A detector-based measurement theory for quantum field theory

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We propose a measurement theory for quantum fields based on measurements made with localized nonrelativistic systems that couple covariantly to quantum fields (like the Unruh-DeWitt detector). Concretely, we analyze the positive operator-valued measure induced on the field when an idealized measurement is carried out on the detector after it coupled to the field. Using an information-theoretic approach, we provide a relativistic analogue to the quantum-mechanical Lüders update rule to update the field state following the measurement on the detector. We argue that this proposal has all the desirable characteristics of a proper measurement theory. In particular it does not suffer from the "impossible measurements" problem pointed out by Rafael Sorkin in the 1990s which shows that idealized measurements cannot be used in quantum field theory.

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I. INTRODUCTION

In any physical theory, it is necessary to describe the mechanism that allows us to gather information about the physical systems we are modeling, that is, it is necessary to describe measurements. In classical theories, the description of measurements is frequently not explicit, often hidden under the assumption that we can neglect the effect of measurements on the state of the system of interest. However, in quantum mechanics, describing measurement processes has been problematic and a subject of discussion since its very inception (see e.g., Ref. [1]). Nevertheless, from an operational point of view the problem can be bypassed in the context of nonrelativistic quantum mechanics by employing Lüders rule, also known as the projection postulate [2]. This rule prescribes how to update the state of the system after the measurement in a way consistent with the measurement outcome, through projection-valued measures. This model of measurement is called *projective measurement* or *idealized measurement*.

However, projective measurements are not suitable to describe measurements in quantum field theory, since they are not compatible with relativistic causality, and therefore they are not consistent with the very foundational framework of quantum field theory (QFT). Specifically, there are no local projectors of finite rank in QFT [3–6]. Any finite rank projector in QFT, such as a projector onto some single-particle wave-packet state, is inherently nonlocal, and so any attempt to generalize the projection postulate with such a projector leads to spurious faster-than-light signaling [7–11]. It should be clear from the beginning that when we talk about the causality issues of the projection postulate, we are indeed referring to superluminal causation, and not the nonlocality that arises from entanglement, which can be present even between nonrelativistic systems [12]. The latter is perfectly compatible with causality as long as it does not enable signaling; it just tells us that quantum theories are nonlocal in nature, and correlations can be present between quantum systems that are spacelike separated even in QFT [13,14].

The impossibility of naively generalizing the projection postulate to QFT has been addressed mainly in three different ways.

First, one could consider localized ideal measurements[1] (in the form of infinite rank projectors) and try to modify the projection postulate in a covariant way, as in Hellwig and Kraus's proposal [5,15,16]. This prescription however suffers from the same faster-than-light signaling that Sorkin pointed out in Ref. [7], as discussed in Ref. [10].

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[1]For a more thorough analysis where general completely positive trace-preserving maps (not necessarily ideal measurements) on quantum fields are characterized according to its causal behaviour, see Ref. [11].

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Second, another way consists of formulating a measurement theory completely within quantum field theory, such as Fewster and Verch's framework [17–19]. By giving a covariant update rule, they obtained a measurement scheme consistent with QFT and therefore completely safe from any causality issues. In this framework, however, being entirely within QFT, the localized measurement probes are also quantum fields, and we are still left with the problem of how to access the information of that second ancillary field [20]. This is because low-energy measurement apparatuses (like atoms, photodetectors, photomultipliers, human retinas, etc.) are not well described by a free field theory, and the treatment of bound states in QFT is still very much an open problem [21].

Finally, the third option relies on coupling so-called *particle detectors*—localized nonrelativistic quantum systems—to quantum fields, such as the Unruh-DeWitt (UDW) model [22,23]. Although pointlike detector models are fully compatible with relativistic causality [24–27], their singular nature leads, in certain contexts, to divergences [28]. However, those divergences are not present for detectors that are adiabatically switched on [29], or that are spatially smeared [30,31]. In the latter case, even though the unitary evolution is perfectly compatible with causality and does not allow faster-than-light signaling with a second detector [24,27], the use of a (nonpointlike) spatial smearing along with the nonrelativistic approximation can indeed enable some degree of faster-than-light signaling between two detectors with the help of a third ancillary one in between [27]. However, unlike in the case of projective measurements, in the smeared setups that show any degree of superluminal signaling, its impact is bounded by the smearing length scale of the ancillary detector (which by approximating it to be nonrelativistic we already neglected in our frame of reference, to start with) and moreover does not show up at leading orders in perturbation theory [27]. These results, together with the fact that detector models can realistically represent the way fields are measured experimentally [32–34], make this option especially appealing for modeling measurements in QFT.

In the particle detector approach, however, not every step of the process is already well understood. We still need to describe the mechanism through which we go from a field state and a detector that are originally decoupled and uncorrelated, to a measurement outcome that an experimentalist can put in a plot or write on a notepad. After the experiment is performed and some classical information has been obtained about the field, how does one take into account this information for the description of future experiments involving the field?

This question is particularly relevant for the field of relativistic quantum information. Indeed, there are several landmark protocols and experimental proposals in the context of quantum information (e.g., the quantum Zeno effect [35,36], the delayed choice quantum eraser experiment [37–41], or the Wigner's friend experiment [42,43], among others) in which the ability to perform measurements and using the information of the outcomes to update the state is essential for their implementation. To be able to formulate these scenarios in relativistic contexts, it is necessary to have a well-understood measurement theory that works in the context of quantum field theory and that connects to experimentally measurable quantities.

In this paper we aim to formulate consistently a measurement theory for QFT using detectors as measuring tools. First, in Sec. II we present our working model. In Sec. III we describe the measurement process (including field-detector interaction and idealized measurement of the detector) and obtain the field state update according to the measurement outcome. In Sec. IV we analyze this update in order to determine whether this kind of measurement abides by relativistic causality. In Sec. V we present a context-dependent update rule consistent with QFT. In Sec. VI we explicitly formulate it in terms of n-point functions and in Sec. VII we analyze the most general initial scenario. Section IX is devoted to discussing how the framework presented in this manuscript constitutes a valid measurement theory for QFT. Finally we present our conclusions in Sec. X.

II. SETUP

In this work we consider a spatially smeared Unruh-DeWitt model [22,23] for a detector coupled to a real scalar field in a $(1+d)$-dimensional flat spacetime. This is a simplified model that is both covariant [25,26] and yet captures the phenomenology of light-matter interaction neglecting angular momentum exchange but without any further common quantum optics approximations—such as the rotating-wave or single (or few) mode approximation [32–34,44]. For our purposes, let us consider that the detector is inertial and at rest in the frame of coordinates $(t,\boldsymbol{x})$ so that its proper time coincides with the coordinate time t. Then, in the interaction picture, the interaction Hamiltonian is [31]

$$\hat{H}_I(t) = \lambda\chi(t)\hat{\mu}(t)\int \mathrm{d}^d\boldsymbol{x}\, F(\boldsymbol{x})\hat{\phi}(t,\boldsymbol{x}). \tag{1}$$

In this equation, the scalar field $\hat{\phi}$ can be expanded in terms of plane-wave solutions in the quantization frame $(t,\boldsymbol{x})$ as

$$\hat{\phi}(t,\boldsymbol{x}) = \int \frac{\mathrm{d}^d\boldsymbol{k}}{\sqrt{(2\pi)^d 2\omega_{\boldsymbol{k}}}}\left(\hat{a}_{\boldsymbol{k}}e^{-\mathrm{i}(\omega_{\boldsymbol{k}}t-\boldsymbol{k}\cdot\boldsymbol{x})} + \hat{a}^\dagger_{\boldsymbol{k}}e^{\mathrm{i}(\omega_{\boldsymbol{k}}t-\boldsymbol{k}\cdot\boldsymbol{x})}\right), \tag{2}$$

where $\hat{a}^\dagger_{\boldsymbol{k}}$ and $\hat{a}_{\boldsymbol{k}}$ are canonical creation and annihilation operators satisfying the commutation relations $[\hat{a}_{\boldsymbol{k}},\hat{a}^\dagger_{\boldsymbol{k}'}] = \delta(\boldsymbol{k}-\boldsymbol{k}')$. The internal degree of freedom (monopole moment) of the detector, which we choose to have two levels (ground $|g\rangle$ and excited $|e\rangle$) with an energy gap Ω between them is given by

$$\hat{\mu}(t) = |g\rangle\langle e|e^{-\mathrm{i}\Omega t} + |e\rangle\langle g|e^{\mathrm{i}\Omega t}. \tag{3}$$

λ is the coupling strength and $\chi(t)$ is the switching function controlling the time dependence of the coupling. The interaction is on only for times in the support of $\chi(t)$. For simplicity we assume this to be a finite interval $[t_{\mathrm{on}}, t_{\mathrm{off}}]$ [i.e., $\chi(t)$ is compactly supported]. $F(\boldsymbol{x})$ is the spatial smearing function that models the localization of the detector, and therefore the support of the product of χ and F,

$$\mathcal{D} \coloneqq \mathrm{supp}\{\chi(t)\cdot F(\boldsymbol{x})\}, \tag{4}$$

is what we will call the *interaction region* or, slightly abusing nomenclature, *detector*. Its *causal future/past* $\mathcal{J}^{\pm}(\mathcal{D})$ is the union of the future/past light cones of all its points and their interiors. In Minkowski spacetime[2] $\mathcal{M}$,

$$\mathcal{J}^{\pm}(\mathcal{D}) \coloneqq \{\mathsf{y}\in\mathcal{M} : \exists \mathsf{x}\in\mathcal{D}, \mathsf{x}\cdot\mathsf{y}\le 0, \pm(y^0 - x^0)\ge 0\}. \tag{5}$$

The *causal support* of $\mathcal{D}$ is the union of both its causal past and its causal future,

$$\mathcal{J}(\mathcal{D}) \coloneqq \mathcal{J}^{+}(\mathcal{D}) \cup \mathcal{J}^{-}(\mathcal{D}). \tag{6}$$

III. THE UPDATED STATE OF THE FIELD

In this section we will compute the update on the field state after an observable of the detector is measured by an experimenter through an idealized measurement. In other words, we will compute the positive operator-valued measure (POVM) that is applied to update the field state if the detector is updated by a projective measurement. Although we will consider the most general case, a physical example of this kind of situation could be an experiment in the lab where we check whether the detector clicks (gets excited) or not (stays in the ground state) after interacting with an excited state of the electromagnetic field.

It is reasonable to consider that the detector and the field are initially uncorrelated. That is, in the absence of third parties,[3] the state of the system before the interaction is $\hat{\rho} = \hat{\rho}_{\mathrm{d}} \otimes \hat{\rho}_{\phi}$. At time $t = T$, the experimenter performs a rank-one projective measurement $P = |s\rangle\langle s|$ of an arbitrary observable of the detector. One can think of the idealized measurement as performing a measurement of an observable of the detector, and of $|s\rangle\langle s|$ as being the eigenprojector associated to the obtained measurement outcome. For simplicity we assume that the measurement takes place after the interaction between the field and the detector has been switched off, that is, $T \ge t_{\mathrm{off}}$. Then, after the projective measurement, the updated state of the joint system is [45]

$$\hat{\rho}^{P} = \frac{(P\otimes\mathbb{1})\hat{U}\,\hat{\rho}\,\hat{U}^{\dagger}(P\otimes\mathbb{1})}{\mathrm{tr}[(P\otimes\mathbb{1})\hat{U}\,\hat{\rho}\,\hat{U}^{\dagger}]}, \tag{7}$$

where the unitary evolution operator $\hat{U}$ is given by

$$\hat{U} = \mathcal{T}\exp\left(-\mathrm{i}\int_{-\infty}^{\infty}\mathrm{d}t'\,\hat{H}_I(t')\right). \tag{8}$$

The assumption that $\chi(t) = 0$ for all $t \ge t_{\mathrm{off}}$ allows us to safely extend the integration range to infinity. From now on, we will use the integral sign without specifying limits whenever the integral is carried out in the whole domain of the integrand. We thus have, for the updated state of the field,

$$\hat{\rho}_{\phi}^{P} = \mathrm{tr}_{\mathrm{d}}(\hat{\rho}^{P}) \propto \mathrm{tr}_{\mathrm{d}}[(P\otimes\mathbb{1})\hat{U}\,\hat{\rho}\,\hat{U}^{\dagger}(P\otimes\mathbb{1})], \tag{9}$$

where $\mathrm{tr}_{\mathrm{d}}(\cdot)$ stands for the partial trace over the Hilbert space of the detector. We note that the matrix elements of $\hat{\rho}_{\phi}^{P}$ satisfy that

$$\langle\varphi_1|\hat{\rho}_{\phi}^{P}|\varphi_2\rangle \propto \langle s,\varphi_1|\hat{U}\,\hat{\rho}\,\hat{U}^{\dagger}|s,\varphi_2\rangle. \tag{10}$$

where $|s,\varphi_i\rangle \equiv |s\rangle\otimes|\varphi_i\rangle$ and $|\varphi_i\rangle$ is a vector in the field Hilbert space. From now on, we will use the following notation: if $\hat{\mathcal{O}}$ is an operator acting on the detector-field Hilbert space and $|\psi_1\rangle, |\psi_2\rangle \in \mathcal{H}_{\mathrm{d}}$ are detector states, then we will understand $\langle\psi_1|\hat{\mathcal{O}}|\psi_2\rangle$ to be the field operator that satisfies

$$\langle\varphi_1|\langle\psi_1|\hat{\mathcal{O}}|\psi_2\rangle|\varphi_2\rangle = \langle\psi_1,\varphi_1|\hat{\mathcal{O}}|\psi_2,\varphi_2\rangle \tag{11}$$

for any field states $|\varphi_1\rangle, |\varphi_2\rangle \in \mathcal{H}_{\phi}$. Finally, let us assume that the initial state of the detector is pure,[4] $\hat{\rho}_{\mathrm{d}} = |\psi\rangle\langle\psi|$. Thus, using the convention in Eq. (11),

$$\langle s,\varphi_1|\hat{U}\,\hat{\rho}\,\hat{U}^{\dagger}|s,\varphi_2\rangle = \langle s|\hat{U}|\psi\rangle\hat{\rho}_{\phi}\langle\psi|\hat{U}^{\dagger}|s\rangle. \tag{12}$$

We can therefore write the updated state of the field for the projection over the state $|s\rangle$ of the detector as

$$\hat{\rho}_{\phi}^{s,\psi} = \frac{\hat{M}_{s,\psi}\hat{\rho}_{\phi}\hat{M}_{s,\psi}^{\dagger}}{\mathrm{tr}_{\phi}(\hat{\rho}_{\phi}\hat{E}_{s,\psi})}, \tag{13}$$

where

[2]The metric in a coordinate system associated with inertial observers is $\eta_{\mu\nu} = \mathrm{diag}(-1,1,\ldots,1)$, $\{x^0,\ldots,x^d\}$ are the coordinates of the event x, and $\mathsf{x}\cdot\mathsf{y} \coloneqq \eta_{\mu\nu}x^{\mu}y^{\nu}$.

[3]We will generalize to cases where the field is entangled with third parties in Sec. VII.

[4]This simplification can be easily dropped, and the result straightforwardly generalized, in the same way that happens with POVMs in nonrelativistic quantum mechanics [45].

$$\hat{M}_{s,\psi} = \langle s|\hat{U}|\psi\rangle \tag{14}$$

is an operator acting on the field Hilbert space, and the POVM elements [45] are

$$\hat{E}_{s,\psi} = \hat{M}^{\dagger}_{s,\psi}\hat{M}_{s,\psi}. \tag{15}$$

For our system, we can get a tractable expression for the $\hat{M}_{s,\psi}$ operators proceeding perturbatively in λ. First, the unitary $\hat{U}$ in Eq. (8) can be written as

$$\hat{U} = \mathbb{1} + \sum_{n=1}^{\infty} \lambda^n \hat{U}^{(n)}. \tag{16}$$

For the first two orders, substituting Eq. (1), we have

$$\hat{U}^{(1)} = -\mathrm{i}\int \mathrm{d}t\, \mathrm{d}^d\boldsymbol{x}\,\chi(t)F(\boldsymbol{x})\hat{\mu}(t)\hat{\phi}(t,\boldsymbol{x}) \tag{17}$$

and

$$\hat{U}^{(2)} = -\int \mathrm{d}t\,\mathrm{d}t'\,\mathrm{d}^d\boldsymbol{x}\,\mathrm{d}^d\boldsymbol{x}'\,\theta(t-t')\chi(t)\chi(t') \times F(\boldsymbol{x})F(\boldsymbol{x}')\hat{\mu}(t)\hat{\mu}(t')\hat{\phi}(t,\boldsymbol{x})\hat{\phi}(t',\boldsymbol{x}'). \tag{18}$$

As a result, we can apply the same expansion to $\hat{M}_{s,\psi}$,

$$\hat{M}_{s,\psi} = \hat{M}^{(0)}_{s,\psi} + \lambda\hat{M}^{(1)}_{s,\psi} + \lambda^2\hat{M}^{(2)}_{s,\psi} + \cdots, \tag{19}$$

where we are denoting $\hat{M}^{(n)}_{s,\psi} = \langle s|\hat{U}^{(n)}|\psi\rangle$. In particular,

$$\hat{M}^{(0)}_{s,\psi} = \langle s|\psi\rangle \mathbb{1}_{\phi}, \tag{20}$$

$$\hat{M}^{(1)}_{s,\psi} = -\mathrm{i}\int \mathrm{d}t\,\mathrm{d}^d\boldsymbol{x}\,\chi(t)F(\boldsymbol{x})\langle s|\hat{\mu}(t)|\psi\rangle\hat{\phi}(t,\boldsymbol{x}), \tag{21}$$

$$\hat{M}^{(2)}_{s,\psi} = -\int \mathrm{d}t\,\mathrm{d}t'\,\mathrm{d}\boldsymbol{x}\,\mathrm{d}^d\boldsymbol{x}'\,\theta(t-t')\chi(t)\chi(t') \times F(\boldsymbol{x})F(\boldsymbol{x}')\langle s|\hat{\mu}(t)\hat{\mu}(t')|\psi\rangle\hat{\phi}(t,\boldsymbol{x})\hat{\phi}(t',\boldsymbol{x}'). \tag{22}$$

IV. CAUSAL BEHAVIOR

Once the form of the POVM that updates the state of the field (13) has been obtained, we want to analyze whether this update respects relativistic causality. In this section we will study whether the measurement defined in the previous section influences the field state outside of the causal future of the measurement.

Concretely, in order to understand the causal behavior of the update of the field state that arises from performing a projective measurement *on the detector*, we need to compare the updated state of the field $\hat{\rho}^{\mathrm{u}}_{\phi}$ (post-measurement) and the initial state of the field $\hat{\rho}_{\phi}$ (pre-measurement) and see that there is no influence on the field state outside the causal future of the interaction region. Since the state of the field is fully characterized by its n-point functions, the analysis can be reduced to studying how the n-point functions change after the measurement process [consisting of (i) interaction with the detector, and—after switching off the interaction—(ii) idealized measurement of the detector and corresponding POVM update on the field] in the region that is spacelike separated from the detector.

Regarding the updated field state $\hat{\rho}^{\mathrm{u}}_{\phi}$, note that the update given by Eqs. (13) and (14) corresponds to a *selective measurement* [2,5]: the measurement is performed and its outcome is checked, updating the state of the field accordingly. However, if an observer is spacelike separated from the detector, then they might know that the measurement was prearranged to be performed, but they cannot know the outcome of such a measurement since information cannot be transmitted to them. Thus, from that observer's perspective, the update of the state has to be the one associated with a *nonselective measurement* [2,5]: the state of the field is updated taking into account that the measurement has been carried out, but without knowing its outcome. This measurement model respects causality if the spacelike separated observer cannot tell with local operations whether the measurement was carried out or not, i.e., if the nonselective update does not impact the outcome of local operations performed outside the causal support of the measurement.

A nonselective measurement has to be understood as having made the projective measurement on the detector when the outcome is not made concrete. Therefore, to update the state we apply a convex mixture of all the projectors over all the possible proper subspaces associated with every potential outcome of the measurement, weighted by its associated probabilities given by Born's rule (see again Refs. [2,5]).

Since we are considering a two-level Unruh-DeWitt detector, the most general nonselective projective measurement can be described by two complementary rank-one projections, $|s\rangle\langle s|$ and $|\bar{s}\rangle\langle\bar{s}|$, such that

$$\mathbb{1}_{\mathrm{d}} - |s\rangle\langle s| = |\bar{s}\rangle\langle\bar{s}|, \tag{23}$$

where $|s\rangle$ and $|\bar{s}\rangle$ are two orthonormal vectors in the detector's Hilbert space, $\mathcal{H}_{\mathrm{d}}$. The state of the field updated by a nonselective measurement can then be written as the mixture of the updates for each projection $\hat{\rho}^{s,\psi}_{\phi}$ and $\hat{\rho}^{\bar{s},\psi}_{\phi}$ given by Eq. (13), weighted by their respective probabilities, $\langle\hat{E}_{s,\psi}\rangle_{\hat{\rho}_{\phi}}$ and $\langle\hat{E}_{\bar{s},\psi}\rangle_{\hat{\rho}_{\phi}}$,

$$\hat{\rho}^{\mathrm{u}}_{\phi} = \langle\hat{E}_{s,\psi}\rangle_{\hat{\rho}_{\phi}}\hat{\rho}^{s,\psi}_{\phi} + \langle\hat{E}_{\bar{s},\psi}\rangle_{\hat{\rho}_{\phi}}\hat{\rho}^{\bar{s},\psi}_{\phi} = \hat{M}_{s,\psi}\hat{\rho}_{\phi}\hat{M}^{\dagger}_{s,\psi} + \hat{M}_{\bar{s},\psi}\hat{\rho}_{\phi}\hat{M}^{\dagger}_{\bar{s},\psi}. \tag{24}$$

By Eq. (14), recalling the notation described in Eq. (11), from Eq. (24),

$$\begin{aligned}\hat{\rho}_\phi^{\rm NS} &= \langle s|\hat{U}|\psi\rangle\hat{\rho}_\phi\langle\psi|\hat{U}^\dagger|s\rangle + \langle\bar{s}|\hat{U}|\psi\rangle\hat{\rho}_\phi\langle\psi|\hat{U}^\dagger|\bar{s}\rangle\\ &= {\rm tr}_{\rm d}[\hat{U}(|\psi\rangle\langle\psi|\otimes\hat{\rho}_\phi)\hat{U}^\dagger],\end{aligned} \tag{25}$$

where we have used Eq. (23) to reduce the sum to a trace over the detector Hilbert space.

Let $\hat{A}$ be a field observable. Then we get that its expectation value for the nonselective update of the field state is

$$\begin{aligned}\langle\hat{A}\rangle_{\hat{\rho}_\phi^{\rm NS}} &= {\rm tr}_\phi[{\rm tr}_{\rm d}[\hat{U}(|\psi\rangle\langle\psi|\otimes\hat{\rho}_\phi)\hat{U}^\dagger]\hat{A}]\\ &= {\rm tr}[\hat{U}(|\psi\rangle\langle\psi|\otimes\hat{\rho}_\phi)\hat{U}^\dagger\hat{A}]\\ &= {\rm tr}[(|\psi\rangle\langle\psi|\otimes\hat{\rho}_\phi)\hat{U}^\dagger\hat{A}\,\hat{U}],\end{aligned} \tag{26}$$

where $\langle\hat{\mathcal{O}}\rangle_{\hat{\rho}} = {\rm tr}(\hat{\rho}\,\hat{\mathcal{O}})$ as usual. We have used the cyclic property of the trace and we have denoted the detector-field operator $\mathbb{1}_{\rm d}\otimes\hat{A}$ simply as $\hat{A}$, omitting the identity of the detector. Now, taking into account the form of the UDW interaction Hamiltonian (1) and the unitary evolution operator (8), if $\hat{A}$ is a field observable supported outside the causal support of the interaction region, then microcausality ensures that

$$[\hat{A},\hat{\phi}(t,\boldsymbol{x})] = 0 \tag{27}$$

for every $(t,\boldsymbol{x})\in\mathcal{D}$, and therefore

$$[\hat{A},\hat{U}] = 0. \tag{28}$$

This means that Eq. (26) yields

$$\begin{aligned}\langle\hat{A}\rangle_{\hat{\rho}_\phi^{\rm NS}} &= {\rm tr}_\phi(\hat{\rho}_\phi^{\rm NS}\hat{A}) = {\rm tr}[(|\psi\rangle\langle\psi|\otimes\hat{\rho}_\phi)\hat{U}^\dagger\hat{A}\,\hat{U}]\\ &= {\rm tr}[(|\psi\rangle\langle\psi|\otimes\hat{\rho}_\phi)\hat{U}^\dagger\hat{U}\,\hat{A}] = {\rm tr}_\phi(\hat{\rho}_\phi\hat{A})\\ &= \langle\hat{A}\rangle_{\hat{\rho}_\phi}.\end{aligned} \tag{29}$$

We conclude that the nonselective POVM does not affect the expectation value of any local observable outside the causal influence region of the detector. Of particular importance is the case when we take $\hat{A} = \hat{\phi}(t_1,\boldsymbol{x}_1)\cdots\hat{\phi}(t_n,\boldsymbol{x}_n)$, with all $(t_1,\boldsymbol{x}_1),\ldots,(t_n,\boldsymbol{x}_n)$ outside the causal support of the interaction region, i.e., spacelike separated from the interaction region. Then Eq. (29) allows us to conclude that the corresponding n-point functions do not change under the nonselective update.

The effect of the rank-one projective measurement performed *on the detector* is thus restricted to the causal future of the interaction region between the detector and the field. In particular, it is bounded in every spacelike hypersurface of the Minkowski spacetime. This feature contrasts with the effect (studied by Sorkin in Ref. [7]) of a finite-rank projective measurement performed *on the field*, which affects the whole future half of the spacetime determined by the spacelike hypersurface in which the measurement is considered to be performed [7]. Therefore, if we are in the regimes where causality is respected by the coupling between detector and field (e.g., pointlike detectors in any scenario or spatially smeared detectors in the scales identified in Ref. [27]), the projective measurement performed on the detector is safe from any causality issues[5] of the kind exposed in "Impossible measurements on quantum fields"[7]. The existence of physically motivated regimes that set the limits of validity of the particle detector model distinguishes this approach from the performance of infinite-rank projective measurements on the field, where faster-than-light signaling is allowed in general [10]. Moreover, Eq. (26) also shows that, for the nonselective update, expectation values of arbitrary observables only depend on the joint state of the field and the detector after the interaction, and not on the measurement performed on the detector. Indeed, the nonselective measurement eliminates the entanglement that the detector and the field acquired through the interaction, but it does not change the partial state of the field, as Eq. (25) explicitly shows. This is of course a physically reasonable feature of the update: we have specified that the measurement is performed after the interaction is switched off, but it could be some arbitrary amount of time after this. The physical change of the field state due to the measurement is due only to the physical coupling between the detector and the field, and not to the fact that we decide to do a projective measurement on the detector after this interaction. This is because the projective measurement acts only on the detector once the interaction has been switched off, and it does not provide additional information since being nonselective the outcome is not known. This important interpretational point will be revisited when we consider the update rule for selective measurements, where the state of the field has to be updated consistently with the concrete outcome of the measurement.

V. THE UPDATE RULE

In the previous section we have shown that the process of measuring a quantum field through locally coupling an Unruh-DeWitt detector and then carrying out an idealized measurement on the detector—which corresponds to

[5]For the sake of brevity, from now on we will not restate the conditions under which particle detector models behave causally and will just state that the particle detector models are causal. The facts that should be acknowledged throughout this manuscript are that a pointlike detector is fully causal, that its quantum dynamics is nonsingular (when switched on carefully) and that the causality violations (if any) of the model come through (well-controlled) smearing scales. For a more careful recapitulation of these conditions, see Sec. I.

a field state update with the appropriate (nonselective) POVM—does not introduce causality violations. We are now in a position to build an update rule for the field when we assume that the experimenter knows the concrete outcome of the idealized measurement carried out on the detector, which is akin to considering what is the field state update induced by a selective measurement on the detector after the detector finished interacting with the field.

A. Issues of a noncontextual update

Prescribing an update rule for selective measurements in a way that is compatible with the relativistic nature of QFT requires more care than in regular quantum mechanics. An update rule for selective measurements based on particle detectors should:

(1) Include the knowledge of the measurement outcome in the description of the field and implement the compatibility between measurements that are sequentially applied to the field, in the spirit of Lüders rule [2].

(2) Be compatible with relativity.

To guarantee that condition (1) is fulfilled, it is necessary to use the update of the state of the field given by Eq. (13). However, in Appendix A we show that this update cannot be applied outside the causal future of the detector in a way consistent with relativity. Hence, we see that any noncontextual update (i.e., an update where one gives the density operator a global nature and its change affects all observers regardless of whether they are in causal contact with the detector or not) cannot satisfy conditions (1) and (2) simultaneously. To bypass this difficulty, a first attempt that one could try is to prescribe that the selective update given by Eq. (13) should only be used in the causal future of the measurement. This prescription, however, is ill-defined since the density operator does not naturally depend on points of the spacetime manifold. In particular, this kind of prescription does not provide a way to calculate arbitrary n-point functions, since by naively looking at the formula $w_n(\mathsf{x},\mathsf{x}',\cdots) = \mathrm{tr}_\phi(\hat{\sigma}_\phi\hat{\phi}(\mathsf{x})\hat{\phi}(\mathsf{x}')\cdots)$—where $\hat{\sigma}_\phi$ is an arbitrary field state—it is not clear what density matrix $\hat{\sigma}_\phi$ we should use when considering points x,x' in regions of spacetime with different updates.

We conclude that a noncontextual update that includes the information obtained from a selective measurement performed on the detector is at odds with relativistic causality. Instead, in order to satisfy conditions (1) and (2) we must partially give up on the physical significance of density operators $\hat{\rho}_\phi$ and $\hat{\rho}_\phi^{\mathrm{u}}$ as representatives of observer-independent field states and simply treat them as states of information about the field (much like it is done in quantum-informational approaches to the measurement problem in quantum mechanics [46–51]). This is precisely what the next subsection focuses on.

B. A contextual update rule

As we just concluded, to properly formulate an update rule that is respectful of relativity we need to consider the field density operators to be observer dependent. In particular, we propose that the update depends on the context of the observer, i.e., the information available to them according to their position in spacetime. Moreover, because it depends on the observer, once they receive information about a measurement the update only takes place inside their causal future. It is perhaps interesting to remark here the distinction between *the measurement*, that is performed by the experimenter, and *the update*, that is performed by each observer according to the information they have about the field. It is in this sense that we say that the update is observer dependent. As such, when we write that a certain observer updates their field state, we mean that they are updating their information about the field and changing the field density operator that describes the field state for them, without acting upon the field in any way whatsoever. This operational approach can be summarized as follows:

(1) After an experimenter provided with a detector performs a projective measurement on the detector, an observer that becomes aware that the measurement has been performed can either have information about its outcome or not. If they do, they apply the selective update (13); if they do not, they apply the nonselective update (24). Both updates take place in the causal future of the observer.

(2) If an observer is spacelike separated from the interaction region, at most they can be aware of the measurement being performed, but they do not have access to the outcome of the measurement. Their update, if anything, should be nonselective, and we have already seen that the nonselective update does not have any effect on the outcome of spacelike separated operations. Hence, the spacelike separated observer does not have to take into account at all that a measurement has been performed. As it is desirable in a relativistic measurement theory, spacelike operations do not affect each other.

(3) To update the n-point functions we need to take into account where the information of the measurement is accessible. As such, the n-point functions will only be nontrivially updated (selectively or nonselectively) if any point of their n arguments is in the causal future of the measurement region. This will be addressed in depth in Sec. VI.

This update rule respects causality by fiat, and its consistency for spacelike separated observers is guaranteed by the fact that the nonselective update is causal. However, since it only updates the state in the causal future of the detector, one could legitimately wonder if the measurement prescription takes into account the correlations present in spacelike separated regions of the field that are well known to exist [3,6,13,14]. Condition 3 tells us how to proceed in order to ensure that this is the case. Consider two experimenters, Alba and Blanca, each provided with their own detector. The initial state of Alba's detector is $\hat{\rho}_{\text{A}} = |\xi\rangle\langle\xi|$, while Blanca's is $\hat{\rho}_{\text{B}} = |\zeta\rangle\langle\zeta|$. While being spacelike separated, they perform measurements, i.e., 1) they couple their detectors to the field and 2) after switching off the interaction they perform a projective measurement on the detectors and update their field states selectively with the information obtained in their local measurements. Alba gets a result associated to state $|a\rangle$ of her detector, while Blanca gets another associated to $|b\rangle$. Their corresponding updates are

$$\hat{\rho}_\phi^{\text{A}} = \frac{\hat{M}_{a,\xi}\hat{\rho}_\phi\hat{M}_{a,\xi}^\dagger}{\mathrm{tr}_\phi(\hat{\rho}_\phi\hat{E}_{a,\xi})} \quad \text{and} \quad \hat{\rho}_\phi^{\text{B}} = \frac{\hat{M}_{b,\zeta}\hat{\rho}_\phi\hat{M}_{b,\zeta}^\dagger}{\mathrm{tr}_\phi(\hat{\rho}_\phi\hat{E}_{b,\zeta})}. \tag{30}$$

In the future, they eventually meet and inform each other of their results. Their final updates based on the exchanged information are as follows: for Alba,

$$\hat{\rho}_\phi^{\text{AB}} = \frac{\hat{M}_{b,\zeta}\hat{\rho}_\phi^{\text{A}}\hat{M}_{b,\zeta}^\dagger}{\mathrm{tr}_\phi(\hat{\rho}_\phi^{\text{A}}\hat{E}_{b,\zeta})} = \frac{\hat{M}_{b,\zeta}\hat{M}_{a,\xi}\hat{\rho}_\phi\hat{M}_{a,\xi}^\dagger\hat{M}_{b,\zeta}^\dagger}{\mathrm{tr}_\phi(\hat{\rho}_\phi\hat{E}_{a,\xi})\mathrm{tr}_\phi(\hat{\rho}_\phi^{\text{A}}\hat{E}_{b,\zeta})}, \tag{31}$$

while for Blanca

$$\hat{\rho}_\phi^{\text{BA}} = \frac{\hat{M}_{a,\xi}\hat{\rho}_\phi^{\text{B}}\hat{M}_{a,\xi}^\dagger}{\mathrm{tr}_\phi(\hat{\rho}_\phi^{\text{B}}\hat{E}_{a,\xi})} = \frac{\hat{M}_{a,\xi}\hat{M}_{b,\zeta}\hat{\rho}_\phi\hat{M}_{b,\zeta}^\dagger\hat{M}_{a,\xi}^\dagger}{\mathrm{tr}_\phi(\hat{\rho}_\phi\hat{E}_{b,\zeta})\mathrm{tr}_\phi(\hat{\rho}_\phi^{\text{B}}\hat{E}_{a,\xi})}. \tag{32}$$

Now, taking into account the form of the $\hat{M}$ operators (14), in terms of the unitary (8) and therefore the Hamiltonian (1), it is straightforward to prove that if Alba's and Blanca's measurements are carried out in spacelike separated regions, then

$$[\hat{M}_{a,\xi}, \hat{M}_{b,\zeta}] = [\hat{M}_{a,\xi}, \hat{M}_{b,\zeta}^\dagger] = 0. \tag{33}$$

This means that the numerators in Eqs. (31) and (32) are the same. Since the denominators are normalization factors, we first conclude that the updates are consistent. Once they meet they have the same information, and indeed it holds that

$$\hat{\rho}_\phi^{\text{AB}} = \hat{\rho}_\phi^{\text{BA}}. \tag{34}$$

Moreover, by Eq. (33)

$$\mathrm{tr}_\phi(\hat{M}_{b,\zeta}\hat{M}_{a,\xi}\hat{\rho}_\phi\hat{M}_{a,\xi}^\dagger\hat{M}_{b,\zeta}^\dagger) = \mathrm{tr}_\phi(\hat{\rho}_\phi\hat{E}_{a,\xi}\hat{E}_{b,\zeta}) \tag{35}$$

so that we can write

$$\mathrm{tr}_\phi(\hat{\rho}_\phi^{\text{A}}\hat{E}_{b,\xi}) = \frac{\mathrm{tr}_\phi(\hat{\rho}_\phi\hat{E}_{a,\xi}\hat{E}_{b,\zeta})}{\mathrm{tr}_\phi(\hat{\rho}_\phi\hat{E}_{a,\xi})} \neq \mathrm{tr}_\phi(\hat{\rho}_\phi\hat{E}_{b,\xi}). \tag{36}$$

For a POVM, the probability of getting an outcome r from a generic state $\hat{\sigma}_\phi$ is the trace $\mathrm{tr}_\phi(\hat{\sigma}_\phi\hat{E}_r)$, where $\hat{E}_r$ is the POVM element associated to the outcome r [45]. This means that Eq. (36) displays the correlations between the measurements due to the initial correlations in the field state. Indeed, Eq. (36) can be read in terms of probabilities as

$$\begin{aligned}
&\mathrm{Prob}(\text{Blanca gets } b|\text{Alba gets } a)\\
&\quad = \frac{\mathrm{Prob}(\text{Alba gets } a \text{ and Blanca gets } b)}{\mathrm{Prob}(\text{Alba gets } a)}\\
&\quad \neq \mathrm{Prob}(\text{Blanca gets } b),
\end{aligned} \tag{37}$$

where the first equality is the formula for conditional probability, and in particular shows that Alba's and Blanca's outcomes are not independent.

We conclude that the proposed update rule respects causality, is consistent for spacelike separated measurements (and trivially for timelike separated measurements), and accurately accounts for spacelike correlations. Therefore it is a suitable contextual rule for updating the state of the field after measuring with detectors in QFT.

It is remarkable that the proposed update rule, for a particular observer, is somewhat similar in its structure to that proposed by Hellwig and Kraus [5]. The formalism proposed in our work, however, relies on particle detectors instead of on local projections, establishing a direct connection with experiments [32–34,44].

It should be noted that by giving up on density operators as global descriptions of the field state we are displacing the focus from the Hilbert space description to another based on what experimenters measure and the correlations between the possible measurements. This is precisely the approach adopted in algebraic quantum field theory (see e.g., Refs. [52–55]), where the *algebraic state* is interpreted to be the complex linear form that associates to each observable[6] its expectation value. Indeed, the contextual update rule described above should be given just in terms of updated n-point functions, as we will show in the next section.

VI. UPDATE OF *n*-POINT FUNCTIONS

In a free quantum field theory, the state of the field can be described in two interchangeable ways: either by a density operator in some particular Hilbert space representation or, equivalently, by the set of the field n-point functions. However, in Sec. V, we have argued that there are serious

[6] As an element of the direct limit of the net of local algebras [55].

difficulties to apply a selective update to a field density operator because of the incompatibility with a context-independent description. Fortunately, the formalism of n-point functions is still adequate for describing the contextual update rule proposed in the previous section. In the present section, we will formulate the update rule proposed in the previous section explicitly in terms of the n-point functions that fully characterize the state of the field. The n-point functions directly appear in the most common expressions for the response of particle detectors (see e.g., Refs. [29,30,56] among others), so having an update rule for all the n-point functions not only fully characterizes the updated state but is also of practical interest for any calculations involving particle detectors.

Notice that for the update after the measurement to be given *just* as an update of n-point functions, we need to initially assume that in our particular experiment the only relevant systems are the field and the detector. If the field is entangled with a third party in the past of the detector, we will assume for now that this third party will not be addressed in this measurement experiment, leaving the more complicated case for future sections.[7]

For this section, this simplifying constraint will allow us to "forget" about the causal past of the measurement and define the update only in the region of spacetime outside of it. In the spirit of the discussion in the previous section, we will distinguish whether the measurement performed on the detector is nonselective or selective.

A. Nonselective update

The nonselective update is straightforward to implement from the state update in Sec. IV. Since, as we showed, nonselective updates do not affect the state in regions spacelike separated from the measurement, there is no need to prescribe different updates whether the arguments are in the causal future of the detector or in the spacelike separated region. Hence, the updated n-point function is

$$\begin{aligned} w_n^{\mathrm{NS}}(\mathsf{x}_1,\dots,\mathsf{x}_n) &= \mathrm{tr}_\phi(\hat{\rho}_\phi^{\mathrm{u}}\hat{\phi}(\mathsf{x}_1)\cdots\hat{\phi}(\mathsf{x}_n)) \\ &= \langle \hat{M}_{s,\psi}^\dagger\hat{\phi}(\mathsf{x}_1)\cdots\hat{\phi}(\mathsf{x}_n)\hat{M}_{s,\psi}\rangle_{\hat{\rho}_\phi} \\ &\quad + \langle \hat{M}_{\bar{s},\psi}^\dagger\hat{\phi}(\mathsf{x}_1)\cdots\hat{\phi}(\mathsf{x}_n)\hat{M}_{\bar{s},\psi}\rangle_{\hat{\rho}_\phi} \end{aligned} \tag{38}$$

for every $\mathsf{x}_1,\dots,\mathsf{x}_n\in\mathcal{M}$ outside the causal past of the interaction region. This update can be given explicitly in terms of the n-point functions of the initial state of the field. In particular, for the one-point function and to first order in λ,

$$w_1^{\mathrm{NS}}(t_1,\boldsymbol{x}_1) = w_1(t_1,\boldsymbol{x}_1) + 2\lambda\int \mathrm{d}t\,\mathrm{d}^d\boldsymbol{x}\,\chi(t)F(\boldsymbol{x})\langle\psi|\hat{\mu}(t)|\psi\rangle \mathrm{Im}(w_2(t_1,\boldsymbol{x}_1,t,\boldsymbol{x})) + O(\lambda^2) \tag{39}$$

where w_n is the n-point function of the initial state of the field $\hat{\rho}_\phi$. Analogously, for the two-point function,

$$\begin{aligned} w_2^{\mathrm{NS}}(t_1,\boldsymbol{x}_1,t_2,\boldsymbol{x}_2) &= w_2(t_1,\boldsymbol{x}_1,t_2,\boldsymbol{x}_2) + \mathrm{i}\lambda\int \mathrm{d}t\,\mathrm{d}^d\boldsymbol{x}\,\chi(t)F(\boldsymbol{x})\langle\psi|\hat{\mu}(t)|\psi\rangle(w_3(t,\boldsymbol{x},t_1,\boldsymbol{x}_1,t_2,\boldsymbol{x}_2) - w_3(t_1,\boldsymbol{x}_1,t_2,\boldsymbol{x}_2,t,\boldsymbol{x})) \\ &\quad + O(\lambda^2). \end{aligned} \tag{40}$$

And in general,

$$\begin{aligned} w_n^{\mathrm{NS}}(t_1,\boldsymbol{x}_1,\dots,t_n,\boldsymbol{x}_n) &= w_n(t_1,\boldsymbol{x}_1,\dots,t_n,\boldsymbol{x}_n) + \mathrm{i}\lambda\int \mathrm{d}t\,\mathrm{d}^d\boldsymbol{x}\,\chi(t)F(\boldsymbol{x})\langle\psi|\hat{\mu}(t)|\psi\rangle(w_{n+1}(t,\boldsymbol{x},t_1,\boldsymbol{x}_1,\dots,t_n,\boldsymbol{x}_n) \\ &\quad - w_{n+1}(t_1,\boldsymbol{x}_1,\dots,t_n,\boldsymbol{x}_n,t,\boldsymbol{x})) + O(\lambda^2). \end{aligned} \tag{41}$$

The details of these calculations can be seen in Appendix B. The second-order terms in λ for the previous perturbative expressions are also displayed in Eq. (B12) of Appendix B. It is worth remarking that microcausality ensures that Eqs. (39), (40) and (41) reduce to the unchanged n-point function whenever their arguments are outside the causal future of the detector, since in that case $[\hat{\phi}(t,\boldsymbol{x}),\hat{\phi}(t_j,\boldsymbol{x}_j)]=0$ for every $j\in\{1,\dots,n\}$ and therefore

[7]This initial assumption can indeed be relaxed: treated with some care, the update rule for n-point functions which we are about to formulate can also be used in arbitrarily general scenarios. The reason is that the scheme given in Sec. V applies to arbitrary states $\hat{\rho}_\phi$ that may be extended to include third-party systems in addition to the field. We will show how this more general scenario can be straightforwardly dealt with in Sec. VII.

$$\begin{aligned} & w_{n+1}(t,\boldsymbol{x},t_1,\boldsymbol{x}_1,\ldots,t_n,\boldsymbol{x}_n) \\ &\quad = \mathrm{tr}_\phi\Big(\hat{\rho}_\phi\hat{\phi}(t,\boldsymbol{x})\hat{\phi}(t_1,\boldsymbol{x}_1)\cdots\hat{\phi}(t_n,\boldsymbol{x}_n)\Big) \\ &\quad = \mathrm{tr}_\phi\Big(\hat{\rho}_\phi\hat{\phi}(t_1,\boldsymbol{x}_1)\cdots\hat{\phi}(t_n,\boldsymbol{x}_n)\hat{\phi}(t,\boldsymbol{x})\Big) \\ &\quad = w_{n+1}(t_1,\boldsymbol{x}_1,\ldots,t_n,\boldsymbol{x}_n,t,\boldsymbol{x}). \end{aligned} \tag{42}$$

B. Selective update

For selective measurements, we will first present the update for the one-point function. Second, we will consider the two-point function, which involves a few subtleties that deserve attention. And finally, with the one-point and two-point functions as landmarks, we will generalize the update scheme to n-point functions. As before, the details of the calculations (as well as results at higher orders in the coupling strength) can be found in Appendix B.

1. One-point function

As shown in Appendix A, when dealing with selective measurements the update cannot be applied outside the causal future of the detector. Moreover, it should be noticed that in full rigor, and unlike for nonselective measurements, the selective update does depend on the region of spacetime in which the projective measurement on the detector is performed, whose future we shall denote $\mathcal{P}$. Therefore we have to distinguish three cases depending on the argument $\mathsf{x}_1 \in \mathcal{M}$ of the one-point function:

(a) If $\mathsf{x}_1 \in \mathcal{P}$, then we should consider the state of the field to be updated by the selective rule (13).

(b) If $\mathsf{x}_1 \in \mathcal{J}^+(\mathcal{D})\backslash\mathcal{P}$, with $\mathcal{J}^+(\mathcal{D})$ being the causal future of the interaction region[8] as defined in Eq. (5), then we only have to take into account the interaction, which as shown in Eq. (25) yields the same update as the nonselective rule (24).

(c) If x_1 is spacelike separated from $\mathcal{D}$ [i.e., it is outside the causal support of the interaction region, $\mathsf{x}_1 \notin \mathcal{J}(\mathcal{D})$], then we should use the initial state of the field, or equivalently the nonselective update.

However, we saw in Sec. VI A that the nonselective update can be safely applied to spacelike separated regions. Therefore, we can consider cases (b) and (c) jointly when prescribing the update rule. All together,

$$w_1^{\mathrm{S}}(\mathsf{x}_1) = \begin{cases} \dfrac{\langle \hat{M}_{s,\psi}^\dagger \hat{\phi}(\mathsf{x}_1)\hat{M}_{s,\psi}\rangle_{\hat{\rho}_\phi}}{\langle \hat{E}_{s,\psi}\rangle_{\hat{\rho}_\phi}} & \text{if } \mathsf{x}_1 \in \mathcal{P}, \\ w_1^{\mathrm{NS}}(\mathsf{x}_1) & \text{otherwise.} \end{cases} \tag{43}$$

Note that all expectation values are calculated for the initial state of the field, $\hat{\rho}_\phi$. For the case in which $\mathsf{x}_1 \in \mathcal{P}$, we have used Eq. (13) and the cyclic property of the trace. Therefore, the update can be given in terms of the n-point functions of the initial state of the field. In particular, if $\langle s|\psi\rangle \neq 0$, to first order in λ,

$$\begin{aligned} w_1^{\mathrm{S}}(t_1,\boldsymbol{x}_1) = {} & w_1(t_1,\boldsymbol{x}_1) + \frac{2\lambda}{|\langle s|\psi\rangle|^2}\int \mathrm{d}t\,\mathrm{d}^d\boldsymbol{x}\,\chi(t) \\ & \times F(\boldsymbol{x})\mathrm{Im}\Big[\langle\psi|s\rangle\langle s|\hat{\mu}(t)|\psi\rangle\Big(w_2(t_1,\boldsymbol{x}_1,t,\boldsymbol{x}) \\ & - w_1(t_1,\boldsymbol{x}_1)w_1(t,\boldsymbol{x})\Big)\Big] + O(\lambda^2) \end{aligned} \tag{44}$$

whenever $(t_1,\boldsymbol{x}_1) \in \mathcal{P}$. The more cumbersome case in which $\langle s|\psi\rangle = 0$ is displayed in Eq. (B33) of Appendix B, along with the case $\langle s|\psi\rangle \neq 0$ up to order 2 in λ, that can be seen in Eq. (B23).

2. Two-point function

For prescribing the update of the two-point function we also need to distinguish different cases. Following the same spirit of the prescription of the one-point function, when both arguments x_1, $\mathsf{x}_2 \in \mathcal{P}$ are in the causal future of the projective measurement, we consider the field state to be updated by Eq. (13), while if both x_1, x_2 are outside $\mathcal{P}$, the information of the measurement cannot propagate to those points and therefore we should use the nonselective update of the field state to calculate the expectation value. However, what should we do when we have a mixed situation (e.g., if $\mathsf{x}_1 \in \mathcal{P}$ and $\mathsf{x}_2 \notin \mathcal{P}$)? First, note that the two-point function is a nonlocal object that is only relevant in nonlocal experiments (for example, coordinating several labs around the world, or an interaction that is extended in space). However, it is only pertinent to ask about the result of a nonlocal experiment if we assume that the information obtained by the different measurements can be combined in a "processing" region[9] that intersects the causal futures of all the experiments. It is reasonable then that when the two-point function has mixed arguments inside and outside $\mathcal{P}$, the information about the outcome of the selective measurement is accessible to the processing region, as it has to have a nonzero intersection with $\mathcal{P}$. Hence, as long as one of the points of the two-point function is inside $\mathcal{P}$ we must use the selective update of the field state.

This is consistent with treating the field state as a state of information about the field. To update the field in accordance with the outcome of a measurement we need to look at where in spacetime the information obtained in the measurement can be accessed. Conversely, if an observer never accesses the causal future of a region in spacetime, it does not make sense for them to ask about the correlations between the field in that region and the region they have access to.[10] All this considered, we shall prescribe the selective update for the two-point function as

[8]Note that, since the projective measurement on the detector is performed in the causal future of the interaction region, $\mathcal{P} \subset \mathcal{J}^+(\mathcal{D})$.

[9]Notice the similarity with the notion of a processing region introduced in Ref. [57].

[10]For example, if two observers never get to communicate, directly or indirectly, it lacks physical meaning that they can ask any question that involves the correlations between their operations.

$$w_2^{\mathrm{S}}(\mathsf{x}_1,\mathsf{x}_2)=\begin{cases}\frac{\langle\hat{M}_{s,\psi}^\dagger\hat{\phi}(\mathsf{x}_1)\hat{\phi}(\mathsf{x}_2)\hat{M}_{s,\psi}\rangle_{\hat{\rho}_\phi}}{\langle\hat{E}_{s,\psi}\rangle_{\hat{\rho}_\phi}} & \text{if } \mathsf{x}_1 \text{ or } \mathsf{x}_2\in\mathcal{P},\\ w_2^{\mathrm{NS}}(\mathsf{x}_1,\mathsf{x}_2) & \text{otherwise.}\end{cases}\tag{45}$$

Again, the update can be given in terms of the n-point functions of the initial state of the field. In particular, if $\langle s|\psi\rangle\neq 0$, to first order in λ,

$$\begin{aligned}w_2^{\mathrm{S}}(t_1,\boldsymbol{x}_1,t_2,\boldsymbol{x}_2)=&\,w_2(t_1,\boldsymbol{x}_1,t_2,\boldsymbol{x}_2)+\frac{\lambda}{|\langle s|\psi\rangle|^2}\int\mathrm{d}t\,\mathrm{d}^d\boldsymbol{x}\,\chi(t)F(\boldsymbol{x})(\mathrm{i}\langle s|\psi\rangle\langle\psi|\hat{\mu}(t)|s\rangle w_3(t,\boldsymbol{x},t_1,\boldsymbol{x}_1,t_2,\boldsymbol{x}_2)\\&-\mathrm{i}\langle\psi|s\rangle\langle s|\hat{\mu}(t)|\psi\rangle w_3(t_1,\boldsymbol{x}_1,t_2,\boldsymbol{x}_2,t,\boldsymbol{x})-2\mathrm{Im}(\langle\psi|s\rangle\langle s|\hat{\mu}(t)|\psi\rangle)w_2(t_1,\boldsymbol{x}_1,t_2,\boldsymbol{x}_2)w_1(t,\boldsymbol{x}))\\&+O(\lambda^2)\end{aligned}\tag{46}$$

whenever $(t_1,\boldsymbol{x}_1)\in\mathcal{P}$ or $(t_2,\boldsymbol{x}_2)\in\mathcal{P}$ (or both). As before, the case in which $\langle s|\psi\rangle=0$ and the second-order term of the previous expression are left to be displayed in Appendix B.

3. *n*-point function

The arguments given to justify the prescription for the two-point function immediately generalize to arbitrary n-point functions, for which the selective update is

$$w_n^{\mathrm{S}}(\mathsf{x}_1,\ldots,\mathsf{x}_n)=w_n^{\mathrm{NS}}(\mathsf{x}_1,\ldots,\mathsf{x}_n)\tag{47}$$

if all $\mathsf{x}_1,\ldots,\mathsf{x}_n$ are outside $\mathcal{P}$, and

$$w_n^{\mathrm{S}}(\mathsf{x}_1,\ldots,\mathsf{x}_n)=\frac{\langle\hat{M}_{s,\psi}^\dagger\hat{\phi}(\mathsf{x}_1)\cdots\hat{\phi}(\mathsf{x}_n)\hat{M}_{s,\psi}\rangle_{\hat{\rho}_\phi}}{\langle\hat{E}_{s,\psi}\rangle_{\hat{\rho}_\phi}}\tag{48}$$

otherwise. Once again, the update can be given in terms of the n-point functions of the initial state of the field, and in particular, if $\langle s|\psi\rangle\neq 0$, to first order in λ,

$$\begin{aligned}w_n^{\mathrm{S}}(t_1,\boldsymbol{x}_1,\ldots,t_n,\boldsymbol{x}_n)=&\,w_n(t_1,\boldsymbol{x}_1,\ldots,t_n,\boldsymbol{x}_n)+\frac{\lambda}{|\langle s|\psi\rangle|^2}\int\mathrm{d}t\,\mathrm{d}^d\boldsymbol{x}\,\chi(t)F(\boldsymbol{x})\Big(\mathrm{i}\langle s|\psi\rangle\langle\psi|\hat{\mu}(t)|s\rangle w_{n+1}(t,\boldsymbol{x},t_1,\boldsymbol{x}_1,\ldots,t_n,\boldsymbol{x}_n)\\&-\mathrm{i}\langle\psi|s\rangle\langle s|\hat{\mu}(t)|\psi\rangle w_{n+1}(t_1,\boldsymbol{x}_1,\ldots,t_n,\boldsymbol{x}_n,t,\boldsymbol{x})\\&-2\mathrm{Im}(\langle\psi|s\rangle\langle s|\hat{\mu}(t)|\psi\rangle)w_n(t_1,\boldsymbol{x}_1,\ldots,t_n,\boldsymbol{x}_n)w_1(t,\boldsymbol{x})\Big)\\&+O(\lambda^2)\end{aligned}\tag{49}$$

whenever $(t_i,\boldsymbol{x}_i)\in\mathcal{P}$ for some $i\in\{1,\ldots,n\}$. Just as before, the second-order terms and the more tedious case in which $\langle s|\psi\rangle=0$ can be found in Appendix B.

VII. GENERALIZATION TO THE PRESENCE OF ENTANGLED THIRD PARTIES

In the previous sections the analysis was performed considering that the initial entanglement of the field with systems other than the detector is not addressable and hence irrelevant for the scenarios considered. However, the update rule given in Secs. III and V is not restricted to these situations. Generalizing beyond these situations is rather straightforward and conceptually identical to the prescription given in previous sections. For completeness, we will show here how the prescribed update rule can be generalized to the case in which there are other physical systems apart from the field and the detector that are relevant for the experiments under analysis.

First, note that in Sec. III we considered the initial state of the system to be $\hat{\rho}=\hat{\rho}_{\mathrm{d}}\otimes\hat{\rho}_\phi$ for the sake of simplicity, since these two systems are the only ones involved in the measurement. But it should be immediately realized that we can consider general initial states of the form $\hat{\rho}=\hat{\rho}_{\mathrm{d}}\otimes\hat{\rho}_\Phi$, where $\hat{\rho}_\Phi$ is the joint state of the field and all the other physical systems with which it might share entanglement that may be relevant for our experiment. For simplicity of the treatment, let us first assume that all of them are nonrelativistic, in the sense that their individual dynamics can be dealt with using nonrelativistic quantum mechanics and in particular they are localized. We will relax this assumption and allow for the presence of other relativistic fields at the end of the section.

Let us denote the relevant physical systems that are not the field or the detector by $\Sigma = \{\Sigma_1, \Sigma_2, \cdots\}$. The derivation of the updated state for $\hat{\rho}_\Phi$ proceeds as shown in Sec. III. The only difference that we need to take into account is that now, if $\hat{\mathcal{O}}$ is an operator acting on the Hilbert space of the whole system Hilbert space (detector, field and Σ) and $|\psi_1\rangle$, $|\psi_2\rangle$ are detector states, then we should understand $\langle\psi_1|\hat{\mathcal{O}}|\psi_2\rangle$ to be an operator acting on the Hilbert space $\mathcal{H}_\Phi$ of the field and the systems in Σ, such that

$$\langle\Phi_1|\langle\psi_1|\hat{\mathcal{O}}|\psi_2\rangle|\Phi_2\rangle = \langle\psi_1,\Phi_1|\hat{\mathcal{O}}|\psi_2,\Phi_2\rangle \tag{50}$$

for any $|\Phi_1\rangle$, $|\Phi_2\rangle \in \mathcal{H}_\Phi$. Clearly, Eq. (50) is the generalization of Eq. (11). It is straightforward to check that all the prescriptions given in Sec. V still apply after this direct generalization has been made, taking into account the extra systems when keeping track of the available information. However, in this more general setup, giving the update solely in terms of n-point functions as in Sec. VI would no longer be possible. Nevertheless, we can consider "joint" *extended* n-point functions of the joint system as follows: notice, first of all, that just as any observable of the field can be expressed in terms of the field operators, any observable of the systems in a subset $\Gamma \subseteq \Sigma$ can be expressed in terms of the rank-one operators $|\gamma_l\rangle\langle\gamma_m|$, as

$$\hat{\mathcal{O}}_\Gamma = \sum_{l,m}\langle\gamma_l|\hat{\mathcal{O}}_\Gamma|\gamma_m\rangle|\gamma_l\rangle\langle\gamma_m| \tag{51}$$

for an orthonormal basis $\{|\gamma_l\rangle\}$ of the Hilbert space of Γ. Thus, any operator acting on the field and Γ can be expressed in terms of the field operators $\hat{\phi}(\mathsf{x})$ and the rank-one operators $|\gamma_l\rangle\langle\gamma_m|$. We can therefore define the *extended n-point functions* as

$$\begin{aligned}\tilde{w}_{\Gamma,n}(l,m;\mathsf{x}_1,\ldots,\mathsf{x}_n)\\ \coloneqq \operatorname{tr}(\hat{\rho}_\Phi|\gamma_l\rangle\langle\gamma_m|\hat{\phi}(\mathsf{x}_1)\cdots\hat{\phi}(\mathsf{x}_n))\end{aligned} \tag{52}$$

for $n \geq 0$, where

$$\tilde{w}_{\Gamma,0}(l,m) \coloneqq \operatorname{tr}(\hat{\rho}_\Phi|\gamma_l\rangle\langle\gamma_m|). \tag{53}$$

The extended n-point functions characterize $\hat{\rho}_\Phi$. The update rule can now be given in terms of an update of the extended n-point functions, which can be shown to be just a modification of the update for n-point functions given in Sec. VI.

(a) *Nonselective update*. Since the nonselective update (24) is trace preserving and it acts nontrivially only on the Hilbert space of the field, we can simply prescribe the same update of Eq. (38) for each of the extended n-point functions:

$$\begin{aligned}&\tilde{w}^{\mathrm{NS}}_{\Gamma,n}(l,m;\mathsf{x}_1,\ldots,\mathsf{x}_n)\\ &= \operatorname{tr}\Big(\hat{M}_{s,\psi}\hat{\rho}_\Phi\hat{M}^\dagger_{s,\psi}|\gamma_l\rangle\langle\gamma_m|\hat{\phi}(\mathsf{x}_1)\cdots\hat{\phi}(\mathsf{x}_n)\Big)\\ &\quad + \operatorname{tr}\Big(\hat{M}_{\bar{s},\psi}\hat{\rho}_\Phi\hat{M}^\dagger_{\bar{s},\psi}|\gamma_l\rangle\langle\gamma_m|\hat{\phi}(\mathsf{x}_1)\cdots\hat{\phi}(\mathsf{x}_n)\Big).\end{aligned} \tag{54}$$

As in Sec. VI A, this expression can be written in terms of the nonupdated extended n-point functions. Note that, in particular, $\tilde{w}^{\mathrm{NS}}_{\Gamma,0}(l,m) = \tilde{w}_{\Gamma,0}(l,m)$.

(b) *Selective update*. Same as in Sec. VI, the prescription of the update requires to keep track of where the information is accessible. This leads to a piecewise definition as in Eqs. (47) and (48): let $\mathcal{P}$ be the causal future of the region in which the projective measurement on the detector is performed,

$$\tilde{w}^{\mathrm{S}}_{\Gamma,n}(l,m;\mathsf{x}_1,\ldots,\mathsf{x}_n) = \tilde{w}^{\mathrm{NS}}_{\Gamma,n}(l,m;\mathsf{x}_1,\ldots,\mathsf{x}_n) \tag{55}$$

if all $\mathsf{x}_1,\ldots,\mathsf{x}_n$ *and* the systems of Γ are outside $\mathcal{P}$, and

$$\begin{aligned}&\tilde{w}^{\mathrm{S}}_{\Gamma,n}(l,m;\mathsf{x}_1,\ldots,\mathsf{x}_n)\\ &= \frac{\operatorname{tr}\Big(\hat{M}_{s,\psi}\hat{\rho}_\Phi\hat{M}^\dagger_{s,\psi}|\gamma_l\rangle\langle\gamma_m|\hat{\phi}(\mathsf{x}_1)\cdots\hat{\phi}(\mathsf{x}_n)\Big)}{\operatorname{tr}_\Phi(\hat{\rho}_\Phi\hat{E}_{s,\psi})}\end{aligned} \tag{56}$$

otherwise. In particular,

$$\tilde{w}^{\mathrm{S}}_{\Gamma,0}(l,m) = \tilde{w}^{\mathrm{NS}}_{\Gamma,0}(l,m) \tag{57}$$

if the systems of Γ are outside $\mathcal{P}$, and

$$\tilde{w}^{\mathrm{S}}_{\Gamma,0}(l,m) = \frac{\operatorname{tr}(\hat{M}_{s,\psi}\hat{\rho}_\Phi\hat{M}^\dagger_{s,\psi}|\gamma_l\rangle\langle\gamma_m|)}{\operatorname{tr}_\Phi(\hat{\rho}_\Phi\hat{E}_{s,\psi})} \tag{58}$$

otherwise.

To end this section, we can address the case where the third parties sharing entanglement with the probed field are themselves relativistic fields. In that scenario, Eq. (51) is not useful anymore, since for a basis of the Hilbert space of a field, the rank-one operators $|\gamma_l\rangle\langle\gamma_m|$ are not local objects and the update to the extended n-point function has to be defined over local regions of spacetime. Fortunately, the field itself is defined in terms of local observables. The local observables of a field σ in Σ can be expressed in terms of its associated field operators $\hat{\sigma}(\mathsf{x})$. Thus, for the simplest case in which the only system in Σ is a field σ, we define the extended n-point function as an (n',n)-*point function*,

$$\begin{aligned}&\tilde{w}_{n',n}(\mathsf{y}_1,\ldots,\mathsf{y}_{n'};\mathsf{x}_1,\ldots,\mathsf{x}_n)\\ &\quad = \operatorname{tr}(\hat{\rho}_\Phi\hat{\sigma}(\mathsf{y}_1)\cdots\hat{\sigma}(\mathsf{y}_{n'})\hat{\phi}(\mathsf{x}_1)\cdots\hat{\phi}_1(\mathsf{x}_n)).\end{aligned} \tag{59}$$

This expression provides the extended n-point function that substitutes Eq. (52) for the case in which Σ is one relativistic field. If there are more fields present, one can build the extended n-point function in an analogous fashion. Regarding the update rule, in the same spirit of the prescriptions given in Eqs. (54), (55) and (56),

$$\tilde{w}^{\mathrm{NS}}_{n',n}(\mathsf{y}_1,\ldots,\mathsf{y}_{n'};\mathsf{x}_1,\ldots,\mathsf{x}_n) = \mathrm{tr}(\hat{M}_{s,\psi}\hat{\rho}_\Phi\hat{M}^\dagger_{s,\psi}\hat{\sigma}(\mathsf{y}_1)\cdots\hat{\sigma}(\mathsf{y}_{n'})\hat{\phi}(\mathsf{x}_1)\cdots\hat{\phi}(\mathsf{x}_n)) + \mathrm{tr}(\hat{M}_{\bar{s},\psi}\hat{\rho}_\Phi\hat{M}^\dagger_{\bar{s},\psi}\hat{\sigma}(\mathsf{y}_1)\cdots\hat{\sigma}(\mathsf{y}_{n'})\hat{\phi}(\mathsf{x}_1)\cdots\hat{\phi}(\mathsf{x}_n)) \tag{60}$$

for the nonselective case, while for the selective case,

$$\tilde{w}^{\mathrm{S}}_{n',n}(\mathsf{y}_1,\ldots,\mathsf{y}_{n'};\mathsf{x}_1,\ldots,\mathsf{x}_n) = \tilde{w}^{\mathrm{NS}}_{\Gamma,n}(\mathsf{y}_1,\ldots,\mathsf{y}_{n'};\mathsf{x}_1,\ldots,\mathsf{x}_n) \tag{61}$$

if all $\mathsf{x}_1,\ldots,\mathsf{x}_n$ *and* $\mathsf{y}_1,\ldots,\mathsf{y}_{n'}$ are outside $\mathcal{P}$, and

$$\tilde{w}^{\mathrm{S}}_{n',n}(\mathsf{y}_1,\ldots,\mathsf{y}_{n'};\mathsf{x}_1,\ldots,\mathsf{x}_n) = \frac{\mathrm{tr}\left(\hat{M}_{s,\psi}\hat{\rho}_\Phi\hat{M}^\dagger_{s,\psi}\hat{\sigma}(\mathsf{y}_1)\cdots\hat{\sigma}(\mathsf{y}_{n'})\hat{\phi}(\mathsf{x}_1)\cdots\hat{\phi}(\mathsf{x}_n)\right)}{\mathrm{tr}_\Phi(\hat{\rho}_\Phi\hat{E}_{s,\psi})} \tag{62}$$

otherwise.

Finally, for the mixed case in which Σ contains both localized nonrelativistic systems and relativistic fields, we just need to use the natural combination of both formalisms, that includes rank-one operators of the form $|\gamma_l\rangle\langle\gamma_m|$ for the nonrelativistic systems and field operators $\hat{\sigma}(\mathsf{y})$ for the relativistic fields.

VIII. A PRACTICAL EXAMPLE WITH DETECTORS

To further clarify how to use the formalism in a practical implementation we will consider an example involving three stationary experimenters, Alba, Blanca and Clara, each provided with a two-level Unruh-DeWitt detector. The situation, depicted in Fig. 1, is as follows[11]:

(1) Clara performs a measurement with her detector, by first letting it interact with the field and then performing a projective measurement on it, immediately after the interaction is switched off.

(2) Blanca lets her detector interact with the field in the causal future of the projective measurement performed by Clara, $\mathcal{P}$.

(3) Alba lets her detector interact with the field in a region that is spacelike separated from both Blanca's and Clara's interaction regions.

We consider an initial state

$$\hat{\rho} = \hat{\rho}_{\mathrm{A}} \otimes \hat{\rho}_{\mathrm{B}} \otimes \hat{\rho}_{\mathrm{C}} \otimes \hat{\rho}_\phi \tag{63}$$

[11]This configuration is a pretty archetypal setup in relativistic quantum information in scenarios of entanglement harvesting; see e.g., Refs. [58,59] among many others.

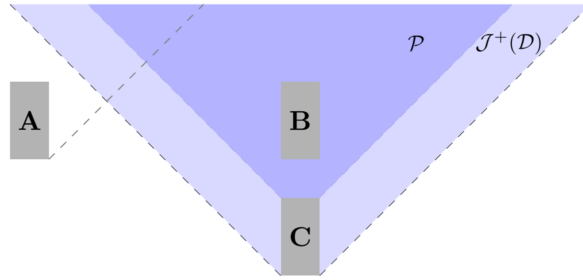

FIG. 1. Configuration in a slice of spacetime of the interaction regions of detectors A, B and C. The causal future of the projective measurement performed on C, $\mathcal{P}$ is shown in blue; the causal future of the measurement that is not already in the future of the projective measurement, $\mathcal{J}^+(\mathcal{D})\backslash\mathcal{P}$ is shown in pale blue.

for the array of detectors and the field. For simplicity we have assumed that the detector that is measured starts out in a pure state, $\hat{\rho}_{\mathrm{C}} = |\psi\rangle\langle\psi|$. In the interaction picture, the interaction of the detectors with the field is given by the Hamiltonian

$$\hat{H}_I(t) = \hat{H}_{\mathrm{A}}(t) + \hat{H}_{\mathrm{B}}(t) + \hat{H}_{\mathrm{C}}(t), \tag{64}$$

where

$$\hat{H}_\nu(t) = \lambda_\nu\chi_\nu(t)\hat{\mu}_\nu(t)\int \mathrm{d}^d\boldsymbol{x}\, F_\nu(\boldsymbol{x})\hat{\phi}(t,\boldsymbol{x}) \tag{65}$$

is the same Unruh-DeWitt Hamiltonian from Eq. (1), for $\nu \in \{\mathrm{A},\mathrm{B},\mathrm{C}\}$. Now, since Clara's operations causally precede Blanca's, and since Alba is spacelike separated from both of them, the unitary operator that describes the evolution of the three detectors and the field

$$\hat{U} = \mathcal{T}\exp\left[-\mathrm{i}\int_{-\infty}^{\infty}\mathrm{d}t'(\hat{H}_{\mathrm{A}}(t') + \hat{H}_{\mathrm{B}}(t') + \hat{H}_{\mathrm{C}}(t'))\right] \tag{66}$$

can in fact be written as [27]

$$\hat{U} = \hat{U}_{\mathrm{A}}\hat{U}_{\mathrm{B}}\hat{U}_{\mathrm{C}} = \hat{U}_{\mathrm{B}}\hat{U}_{\mathrm{C}}\hat{U}_{\mathrm{A}}, \tag{67}$$

where

$$\hat{U}_\nu = \mathcal{T}\exp\left(-\mathrm{i}\int_{-\infty}^{\infty}\mathrm{d}t'\hat{H}_\nu(t')\right) \tag{68}$$

for $\nu \in \{\mathrm{A},\mathrm{B},\mathrm{C}\}$. In particular, we have that

$$[\hat{U}_{\mathrm{A}}, \hat{U}_{\mathrm{B}}] = [\hat{U}_{\mathrm{A}}, \hat{U}_{\mathrm{C}}] = 0, \tag{69}$$

and by Eq. (14),

$$[\hat{U}_{\rm A}, \hat{M}_{c,\psi}] = [\hat{U}_{\rm A}, \hat{M}^{\dagger}_{c,\psi}] = 0 \tag{70}$$

for any $|c\rangle \in \mathcal{H}_{\rm C}$.

We are interested in studying how the measurement performed by Clara affects the joint partial state of Alba and Blanca, $\hat{\rho}_{\rm AB}$, as well as their individual partial states $\hat{\rho}_{\rm A}$ and $\hat{\rho}_{\rm B}$. Along the lines of previous sections, we distinguish whether the measurement performed by Clara is nonselective or selective. For the sake of clarity, in the main body of this section we will use the approach that uses a context-dependent density operator, as presented in Sec. V, instead of the equivalent but more involved formulation based on n-point functions and its extensions, presented in Secs. VI and VII. Nevertheless, we have explicitly computed all the updates using the formulation of extended n-point functions in Appendix C, showing explicitly that both methods give the same results.

A. Nonselective measurement

Since it is less involved from the point of view of the update rule, let us first address the case in which Clara measures nonselectively. We will show that in the nonselective case the updated partial states of Alba and Blanca will coincide with the case where the three detectors interact with the field and we trace out the state of Clara's detector. That is, the only influence that Clara's detector has on $\hat{\rho}_{\rm AB}$ is through its coupling to the field since no information about the measurement is assumed to be known by Alba and Blanca.

We already argued in Sec. IV that all observers are susceptible to being informed of the performance of the measurement without knowing its outcome. Thus, we can consider that both Alba and Blanca have access to the nonselective update of the state.[12] For our purposes, it is simpler to consider the update of the measurement in the first place. Thus, we obtain a final joint state

$$\begin{aligned}\hat{\rho}'_{\rm AB} &= {\rm tr}_{\phi}(\hat{U}_{\rm A}\hat{U}_{\rm B}[\hat{\rho}_{\rm A}\otimes\hat{\rho}_{\rm B}\otimes\hat{M}_{c,\psi}\hat{\rho}_{\phi}\hat{M}^{\dagger}_{c,\psi}]\hat{U}^{\dagger}_{\rm B}\hat{U}^{\dagger}_{\rm A})\\ &\quad+{\rm tr}_{\phi}[\hat{U}_{\rm A}\hat{U}_{\rm B}(\hat{\rho}_{\rm A}\otimes\hat{\rho}_{\rm B}\otimes\hat{M}_{\bar{c},\psi}\hat{\rho}_{\phi}\hat{M}^{\dagger}_{\bar{c},\psi})\hat{U}^{\dagger}_{\rm B}\hat{U}^{\dagger}_{\rm A}]\\ &= {\rm tr}_{{\rm C},\phi}[\hat{U}_{\rm A}\hat{U}_{\rm B}\hat{U}_{\rm C}(\hat{\rho}_{\rm A}\otimes\hat{\rho}_{\rm B}\otimes|\psi\rangle\langle\psi|\otimes\hat{\rho}_{\phi})\hat{U}^{\dagger}_{\rm C}\hat{U}^{\dagger}_{\rm B}\hat{U}^{\dagger}_{\rm A}],\end{aligned} \tag{71}$$

where in the last step we used Eq. (25). As anticipated, this is the same result obtained for $\hat{\rho}_{\rm AB}$ in the case in which Clara does not perform a projective measurement on the detector at all. The partial states are

$$\begin{aligned}\hat{\rho}'_{\rm B} &= {\rm tr}_{\rm A}(\hat{\rho}'_{\rm AB})\\ &= {\rm tr}_{{\rm C},\phi}[\hat{U}_{\rm B}\hat{U}_{\rm C}(\hat{\rho}_{\rm B}\otimes|\psi\rangle\langle\psi|\otimes\hat{\rho}_{\phi})\hat{U}^{\dagger}_{\rm C}\hat{U}^{\dagger}_{\rm B}]\end{aligned} \tag{72}$$

and

$$\hat{\rho}'_{\rm A} = {\rm tr}_{\rm B}(\hat{\rho}'_{\rm AB}) = {\rm tr}_{\phi}[\hat{U}_{\rm A}(\hat{\rho}_{\rm A}\otimes\hat{\rho}_{\phi})\hat{U}^{\dagger}_{\rm A}], \tag{73}$$

where in order to trace out A and B we have used Eq. (69) and the cyclic property of the trace.

The same results of Eqs. (71), (72) and (73) are obtained by using the extended n-point function update formalized in Sec. VII, as can be explicitly seen in the calculations leading to Eqs. (C6), (C8) and (C10) in Appendix C.

Notice in particular that $\hat{\rho}'_{\rm A}$ does not depend on the operations performed by Blanca and Clara. In fact, as we expected, this is the same result that we would have obtained had we updated the state with the interaction of Alba's detector in the first place. Note that both partial states satisfy

$$\hat{\rho}'_{\rm A} = {\rm tr}_{\rm B}(\hat{\rho}'_{\rm AB}) \quad \text{and} \quad \hat{\rho}'_{\rm B} = {\rm tr}_{\rm A}(\hat{\rho}'_{\rm AB}). \tag{74}$$

This is a consequence of the fact that for nonselective measurements, as we saw in Sec. IV, there is no need to make a distinction in the update for observers inside $\mathcal{P}$ and outside $\mathcal{P}$, since they may in principle have access to the same information: a measurement whose outcome is unknown has potentially been performed. More concretely, here both Alba and Blanca are ignorant about the outcome of Clara's measurement, and therefore all three partial density operators, $\hat{\rho}_{\rm AB}$, $\hat{\rho}_{\rm A}$ and $\hat{\rho}_{\rm B}$ are calculated with the same amount of information about the field and its interactions.

B. Selective measurement

The case in which Clara performs a selective measurement requires slightly more care than the nonselective one, since in this case the updated state after the measurement depends on the observer and the information that is available to them (in the language of n-point functions, the update is defined piecewise, unlike the nonselective case). As in the nonselective case, for the sake of formal simplicity, in this derivation we will perform the update due to Clara's measurement in the first place. We will check nevertheless that, as before and as should be required, the results are the same if we evolve the state due to Alba's interaction in the first place.

To calculate the joint state $\hat{\rho}_{\rm AB}$, we need to take into account that the information in this state is only fully accessible by an observer that eventually has access to the information from both systems held by Alba and Blanca.

[12]Note that since Alba is spacelike separated from both Blanca and Clara, it does not matter whether we carry out first the update of Clara's measurement or the evolution due to the interaction of Alba's detector, as we saw in Sec. IV and becomes apparent in Eq. (70).

In particular, such an observer has access to the outcome of Clara's measurement, since Blanca does.[13] Therefore

$$\hat{\rho}'_{\mathrm{AB}} = \frac{\mathrm{tr}_\phi[\hat{U}_{\mathrm{A}}\hat{U}_{\mathrm{B}}(\hat{\rho}_{\mathrm{A}} \otimes \hat{\rho}_{\mathrm{B}} \otimes \hat{M}_{c,\psi}\hat{\rho}_\phi \hat{M}^\dagger_{c,\psi})\hat{U}^\dagger_{\mathrm{B}}\hat{U}^\dagger_{\mathrm{A}}]}{\mathrm{tr}_\phi(\hat{\rho}_\phi \hat{E}_{c,\psi})} = \frac{\mathrm{tr}_\phi[\hat{U}_{\mathrm{A}}\hat{U}_{\mathrm{B}}\hat{M}_{c,\psi}(\hat{\rho}_{\mathrm{A}} \otimes \hat{\rho}_{\mathrm{B}} \otimes \hat{\rho}_\phi)\hat{M}^\dagger_{c,\psi}\hat{U}^\dagger_{\mathrm{B}}\hat{U}^\dagger_{\mathrm{A}}]}{\mathrm{tr}_\phi(\hat{\rho}_\phi \hat{E}_{c,\psi})}, \tag{75}$$

where the last step is simply an abuse of notation. For calculating $\hat{\rho}_{\mathrm{B}}$, observe that since Blanca is in the causal future of the measurement performed by Clara, she has access to its outcome. Thus,

$$\hat{\rho}'_{\mathrm{B}} = \frac{\mathrm{tr}_{\mathrm{A},\phi}[\hat{U}_{\mathrm{A}}\hat{U}_{\mathrm{B}}\hat{M}_{c,\psi}(\hat{\rho}_{\mathrm{A}} \otimes \hat{\rho}_{\mathrm{B}} \otimes \hat{\rho}_\phi)\hat{M}^\dagger_{c,\psi}\hat{U}^\dagger_{\mathrm{B}}\hat{U}^\dagger_{\mathrm{A}}]}{\mathrm{tr}_\phi(\hat{\rho}_\phi \hat{E}_{c,\psi})} = \frac{\mathrm{tr}_\phi[\hat{U}_{\mathrm{B}}\hat{M}_{c,\psi}(\hat{\rho}_{\mathrm{B}} \otimes \hat{\rho}_\phi)\hat{M}^\dagger_{c,\psi}\hat{U}^\dagger_{\mathrm{B}}]}{\mathrm{tr}_\phi(\hat{\rho}_\phi \hat{E}_{c,\psi})} = \mathrm{tr}_{\mathrm{A}}(\hat{\rho}'_{\mathrm{AB}}). \tag{76}$$

Finally, if we want to obtain $\hat{\rho}_{\mathrm{A}}$, we just need to take into account that Alba does not have access to the outcome of Clara's measurement, and hence the state of the field that she deals with is the one updated nonselectively or directly the initial one (since both bring the same result, as we saw in the previous section). The result is therefore the same as in Eq. (73) for the nonselective measurement,

$$\hat{\rho}'_{\mathrm{A}} = \mathrm{tr}_\phi[\hat{U}_{\mathrm{A}}(\hat{\rho}_{\mathrm{A}} \otimes \hat{\rho}_\phi)\hat{U}^\dagger_{\mathrm{A}}]. \tag{77}$$

Notice that

$$\hat{\rho}'_{\mathrm{A}} \neq \mathrm{tr}_{\mathrm{B}}(\hat{\rho}'_{\mathrm{AB}}), \tag{78}$$

since $\hat{\rho}'_{\mathrm{AB}}$ was calculated for an observer that, unlike Alba, knows the outcome of Clara's measurement. In fact, if Alba eventually reaches the causal future of Clara's measurement and learns of its outcome, then the state should be updated as

$$\hat{\rho}''_{\mathrm{A}} = \frac{\mathrm{tr}_{\mathrm{B},\phi}[\hat{U}_{\mathrm{A}}\hat{U}_{\mathrm{B}}\hat{M}_{c,\psi}(\hat{\rho}_{\mathrm{A}} \otimes \hat{\rho}_{\mathrm{B}} \otimes \hat{\rho}_\phi)\hat{M}^\dagger_{c,\psi}\hat{U}^\dagger_{\mathrm{B}}\hat{U}^\dagger_{\mathrm{A}}]}{\mathrm{tr}_\phi(\hat{\rho}_\phi \hat{E}_{c,\psi})} = \frac{\mathrm{tr}_\phi[\hat{U}_{\mathrm{A}}\hat{M}_{c,\psi}(\hat{\rho}_{\mathrm{A}} \otimes \hat{\rho}_\phi)\hat{M}^\dagger_{c,\psi}\hat{U}^\dagger_{\mathrm{A}}]}{\mathrm{tr}_\phi(\hat{\rho}_\phi \hat{E}_{c,\psi})} = \mathrm{tr}_{\mathrm{B}}(\hat{\rho}'_{\mathrm{AB}}), \tag{79}$$

which coincides with what happens to $\hat{\rho}'_{\mathrm{B}}$ in Eq. (76).

The same results of Eqs. (75), (76), (77) and (79) are obtained by using the extended n-point function update formalized in Sec. VII, as can be explicitly seen in the calculations leading to Eqs. (C22), (C24), (C26) and (C28) in Appendix C.

[13]This line of reasoning is completely analogous to the one carried out in Sec. VI B 2 to prescribe the piecewise update of two-point functions.

IX. A MEASUREMENT THEORY

We have proposed a measurement scheme where localized nonrelativistic quantum systems that couple covariantly to the field gather information about its state. We are now in position to argue that this measurement framework has all the characteristics that one should expect from a proper measurement theory for QFT. Namely:

(1) *It is consistent with relativistic QFT*. The measurement process consists of two steps: the interaction between the detector and the field, and the projective measurement on the detector once the interaction has been switched off in order to access the information about the field stored in it. From recently established results, we know that UDW detectors can be coupled fully covariantly to quantum fields [25,26], and that the interaction with the field does not *per se* allow faster-than-light signaling [24,27]. Furthermore, when a detector is smeared, the possible signaling appears only in a restricted and controlled way if there is a third, nonpointlike detector mediating between them. Such a causality violation does not even become apparent in leading orders of perturbation theory [27]. As for the projective measurement on the detector, in this work we have shown that the effect of performing projective measurements on detectors and updating the field state consistently is as safe from causality violations as the interaction with the field itself (Sec. IV).

(2) *It provides an update rule*. As we have explicitly described and discussed in Secs. V, VI and VII, we have given a consistent update rule for the field state after the measurement that respects causality—as explicitly manifested in the update of the (extended) n-point functions—and includes the information obtained from the outcome of the measurement in the spirit of Lüders rule, enforcing the compatibility of sequential measurements.

(3) *It produces definite values for the outcome of single-shot measurements*. Since the detectors are measured through projective measurements, the outcome of a measurement is a real number that can be written down in an experimenter's notepad.

(4) *It is capable of reproducing experiments*. Indeed, particle detector models have been proven to capture the features of experimental setups in quantum optics and the light-matter interaction [32–34,44], as well as the phenomenology of the measurement of other quantum fields such as, e.g., neutrinos [60,61]. Particle detector models are therefore directly connected with experimentally realistic setups where quantum fields are measured.

By satisfying these four characteristics, we conclude that the measurement scheme proposed in this article constitutes a measurement theory for QFT that can still rely on the projection postulate of nonrelativistic quantum mechanics to access the information in the field. This proposed scheme has the additional advantage of combining localized rank-one projective measurements (that return a definite value of a measurement outcome) with compatibility with the relativistic nature of the theory. This is something that we cannot accomplish by performing projective measurements on quantum fields, where localized projective measurements are forced to be infinite-rank [6].

X. CONCLUSIONS

Since Sorkin's seminal paper in 1993, it has been evident that the measurement theory of nonrelativistic quantum mechanics cannot be directly imported to quantum field theory due to relativistic considerations. As Sorkin put it, "*this problem leaves the Hilbert space formulation of quantum field theory with no definite measurement theory*" [7]. In this paper we have proposed a way to build a measurement theory for QFT based on particle detectors that (1) has all the advantages of the measurement theory of nonrelativistic quantum mechanics, in that it provides the values of single-shot experiments and there is a state update enforcing compatibility with future measurements, (2) is compatible with relativity and is safe from gross causality violations, and (3) can be easily connected to experiments.

In order to establish the consistency of the proposed measurement scheme—consisting of (1) interaction of the detector with the probed field and (2) performing an idealized measurement on the detector and updating accordingly—we have relied on previous results establishing the covariance of the UDW detector-field coupling [25,26] and the compatibility of the interaction with relativity [24,27]. In addition, in this work we have shown that the performance of the projective measurement on the detector does not introduce any causality violations, and—effectively subscribing to an epistemic interpretation of the field state—we have provided a contextual update rule for the state of the field after the measurement. This update rule has been given in full detail in terms of (extended) n-point functions of the field for both nonselective and selective measurements on particle detectors, and we have shown how it is implemented in a practical example.

These results provide a formal basis for a measurement theory for QFT. Furthermore, they pave the way to fully relativistic formulations of problems where the role of measurements is central, such as the quantum Zeno effect [35,36], the delayed choice quantum eraser experiment [37–41], and many other similar experiments that can be performed within, e.g., the framework of the light-matter interaction.

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APPENDIX A: CAUSAL BEHAVIOR OF THE SELECTIVE UPDATE

In this appendix we analyze the causal structure of the selective update, confirming that it affects local operations outside the causal future of the interaction region and therefore should not be applied as a global update anywhere outside the causal support of the detector if we want the update rule to be compatible with relativity.

Following a selective measurement, we consider the updated state of the field to be

$$\hat{\rho}_{\phi}^{\mathrm{u}} = \hat{\rho}_{\phi}^{s,\psi} \tag{A1}$$

where $\hat{\rho}_{\phi}^{s,\psi}$ is given by Eq. (13) for a specific $|s\rangle$ coming from the result of the measurement. For this update, we will use the following estimator to evaluate where in spacetime the POVM alters the field:

$$\begin{aligned}\Delta_n(t_1,\boldsymbol{x}_1,\dots,t_n,\boldsymbol{x}_n) &= (\langle\hat{\phi}(t_1,\boldsymbol{x}_1)\cdots\hat{\phi}(t_n,\boldsymbol{x}_n)\rangle_{\hat{\rho}_{\phi}^{\mathrm{u}}} \\ &\quad - \langle\hat{\phi}(t_1,\boldsymbol{x}_1)\cdots\hat{\phi}(t_n,\boldsymbol{x}_n)\rangle_{\hat{\rho}_{\phi}})\langle\hat{E}_{s,\psi}\rangle_{\hat{\rho}_{\phi}} \\ &= \langle\hat{M}_{s,\psi}^{\dagger}\hat{\phi}(t_1,\boldsymbol{x}_1)\cdots\hat{\phi}(t_n,\boldsymbol{x}_n)\hat{M}_{s,\psi}\rangle_{\hat{\rho}_{\phi}} \\ &\quad - \langle\hat{M}_{s,\psi}^{\dagger}\hat{M}_{s,\psi}\rangle_{\hat{\rho}_{\phi}}\langle\hat{\phi}(t_1,\boldsymbol{x}_1)\cdots\hat{\phi}(t_n,\boldsymbol{x}_n)\rangle_{\hat{\rho}_{\phi}}.\end{aligned} \tag{A2}$$

This estimator is the difference between the n-point function of the post-measurement and pre-measurement states of the field, but multiplied by the quantity $\mathrm{tr}(\hat{\rho}_{\phi}\hat{E}_{s,\psi})$ to make the evaluation simpler. Note that this trace is finite and positive if the updated state is well defined. Studying where the estimator Δ_n is nonzero gives us information about the spacetime domain of the effect of the selective update.

We will first perform the analysis without making any assumptions on the pure state of the detector $|\psi\rangle$ or the initial state of the field $\hat{\rho}_\phi$. In this general case we will see that already for the one-point function, $\Delta_1(t_1, \boldsymbol{x}_1)$ can be nonzero out of the causal future of the detector, to first order in perturbation theory. In the next subsection, we show this also happens for the updates associated with eigenstates of the detector Hamiltonian, by going to order λ^2 in the one-point function. We end the appendix by showing that there are certain conditions under which the update does not affect the one-point function out of the causal support of the detector. In this last case we have to use the two-point function to confirm that, as in the rest of the cases, the selective update has an effect over regions of spacetime that are spacelike separated from the measurement.

1. The general case

In order to analyze the change in the one-point function, we use Eq. (A2) with $n = 1$ and the perturbative expansions considered at the end of Sec. III. For the first term in Eq. (A2),

$$\hat{M}^\dagger_{s,\psi}\hat{\phi}(t_1,\boldsymbol{x}_1)\hat{M}_{s,\psi} = \hat{M}^{\dagger(0)}_{s,\psi}\hat{\phi}(t_1,\boldsymbol{x}_1)\hat{M}^{(0)}_{s,\psi} + \lambda\left(\hat{M}^{\dagger(1)}_{s,\psi}\hat{\phi}(t_1,\boldsymbol{x}_1)\hat{M}^{(0)}_{s,\psi} + \hat{M}^{\dagger(0)}_{s,\psi}\hat{\phi}(t_1,\boldsymbol{x}_1)\hat{M}^{(1)}_{s,\psi}\right) + O(\lambda^2). \tag{A3}$$

Term by term, for the zeroth order,

$$\hat{M}^{\dagger(0)}_{s,\psi}\hat{\phi}(t_1,\boldsymbol{x}_1)\hat{M}^{(0)}_{s,\psi} = |\langle s|\psi\rangle|^2\hat{\phi}(t_1,\boldsymbol{x}_1). \tag{A4}$$

For the first order in λ,

$$\hat{M}^{\dagger(1)}_{s,\psi}\hat{\phi}(t_1,\boldsymbol{x}_1)\hat{M}^{(0)}_{s,\psi} = \mathrm{i}\langle s|\psi\rangle\int \mathrm{d}t\,\mathrm{d}^d\boldsymbol{x}\,\chi(t)F(\boldsymbol{x})\langle s|\hat{\mu}(t)|\psi\rangle^*\hat{\phi}(t,\boldsymbol{x})\hat{\phi}(t_1,\boldsymbol{x}_1) \tag{A5}$$

and

$$\hat{M}^{\dagger(0)}_{s,\psi}\hat{\phi}(t_1,\boldsymbol{x}_1)\hat{M}^{(1)}_{s,\psi} = -\mathrm{i}\langle s|\psi\rangle^*\int \mathrm{d}t\,\mathrm{d}^d\boldsymbol{x}\,\chi(t)F(\boldsymbol{x})\langle s|\hat{\mu}(t)|\psi\rangle\hat{\phi}(t_1,\boldsymbol{x}_1)\hat{\phi}(t,\boldsymbol{x}) \tag{A6}$$

which add up to

$$\begin{aligned}(\hat{M}^\dagger_{s,\psi}\hat{\phi}(t_1,\boldsymbol{x}_1)\hat{M}_{s,\psi})^{(1)} &= 2\int \mathrm{d}t\,\mathrm{d}^d\boldsymbol{x}\,\chi(t)F(\boldsymbol{x})\mathrm{Im}(\langle\psi|s\rangle s\hat{\mu}(t)|\psi\rangle)\hat{\phi}(t,\boldsymbol{x})\hat{\phi}(t_1,\boldsymbol{x}_1)\\ &\quad -\mathrm{i}\int \mathrm{d}t\,\mathrm{d}^d\boldsymbol{x}\,\chi(t)F(\boldsymbol{x})\langle\psi|s\rangle\langle s|\hat{\mu}(t)|\psi\rangle[\hat{\phi}(t_1,\boldsymbol{x}_1),\hat{\phi}(t,\boldsymbol{x})].\end{aligned} \tag{A7}$$

On the other hand, for the second term in Eq. (A2) we have

$$\hat{M}^\dagger_{s,\psi}\hat{M}_{s,\psi} = \hat{M}^{\dagger(0)}_{s,\psi}\hat{M}^{(0)}_{s,\psi} + \lambda(\hat{M}^{\dagger(1)}_{s,\psi}\hat{M}^{(0)}_{s,\psi} + \hat{M}^{\dagger(0)}_{s,\psi}\hat{M}^{(1)}_{s,\psi}) + O(\lambda^2) \tag{A8}$$

with

$$(\hat{M}^\dagger_{s,\psi}\hat{M}_{s,\psi})^{(0)} = \hat{M}^{\dagger(0)}_{s,\psi}\hat{M}^{(0)}_{s,\psi} = |\langle s|\psi\rangle|^2 \tag{A9}$$

and

$$(\hat{M}^\dagger_{s,\psi}\hat{M}_{s,\psi})^{(1)} = 2\int \mathrm{d}t\,\mathrm{d}^d\boldsymbol{x}\,\chi(t)F(\boldsymbol{x})\mathrm{Im}(\langle\psi|s\rangle\langle s|\hat{\mu}(t)|\psi\rangle)\hat{\phi}(t,\boldsymbol{x}). \tag{A10}$$

We are now set to examine $\Delta_1(t_1,\boldsymbol{x}_1)$. At zeroth order it is trivial that there is no difference. Up to first order in λ,

$$
\begin{aligned}
\Delta_1(t_1,\boldsymbol{x}_1) &= \lambda\Delta_1(t_1,\boldsymbol{x}_1)^{(1)} + O(\lambda^2) \\
&= 2\lambda\int \mathrm{d}t\,\mathrm{d}^d\boldsymbol{x}\,\chi(t)F(\boldsymbol{x})\mathrm{Im}(\langle\psi|s\rangle\langle s|\hat{\mu}(t)|\psi\rangle)\langle\hat{\phi}(t,\boldsymbol{x})\hat{\phi}(t_1,\boldsymbol{x}_1)\rangle_{\hat{\rho}_\phi} \\
&\quad - \mathrm{i}\lambda\int \mathrm{d}t\,\mathrm{d}^d\boldsymbol{x}\,\chi(t)F(\boldsymbol{x})\langle\psi|s\rangle\langle s|\hat{\mu}(t)|\psi\rangle\langle[\hat{\phi}(t,\boldsymbol{x}),\hat{\phi}(t_1,\boldsymbol{x}_1)]\rangle_{\hat{\rho}_\phi} \\
&\quad - 2\lambda\int \mathrm{d}t\,\mathrm{d}^d\boldsymbol{x}\,\chi(t)F(\boldsymbol{x})\mathrm{Im}(\langle\psi|s\rangle\langle s|\hat{\mu}(t)|\psi\rangle)\langle\hat{\phi}(t,\boldsymbol{x})\rangle_{\hat{\rho}_\phi}\langle\hat{\phi}(t_1,\boldsymbol{x}_1)\rangle_{\hat{\rho}_\phi} + O(\lambda^2) \\
&= 2\lambda\int \mathrm{d}t\,\mathrm{d}^d\boldsymbol{x}\,\chi(t)F(\boldsymbol{x})\mathrm{Im}(\langle\psi|s\rangle\langle s|\hat{\mu}(t)|\psi\rangle)\mathrm{Cov}_{\hat{\rho}_\phi}[\hat{\phi}(t,\boldsymbol{x}),\hat{\phi}(t_1,\boldsymbol{x}_1)] \\
&\quad - \mathrm{i}\lambda\int \mathrm{d}t\,\mathrm{d}^d\boldsymbol{x}\,\chi(t)F(\boldsymbol{x})\langle\psi|s\rangle\langle s|\hat{\mu}(t)|\psi\rangle\langle[\hat{\phi}(t,\boldsymbol{x}),\hat{\phi}(t_1,\boldsymbol{x}_1)]\rangle_{\hat{\rho}_\phi} + O(\lambda^2)
\end{aligned} \tag{A11}
$$

where $\mathrm{Cov}_{\hat{\rho}}[A,B] = \langle AB\rangle_{\hat{\rho}} - \langle A\rangle_{\hat{\rho}}\langle B\rangle_{\hat{\rho}}$. The term in the last line, depending on the commutator, is definitely *not* contributing to Δ_1 when $(t_1,\boldsymbol{x}_1)$ is out of the causal future of the interaction region. However, the term in the penultimate line *will* contribute in general to Δ_1 not being zero in that same case. Indeed, $\langle\psi|s\rangle\langle s|\hat{\mu}(t)|\psi\rangle$ is not real in general, and the correlations of the field $\mathrm{Cov}_{\hat{\rho}_\phi}[\hat{\phi}(t,\boldsymbol{x}),\hat{\phi}(t_1,\boldsymbol{x}_1)]$ will not vanish in general if $(t,\boldsymbol{x})$ and $(t_1,\boldsymbol{x}_1)$ are spacelike separated. The reason why Δ_1 does not vanish everywhere outside the causal support of the detector is that once the detector starts interacting with the field, it gets entangled with it (in a way that respects causality [24,27]). As the state of the field will in general show spacelike correlations [3,6,13,14,62], the projection operator destroys some of these correlations. The entanglement between the detector and the field generated by their interaction thus hinders the possibility of applying the selective update outside the causal future of the detector in a way consistent with the relativistic framework of QFT. Not even in the—singular but generally less problematic in causality-related issues—case in which the detector is considered to be pointlike and the interaction sudden, that is, with $\chi(t)=\delta(t)$ and $F(\boldsymbol{x})=\delta(\boldsymbol{x})$, is the update safe from being noncausal. Indeed, choosing

$$
|\psi\rangle = \frac{1}{\sqrt{2}}(\mathrm{i}|g\rangle + |e\rangle) \quad |s\rangle = |e\rangle \quad \hat{\rho}_\phi = |0\rangle\langle 0| \tag{A12}
$$

we have $\langle\hat{\phi}(t,\boldsymbol{x})\rangle = 0$ for every $(t,\boldsymbol{x})\in\mathcal{M}$ and therefore

$$
\begin{aligned}
\Delta_1(t_1,\boldsymbol{x}_1) &= \lambda\langle\hat{\phi}(0,0)\hat{\phi}(t_1,\boldsymbol{x}_1)\rangle_{\hat{\rho}_\phi} \\
&\quad + \frac{\lambda}{2}\langle[\hat{\phi}(0,0),\hat{\phi}(t_1,\boldsymbol{x}_1)]\rangle_{\hat{\rho}_\phi} + O(\lambda^2).
\end{aligned} \tag{A13}
$$

For spacelike $(t_1,\boldsymbol{x}_1)$, this gives

$$
\Delta_1(t_1,\boldsymbol{x}_1) = \lambda\langle\hat{\phi}(0,0)\hat{\phi}(t_1,\boldsymbol{x}_1)\rangle_{\hat{\rho}_\phi} + O(\lambda^2) \tag{A14}
$$

which in general does not vanish.

2. The ground and excited states

By examining Eq. (A11) one observes that $\Delta_1^{(1)}$ cancels out of the causal future of the interaction region if both the initial state $|\psi\rangle$ and the state associated with the projection $|s\rangle$ are eigenstates of the free Hamiltonian of the detector, that is, if $|\psi\rangle, |s\rangle \in \{|g\rangle, |e\rangle\}$. These are important states, and one could wonder if for these states the selective update could be safe from showing the noncausal features we saw in the previous subsection. The answer is no, as can be checked by simply analyzing the next order of Δ_1 in perturbation theory. Proceeding as before,

$$
\begin{aligned}
&(\hat{M}^\dagger_{s,\psi}\hat{\phi}(t_1,\boldsymbol{x}_1)\hat{M}_{s,\psi})^{(2)} \\
&\quad = \hat{M}^{\dagger(2)}_{s,\psi}\hat{\phi}(t_1,\boldsymbol{x}_1)\hat{M}^{(0)}_{s,\psi} \\
&\qquad + \hat{M}^{\dagger(0)}_{s,\psi}\hat{\phi}(t_1,\boldsymbol{x}_1)\hat{M}^{(2)}_{s,\psi} + \hat{M}^{\dagger(1)}_{s,\psi}\hat{\phi}(t_1,\boldsymbol{x}_1)\hat{M}^{(1)}_{s,\psi} \\
&\quad = -\int \mathrm{d}t\,\mathrm{d}t'\,\mathrm{d}^d\boldsymbol{x}\,\mathrm{d}^d\boldsymbol{x}'\,\chi(t)\chi(t')F(\boldsymbol{x})F(\boldsymbol{x}')\cdot\mathcal{C}
\end{aligned} \tag{A15}
$$

where

$$
\begin{aligned}
\mathcal{C} &= \theta(t-t') \\
&\quad \times (\langle s|\psi\rangle\langle\psi|\hat{\mu}(t')\hat{\mu}(t)|s\rangle\hat{\phi}(t',\boldsymbol{x}')\hat{\phi}(t,\boldsymbol{x})\hat{\phi}(t_1,\boldsymbol{x}_1) \\
&\quad + \langle\psi|s\rangle\langle s|\hat{\mu}(t)\hat{\mu}(t')|\psi\rangle\hat{\phi}(t_1,\boldsymbol{x}_1)\hat{\phi}(t,\boldsymbol{x})\hat{\phi}(t',\boldsymbol{x}')) \\
&\quad - \langle\psi|\hat{\mu}(t)|s\rangle\langle s|\hat{\mu}(t')|\psi\rangle\hat{\phi}(t,\boldsymbol{x})\hat{\phi}(t_1,\boldsymbol{x}_1)\hat{\phi}(t',\boldsymbol{x}').
\end{aligned} \tag{A16}
$$

Now,

$$
\theta(t-t') + \theta(t'-t) = 1 \tag{A17}
$$

almost everywhere, as the diagonal set $\{t=t'\}\subset\mathbb{R}^2$ in which the equality does not hold has zero Lebesgue measure. Therefore, for a smooth switching function χ or, in general, one switching not involving delta functions, we can write

$$\begin{aligned}\mathcal{C} = {}&\theta(t'-t)\hat{\phi}(t,\boldsymbol{x})(\langle s|\psi\rangle\langle\psi|\hat{\mu}(t)\hat{\mu}(t')|s\rangle\hat{\phi}(t',\boldsymbol{x}')\hat{\phi}(t_1,\boldsymbol{x}_1) - \langle\psi|\hat{\mu}(t)|s\rangle\langle s|\hat{\mu}(t')|\psi\rangle\hat{\phi}(t_1,\boldsymbol{x}_1)\hat{\phi}(t',\boldsymbol{x}')) \\ &+ \theta(t-t')(\langle\psi|s\rangle\langle s|\hat{\mu}(t)\hat{\mu}(t')|\psi\rangle\hat{\phi}(t_1,\boldsymbol{x}_1)\hat{\phi}(t,\boldsymbol{x}) - \langle\psi|\hat{\mu}(t)|s\rangle\langle s|\hat{\mu}(t')|\psi\rangle\hat{\phi}(t,\boldsymbol{x})\hat{\phi}(t_1,\boldsymbol{x}_1))\hat{\phi}(t',\boldsymbol{x}')\end{aligned} \tag{A18}$$

and in particular

$$\begin{aligned}&\langle s|\psi\rangle\langle\psi|\hat{\mu}(t)\hat{\mu}(t')|s\rangle\hat{\phi}(t',\boldsymbol{x}')\hat{\phi}(t_1,\boldsymbol{x}_1) - \langle\psi|\hat{\mu}(t)|s\rangle\langle s|\hat{\mu}(t')|\psi\rangle\hat{\phi}(t_1,\boldsymbol{x}_1)\hat{\phi}(t',\boldsymbol{x}') \\ &\quad= \langle s|\psi\rangle\langle\psi|\hat{\mu}(t)\hat{\mu}(t')|s\rangle[\hat{\phi}(t',\boldsymbol{x}'),\hat{\phi}(t_1,\boldsymbol{x}_1)] + (\langle s|\psi\rangle\langle\psi|\hat{\mu}(t)\hat{\mu}(t')|s\rangle - \langle\psi|\hat{\mu}(t)|s\rangle\langle s|\hat{\mu}(t')|\psi\rangle)\hat{\phi}(t_1,\boldsymbol{x}_1)\hat{\phi}(t',\boldsymbol{x}')\end{aligned} \tag{A19}$$

and

$$\begin{aligned}&\langle\psi|s\rangle\langle s|\hat{\mu}(t)\hat{\mu}(t')|\psi\rangle\hat{\phi}(t_1,\boldsymbol{x}_1)\hat{\phi}(t,\boldsymbol{x}) - \langle\psi|\hat{\mu}(t)|s\rangle\langle s|\hat{\mu}(t')|\psi\rangle\hat{\phi}(t,\boldsymbol{x})\hat{\phi}(t_1,\boldsymbol{x}_1) \\ &\quad= \langle\psi|s\rangle\langle s|\hat{\mu}(t)\hat{\mu}(t')|\psi\rangle[\hat{\phi}(t_1,\boldsymbol{x}_1),\hat{\phi}(t,\boldsymbol{x})] + (\langle\psi|s\rangle\langle s|\hat{\mu}(t)\hat{\mu}(t')|\psi\rangle - \langle\psi|\hat{\mu}(t)|s\rangle\langle s|\hat{\mu}(t')|\psi\rangle)\hat{\phi}(t,\boldsymbol{x})\hat{\phi}(t_1,\boldsymbol{x}_1).\end{aligned} \tag{A20}$$

Now, since the factors accompanying $\mathcal{C}$ in the integral of Eq. (A15) are symmetric in t and t', we can safely exchange both time parameters in the $\theta(t'-t)$ term, hence rewriting

$$\begin{aligned}\mathcal{C} = \theta(t-t')\Big[&\langle s|\psi\rangle\langle\psi|\hat{\mu}(t')\hat{\mu}(t)|s\rangle\hat{\phi}(t',\boldsymbol{x}')[\hat{\phi}(t,\boldsymbol{x}),\hat{\phi}(t_1,\boldsymbol{x}_1)] + (\langle s|\psi\rangle\langle\psi|\hat{\mu}(t')\hat{\mu}(t)|s\rangle \\ &- \langle\psi|\hat{\mu}(t')|s\rangle\langle s|\hat{\mu}(t)|\psi\rangle)\hat{\phi}(t',\boldsymbol{x}')\hat{\phi}(t_1,\boldsymbol{x}_1)\hat{\phi}(t,\boldsymbol{x}) + \langle\psi|s\rangle\langle s|\hat{\mu}(t)\hat{\mu}(t')|\psi\rangle[\hat{\phi}(t_1,\boldsymbol{x}_1),\hat{\phi}(t,\boldsymbol{x})]\hat{\phi}(t',\boldsymbol{x}') \\ &+ (\langle\psi|s\rangle\langle s|\hat{\mu}(t)\hat{\mu}(t')|\psi\rangle - \langle\psi|\hat{\mu}(t)|s\rangle\langle s|\hat{\mu}(t')|\psi\rangle)\hat{\phi}(t,\boldsymbol{x})\hat{\phi}(t_1,\boldsymbol{x}_1)\hat{\phi}(t',\boldsymbol{x}')\Big].\end{aligned} \tag{A21}$$

We still have to compute

$$(\hat{M}^\dagger_{s,\psi}\hat{M}_{s,\psi})^{(2)} = \hat{M}^{\dagger(2)}_{s,\psi}\hat{M}^{(0)}_{s,\psi} + \hat{M}^{\dagger(0)}_{s,\psi}\hat{M}^{(2)}_{s,\psi} + \hat{M}^{\dagger(1)}_{s,\psi}\hat{M}^{(1)}_{s,\psi}. \tag{A22}$$

We proceed exactly as above, the difference being that we do not have the field operator $\hat{\phi}(t_1,\boldsymbol{x}_1)$ in between anymore. In the same spirit, we get

$$(\hat{M}^\dagger_{s,\psi}\hat{M}_{s,\psi})^{(2)} = -\int \mathrm{d}t\,\mathrm{d}t'\,\mathrm{d}^d\boldsymbol{x}\,\mathrm{d}^d\boldsymbol{x}'\,\chi(t)\chi(t')F(\boldsymbol{x})F(\boldsymbol{x}')\cdot\mathcal{I} \tag{A23}$$

where

$$\begin{aligned}\mathcal{I} = \theta(t-t')\Big[&(((\langle s|\psi\rangle\psi\hat{\mu}(t')\hat{\mu}(t)|s\rangle - \langle\psi|\hat{\mu}(t')|s\rangle\langle s|\hat{\mu}(t)|\psi\rangle)\hat{\phi}(t',\boldsymbol{x}')\hat{\phi}(t,\boldsymbol{x}) \\ &+ (\langle\psi|s\rangle\langle s|\hat{\mu}(t)\hat{\mu}(t')|\psi\rangle - \langle\psi|\hat{\mu}(t)|s\rangle\langle s|\hat{\mu}(t')|\psi\rangle)\hat{\phi}(t,\boldsymbol{x})\hat{\phi}(t',\boldsymbol{x}')\Big].\end{aligned} \tag{A24}$$

We realize that the terms that are not included in the commutators in Eq. (A21) are conjugate to each other, and that the same happens with the terms in Eq. (A24). Putting everything together, we get

$$\Delta_1^{(2)} = -\int \mathrm{d}t\,\mathrm{d}t'\,\mathrm{d}^d\boldsymbol{x}\,\mathrm{d}^d\boldsymbol{x}'\,\chi(t)\chi(t')F(\boldsymbol{x})F(\boldsymbol{x}')\theta(t-t')\cdot\mathcal{R} \tag{A25}$$

where

$$\begin{aligned}\mathcal{R} = {}&\langle s|\psi\rangle\langle\psi|\hat{\mu}(t')\hat{\mu}(t)|s\rangle\langle\hat{\phi}(t',\boldsymbol{x}')[\hat{\phi}(t,\boldsymbol{x}),\hat{\phi}(t_1,\boldsymbol{x}_1)]\rangle_{\hat{\rho}_\phi} + \langle\psi|s\rangle\langle s|\hat{\mu}(t)\hat{\mu}(t')|\psi\rangle\langle[\hat{\phi}(t_1,\boldsymbol{x}_1),\hat{\phi}(t,\boldsymbol{x})]\hat{\phi}(t',\boldsymbol{x}')\rangle_{\hat{\rho}_\phi} \\ &+ 2\mathrm{Re}(\mathcal{S})\end{aligned} \tag{A26}$$

and

$$\begin{aligned}\mathcal{S} = (&\langle\psi|s\rangle\langle s|\hat{\mu}(t)\hat{\mu}(t')|\psi\rangle \\ &- \langle\psi|\hat{\mu}(t)|s\rangle\langle s|\hat{\mu}(t')|\psi\rangle) \\ &\times (\langle\hat{\phi}(t,\boldsymbol{x})\hat{\phi}(t_1,\boldsymbol{x}_1)\hat{\phi}(t',\boldsymbol{x}')\rangle_{\hat{\rho}_\phi} \\ &- \langle\hat{\phi}(t,\boldsymbol{x})\hat{\phi}(t',\boldsymbol{x}')\rangle_{\hat{\rho}_\phi}\langle\hat{\phi}(t_1,\boldsymbol{x}_1)\rangle_{\hat{\rho}_\phi}). \end{aligned} \tag{A27}$$

Now, the first two terms of $\mathcal{R}$ are proportional to commutators, so that if $(t_1,\boldsymbol{x}_1)$ is spacelike separated from the interaction region, they become zero. However, $\mathcal{S}$ is not zero in general in that case, nor purely imaginary. In fact, this is the term that allows us to confirm that even for the ground and excited states the selective update should not be applied outside the causal future of the detector. For example, consider the case in which the field is initially in a coherent state $\hat{\rho}_\phi = |\varphi\rangle\langle\varphi|$. For these states, the correlation functions of one, two and three points are

$$\langle\hat{\phi}(\mathsf{x})\rangle_{\hat{\rho}_\phi} = \varphi(\mathsf{x}), \tag{A28}$$

$$\langle\hat{\phi}(\mathsf{x})\hat{\phi}(\mathsf{y})\rangle_{\hat{\rho}_\phi} = \varphi(\mathsf{x})\varphi(\mathsf{y}) + w_{\mathrm{vac}}(\mathsf{x},\mathsf{y}), \tag{A29}$$

$$\begin{aligned}\langle\hat{\phi}(\mathsf{x})\hat{\phi}(\mathsf{y})\hat{\phi}(\mathsf{z})\rangle_{\hat{\rho}_\phi} &= \varphi(\mathsf{x})\varphi(\mathsf{y})\varphi(\mathsf{z}) + \varphi(\mathsf{x})w_{\mathrm{vac}}(\mathsf{y},\mathsf{z}) \\ &\quad + \varphi(\mathsf{y})w_{\mathrm{vac}}(\mathsf{x},\mathsf{z}) + \varphi(\mathsf{z})w_{\mathrm{vac}}(\mathsf{x},\mathsf{y}),\end{aligned} \tag{A30}$$

where $\mathsf{x},\mathsf{y},\mathsf{z}\in\mathcal{M}$, $\varphi(\mathsf{x})$ is the field amplitude of the coherent state at x and

$$w_{\mathrm{vac}}(\mathsf{x},\mathsf{y}) = \langle 0|\hat{\phi}(\mathsf{x})\hat{\phi}(\mathsf{y})|0\rangle \tag{A31}$$

is the two-point Wightman function. If we additionally consider $|\psi\rangle = |g\rangle$ and $|s\rangle = |e\rangle$, then

$$\begin{aligned}&\langle\psi|s\rangle\langle s|\mu(t)\mu(t')|\psi\rangle - \langle\psi|\mu(t)|s\rangle\langle s|\mu(t')|\psi\rangle \\ &\quad = -e^{-\mathrm{i}\Omega(t-t')}\end{aligned} \tag{A32}$$

and

$$\begin{aligned}&\langle\phi(t,\boldsymbol{x})\hat{\phi}(t_1,\boldsymbol{x}_1)\phi(t',\boldsymbol{x}')\rangle_{\hat{\rho}_\phi} - \langle\phi(t,\boldsymbol{x})\phi(t',\boldsymbol{x}')\rangle\langle\hat{\phi}(t_1,\boldsymbol{x}_1)\rangle_{\hat{\rho}_\phi} \\ &\quad = \varphi(t,\boldsymbol{x})w_{\mathrm{vac}}(t_1,\boldsymbol{x}_1,t',\boldsymbol{x}') + \varphi(t',\boldsymbol{x}')w_{\mathrm{vac}}(t,\boldsymbol{x},t_1,\boldsymbol{x}_1).\end{aligned} \tag{A33}$$

The product of both terms is not always purely imaginary, as the sum of it and its conjugate is nonzero in general. We conclude that, under these conditions, $\Delta_1^{(2)}(t_1,\boldsymbol{x}_1)$ does not cancel out for every $(t_1,\boldsymbol{x}_1)$ spacelike separated from the interaction region.

3. Gaussian states

Observation of Eq. (A27) reveals that still, if the initial state of the field is the vacuum, or a thermal state, the selective update does not affect the one-point function outside the causal support of the detector. What these states share that makes $\mathcal{S}$ in Eq. (A27) cancel out are its vanishing one-point and three-point functions. In particular, we can prove the following:

In the previous setting, if $|s\rangle$ and $|\psi\rangle$ are in $\{|g\rangle,|e\rangle\}$ and $\hat{\rho}_\phi$ is a Gaussian state with $\langle\hat{\phi}(\mathsf{x})\rangle_{\hat{\rho}_\phi} = 0$ for every $\mathsf{x}\in\mathcal{M}$, then the selective POVM update does not affect the one-point function outside the causal future of the interaction region.

To prove this claim, we proceed simply by examination of the general term

$$\begin{aligned}\Delta_1^{(p,q)} \coloneqq &\langle\hat{M}^{\dagger(p)}\hat{\phi}(t_1,\boldsymbol{x}_1)\hat{M}^{(q)}_{s,\psi}\rangle_{\hat{\rho}_\phi} \\ &- \langle\hat{M}^{\dagger(p)}_{s,\psi}\hat{M}^{(q)}_{s,\psi}\rangle\langle\hat{\phi}(t_1,\boldsymbol{x}_1)\rangle_{\hat{\rho}_\phi},\end{aligned} \tag{A34}$$

where the second term vanishes because of the assumption $\langle\hat{\phi}(\mathsf{x})\rangle_{\hat{\rho}_\phi} = 0$. The exact same kind of calculation carried out before yields

$$\begin{aligned}\Delta_1^{(p,q)} = \mathrm{i}^{p+3q}\int &\mathrm{d}\mathsf{t}_1\cdots\mathrm{d}\mathsf{t}_p\mathrm{d}\mathsf{t}'_1\cdots\mathrm{d}\mathsf{t}'_q\mathrm{d}^d z_1\cdots\mathrm{d}^d z_p\mathrm{d}^d z'_1\cdots\mathrm{d}^d z'_q\theta(\mathsf{t}_1-\mathsf{t}_2)\cdots\theta(\mathsf{t}_{p-1}-\mathsf{t}_p) \\ &\times\theta(\mathsf{t}'_1-\mathsf{t}'_2)\cdots\theta(\mathsf{t}_{q-1}'-\mathsf{t}_q)\chi(\mathsf{t}_1)\cdots\chi(\mathsf{t}'_q) \\ &\times F(z_1)\cdots F(z'_q)\langle\psi|\hat{\mu}(\mathsf{t}_p)\cdots\hat{\mu}(\mathsf{t}_1)|s\rangle\langle s|\hat{\mu}(\mathsf{t}'_1)\cdots\hat{\mu}(\mathsf{t}'_q)|\psi\rangle\langle\hat{\phi}(\mathsf{t}_p,z_p)\cdots\hat{\phi}(\mathsf{t}_1,z_1) \\ &\times\hat{\phi}(t_1,\boldsymbol{x}_1)\hat{\phi}(\mathsf{t}'_1,z'_1)\cdots\hat{\phi}(\mathsf{t}_q,z_q)\rangle_{\hat{\rho}_\phi}.\end{aligned} \tag{A35}$$

In order to analyze this expression, we first calculate the general form of the operator $\hat{\mu}(\mathsf{t}_1)\cdots\hat{\mu}(\mathsf{t}_N)$: let us define

$$T \equiv \sum_{n=1}^{N}(-1)^{n-1}\mathsf{t}_n; \tag{A36}$$

then if N is odd, in the ordered basis $\{|g\rangle,|e\rangle\}$,

$$\hat{\mu}(\mathsf{t}_1)\cdots\hat{\mu}(\mathsf{t}_N) = \begin{pmatrix} 0 & e^{-\mathrm{i}\Omega T} \\ e^{\mathrm{i}\Omega T} & 0\end{pmatrix} \tag{A37}$$

while if N is even,

$$\hat{\mu}(\mathsf{t}_1)\cdots\hat{\mu}(\mathsf{t}_N) = \begin{pmatrix} e^{-\mathrm{i}\Omega T} & 0 \\ 0 & e^{\mathrm{i}\Omega T} \end{pmatrix}. \tag{A38}$$

Thus, if $|s\rangle$ and $|\psi\rangle$ are in $\{|g\rangle, |e\rangle\}$, the term $\langle\psi|\hat{\mu}(\mathsf{t}_p)\cdots\hat{\mu}(\mathsf{t}_1)|s\rangle\langle s|\hat{\mu}(\mathsf{t}'_1)\cdots\hat{\mu}(\mathsf{t}'_q)|\psi\rangle$ in Eq. (A35) only survives if

(1) $|s\rangle \neq |\psi\rangle$ and both p and q are odd, or.

(2) $|s\rangle = |\psi\rangle$ and both p and q are even.

Now, the last factor of the integral in Eq. (A35) is the $(p+q+1)$-point correlation function

$$\langle\phi(\mathsf{t}_p, z_p)\cdots\phi(\mathsf{t}_1, z_1)\hat{\phi}(t_1, \boldsymbol{x}_1)\phi(\mathsf{t}'_1, z'_1)\cdots\phi(\mathsf{t}_q, z_q)\rangle_{\hat{\rho}_\phi}. \tag{A39}$$

But for both cases 1) and 2), the parities of p and q are the same, so that $p+q+1$ is odd, and therefore the correlation function above is zero, since the state $\hat{\rho}_\phi$ is Gaussian [52,53]. We conclude that under the stated hypotheses,

$$\Delta_1^{(p,q)}(t_1, \boldsymbol{x}_1) = 0 \tag{A40}$$

for every (p,q) and therefore

$$\Delta_1(t_1, \boldsymbol{x}_1) = 0 \tag{A41}$$

for *every* point $(t_1, \boldsymbol{x}_1)$.

Observe that rather than showing that the conditions of the claim guarantee that the selective update does not affect the one-point function outside the causal support of the interaction region, we have proven that it does not affect it *at all*. As a consequence, to show that the selective update alters the state of the field outside the causal future of the detector under the hypotheses of the claim's statement, we need to consider the two-point function of the updated state.

First, we have by Eq. (A2) for $n=2$ that

$$\begin{aligned}\Delta_2(\mathsf{x}_1, \mathsf{x}_2) &= \langle\hat{M}^\dagger_{s,\psi}\hat{\phi}(\mathsf{x}_1)\hat{\phi}(\mathsf{x}_2)\hat{M}_{s,\psi}\rangle_{\hat{\rho}_\phi} \\ &\quad - \langle\hat{M}^\dagger_{s,\psi}\hat{M}_{s,\psi}\rangle_{\hat{\rho}_\phi}\langle\hat{\phi}(\mathsf{x}_1)\hat{\phi}(\mathsf{x}_2)\rangle_{\hat{\rho}_\phi}.\end{aligned} \tag{A42}$$

Following the same calculations as in Appendix A 1, but with two field operators instead of one, it is straightforward to see that, up to second order in λ,

$$\begin{aligned}\Delta_2(t_1, \boldsymbol{x}_1, t_2, \boldsymbol{x}_2) &= 2\lambda\int \mathrm{d}t\,\mathrm{d}^d\boldsymbol{x}\,\chi(t)F(\boldsymbol{x})\mathrm{Im}(\langle\psi|s\rangle\langle s|\hat{\mu}(t)|\psi\rangle)\mathrm{Cov}_{\hat{\rho}_\phi}[\hat{\phi}(t,\boldsymbol{x}), \hat{\phi}(t_1,\boldsymbol{x}_1)\hat{\phi}(t_2,\boldsymbol{x}_2)] \\ &\quad - \mathrm{i}\lambda\int \mathrm{d}t\,\mathrm{d}^d\boldsymbol{x}\,\chi(t)F(\boldsymbol{x})\langle\psi|s\rangle\langle s|\hat{\mu}(t)|\psi\rangle\langle[\hat{\phi}(t,\boldsymbol{x}), \hat{\phi}(t_1,\boldsymbol{x}_1)\hat{\phi}(t_2,\boldsymbol{x}_2)]\rangle_{\hat{\rho}_\phi} \\ &\quad - \lambda^2\int \mathrm{d}t\,\mathrm{d}t'\,\mathrm{d}^d\boldsymbol{x}\,\mathrm{d}^d\boldsymbol{x}'\,\chi(t)\chi(t')F(\boldsymbol{x})F(\boldsymbol{x}')\theta(t-t')\cdot\mathcal{R}_2 + O(\lambda^3)\end{aligned} \tag{A43}$$

where

$$\begin{aligned}\mathcal{R}_2 &= \langle s|\psi\rangle\langle\psi|\hat{\mu}(t')\hat{\mu}(t)|s\rangle \\ &\quad \times\langle\hat{\phi}(t',\boldsymbol{x}')[\hat{\phi}(t,\boldsymbol{x}), \hat{\phi}(t_1,\boldsymbol{x}_1)\hat{\phi}(t_2,\boldsymbol{x}_2]\rangle_{\hat{\rho}_\phi} \\ &\quad + \langle\psi|s\rangle\langle s|\hat{\mu}(t)\hat{\mu}(t')|\psi\rangle \\ &\quad \times\langle[\hat{\phi}(t_1,\boldsymbol{x}_1)\hat{\phi}(t_2,\boldsymbol{x}_2, \hat{\phi}(t,\boldsymbol{x})]\hat{\phi}(t',\boldsymbol{x}')\rangle_{\hat{\rho}_\phi} \\ &\quad + 2\mathrm{Re}(\mathcal{S}_2)\end{aligned} \tag{A44}$$

with

$$\begin{aligned}\mathcal{S}_2 &= (\langle\psi|s\rangle\langle s|\hat{\mu}(t)\hat{\mu}(t')|\psi\rangle \\ &\quad - \langle\psi|\hat{\mu}(t)|s\rangle\langle s|\hat{\mu}(t')|\psi\rangle) \\ &\quad \times(\langle\hat{\phi}(t,\boldsymbol{x})\hat{\phi}(t_1,\boldsymbol{x}_1)\hat{\phi}(t_2,\boldsymbol{x}_2)\hat{\phi}(t',\boldsymbol{x}')\rangle_{\hat{\rho}_\phi} \\ &\quad - \langle\hat{\phi}(t,\boldsymbol{x})\hat{\phi}(t',\boldsymbol{x}')\rangle_{\hat{\rho}_\phi}\langle\hat{\phi}(t_1,\boldsymbol{x}_1)\hat{\phi}(t_2,\boldsymbol{x}_2)\rangle_{\hat{\rho}_\phi}).\end{aligned} \tag{A45}$$

When we consider $|s\rangle, |\psi\rangle \in \{|g\rangle, |e\rangle\}$, the first order in λ becomes zero. Moreover, when $(t_1, \boldsymbol{x}_1)$ and $(t_2, \boldsymbol{x}_2)$ are spacelike separated from the detector, the first two terms of $\mathcal{R}_2$, that depend on commutators, are zero, and only the real part of $\mathcal{S}_2$ remains. When $|s\rangle$ and $|\psi\rangle$ are eigenstates of the detector's Hamiltonian, then it can be checked that the first factor in $\mathcal{S}_2$ is $\pm e^{\pm\mathrm{i}\Omega(t-t')}$, where the signs depend on whether we take the ground or the excited state for each of $|s\rangle$ and $|\psi\rangle$. For the second factor, because the field state is Gaussian, it holds that

$$\begin{aligned}&w_4(\mathsf{x}, \mathsf{x}_1, \mathsf{x}_2, \mathsf{x}') - w_2(\mathsf{x}, \mathsf{x}')w_2(\mathsf{x}_1, \mathsf{x}_2) \\ &\quad = w_2(\mathsf{x}, \mathsf{x}_1)w_2(\mathsf{x}_2, \mathsf{x}) + w_2(\mathsf{x}, \mathsf{x}_2)w_2(\mathsf{x}_1, \mathsf{x}').\end{aligned} \tag{A46}$$

This makes apparent that if we exchange t and t', the modified $\mathcal{S}_2$ is the complex conjugate of the original. Taking advantage of the fact that only the real part of $\mathcal{S}_2$ contributes, and proceeding as in Appendix A 2 to get rid of the Heaviside step functions θ, we conclude that under the conditions of the claim above,

$$\Delta_2(t_1,\boldsymbol{x}_1,t_2,\boldsymbol{x}_2) = \pm\lambda^2\int \mathrm{d}t\,\mathrm{d}t'\,\mathrm{d}^d\boldsymbol{x}\,\mathrm{d}^d\boldsymbol{x}'\,\chi(t)\chi(t')F(\boldsymbol{x})F(\boldsymbol{x}')e^{\pm \mathrm{i}\Omega(t-t')} \times (w_2(t,\boldsymbol{x},t_1,\boldsymbol{x}_1)w_2(t_2,\boldsymbol{x}_2,t',\boldsymbol{x}') + w_2(t,\boldsymbol{x},t_2,\boldsymbol{x}_2)w_2(t_1,\boldsymbol{x}_1,t',\boldsymbol{x}')) + O(\lambda^3), \tag{A47}$$

which as a distribution is nonzero in general. Consider for example the case in which the initial state of the field is the vacuum. In that case the last factor of the integrand involving the two-point functions is

$$\lambda^2\int\frac{\mathrm{d}^d\boldsymbol{k}}{2(2\pi)^d\omega_{\boldsymbol{k}}}\int\frac{\mathrm{d}^d\boldsymbol{k}'}{2(2\pi)^d\omega_{\boldsymbol{k}'}}(e^{-\mathrm{i}[\omega_{\boldsymbol{k}}(t-t_1)-\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{x}_1)]} \times e^{-\mathrm{i}[\omega_{\boldsymbol{k}'}(t_2-t')-\boldsymbol{k}'\cdot(\boldsymbol{x}_2-\boldsymbol{x}')]} + e^{-\mathrm{i}[\omega_{\boldsymbol{k}}(t-t_2)-\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{x}_2)]} \times e^{-\mathrm{i}[\omega_{\boldsymbol{k}'}(t_1-t')-\boldsymbol{k}'\cdot(\boldsymbol{x}_1-\boldsymbol{x}')]}). \tag{A48}$$

Particularizing for $|\psi\rangle = |g\rangle$ and $|s\rangle = |e\rangle$ and using Eq. (A47), we get that if $(t_1,\boldsymbol{x}_1)$ and $(t_2,\boldsymbol{x}_2)$ are spacelike separated from the detector,

$$\Delta_2(t_1,\boldsymbol{x}_1,t_2,\boldsymbol{x}_2) = \frac{\lambda^2}{4(2\pi)^{d-1}}\int\frac{\mathrm{d}^d\boldsymbol{k}}{\omega_{\boldsymbol{k}}}\int\frac{\mathrm{d}^d\boldsymbol{k}'}{\omega_{\boldsymbol{k}'}} \times \tilde{\chi}(\omega_{\boldsymbol{k}}+\Omega)\tilde{\chi}(\omega_{\boldsymbol{k}'}+\Omega)^*\tilde{F}(\boldsymbol{k})^*\tilde{F}(\boldsymbol{k}') \times (e^{\mathrm{i}(\omega_{\boldsymbol{k}}t_1-\omega_{\boldsymbol{k}'}t_2-\boldsymbol{k}\cdot\boldsymbol{x}_1+\boldsymbol{k}'\cdot\boldsymbol{x}_2)} + e^{-\mathrm{i}(\omega_{\boldsymbol{k}}t_2-\omega_{\boldsymbol{k}'}t_1-\boldsymbol{k}\cdot\boldsymbol{x}_2+\boldsymbol{k}'\cdot\boldsymbol{x}_1)}) + O(\lambda^3). \tag{A49}$$

This expression does not cancel out in general, as can be checked considering for example Gaussian smearings and switchings.

With these calculations we have discarded the last of the cases that remained open to the possibility of applying the selective update globally in a way compatible with causality. We thus conclude that the selective update cannot be applied outside the causal future of the interaction region if we want the update to be consistent with the relativistic nature of QFT.

APPENDIX B: UPDATE RULES FOR *n*-POINT FUNCTIONS

In this appendix we give some of the details behind the perturbative results of Secs. VI A and VI B, where we formulated the update rule for the n-point functions explicitly in terms of the initial n-point functions to first order in λ. We will reuse some of the calculations already performed in Appendix A.

1. Nonselective case

After a nonselective measurement, we consider the update w_n^{NS} given in Eq. (38) for the n-point function. By Eq. (A4), we have that

$$(\hat{M}^\dagger_{l,\psi}\hat{\phi}(t_1,\boldsymbol{x}_1)\cdots\hat{\phi}(t_n,\boldsymbol{x}_n)\hat{M}_{l,\psi})^{(0)} = \langle\psi|l\rangle\langle l|\psi\rangle\hat{\phi}(t_1,\boldsymbol{x}_1)\cdots\hat{\phi}(t_n,\boldsymbol{x}_n) \tag{B1}$$

for $l = s,\bar{s}$. Since $|s\rangle\langle s| + |\bar{s}\rangle\langle\bar{s}| = \mathbb{1}_{\mathrm{d}}$ and $|\psi\rangle$ is normalized, the zeroth order of w_n^{NS} is

$$w_n^{\mathrm{NS}}(t_1,\boldsymbol{x}_1,\ldots,t_n,\boldsymbol{x}_n)^{(0)} = \langle\hat{\phi}(t_1,\boldsymbol{x}_1)\cdots\hat{\phi}(t_n,\boldsymbol{x}_n)\rangle_{\hat{\rho}_\phi} = w_n(t_1,\boldsymbol{x}_1,\ldots,t_n,\boldsymbol{x}_n). \tag{B2}$$

For the first order, by Eq. (A5) we have

$$\hat{M}^{\dagger(1)}_{l,\psi}\hat{\phi}(t_1,\boldsymbol{x}_1)\cdots\hat{\phi}(t_n,\boldsymbol{x}_n)\hat{M}^{(0)}_{l,\psi} = \mathrm{i}\langle\psi|l\rangle^*\int \mathrm{d}t\,\mathrm{d}^d\boldsymbol{x}\,\chi(t)F(\boldsymbol{x})\langle l|\hat{\mu}(t)|\psi\rangle^* \times \hat{\phi}(t,\boldsymbol{x})\hat{\phi}(t_1,\boldsymbol{x}_1)\cdots\hat{\phi}(t_n,\boldsymbol{x}_n) \tag{B3}$$

and by Eq. (A6)

$$\hat{M}^{\dagger(0)}_{l,\psi}\hat{\phi}(t_1,\boldsymbol{x}_1)\cdots\hat{\phi}(t_n,\boldsymbol{x}_n)\hat{M}^{(1)}_{l,\psi} = -\mathrm{i}\langle\psi|l\rangle\int \mathrm{d}t\,\mathrm{d}^d\boldsymbol{x}\,\chi(t)F(\boldsymbol{x})\langle l|\hat{\mu}\rangle(t)|\psi\rangle \times \hat{\phi}(t_1,\boldsymbol{x}_1)\cdots\hat{\phi}(t_n,\boldsymbol{x}_n)\hat{\phi}(t,\boldsymbol{x}), \tag{B4}$$

for $l = s,\bar{s}$. Taking the expectation values and taking into account again that $\{|s\rangle,|\bar{s}\rangle\}$ form an orthonormal basis of the Hilbert space of the detector,

$$w_n^{\mathrm{NS}}(t_1,\boldsymbol{x}_1,\ldots,t_n,\boldsymbol{x}_n)^{(1)} = \mathrm{i}\int \mathrm{d}t\,\mathrm{d}^d\boldsymbol{x}\,\chi(t)F(\boldsymbol{x})\langle\psi|\hat{\mu}(t)|\psi\rangle \times (\langle\hat{\phi}(t,\boldsymbol{x})\hat{\phi}(t_1,\boldsymbol{x}_1)\cdots\hat{\phi}(t_n,\boldsymbol{x}_n)\rangle_{\hat{\rho}_\phi} - \langle\hat{\phi}(t_1,\boldsymbol{x}_1)\cdots\hat{\phi}(t_n,\boldsymbol{x}_n)\hat{\phi}(t,\boldsymbol{x})\rangle_{\hat{\rho}_\phi}). \tag{B5}$$

This equation, along with the zeroth order and the definition of n-point functions, yields Eq. (41). The particularization to $n=2$ leads immediately to Eq. (40). For $n=1$, we can use the property

$$w_2(t,\boldsymbol{x},t_1,\boldsymbol{x}_1)^* = w_2(t_1,\boldsymbol{x}_1,t,\boldsymbol{x}). \tag{B6}$$

Thence,

$$\mathrm{i}w_2(t,\boldsymbol{x},t_1,\boldsymbol{x}_1) - \mathrm{i}w_2(t_1,\boldsymbol{x}_1,t,\boldsymbol{x}) = \mathrm{Im}[w_2(t_1,\boldsymbol{x}_1,t,\boldsymbol{x})], \tag{B7}$$

which yields Eq. (39).

In a completely analogous way, we can perform the somewhat more tedious calculations leading to the expression for w_n^{NS} to second order in λ. Using the expansion in Eq. (A15) and Eqs. (20), (21) and (22),

$$\hat{M}^{\dagger(2)}_{l,\psi}\hat{\phi}(t_1,\boldsymbol{x}_1)\cdots\hat{\phi}(t_n,\boldsymbol{x}_n)\hat{M}^{(0)}_{l,\psi} = -\langle\psi|l\rangle^* \int \mathrm{d}t\,\mathrm{d}t'\,\mathrm{d}^d\boldsymbol{x}\,\mathrm{d}^d\boldsymbol{x}'\,\theta(t-t')\chi(t)\chi(t')F(\boldsymbol{x})F(\boldsymbol{x}')\langle s|\hat{\mu}(t)\hat{\mu}(t')|\psi\rangle^* \times\hat{\phi}(t',\boldsymbol{x}')\hat{\phi}(t,\boldsymbol{x})\hat{\phi}(t_1,\boldsymbol{x}_1)\cdots\hat{\phi}(t_n,\boldsymbol{x}_n), \tag{B8}$$

$$\hat{M}^{\dagger(0)}_{l,\psi}\hat{\phi}(t_1,\boldsymbol{x}_1)\cdots\hat{\phi}(t_n,\boldsymbol{x}_n)\hat{M}^{(2)}_{l,\psi} = -\langle\psi|l\rangle \int \mathrm{d}t\,\mathrm{d}t'\,\mathrm{d}^d\boldsymbol{x}\,\mathrm{d}^d\boldsymbol{x}'\,\theta(t-t')\chi(t)\chi(t')F(\boldsymbol{x})F(\boldsymbol{x}')\langle s|\hat{\mu}(t)\hat{\mu}(t')|\psi\rangle \times\hat{\phi}(t_1,\boldsymbol{x}_1)\cdots\hat{\phi}(t_n,\boldsymbol{x}_n)\hat{\phi}(t,\boldsymbol{x})\hat{\phi}(t',\boldsymbol{x}'), \tag{B9}$$

$$\hat{M}^{\dagger(1)}_{l,\psi}\hat{\phi}(t_1,\boldsymbol{x}_1)\cdots\hat{\phi}(t_n,\boldsymbol{x}_n)\hat{M}^{(1)}_{l,\psi} = \langle\psi|l\rangle \int \mathrm{d}t\,\mathrm{d}t'\,\mathrm{d}^d\boldsymbol{x}\,\mathrm{d}^d\boldsymbol{x}'\,\chi(t)\chi(t')F(\boldsymbol{x})F(\boldsymbol{x}')\langle\psi|\hat{\mu}(t)|s\rangle\langle s|\hat{\mu}(t')|\psi\rangle \times\hat{\phi}(t,\boldsymbol{x})\hat{\phi}(t_1,\boldsymbol{x}_1)\cdots\hat{\phi}(t_n,\boldsymbol{x}_n)\hat{\phi}(t',\boldsymbol{x}'), \tag{B10}$$

for $l=s,\bar{s}$. Taking again the expectation values, we arrive at the second-order contribution to w_n^{NS},

$$\begin{aligned} w_n^{\mathrm{NS}}(t_1,\boldsymbol{x}_1,\ldots,t_n,\boldsymbol{x}_n)^{(2)} = -\int \mathrm{d}t\,\mathrm{d}t'\,\mathrm{d}^d\boldsymbol{x}\,\mathrm{d}^d\boldsymbol{x}'\,\chi(t)\chi(t')F(\boldsymbol{x})F(\boldsymbol{x}')\Big[&\theta(t-t')(\langle\psi|\hat{\mu}(t')\hat{\mu}(t)|\psi\rangle w_{n+2}(t',\boldsymbol{x}',t,\boldsymbol{x},t_1,\boldsymbol{x}_1,\ldots,t_n,\boldsymbol{x}_n)\\ &+\langle\psi|\hat{\mu}(t')\hat{\mu}(t)|\psi\rangle w_{n+2}(t_1,\boldsymbol{x}_1,\ldots,t_n,\boldsymbol{x}_n,t,\boldsymbol{x},t',\boldsymbol{x}'))\\ &-\langle\psi|\hat{\mu}(t)\hat{\mu}(t')|\psi\rangle w_{n+2}(t,\boldsymbol{x},t_1,\boldsymbol{x}_1,\ldots,t_n,\boldsymbol{x}_n,t',\boldsymbol{x}')\Big]. \end{aligned} \tag{B11}$$

All together,

$$\begin{aligned} w_n^{\mathrm{NS}}(t_1,\boldsymbol{x}_1,\ldots,t_n,\boldsymbol{x}_n) = w_n(t_1,\boldsymbol{x}_1,\ldots,t_n,\boldsymbol{x}_n) + \mathrm{i}\lambda\int &\mathrm{d}t\,\mathrm{d}^d\boldsymbol{x}\,\chi(t)F(\boldsymbol{x})\langle\psi|\hat{\mu}(t)|\psi\rangle\\ &\times(w_{n+1}(t,\boldsymbol{x},t_1,\boldsymbol{x}_1,\ldots,t_n,\boldsymbol{x}_n) - w_{n+1}(t_1,\boldsymbol{x}_1,\ldots,t_n,\boldsymbol{x}_n,t,\boldsymbol{x}))\\ &-\lambda^2\int \mathrm{d}t\,\mathrm{d}t'\,\mathrm{d}^d\boldsymbol{x}\,\mathrm{d}^d\boldsymbol{x}'\,\chi(t)\chi(t')F(\boldsymbol{x})F(\boldsymbol{x}')\\ &\times\Big[\theta(t-t')(\langle\psi|\hat{\mu}(t')\hat{\mu}(t)|\psi\rangle w_{n+2}(t',\boldsymbol{x}',t,\boldsymbol{x},t_1,\boldsymbol{x}_1,\ldots,t_n,\boldsymbol{x}_n)\\ &+\langle\psi|\hat{\mu}(t')\hat{\mu}(t)|\psi\rangle w_{n+2}(t_1,\boldsymbol{x}_1,\ldots,t_n,\boldsymbol{x}_n,t,\boldsymbol{x},t',\boldsymbol{x}'))\\ &-\langle\psi|\hat{\mu}(t)\hat{\mu}(t')|\psi\rangle w_{n+2}(t,\boldsymbol{x},t_1,\boldsymbol{x}_1,\ldots,t_n,\boldsymbol{x}_n,t',\boldsymbol{x}')\Big] + O(\lambda^3). \end{aligned} \tag{B12}$$

2. Selective case

For the selective update w_n^{S}, we consider the update corresponding to the case in which not all the points in the argument are outside the causal future of the region in which the projective measurement on the detector is performed, $\mathcal{P}$ (otherwise the update is just the nonselective one). Recalling Eq. (48), we need to consider two expansions. First, by Eq. (A3),

$$\langle\hat{M}^{\dagger}_{s,\psi}\hat{\phi}(t_1,\boldsymbol{x}_1)\cdots\hat{\phi}(t_n,\boldsymbol{x}_n)\hat{M}_{s,\psi}\rangle_{\hat{\rho}_\phi} = \langle\hat{M}^{\dagger(0)}_{s,\psi}\hat{\phi}(t_1,\boldsymbol{x}_1)\cdots\hat{\phi}(t_n,\boldsymbol{x}_n)\hat{M}^{(0)}_{s,\psi}\rangle_{\hat{\rho}_\phi} + \lambda(\langle\hat{M}^{\dagger(1)}_{s,\psi}\hat{\phi}(t_1,\boldsymbol{x}_1)\cdots\hat{\phi}(t_n,\boldsymbol{x}_n)\hat{M}^{(0)}_{s,\psi}\rangle_{\hat{\rho}_\phi} + \langle\hat{M}^{\dagger(0)}_{s,\psi}\hat{\phi}(t_1,\boldsymbol{x}_1)\cdots\hat{\phi}(t_n,\boldsymbol{x}_n)\hat{M}^{(1)}_{s,\psi}\rangle_{\hat{\rho}_\phi}) + O(\lambda^2). \tag{B13}$$

And second, by Eq. (A8),

$$\begin{aligned} \langle\hat{E}_{s,\phi}\rangle_{\hat{\rho}_\phi} &= \langle\hat{M}^{\dagger}_{s,\psi}\hat{M}_{s,\psi}\rangle_{\hat{\rho}_\phi}\\ &= \langle\hat{M}^{\dagger(0)}_{s,\psi}\hat{M}^{(0)}_{s,\psi}\rangle_{\hat{\rho}_\phi} + \lambda(\langle\hat{M}^{\dagger(1)}_{s,\psi}\hat{M}^{(0)}_{s,\psi}\rangle_{\hat{\rho}_\phi} + \langle\hat{M}^{\dagger(0)}_{s,\psi}\hat{M}^{(1)}_{s,\psi}\rangle_{\hat{\rho}_\phi}) + O(\lambda^2). \end{aligned} \tag{B14}$$

If the zeroth-order term is not zero,

$$\langle\hat{M}^{\dagger(0)}_{s,\psi}\hat{M}^{(0)}_{s,\psi}\rangle_{\hat{\rho}_\phi} = |\langle s|\psi\rangle|^2 \neq 0, \tag{B15}$$

we can give an expansion for the inverse

$$(\langle\hat{E}_{s,\psi}\rangle_{\hat{\rho}_\phi})^{-1} = (\langle\hat{M}^{\dagger(0)}_{s,\psi}\hat{M}^{(0)}_{s,\psi}\rangle_{\hat{\rho}_\phi})^{-1} - \lambda(\langle\hat{M}^{\dagger(0)}_{s,\psi}\hat{M}^{(0)}_{s,\psi}\rangle_{\hat{\rho}_\phi})^{-2}(\langle\hat{M}^{\dagger(1)}_{s,\psi}\hat{M}^{(0)}_{s,\psi}\rangle_{\hat{\rho}_\phi} + \langle\hat{M}^{\dagger(0)}_{s,\psi}\hat{M}^{(1)}_{s,\psi}\rangle_{\hat{\rho}_\phi}) + O(\lambda^2). \tag{B16}$$

Thus, by Eqs. (A4) and (A9), if $\langle s|\psi\rangle \neq 0$, the zeroth order of w_n^{S} is

$$\begin{aligned} w_n^{\mathrm{S}}(t_1,\boldsymbol{x}_1,\ldots,t_n,\boldsymbol{x}_n)^{(0)} &= (\langle\hat{M}^{\dagger(0)}_{s,\psi}\hat{M}^{(0)}_{s,\psi}\rangle_{\hat{\rho}_\phi})^{-1}\langle\hat{M}^{\dagger(0)}_{s,\psi}\hat{\phi}(t_{1,\boldsymbol{x}_1})\cdots\hat{\phi}(t_n,\boldsymbol{x}_n)\hat{M}^{(0)}_{s,\psi}\rangle_{\hat{\rho}_\phi} \\ &= w_n(t_1,\boldsymbol{x}_1,\ldots,t_n,\boldsymbol{x}_n). \end{aligned} \tag{B17}$$

Also, for the first order,

$$\begin{aligned} w_n^{\mathrm{S}}(t_1,\boldsymbol{x}_1,\ldots,t_n,\boldsymbol{x}_n)^{(1)} &= (\langle\hat{M}^{\dagger(0)}_{s,\psi}\hat{M}^{(0)}_{s,\psi}\rangle_{\hat{\rho}_\phi})^{-1}(\langle\hat{M}^{\dagger(1)}_{s,\psi}\hat{\phi}(t_1,\boldsymbol{x}_1)\cdots\hat{\phi}(t_n,\boldsymbol{x}_n)\hat{M}^{(0)}_{s,\psi}\rangle_{\hat{\rho}_\phi} \\ &\quad + \langle\hat{M}^{\dagger(0)}_{s,\psi}\hat{\phi}(t_1,\boldsymbol{x}_1)\cdots\hat{\phi}(t_n,\boldsymbol{x}_n)\hat{M}^{(1)}_{s,\psi}\rangle_{\hat{\rho}_\phi}) \\ &\quad - (\langle\hat{M}^{\dagger(0)}_{s,\psi}\hat{M}^{(0)}_{s,\psi}\rangle_{\hat{\rho}_\phi})^{-2}(\langle\hat{M}^{\dagger(1)}_{s,\psi}\hat{M}^{(0)}_{s,\psi}\rangle_{\hat{\rho}_\phi} + \langle\hat{M}^{\dagger(0)}_{s,\psi}\hat{M}^{(1)}_{s,\psi}\rangle_{\hat{\rho}_\phi}) \\ &\quad \times \langle\hat{M}^{\dagger(0)}_{s,\psi}\hat{\phi}(t_1,\boldsymbol{x}_1)\cdots\hat{\phi}(t_n,\boldsymbol{x}_n)\hat{M}^{(0)}_{s,\psi}\rangle_{\hat{\rho}_\phi}. \end{aligned} \tag{B18}$$

By Eqs. (A7) and (A10),

$$\begin{aligned} w_n^{\mathrm{S}}(t_1,\boldsymbol{x}_1,\ldots,t_n,\boldsymbol{x}_n)^{(1)} &= \frac{1}{|\langle s|\psi\rangle|^2}\int \mathrm{d}t\,\mathrm{d}^d\boldsymbol{x}\,\chi(t)F(\boldsymbol{x})\Big(\mathrm{i}\langle s|\psi\rangle\langle\psi|\hat{\mu}(t)|s\rangle \\ &\quad \times \langle\hat{\phi}(t,\boldsymbol{x})\hat{\phi}(t_1,\boldsymbol{x}_1)\cdots\hat{\phi}(t_n,\boldsymbol{x}_n)\rangle_{\hat{\rho}_\phi} - \mathrm{i}\langle\psi|s\rangle \\ &\quad \times \langle s|\hat{\mu}(t)|\psi\rangle\langle\hat{\phi}(t_1,\boldsymbol{x}_1)\cdots\hat{\phi}(t_n,\boldsymbol{x}_n)\hat{\phi}(t,\boldsymbol{x})\rangle_{\hat{\rho}_\psi} \\ &\quad - 2\mathrm{Im}(\langle\psi|s\rangle\langle s|\hat{\mu}(t)|\psi\rangle)\langle\hat{\phi}(t_1,\boldsymbol{x}_1)\cdots\hat{\phi}(t_n,\boldsymbol{x}_n)\rangle_{\hat{\rho}_\phi}\langle\hat{\phi}(t,\boldsymbol{x})\rangle_{\hat{\rho}_\phi}\Big). \end{aligned} \tag{B19}$$

This equation, along with the one for the zeroth order, yields Eq. (49). Particularization for $n=2$ gives Eq. (46) immediately. For the one-point function, it only remains to use

$$w_2(t,\boldsymbol{x},t_1,\boldsymbol{x}_1)^* = w_2(t_1,\boldsymbol{x}_1,t,\boldsymbol{x}) \tag{B20}$$

as for the nonselective case, to get the final expression in Eq. (44).

We can get involved in more cumbersome calculations in order to arrive at the second-order terms of the expression for w_n^{S} when $\langle s|\psi\rangle \neq 0$. First, by Eq. (A15),

$$\begin{aligned} \langle\hat{M}^\dagger_{s,\psi}\hat{\phi}(t_1,\boldsymbol{x}_1)\cdots\hat{\phi}(t_n,\boldsymbol{x}_n)\hat{M}_{s,\psi}\rangle^{(2)}_{\hat{\rho}_\phi} &= \langle\hat{M}^{\dagger(2)}_{s,\psi}\hat{\phi}(t_1,\boldsymbol{x}_1)\cdots\hat{\phi}(t_n,\boldsymbol{x}_n)\hat{M}^{(0)}_{s,\psi}\rangle_{\hat{\rho}_\phi} + \langle\hat{M}^{\dagger(0)}_{s,\psi}\hat{\phi}(t_1,\boldsymbol{x}_1)\cdots\hat{\phi}(t_n,\boldsymbol{x}_n)\hat{M}^{(2)}_{s,\psi}\rangle_{\hat{\rho}_\phi} \\ &\quad + \langle\hat{M}^{\dagger(1)}_{s,\psi}\hat{\phi}(t_1,\boldsymbol{x}_1)\cdots\hat{\phi}(t_n,\boldsymbol{x}_n)\hat{M}^{(1)}_{s,\psi}\rangle_{\hat{\rho}_\phi}. \end{aligned} \tag{B21}$$

Removing the field operators in this last equation we get the expansion to second order of $\langle\hat{E}_{s,\psi}\rangle_{\hat{\rho}_\phi}$, and thus for its inverse the second-order contribution is

$$\begin{aligned} (\langle\hat{E}_{s,\psi}\rangle^{-1}_{\hat{\rho}_\phi})^{(2)} &= \langle\hat{M}^{\dagger(0)}_{s,\psi}\hat{M}^{(0)}_{s,\psi}\rangle^{-3}_{\hat{\rho}_\phi}[(\langle\hat{M}^{\dagger(1)}_{s,\psi}\hat{M}^{(0)}_{s,\psi}\rangle_{\hat{\rho}_\phi} + \langle\hat{M}^{\dagger(0)}_{s,\psi}\hat{M}^{(1)}_{s,\psi}\rangle_{\hat{\rho}_\phi})^2 - \langle\hat{M}^{\dagger(0)}_{s,\psi}\hat{M}^{(0)}_{s,\psi}\rangle_{\hat{\rho}_\phi}(\langle\hat{M}^{\dagger(2)}_{s,\psi}\hat{M}^{(0)}_{s,\psi}\rangle_{\hat{\rho}_\phi} \\ &\quad + \langle\hat{M}^{\dagger(0)}_{s,\psi}\hat{M}^{(2)}_{s,\psi}\rangle_{\hat{\rho}_\phi} + \langle\hat{M}^{\dagger(1)}_{s,\psi}\hat{M}^{(1)}_{s,\psi}\rangle_{\hat{\rho}_\phi})]. \end{aligned} \tag{B22}$$

Taking also into account the first-order contributions in Eqs. (B13) and (B16), and proceeding along the lines of the calculations for $w_n^{\mathrm{S}(0)}$ and $w_n^{\mathrm{S}(1)}$ we get that

$$w_n^{\mathrm{S}}(t_1,\boldsymbol{x}_1,\ldots,t_n,\boldsymbol{x}_n)^{(2)} = -\frac{1}{|\langle s|\psi\rangle|^2}\int \mathrm{d}t\,\mathrm{d}t'\,\mathrm{d}^d\boldsymbol{x}\,\mathrm{d}^d\boldsymbol{x}'\,\chi(t)\chi(t')F(\boldsymbol{x})F(\boldsymbol{x}')\left(\theta(t-t')\mathcal{J} + \mathcal{K} - \frac{\mathcal{L}}{|\langle s|\psi\rangle|^2}\right), \tag{B23}$$

where

$$\mathcal{J} = \langle s|\psi\rangle\langle\psi|\hat{\mu}(t')\mu(t)|s\rangle\Big(w_{n+2}(t',\boldsymbol{x}',t,\boldsymbol{x},t_1,\boldsymbol{x}_1,\ldots,t_n,\boldsymbol{x}_n) - w_2(t',\boldsymbol{x}',t,\boldsymbol{x})w_n(t_1,\boldsymbol{x}_1,\ldots,t_n,\boldsymbol{x}_n)\Big)$$
$$+ \langle\psi|s\rangle\langle s|\hat{\mu}(t)\hat{\mu}(t')|\psi\rangle\Big(w_{n+2}(t_1,\boldsymbol{x}_1,\ldots,t_n,\boldsymbol{x}_n,t,\boldsymbol{x},t',\boldsymbol{x}') - w_n(t_1,\boldsymbol{x}_1,\ldots,t_n,\boldsymbol{x}_n)w_2(t,\boldsymbol{x},t',\boldsymbol{x}')\Big), \tag{B24}$$

as well as

$$\mathcal{K} = \langle\psi|\hat{\mu}(t)|s\rangle\langle s|\hat{\mu}(t')|\psi\rangle\Big(w_{n+2}(t,\boldsymbol{x},t_1,\boldsymbol{x}_1,\ldots,t_n,\boldsymbol{x}_n,t',\boldsymbol{x}')$$
$$- w_1(t,\boldsymbol{x})w_{n+1}(t_1,\boldsymbol{x}_1,\ldots,t_n,\boldsymbol{x}_n,t',\boldsymbol{x}') - w_{n+1}(t,\boldsymbol{x},t_1,\boldsymbol{x}_1,\ldots,t_n,\boldsymbol{x}_n)w_1(t',\boldsymbol{x}')$$
$$- w_2(t,\boldsymbol{x},t',\boldsymbol{x}')w_n(t_1,\boldsymbol{x}_1,\ldots,t_n,\boldsymbol{x}_n) + 2w_1(t,\boldsymbol{x})w_1(t',\boldsymbol{x}')w_n(t_1,\boldsymbol{x}_1,\ldots,t_n,\boldsymbol{x}_n)\Big) \tag{B25}$$

and

$$\mathcal{L} = 2\mathrm{Re}\Big(\langle s|\psi\rangle^2\langle\psi|\hat{\mu}(t)|s\rangle\langle\psi|\hat{\mu}(t')|s\rangle\Big)w_1(t,\boldsymbol{x})w_1(t',\boldsymbol{x}')w_n(t_1,\boldsymbol{x}_1,\ldots,t_n,\boldsymbol{x}_n)$$
$$+ \langle s|\psi\rangle^2\langle\psi|\hat{\mu}(t)|s\rangle\langle\psi|\hat{\mu}(t')|s\rangle w_1(t,\boldsymbol{x})w_{n+1}(t',\boldsymbol{x}',t_1,\boldsymbol{x}_1,\ldots,t_n,\boldsymbol{x}_n)$$
$$+ \langle\psi|s\rangle^2\langle s|\hat{\mu}(t)|\psi\rangle\langle s|\hat{\mu}(t')|\psi\rangle w_1(t,\boldsymbol{x})w_{n+1}(t_1,\boldsymbol{x}_1,\ldots,t_n,\boldsymbol{x}_n,t',\boldsymbol{x}'). \tag{B26}$$

We finish this appendix by addressing the selective update when $\langle s|\psi\rangle = 0$. In this case, $\hat{M}^{(0)}_{s,\psi} = 0$, and therefore, by Eqs. (B13) and (B14), the zeroth-order and first-order contributions of both the numerator and the denominator in Eq. (48) cancel out. Hence,

$$\langle\hat{M}^{\dagger}_{s,\psi}\hat{\phi}(t_1,\boldsymbol{x}_1)\cdots\hat{\phi}(t_n,\boldsymbol{x}_n)\hat{M}_{s,\psi}\rangle_{\hat{\rho}_\phi} = \lambda^2\langle\hat{M}^{\dagger(1)}_{s,\psi}\hat{\phi}(t_1,\boldsymbol{x}_1)\cdots\hat{\phi}(t_n,\boldsymbol{x}_n)\hat{M}^{(1)}_{s,\psi}\rangle_{\hat{\rho}_\phi} + \lambda^3(\langle\hat{M}^{\dagger(2)}_{s,\psi}\hat{\phi}(t_1,\boldsymbol{x}_1)\cdots\hat{\phi}(t_n,\boldsymbol{x}_n)\hat{M}^{(1)}_{s,\psi}\rangle_{\hat{\rho}_\phi}$$
$$+ \langle\hat{M}^{\dagger(1)}_{s,\psi}\hat{\phi}(t_1,\boldsymbol{x}_1)\cdots\hat{\phi}(t_n,\boldsymbol{x}_n)\hat{M}^{(2)}_{s,\psi}\rangle_{\hat{\rho}_\phi}) + O(\lambda^4), \tag{B27}$$

and in particular,

$$\langle\hat{E}_{s,\phi}\rangle_{\hat{\rho}_\phi} = \langle\hat{M}^{\dagger}_{s,\psi}\hat{M}_{s,\psi}\rangle_{\hat{\rho}_\phi}$$
$$= \lambda^2\langle\hat{M}^{\dagger(1)}_{s,\psi}\hat{M}^{(1)}_{s,\psi}\rangle_{\hat{\rho}_\phi} + \lambda^3(\langle\hat{M}^{\dagger(2)}_{s,\psi}\hat{M}^{(1)}_{s,\psi}\rangle_{\hat{\rho}_\phi} + \langle\hat{M}^{\dagger(1)}_{s,\psi}\hat{M}^{(2)}_{s,\psi}\rangle_{\hat{\rho}_\phi}) + O(\lambda^4). \tag{B28}$$

Since the update depends on the quotient of both expressions, we can drop the factor λ^2 and proceed as we did formerly with the case $\langle s|\psi\rangle \neq 0$. For the zeroth order, as in Eq. (17),

$$w^{\mathrm{S}}_n(t_1,\boldsymbol{x}_1,\ldots,t_n,\boldsymbol{x}_n)^{(0)} = (\langle\hat{M}^{\dagger(1)}_{s,\psi}\hat{M}^{(1)}_{s,\psi}\rangle_{\hat{\rho}_\phi})^{-1}\langle\hat{M}^{\dagger(1)}_{s,\psi}\hat{\phi}(t_{1,\boldsymbol{x}_1})\cdots\hat{\phi}(t_n,\boldsymbol{x}_n)\hat{M}^{(1)}_{s,\psi}\rangle_{\hat{\rho}_\phi}. \tag{B29}$$

And for the first order, as in Eq. (18),

$$w^{\mathrm{S}}_n(t_1,\boldsymbol{x}_1,\ldots,t_n,\boldsymbol{x}_n)^{(1)} = \Big(\langle\hat{M}^{\dagger(1)}_{s,\psi}\hat{M}^{(1)}_{s,\psi}\rangle_{\hat{\rho}_\phi}\Big)^{-1}\Big(\langle\hat{M}^{\dagger(2)}_{s,\psi}\hat{\phi}(t_1,\boldsymbol{x}_1)\cdots\hat{\phi}(t_n,\boldsymbol{x}_n)\hat{M}^{(1)}_{s,\psi}\rangle_{\hat{\rho}_\phi} + \langle\hat{M}^{\dagger(1)}_{s,\psi}\hat{\phi}(t_1,\boldsymbol{x}_1)\cdots\hat{\phi}(t_n,\boldsymbol{x}_n)\hat{M}^{(2)}_{s,\psi}\rangle_{\hat{\rho}_\phi}\Big)$$
$$- \Big(\langle\hat{M}^{\dagger(1)}_{s,\psi}\hat{M}^{(1)}_{s,\psi}\rangle_{\hat{\rho}_\phi}\Big)^{-2}\Big(\langle\hat{M}^{\dagger(2)}_{s,\psi}\hat{M}^{(1)}_{s,\psi}\rangle_{\hat{\rho}_\phi} + \langle\hat{M}^{\dagger(1)}_{s,\psi}\hat{M}^{(2)}_{s,\psi}\rangle_{\hat{\rho}_\phi}\Big)\langle\hat{M}^{\dagger(1)}_{s,\psi}\hat{\phi}(t_1,\boldsymbol{x}_1)\cdots\hat{\phi}(t_n,\boldsymbol{x}_n)\hat{M}^{(1)}_{s,\psi}\rangle_{\hat{\rho}_\phi}. \tag{B30}$$

For the sake of clarity, let us denote

$$\mathcal{F}_n = \int \mathrm{d}t\,\mathrm{d}t'\,\mathrm{d}^d\boldsymbol{x}\,\mathrm{d}^d\boldsymbol{x}'\,\chi(t)\chi(t')F(\boldsymbol{x})F(\boldsymbol{x}')\langle\psi|\hat{\mu}(t)|s\rangle\langle s|\hat{\mu}(t')|\psi\rangle w_{n+2}(t,\boldsymbol{x},t_1,\boldsymbol{x}_1,\ldots,t_n,\boldsymbol{x}_n,t',\boldsymbol{x}') \tag{B31}$$

and

$$
\begin{aligned}
\mathcal{G}_n = \mathrm{i} \int & \mathrm{d}t\, \mathrm{d}t'\, \mathrm{d}t''\, \mathrm{d}^d\boldsymbol{x}\, \mathrm{d}^d\boldsymbol{x}'\, \mathrm{d}^d\boldsymbol{x}''\, \chi(t)\chi(t')\chi(t'')F(\boldsymbol{x})F(\boldsymbol{x}')F(\boldsymbol{x}'')\theta(t-t') \\
& \times \Big(\langle s|\hat{\mu}(t'')|\psi\rangle\langle\psi|\hat{\mu}(t')\hat{\mu}(t)|s\rangle w_{n+3}(t',\boldsymbol{x}',t,\boldsymbol{x},t_1,\boldsymbol{x}_1,\dots,t_n,\boldsymbol{x}_n,t'',\boldsymbol{x}'') \\
& - \langle\psi|\hat{\mu}(t'')|s\rangle\langle s|\hat{\mu}(t)\hat{\mu}(t')|\psi\rangle w_{n+3}(t'',\boldsymbol{x}'',t_1,\boldsymbol{x}_1,\dots,t_n,\boldsymbol{x}_n,t,\boldsymbol{x},t',\boldsymbol{x}') \Big).
\end{aligned} \tag{B32}
$$

Thus, by Eqs. (B29) and (B30), when $\langle s|\psi\rangle = 0$ we have

$$
w_n^{\mathrm{s}}(t_1,\boldsymbol{x}_1,\dots,t_n,\boldsymbol{x}_n) = \frac{\mathcal{F}_n}{\mathcal{F}_0} + \frac{\lambda\mathcal{G}_n}{\mathcal{F}_0} - \frac{\lambda\mathcal{G}_0\mathcal{F}_n}{\mathcal{F}_0^2} + O(\lambda^2). \tag{B33}
$$

APPENDIX C: A PRACTICAL EXAMPLE USING n-POINT FUNCTIONS

In this appendix we study the practical example considered in Sec. VIII using the approach based on n-point functions to implement the update rule. In particular, we will calculate the joint partial state $\hat{\rho}_{\mathrm{AB}}$, and the partial states $\hat{\rho}_{\mathrm{A}}$ and $\hat{\rho}_{\mathrm{B}}$, using n-point functions and its extensions as presented in Secs. VI and VII. As we will show, the results are the same as those obtained in Sec. VIII using a context-dependent density operator.

1. Nonselective case

We consider the case in which Clara performs a nonselective measurement in the first place. For an initial state of the field $\hat{\rho}_\phi$, the initial n-point functions are

$$
w_n(\mathsf{x}_1,\dots,\mathsf{x}_n) = \mathrm{tr}_\phi(\hat{\rho}_\phi\hat{\phi}(\mathsf{x}_1)\cdots\hat{\phi}(\mathsf{x}_n)). \tag{C1}
$$

After the measurement, we update w_n to w_n^{NS} following the prescription in Eq. (38),

$$
\begin{aligned}
w_n^{\mathrm{NS}}(\mathsf{x}_1,\dots,\mathsf{x}_n) &= \mathrm{tr}_\phi(\hat{M}_{c,\psi}\hat{\rho}_\phi\hat{M}_{c,\psi}^\dagger\hat{\phi}(\mathsf{x}_1)\cdots\hat{\phi}(\mathsf{x}_n)) + \mathrm{tr}_\phi(\hat{M}_{\bar{c},\psi}\hat{\rho}_\phi\hat{M}_{\bar{c},\psi}^\dagger\hat{\phi}(\mathsf{x}_1)\cdots\hat{\phi}(\mathsf{x}_n)) \\
&= \mathrm{tr}_{\mathrm{C},\phi}[\hat{U}_{\mathrm{C}}(|\psi\rangle\langle\psi|\otimes\hat{\rho}_\phi)\hat{U}_{\mathrm{C}}^\dagger\hat{\phi}(\mathsf{x}_1)\cdots\hat{\phi}(\mathsf{x}_n)],
\end{aligned} \tag{C2}
$$

where $\hat{M}_{c,\psi}$ is the $\hat{M}$ operator as defined in Eq. (14), for $|c\rangle$ and $|\psi\rangle$ states of the Hilbert space of Clara's detector. Now, we need to include the knowledge about the initial states of detectors A and B in the n-point functions using the extended formalism described in Sec. VII,

$$
\tilde{w}_{\Gamma,n}(k,l;\mathsf{x}_1,\dots,\mathsf{x}_n) = \mathrm{tr}[\hat{U}_{\mathrm{C}}(\hat{\rho}_{\mathrm{A}}\otimes\hat{\rho}_{\mathrm{B}}\otimes|\psi\rangle\langle\psi|\otimes\hat{\rho}_\phi)\hat{U}_{\mathrm{C}}^\dagger|k\rangle\langle l|\hat{\phi}(\mathsf{x}_1)\cdots\hat{\phi}(\mathsf{x}_n)], \tag{C3}
$$

where $\Gamma \subseteq \{A,B\}$ and $|k\rangle, |l\rangle$ are elements of an orthonormal basis of the Hilbert space of Γ. We now update $\tilde{w}_{\Gamma,n}$ taking into account the time evolution of the detectors A and B coupled to the field as

$$
\tilde{w}'_{\Gamma,n}(k,l;\mathsf{x}_1,\dots,\mathsf{x}_n) = \mathrm{tr}[\hat{U}_{\mathrm{A}}\hat{U}_{\mathrm{B}}\hat{U}_{\mathrm{C}}(\hat{\rho}_{\mathrm{A}}\otimes\hat{\rho}_{\mathrm{B}}\otimes|\psi\rangle\langle\psi|\otimes\hat{\rho}_\phi)\hat{U}_{\mathrm{C}}^\dagger\hat{U}_{\mathrm{B}}^\dagger\hat{U}_{\mathrm{A}}^\dagger|k\rangle\langle l|\hat{\phi}(\mathsf{x}_1)\cdots\hat{\phi}(\mathsf{x}_n)]. \tag{C4}
$$

Once we have obtained the extended n-point function (C4), we are in position to calculate the different partial states we are interested in. In particular,

$$
\begin{aligned}
\langle l_{\mathrm{A}}, l_{\mathrm{B}}|\hat{\rho}'_{\mathrm{AB}}|k_{\mathrm{A}}, k_{\mathrm{B}}\rangle &= \tilde{w}_{\{\mathrm{A,B}\},0}{}'((k_{\mathrm{A}},k_{\mathrm{B}}),(l_{\mathrm{A}},l_{\mathrm{B}})) \\
&= \mathrm{tr}[\hat{U}_{\mathrm{A}}\hat{U}_{\mathrm{B}}\hat{U}_{\mathrm{C}}(\hat{\rho}_{\mathrm{A}}\otimes\hat{\rho}_{\mathrm{B}}\otimes|\psi\rangle\langle\psi|\otimes\hat{\rho}_\phi)\hat{U}_{\mathrm{C}}^\dagger\hat{U}_{\mathrm{B}}^\dagger\hat{U}_{\mathrm{A}}^\dagger|k_{\mathrm{A}},k_{\mathrm{B}}\rangle\langle l_{\mathrm{A}},l_{\mathrm{B}}|] \\
&= \langle l_{\mathrm{A}}, l_{\mathrm{B}}|\mathrm{tr}_{\mathrm{C},\phi}[\hat{U}_{\mathrm{A}}\hat{U}_{\mathrm{B}}\hat{U}_{\mathrm{C}}(\hat{\rho}_{\mathrm{A}}\otimes\hat{\rho}_{\mathrm{B}}\otimes|\psi\rangle\langle\psi|\otimes\hat{\rho}_\phi)\hat{U}_{\mathrm{C}}^\dagger\hat{U}_{\mathrm{B}}^\dagger\hat{U}_{\mathrm{A}}^\dagger]|k_{\mathrm{A}},k_{\mathrm{B}}\rangle,
\end{aligned} \tag{C5}
$$

where $|k_\nu\rangle, |l_\nu\rangle \in \{|g_\nu\rangle, |e_\nu\rangle\}$ for $\nu \in \{\mathrm{A,B}\}$. Therefore,

$$
\hat{\rho}'_{\mathrm{AB}} = \mathrm{tr}_{\mathrm{C},\phi}[\hat{U}_{\mathrm{A}}\hat{U}_{\mathrm{B}}\hat{U}_{\mathrm{C}}(\hat{\rho}_{\mathrm{A}}\otimes\hat{\rho}_{\mathrm{B}}\otimes|\psi\rangle\langle\psi|\otimes\hat{\rho}_\phi)\hat{U}_{\mathrm{C}}^\dagger\hat{U}_{\mathrm{B}}^\dagger\hat{U}_{\mathrm{A}}^\dagger] \tag{C6}
$$

as we obtained in Sec. VIII. For the partial state $\hat{\rho}'_{\mathrm{B}}$,

$$\begin{aligned}\langle l_{\rm B}|\hat{\rho}'_{\rm B}|k_{\rm B}\rangle &= \tilde{w}'_{{\rm B},0}(k_{\rm B},l_{\rm B})\\ &= {\rm tr}[\hat{U}_{\rm A}\hat{U}_{\rm B}\hat{U}_{\rm C}(\hat{\rho}_{\rm A}\otimes\hat{\rho}_{\rm B}\otimes|\psi\rangle\langle\psi|\otimes\hat{\rho}_\phi)\\ &\quad\times\hat{U}^\dagger_{\rm C}\hat{U}^\dagger_{\rm B}\hat{U}^\dagger_{\rm A}|k_{\rm B}\rangle\langle l_{\rm B}|] \end{aligned}\tag{C7}$$

so that, proceeding as for $\hat{\rho}'_{\rm AB}$ above,

$$\begin{aligned}\hat{\rho}'_{\rm B} &= {\rm tr}_{{\rm A,C},\phi}[\hat{U}_{\rm A}\hat{U}_{\rm B}\hat{U}_{\rm C}\\ &\quad\times(\hat{\rho}_{\rm A}\otimes\hat{\rho}_{\rm B}\otimes|\psi\rangle\langle\psi|\otimes\hat{\rho}_\phi)\hat{U}^\dagger_{\rm C}\hat{U}^\dagger_{\rm B}\hat{U}^\dagger_{\rm A}]\\ &= {\rm tr}_{{\rm C},\phi}[\hat{U}_{\rm B}\hat{U}_{\rm C}(\hat{\rho}_{\rm B}\otimes|\psi\rangle\langle\psi|\otimes\hat{\rho}_\phi)\hat{U}^\dagger_{\rm C}\hat{U}^\dagger_{\rm B}]. \end{aligned}\tag{C8}$$

Analogously,

$$\begin{aligned}\langle l_{\rm A}|\hat{\rho}'_{\rm A}|k_{\rm A}\rangle &= \tilde{w}'_{{\rm A},0}(k_{\rm A},l_{\rm A})\\ &= {\rm tr}[\hat{U}_{\rm A}\hat{U}_{\rm B}\hat{U}_{\rm C}(\hat{\rho}_{\rm A}\otimes\hat{\rho}_{\rm B}\otimes|\psi\rangle\langle\psi|\otimes\hat{\rho}_\phi)\\ &\quad\times\hat{U}^\dagger_{\rm C}\hat{U}^\dagger_{\rm B}\hat{U}^\dagger_{\rm A}|k_{\rm B}\rangle\langle l_{\rm B}|], \end{aligned}\tag{C9}$$

thus getting

$$\begin{aligned}\hat{\rho}'_{\rm A} &= {\rm tr}_{{\rm B,C},\phi}[\hat{U}_{\rm A}\hat{U}_{\rm B}\hat{U}_{\rm C}\\ &\quad\times(\hat{\rho}_{\rm A}\otimes\hat{\rho}_{\rm B}\otimes|\psi\rangle\langle\psi|\otimes\hat{\rho}_\phi)\hat{U}^\dagger_{\rm C}\hat{U}^\dagger_{\rm B}\hat{U}^\dagger_{\rm A}]\\ &= {\rm tr}_{\phi}[\hat{U}_{\rm A}(\hat{\rho}_{\rm A}\otimes\hat{\rho}_\phi)\hat{U}^\dagger_{\rm A}]. \end{aligned}\tag{C10}$$

We see that the results obtained for the density operators associated with the partial states $\hat{\rho}'_{\rm AB}$, $\hat{\rho}'_{\rm A}$ and $\hat{\rho}'_{\rm B}$ using the n-point function formalism for implementing the update rule are the same as those obtained with the context-dependent density operator formalism.

2. Selective case

Let us now analyze the case in which the measurement performed by Clara is selective. In this case, the approach based on n-point functions has the advantage that the analysis of where the information is accessible is already contained in the piecewise definition of the selective update, so the calculations are more systematic. As a downside, it is more cumbersome than the density operator approach. Starting from the same initial n-point functions of Eq. (C1), after the measurement we update w_n to $w_n^{\rm S}$ following the prescription of Eqs. (47) and (48):

$$\begin{aligned}&w_n^{\rm S}(\mathsf{x}_1,\ldots,\mathsf{x}_n)\\ &\quad={\rm tr}_{{\rm C},\phi}[\hat{U}_{\rm C}(|\psi\rangle\langle\psi|\otimes\hat{\rho}_\phi)\hat{U}^\dagger_{\rm C}\hat{\phi}(\mathsf{x}_1)\cdots\hat{\phi}(\mathsf{x}_n)]\end{aligned}\tag{C11}$$

if all $\mathsf{x}_1,\ldots,\mathsf{x}_n$ are outside $\mathcal{P}$, and

$$w_n^{\rm S}(\mathsf{x}_1,\ldots,\mathsf{x}_n)=\frac{{\rm tr}(\hat{M}_{c,\psi}\hat{\rho}_\phi\hat{M}^\dagger_{c,\psi}\hat{\phi}(\mathsf{x}_1)\cdots\hat{\phi}(\mathsf{x}_n))}{{\rm tr}_\phi(\hat{\rho}_\phi\hat{E}_{c,\psi})}\tag{C12}$$

otherwise. Now, to include detectors A and B in the picture we use the extended formalism we introduced in Sec. VII.

Since B is in the causal future of the measurement, we have

$$\begin{aligned}&\tilde{w}^{\rm S}_{\{\rm A,B\},n}((k_{\rm A},k_{\rm B}),(l_{\rm A},l_{\rm B});\mathsf{x}_1,\ldots,\mathsf{x}_n)\\ &\quad={\rm tr}_{{\rm A,B},\phi}[\hat{M}_{c,\psi}(\hat{\rho}_{\rm A}\otimes\hat{\rho}_{\rm B}\otimes\hat{\rho}_\phi)\hat{M}^\dagger_{c,\psi}\\ &\qquad\times|k_{\rm A},k_{\rm B}\rangle\langle l_{\rm A},l_{\rm B}|\hat{\phi}(\mathsf{x}_1)\cdots\hat{\phi}(\mathsf{x}_n)]{\rm tr}_\phi(\hat{\rho}_\phi\hat{E}_{c,\psi})^{-1}\end{aligned}\tag{C13}$$

and

$$\begin{aligned}&\tilde{w}^{\rm S}_{{\rm B},n}((k_{\rm B},l_{\rm B});\mathsf{x}_1,\ldots,\mathsf{x}_n)\\ &\quad={\rm tr}_{{\rm A,B},\phi}[\hat{M}_{c,\psi}(\hat{\rho}_{\rm A}\otimes\hat{\rho}_{\rm B}\otimes\hat{\rho}_\phi)\hat{M}^\dagger_{c,\psi}\\ &\qquad\times|k_{\rm B}\rangle\langle l_{\rm B}|\hat{\phi}(\mathsf{x}_1)\cdots\hat{\phi}(\mathsf{x}_n)]{\rm tr}_\phi(\hat{\rho}_\phi\hat{E}_{c,\psi})^{-1}\\ &\quad={\rm tr}_{{\rm B},\phi}[\hat{M}_{c,\psi}(\hat{\rho}_{\rm B}\otimes\hat{\rho}_\phi)\hat{M}^\dagger_{c,\psi}\\ &\qquad\times|k_{\rm B}\rangle\langle l_{\rm B}|\hat{\phi}(\mathsf{x}_1)\cdots\hat{\phi}(\mathsf{x}_n)]{\rm tr}_\phi(\hat{\rho}_\phi\hat{E}_{c,\psi})^{-1}.\end{aligned}\tag{C14}$$

Now, because A is not in the causal future of the measurement,

$$\begin{aligned}&\tilde{w}^{\rm S}_{{\rm A},n}((k_{\rm A},l_{\rm A});\mathsf{x}_1,\ldots,\mathsf{x}_n)\\ &\quad={\rm tr}[\hat{U}_{\rm C}(\hat{\rho}_{\rm A}\otimes\hat{\rho}_{\rm B}\otimes|\psi\rangle\langle\psi|\otimes\hat{\rho}_\phi)\hat{U}^\dagger_{\rm C}\\ &\qquad\times|k_{\rm A}\rangle\langle l_{\rm A}|\hat{\phi}(\mathsf{x}_1)\cdots\hat{\phi}(\mathsf{x}_n)]\end{aligned}\tag{C15}$$

if $\mathsf{x}_1,\ldots,\mathsf{x}_n\notin\mathcal{P}$, and

$$\begin{aligned}&\tilde{w}^{\rm S}_{{\rm A},n}((k_{\rm A},l_{\rm A});\mathsf{x}_1,\ldots,\mathsf{x}_n)\\ &\quad={\rm tr}_{{\rm A},\phi}[\hat{M}_{c,\psi}(\hat{\rho}_{\rm A}\otimes\hat{\rho}_\phi)\hat{M}^\dagger_{c,\psi}\\ &\qquad\times|k_{\rm A}\rangle\langle l_{\rm A}|\hat{\phi}(\mathsf{x}_1)\cdots\hat{\phi}(\mathsf{x}_n)]{\rm tr}_\phi(\hat{\rho}_\phi\hat{E}_{c,\psi})^{-1}\end{aligned}\tag{C16}$$

otherwise.

After taking into account the interaction of the field with A and B, we update the extended n-point functions accordingly: for the n-point functions involving both detectors A and B,

$$\begin{aligned}&\tilde{w}'_{\{\rm A,B\},n}((k_{\rm A},k_{\rm B}),(l_{\rm A},l_{\rm B});\mathsf{x}_1,\ldots,\mathsf{x}_n)\\ &\quad={\rm tr}_{{\rm A,B},\phi}[\hat{U}_{\rm A}\hat{U}_{\rm B}\hat{M}_{c,\psi}(\hat{\rho}_{\rm A}\otimes\hat{\rho}_{\rm B}\otimes\hat{\rho}_\phi)\hat{M}^\dagger_{c,\psi}\\ &\qquad\times\hat{U}^\dagger_{\rm B}\hat{U}^\dagger_{\rm A}|k_{\rm A},k_{\rm B}\rangle\langle l_{\rm A},l_{\rm B}|\hat{\phi}(\mathsf{x}_1)\cdots\hat{\phi}(\mathsf{x}_n)]\\ &\qquad\times{\rm tr}_\phi(\hat{\rho}_\phi\hat{E}_{c,\psi})^{-1},\end{aligned}\tag{C17}$$

and for the ones involving only B,

$$\begin{aligned}&\tilde{w}'_{{\rm B},n}((k_{\rm B},l_{\rm B});\mathsf{x}_1,\ldots,\mathsf{x}_n)\\ &\quad={\rm tr}_{{\rm A,B},\phi}[\hat{U}_{\rm A}\hat{U}_{\rm B}\hat{M}_{c,\psi}(\hat{\rho}_{\rm A}\otimes\hat{\rho}_{\rm B}\otimes\hat{\rho}_\phi)\hat{M}^\dagger_{c,\psi}\\ &\qquad\times\hat{U}^\dagger_{\rm B}\hat{U}^\dagger_{\rm A}|k_{\rm B}\rangle\langle l_{\rm B}|\hat{\phi}(\mathsf{x}_1)\cdots\hat{\phi}(\mathsf{x}_n)]{\rm tr}_\phi(\hat{\rho}_\phi\hat{E}_{c,\psi})^{-1}.\end{aligned}\tag{C18}$$

For the extended n-point functions involving A, however, we get

$$\begin{aligned}&\tilde{w}'_{\mathrm{A},n}((k_{\mathrm{A}},l_{\mathrm{A}});\mathsf{x}_1,\dots,\mathsf{x}_n)\\&\quad=\mathrm{tr}[\hat{U}_{\mathrm{A}}\hat{U}_{\mathrm{B}}\hat{U}_{\mathrm{C}}(\hat{\rho}_{\mathrm{A}}\otimes\hat{\rho}_{\mathrm{B}}\otimes|\psi\rangle\langle\psi|\otimes\hat{\rho}_{\phi})\\&\qquad\times\hat{U}_{\mathrm{C}}^\dagger\hat{U}_{\mathrm{B}}^\dagger\hat{U}_{\mathrm{A}}^\dagger|k_{\mathrm{A}}\rangle\langle l_{\mathrm{A}}|\hat{\phi}(\mathsf{x}_1)\cdots\hat{\phi}(\mathsf{x}_n)]\end{aligned}\tag{C19}$$

if $\mathsf{x}_1,\dots,\mathsf{x}_n$ and Alba are outside $\mathcal{P}$, and

$$\begin{aligned}&\tilde{w}'_{\mathrm{A},n}((k_{\mathrm{A}},l_{\mathrm{A}});\mathsf{x}_1,\dots,\mathsf{x}_n)\\&\quad=\mathrm{tr}_{\mathrm{A,B},\phi}[\hat{U}_{\mathrm{A}}\hat{U}_{\mathrm{B}}\hat{M}_{c,\psi}(\hat{\rho}_{\mathrm{A}}\otimes\hat{\rho}_{\mathrm{B}}\otimes\hat{\rho}_{\phi})\hat{M}_{c,\psi}^\dagger\\&\qquad\times\hat{U}_{\mathrm{B}}^\dagger\hat{U}_{\mathrm{A}}^\dagger|k_{\mathrm{A}}\rangle\langle l_{\mathrm{A}}|\hat{\phi}(\mathsf{x}_1)\cdots\hat{\phi}(\mathsf{x}_n)]\\&\qquad\times\mathrm{tr}_{\phi}(\hat{\rho}_{\phi}\hat{E}_{c,\psi})^{-1}\end{aligned}\tag{C20}$$

otherwise. This is a complete account of the relevant extended n-point functions for the practical example we are dealing with, so that now we can calculate the partial states. In particular,

$$\begin{aligned}\langle l_{\mathrm{A}},l_{\mathrm{B}}|\hat{\rho}'_{\mathrm{AB}}|k_{\mathrm{A}},k_{\mathrm{B}}\rangle&=\tilde{w}'_{\{\mathrm{A,B}\},0}((k_{\mathrm{A}},k_{\mathrm{B}}),(l_{\mathrm{A}},l_{\mathrm{B}}))\\&=\mathrm{tr}_{\mathrm{A,B},\phi}[\hat{U}_{\mathrm{A}}\hat{U}_{\mathrm{B}}\hat{M}_{c,\psi}(\hat{\rho}_{\mathrm{A}}\otimes\hat{\rho}_{\mathrm{B}}\otimes\hat{\rho}_{\phi})\hat{M}_{c,\psi}^\dagger\\&\quad\times\hat{U}_{\mathrm{B}}^\dagger\hat{U}_{\mathrm{A}}^\dagger|k_{\mathrm{A}},k_{\mathrm{B}}\rangle\langle l_{\mathrm{A}},l_{\mathrm{B}}|]\mathrm{tr}_{\phi}(\hat{\rho}_{\phi}\hat{E}_{c,\psi})^{-1}\end{aligned}\tag{C21}$$

giving

$$\hat{\rho}'_{\mathrm{AB}}=\frac{\mathrm{tr}_{\phi}[\hat{U}_{\mathrm{A}}\hat{U}_{\mathrm{B}}\hat{M}_{c,\psi}(\hat{\rho}_{\mathrm{A}}\otimes\hat{\rho}_{\mathrm{B}}\otimes\hat{\rho}_{\phi})\hat{M}_{c,\psi}^\dagger\hat{U}_{\mathrm{B}}^\dagger\hat{U}_{\mathrm{A}}^\dagger]}{\mathrm{tr}_{\phi}(\hat{\rho}_{\phi}\hat{E}_{c,\psi})}.\tag{C22}$$

Analogously,

$$\begin{aligned}\langle l_{\mathrm{B}}|\hat{\rho}'_{\mathrm{B}}|k_{\mathrm{B}}\rangle&=\tilde{w}'_{\mathrm{B},0}(k_{\mathrm{B}},l_{\mathrm{B}})\\&=\mathrm{tr}_{\mathrm{A,B},\phi}[\hat{U}_{\mathrm{A}}\hat{U}_{\mathrm{B}}\hat{M}_{c,\psi}(\hat{\rho}_{\mathrm{A}}\otimes\hat{\rho}_{\mathrm{B}}\otimes\hat{\rho}_{\phi})\hat{M}_{c,\psi}^\dagger\\&\quad\times\hat{U}_{\mathrm{B}}^\dagger\hat{U}_{\mathrm{A}}^\dagger|k_{\mathrm{B}}\rangle\langle l_{\mathrm{B}}|]\mathrm{tr}_{\phi}(\hat{\rho}_{\phi}\hat{E}_{c,\psi})^{-1}\end{aligned}\tag{C23}$$

gives

$$\hat{\rho}'_{\mathrm{B}}=\frac{\mathrm{tr}_{\phi}[\hat{U}_{\mathrm{B}}\hat{M}_{c,\psi}(\hat{\rho}_{\mathrm{B}}\otimes\hat{\rho}_{\phi})\hat{M}_{c,\psi}^\dagger\hat{U}_{\mathrm{B}}^\dagger]}{\mathrm{tr}_{\phi}(\hat{\rho}_{\phi}\hat{E}_{c,\psi})}.\tag{C24}$$

On the other hand, since A stays spacelike separated from the causal future of Clara's measurement $\mathcal{P}$,

$$\begin{aligned}\langle l_{\mathrm{A}}|\hat{\rho}'_{\mathrm{A}}|k_{\mathrm{A}}\rangle&=\tilde{w}'_{\mathrm{A},0}(k_{\mathrm{A}},l_{\mathrm{A}})\\&=\langle l_{\mathrm{A}}|\mathrm{tr}_{\mathrm{B,C},\phi}[\hat{U}_{\mathrm{A}}\hat{U}_{\mathrm{B}}\hat{U}_{\mathrm{C}}\\&\quad\times(\hat{\rho}_{\mathrm{A}}\otimes\hat{\rho}_{\mathrm{B}}\otimes|\psi\rangle\langle\psi|\otimes\hat{\rho}_{\phi})\hat{U}_{\mathrm{C}}^\dagger\hat{U}_{\mathrm{B}}^\dagger\hat{U}_{\mathrm{A}}^\dagger]|k_{\mathrm{A}}\rangle\\&=\langle l_{\mathrm{A}}|\mathrm{tr}_{\phi}[\hat{U}_{\mathrm{A}}(\hat{\rho}_{\mathrm{A}}\otimes\hat{\rho}_{\phi})\hat{U}_{\mathrm{A}}^\dagger]|k_{\mathrm{A}}\rangle\end{aligned}\tag{C25}$$

yielding

$$\hat{\rho}'_{\mathrm{A}}=\mathrm{tr}_{\phi}[\hat{U}_{\mathrm{A}}(\hat{\rho}_{\mathrm{A}}\otimes\hat{\rho}_{\phi})\hat{U}_{\mathrm{A}}^\dagger]\neq\mathrm{tr}_{\mathrm{B}}(\hat{\rho}'_{\mathrm{AB}}),\tag{C26}$$

as expected. Finally, if Alba eventually reaches $\mathcal{P}$, then following the prescribed update rule for the case when A is inside $\mathcal{P}$,

$$\begin{aligned}&\langle l_{\mathrm{A}}|\hat{\rho}''_{\mathrm{A}}|k_{\mathrm{A}}\rangle\\&\quad=\tilde{w}'_{\mathrm{A},0}(k_{\mathrm{A}},l_{\mathrm{A}})\\&\quad=\frac{\langle l_{\mathrm{A}}|\mathrm{tr}_{\mathrm{B},\phi}[\hat{U}_{\mathrm{A}}\hat{U}_{\mathrm{B}}\hat{M}_{c,\psi}(\hat{\rho}_{\mathrm{A}}\otimes\hat{\rho}_{\mathrm{B}}\otimes\hat{\rho}_{\phi})\hat{M}_{c,\psi}^\dagger\hat{U}_{\mathrm{B}}^\dagger\hat{U}_{\mathrm{A}}^\dagger]|k_{\mathrm{A}}\rangle}{\mathrm{tr}_{\phi}(\hat{\rho}_{\phi}\hat{E}_{c,\psi})}\\&\quad=\frac{\langle l_{\mathrm{A}}|\mathrm{tr}_{\phi}[\hat{U}_{\mathrm{A}}\hat{M}_{c,\psi}(\hat{\rho}_{\mathrm{A}}\otimes\hat{\rho}_{\phi})\hat{M}_{c,\psi}^\dagger\hat{U}_{\mathrm{A}}^\dagger]|k_{\mathrm{A}}\rangle}{\mathrm{tr}_{\phi}(\hat{\rho}_{\phi}\hat{E}_{c,\psi})}\end{aligned}\tag{C27}$$

yielding

$$\hat{\rho}''_{\mathrm{A}}=\frac{\mathrm{tr}_{\phi}(\hat{U}_{\mathrm{A}}\hat{M}_{c,\psi}[\hat{\rho}_{\mathrm{A}}\otimes\hat{\rho}_{\phi}]\hat{M}_{c,\psi}^\dagger\hat{U}_{\mathrm{A}}^\dagger)}{\mathrm{tr}_{\phi}(\hat{\rho}_{\phi}\hat{E}_{c,\psi})}=\mathrm{tr}_{\mathrm{B}}(\hat{\rho}_{\mathrm{AB}'}).\tag{C28}$$

These results are again identical to those obtained in Sec. VIII, giving some insight into the equivalence of both formalisms, while showing a practical example of how to perform the calculations in each of them.

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