduction in the momentum space. For this purpose we consider (21) for a massless scalar field in a scaling regime where d_s is constant but not necessarily identical to the topological dimension d of the spacetime. Convergence of the integral requires $d_s > 1$. Assuming convergence, $I_{d_s} = \tilde{c} M^{-d_s}$, where $\tilde{c}$ is a numerical constant. In combination with the classical horizon area (4) the luminosity obtained from the spectral dimension is

$$L_{d_s} = 16\pi\tilde{c} M^{2-d_s}. \tag{22}$$

The mass-loss formula (9) then shows that the generation of a remnant for which $t_{\rm evap}$ is infinite requires $d_s - 2 \leq -1$ or, equivalently, $d_s \leq 1$. This is, however, in conflict with requiring convergence of I_{d_s}. Thus just modifying the spectral dimension for the fields constituting the Hawking radiation is not sufficient to create a long-lived remnant. In particular, the black hole evaporation does not stop if d_s drops below three.

B. Dynamical dimensional reduction

Notably, the luminosity (6) is linear in the two-point correlation function of the corresponding fields. The Euclidean multiscale model following from (20) then suggests the following generalization to the black hole context. First, the Euclidean flat-space propagators are analytically continued to Lorentzian signature using a standard Wick rotation. Subsequently, the principle of covariance is used to promote the derivatives appearing in the position space representation to covariant derivatives. In this way one naturally arrives at the conclusion that the luminosity L_{D_s} of a black hole, in a situation where the scalar field modeling the Hawking radiation exhibits

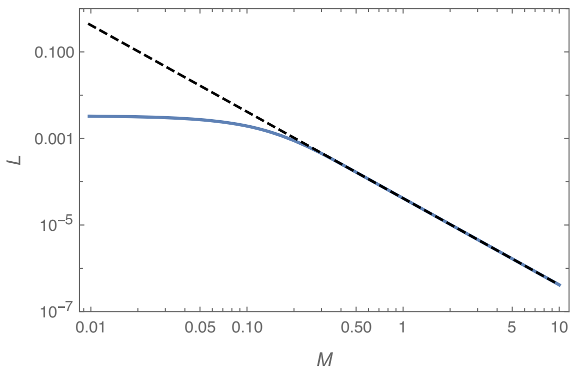

FIG. 3. Luminosity L_{D_s} of a Schwarzschild black hole with mass M arising from the two-scale model (20) with $m^2 = 1$ (blue line). The inclusion of the massive mode triggering the dynamical dimensional reduction renders the luminosity finite as $M \to 0$. The massless case is added as the dashed line for reference.

dynamical dimensional reduction, is given by the sum of a massless and massive contribution weighted by a relative minus sign:

$$L_{D_s}(M; m) = L_{\text{massless}}(M) - L_{\text{massive}}(M; m). \tag{23}$$

From the general analysis in Sec. II we then conclude that the contribution of the second term is exponentially suppressed for $M \gg m$. As a result, $L_{D_s}(M; m)$ agrees with the semiclassical analysis in this regime. Conversely, for $M \lesssim m$ both terms contribute with equal magnitude. As a result L_{D_s} remains finite as $M \to 0$. The crossover between these two regimes together with the removal of the thunderbolt singularity is illustrated in Fig. 3, where L_{D_s} has been evaluated numerically for $m^2 = 1$.

The taming of the luminosity for light black holes arises from the interplay of the massless and massive degrees of freedom that provides a Pauli-Villars-type regularization for L_{massless}. We stress that this feature does not result from introducing an additional ghost field. It is a direct result of the reduced number of degrees of freedom exhibited by the theory at scales $|\vec{p}| \gtrsim m$.

The result shown in Fig. 3 together with a detailed numerical analysis reveals that $L_{D_s}(M; m)$ can be very well approximated by a simple interpolating function

$$L_{D_s}(M; m) = \frac{c}{bm^{-2} + M^2}, \tag{24}$$

where

$$c = \frac{1}{7680\pi}, \qquad b \approx 0.0125. \tag{25}$$

The value for b has been obtained from fitting the ansatz (24) to $L_{D_s}(M, m)$, obtained via numerical integration, at values $M \ll 1$.

The analytic formula (24) again allows us to compute the Hawking evaporation time of the black hole analytically. Integrating Eq. (9) yields

$$t_{\text{evap}} = 2560\pi M_0^3 + \frac{b}{c}\frac{M_0}{m^2}. \tag{26}$$

Thus the dynamical dimensional reduction leads to an increase of the black hole lifetime. Since the scale m where the dynamical dimensional reduction sets in is expected to be the Planck scale, this is a rather tiny effect though. Evaluating the term linear in M_0 for $m = m_{\text{Planck}} = 2.18 \times 10^{-5}$ g and $M_0 = 10^9$ g yields that the change in the lifetime of the black hole is given by $\Delta t_{\text{evap}} = 7.46 \times 10^{-28}$ s. Thus the structure of (26) indicates that a luminosity that is constant as $M \to 0$ does not lead to a long-lived remnant with a lifetime comparable to cosmic timescales.

V. CONCLUSIONS AND DISCUSSION

Motivated by the observation that light black holes with a mass given by the Planck mass $M_{\text{Pl}} \approx 10^{-5}$ g may constitute valid dark matter candidates [17–19,80], we used black hole thermodynamics to investigate the luminosity and lifetime of spherically symmetric black hole solutions. Our work stepped out of the perturbative framework of quantum field theory in curved spacetime by including the effect of a dynamical dimensional reduction of the theories momentum space. As this is a feature shared by many approaches to quantum gravity [41,42], it is intriguing to investigate whether this mechanism leads to the formation of long-lived black hole remnants. While we showed that the dynamical dimensional reduction mechanism generically removes the divergences in the black hole luminosity encountered in the last stage of the evaporation process, the results do not provide any evidence supporting the formation of long-lived remnants.

At this point the following remarks on the scope and limitations of our analysis are in order. Our work implemented the mechanism of dynamical dimensional reduction at the level of the degrees of freedom constituting the Hawking radiation. In this course we did not modify the topology of the background black hole solution that is given by the Schwarzschild solution. This entails that our spacetimes exhibit just one horizon, the event horizon, and that there is no inner horizon. This feature is at variance with many proposals for quantum gravity inspired black hole metrics as, e.g., the Hayward metric [81], the renormalization group improved black hole solutions constructed by Bonanno and Reuter [82], or the Planck stars inspired by loop quantum gravity [83]. A direct consequence of our topology is that the black holes in our work do not have a critical mass where the two horizons coincide and the surface gravity (and hence the Hawking temperature) is zero. By disentangling the effects of dynamical dimensional reduction and the topology of spacetime, our analysis clearly reveals that it is the latter ingredient that is decisive for forming a light black hole remnant during the final stages of black hole evaporation.

This observation is also the key for reconciling our results with the ones reported by Carlip and Grummiller [63]. In this case the effect of dynamical dimensional reduction was essentially incorporated through generalizing the scaling law for the event horizon,

$$A_h = 4\pi r_h^{d_h - 2}, \tag{27}$$

and subsequently identifying $d_h = d_s$. Within the single-scale analysis of Sec. IV A this modification with $d_h = 2$ or $d_h = 3$ would not lead to a stop of the Hawking evaporation process. A careful analysis of the dilaton model used in [63] shows, however, that the underlying black hole solutions must come with a second horizon: in this way one can approach a critical configuration if the generalized dimension of the dilaton model $D(X) = 3$. This picture then corroborates our conclusion that it is actually the topology of the (quantum) black hole and not the effect of a dynamical dimensional reduction that is crucial for forming light long-lived remnants.

Based on our findings, it would be interesting to extend our work in two directions. First, one may seek to understand under which conditions quantum gravity corrections lead to a change in the horizon structure of a Schwarzschild black hole, generating a second (inner) horizon. A detailed understanding of this effect will be essential for determining whether the theory can give rise to light, long-lived remnants. Complementary, one could derive the dynamical dimensional reduction scenario underlying our work from a first-principle derivation. This will require detailed knowledge about the momentum dependence of the theories two-point functions. Notably, the form factor program for asymptotic safety [84] has recently made substantial progress along these lines [75,85–89]. The black hole luminosity then constitutes an interesting observable which is sensitive to a nontrivial momentum dependence in the propagators of the fields. We hope to come back to these points in the future.

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[1] S. W. Hawking, Commun. Math. Phys. **43**, 199 (1975); **46**, 206(E) (1976).
[2] S. W. Hawking, Phys. Rev. D **14**, 2460 (1976).
[3] R. M. Wald, Commun. Math. Phys. **45**, 9 (1975).
[4] W. G. Unruh, Phys. Rev. D **15**, 365 (1977).
[5] W. G. Unruh, Phys. Rev. D **14**, 870 (1976).
[6] W. G. Unruh and R. M. Wald, Phys. Rev. D **29**, 1047 (1984).
[7] B. S. DeWitt, Gravitational radiation, in *General Relativity: An Einstein Centenary Survey*, edited by S. W. Hawking and W. Israel (Cambridge University Press, Cambridge, UK, 1979).
[8] I. Agullo, J. Navarro-Salas, G. J. Olmo, and L. Parker, New J. Phys. **12**, 095017 (2010).
[9] J. Louko and A. Satz, Classical Quantum Gravity **25**, 055012 (2008).
[10] W. G. Unruh, Phys. Rev. D **14**, 870 (1976).
[11] P. Chen, Y. C. Ong, and D. H. Yeom, Phys. Rep. **603**, 1 (2015).
[12] S. B. Giddings, Phys. Rev. D **49**, 4078 (1994).
[13] D. Harlow, Rev. Mod. Phys. **88**, 015002 (2016).
[14] D. N. Page and S. W. Hawking, Astrophys. J. **206**, 1 (1976).
[15] M. Ackermann *et al.* (Fermi-LAT Collaboration), Astrophys. J. **857**, 49 (2018).
[16] S. Clark, B. Dutta, Y. Gao, L. E. Strigari, and S. Watson, Phys. Rev. D **95**, 083006 (2017).
[17] J. H. MacGibbon, Nature (London) **329**, 308 (1987).
[18] C. Rovelli and F. Vidotto, arXiv:1804.04147.
[19] T. Nakama and J. Yokoyama, Phys. Rev. D **99**, 061303 (2019).
[20] B. Carr, F. Kuhnel, and M. Sandstad, Phys. Rev. D **94**, 083504 (2016).
[21] K. Becker, M. Becker, and J. H. Schwarz, *String Theory and M-Theory: A Modern Introduction* (Cambridge University Press, Cambridge, UK, 2006).
[22] B. Zwiebach, *A First Course in String Theory* (Cambridge University Press, Cambridge, UK, 2009).
[23] E. Palti, Fortschr. Phys. **67**, 1900037 (2019).
[24] A. Perez, Living Rev. Relativity **16**, 3 (2013).
[25] C. Rovelli and F. Vidotto, *Covariant Loop Quantum Gravity: An Elementary Introduction to Quantum Gravity and Spinfoam Theory* (Cambridge University Press, Cambridge, UK, 2014).
[26] A. Ashtekar and E. Bianchi, Rep. Prog. Phys. **84**, 042001 (2021).
[27] T. Thiemann, *Modern Canonical Quantum General Relativity* (Cambridge University Press, Cambridge, UK, 2008).
[28] M. Niedermaier and M. Reuter, Living Rev. Relativity **9**, 5 (2006).
[29] M. Reuter and F. Saueressig, New J. Phys. **14**, 055022 (2012).
[30] R. Percacci, *An Introduction to Covariant Quantum Gravity and Asymptotic Safety* (World Scientific, Singapore, 2017).
[31] A. Eichhorn, Front. Astron. Space Sci. **5**, 47 (2019).
[32] M. Reuter and F. Saueressig, *Quantum Gravity and the Functional Renormalization Group: The Road towards*

Asymptotic Safety (Cambridge University Press, Cambridge, UK, 2019).
[33] A. D. Pereira, arXiv:1904.07042.
[34] M. Reichert, Proc. Sci., 384 (**2020**) 005.
[35] J. M. Pawlowski and M. Reichert, Front. Phys. **8**, 551848 (2021).
[36] J. Ambjørn, A. Goerlich, J. Jurkiewicz, and R. Loll, Phys. Rep. **519**, 127 (2012).
[37] R. Loll, Classical Quantum Gravity **37**, 013002 (2020).
[38] P. Hořava, Phys. Rev. D **79**, 084008 (2009).
[39] A. Wang, Int. J. Mod. Phys. D **26**, 1730014 (2017).
[40] A. O. Barvinsky, A. V. Kurov, and S. M. Sibiryakov, Phys. Rev. D **105**, 044009 (2022).
[41] S. Carlip, Classical Quantum Gravity **34**, 193001 (2017).
[42] S. Carlip, Universe **5**, 83 (2019).
[43] G. 't Hooft, Conf. Proc. C **930308**, 284 (1993).
[44] J. Ambjørn, J. Jurkiewicz, and R. Loll, Phys. Rev. Lett. **95**, 171301 (2005).
[45] J. Laiho, S. Bassler, D. Du, J. T. Neelakanta, and D. Coumbe, Acta Phys. Pol. B Proc. Suppl. **10**, 317 (2017).
[46] O. Lauscher and M. Reuter, J. High Energy Phys. 10 (2005) 050.
[47] M. Reuter and F. Saueressig, J. High Energy Phys. 12 (2011) 012.
[48] M. Reuter and F. Saueressig, Lect. Notes Phys. **863**, 185 (2013).
[49] D. Stojkovic, Mod. Phys. Lett. A **28**, 1330034 (2013).
[50] N. Afshordi and D. Stojkovic, Phys. Lett. B **739**, 117 (2014).
[51] L. Barcaroli, L. K. Brunkhorst, G. Gubitosi, N. Loret, and C. Pfeifer, Phys. Rev. D **92**, 084053 (2015).
[52] G. Calcagni, G. Nardelli, and D. R. Fernandez, Phys. Rev. D **94**, 045018 (2016).
[53] G. Calcagni, J. High Energy Phys. 03 (2017) 138.
[54] G. Calcagni, G. Nardelli, and M. Scalisi, J. Math. Phys. (N.Y.) **53**, 102110 (2012).
[55] G. Calcagni, A. Eichhorn, and F. Saueressig, Phys. Rev. D **87**, 124028 (2013).
[56] A. Addazi, G. Calcagni, and A. Marcianò, J. High Energy Phys. 12 (2018) 130.
[57] G. Calcagni and A. De Felice, Phys. Rev. D **102**, 103529 (2020).
[58] G. Calcagni, D. Rodríguez Fernández, and M. Ronco, Eur. Phys. J. C **77**, 335 (2017).
[59] N. Alkofer, G. D'Odorico, F. Saueressig, and F. Versteegen, Phys. Rev. D **94**, 104055 (2016).
[60] G. Calcagni, Mod. Phys. Lett. A **36**, 2140006 (2021).
[61] P. Gaete, J. Helayel-Neto, and E. Spallucci, Phys. Lett. B **693**, 155 (2010).
[62] P. Nicolini and E. Spallucci, Phys. Rev. D **695**, 041 (2011).
[63] S. Carlip and D. Grumiller, Phys. Rev. D **84**, 084029 (2011).
[64] N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, UK, 1982).
[65] R. M. Wald, *Quantum Field Theory in Curved Space-Time and Black Hole Thermodynamics* (University of Chicago Press, Chicago, USA, 1995).
[66] D. Raine and E. Thomas, *Black Holes: A Student Text* (Imperial College Press, London, UK, 2015).
[67] S. W. Hawking and J. M. Stewart, Nucl. Phys. **B400**, 393 (1993).
[68] T. Piran and A. Strominger, Phys. Rev. D **48**, 4729 (1993).
[69] A. Ashtekar and F. M. Ramazanoglu, Phys. Rev. Lett. **106**, 161303 (2011).
[70] A. D. Lowe and M. O'Loughlin, Phys. Rev. D **48**, 3735 (1993).
[71] D. ben-Avraham and S. Havlin, *Diffusion and Reactions in Fractals and Disordered Systems* (Cambridge Universtiy Press, Cambridge, UK, 2004).
[72] K. S. Stelle, Gen. Relativ. Gravit. **9**, 353 (1978).
[73] D. Anselmi, J. High Energy Phys. 12 (2019) 027.
[74] J. F. Donoghue and G. Menezes, Phys. Rev. D **100**, 105006 (2019).
[75] T. Draper, B. Knorr, C. Ripken, and F. Saueressig, Phys. Rev. Lett. **125**, 181301 (2020).
[76] S. L. Dubovsky and S. M. Sibiryakov, Phys. Lett. B **638**, 509 (2006).
[77] R. Brout, S. Massar, R. Parentani, and P. Spindel, Phys. Rev. D **52**, 4559 (1995).
[78] W. G. Unruh, Phys. Rev. D **51**, 2827 (1995).
[79] I. Agullo, J. Navarro-Salas, G. J. Olmo, and L. Parker, Phys. Rev. D **76**, 044018 (2007).
[80] J. R. Mureika, Phys. Lett. B **716**, 171 (2012).
[81] S. A. Hayward, Phys. Rev. Lett. **96**, 031103 (2006).
[82] A. Bonanno and M. Reuter, Phys. Rev. D **62**, 043008 (2000).
[83] C. Rovelli and F. Vidotto, Int. J. Mod. Phys. D **23**, 1442026 (2014).
[84] B. Knorr, C. Ripken, and F. Saueressig, Classical Quantum Gravity **36**, 234001 (2019).
[85] B. Knorr and F. Saueressig, Phys. Rev. Lett. **121**, 161304 (2018).
[86] L. Bosma, B. Knorr, and F. Saueressig, Phys. Rev. Lett. **123**, 101301 (2019).
[87] A. Platania and C. Wetterich, Phys. Lett. B **811**, 135911 (2020).
[88] A. Bonanno, T. Denz, J. M. Pawlowski, and M. Reichert, SciPost Phys. **12**, 001 (2022).
[89] B. Knorr and M. Schiffer, Universe **7**, 216 (2021).