Electric field and voltage fluctuations in the Casimir effect

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The effects of reflecting boundaries on vacuum electric field fluctuations are treated. The presence of the boundaries can enhance these fluctuations and possibly lead to observable effects. The electric field fluctuations lead to voltage fluctuations along the worldline of a charged particle moving perpendicularly to a pair of reflecting plates. These voltage fluctuations in turn lead to fluctuations in the kinetic energy of the particle, which may enhance the probability of quantum barrier penetration by the particle. A recent experiment by Moddel *et al.* is discussed as a possible example of this enhanced barrier penetration probability.

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I. INTRODUCTION

The Casimir effect has become the topic of extensive theoretical and experimental work in recent years [1]. In its original form, it is a force of attraction between a pair of perfectly reflecting plates due to modification of the electromagnetic vacuum fluctuations. The presence of the plates modifies vacuum modes whose wavelengths are of the order of the plate separation and shifts the energy of the vacuum state by an amount which is proportional to an inverse power of the separation. The plates also modify local observables in the region between the plates. These observables can include the energy density and the electric field correlation functions. The shifts in the electric field fluctuations can in principle be detected by charged particles moving between the plates in the form of modified Brownian motion. This has been the topic of several investigations in recent years [2–7]. For example, Ref. [2] examined the shift in the mean squared components of the velocity of a charge moving parallel to a plate and found that this shift can be negative, which was interpreted as a small reduction in the quantum uncertainty.

In this paper, this problem will be reexamined, with particular attention paid to particles moving perpendicular to one or two plates. The motivation for this study is a recent experimental result by Moddel and co-workers [8,9], who found that the current flowing through a metal-insulator-metal (*M*-*I*-*M*) interface can be very sensitive to the distance between the interface and an aluminum mirror. The authors conjectured that this dependence may be related to the Casimir effect. The purpose of this paper is to explore this conjecture in more detail. The outline of the paper is as follows: The electric field correlation functions in the presence of two parallel perfectly reflecting plates will be reviewed in Sec. II. These results will be used in Sec. III to compute voltage fluctuations along a segment of a particle's worldline, which yield fluctuations in the particle's kinetic energy. The Moddel *et al.* experiment is reviewed in Sec. IV, and the extent to which its results might be explained by the modified electric field fluctuations is discussed. The effects of finite temperature are discussed in Sec. V. The paper is summarized in Sec. VI.

Lorentz-Heaviside units in which $\hbar = c = 1$ are used throughout the paper, except as otherwise noted.

II. ELECTRIC FIELD CORRELATION FUNCTIONS

We may define a vacuum correlation function for the Cartesian components of the quantized electric field operator $\mathbf{E}(t,\mathbf{x})$ as $\langle E^i(t,\mathbf{x})E^j(t',\mathbf{x}')\rangle$. Here we are interested primarily in the shift in the correlation functions due to the presence of mirrors, which is described by the renormalized function

$$\langle E^i(t,\mathbf{x})E^j(t',\mathbf{x}')\rangle_R = \langle E^i(t,\mathbf{x})E^j(t',\mathbf{x}')\rangle - \langle E^i(t,\mathbf{x})E^j(t',\mathbf{x}')\rangle_0, \quad (1)$$

where $\langle E^i(t,\mathbf{x})E^j(t',\mathbf{x}')\rangle$ is an expectation value in the Casimir vacuum with the mirrors present and $\langle E^i(t,\mathbf{x})E^j(t',\mathbf{x}')\rangle_0$ is an expectation value in the Minkowski vacuum without mirrors. We consider the Casimir geometry of two parallel, perfectly reflecting mirrors, one located at $z = 0$ and the other at $z = a$. The correlation functions for this geometry were calculated by Brown and Maclay [10] using an image sum method. We are especially interested in the case $i = j = z$, the correction function between the z component of the electric

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field at two different spacetime points which lie along a line perpendicular to the mirrors, so $x = x'$ and $y = y'$. In this case, the results of Ref. [10] may be used to show that

$$\begin{aligned}
&\langle E^z(t,z)E^z(t',z')\rangle_R \\
&= \frac{1}{\pi^2[(t-t')^2-(z+z')^2]^2} \\
&\quad + \frac{1}{\pi^2}\sum_{n=-\infty}^{\infty}{}' \left\{ \frac{1}{[(t-t')^2-(z-z'-2an)^2]^2} \right. \\
&\quad \left. + \frac{1}{[(t-t')^2-(z+z'-2an)^2]^2} \right\}, \qquad (2)
\end{aligned}$$

where the prime on the summation denotes that the $n = 0$ term is omitted. In the limit in which $a \to \infty$, we obtain the following result for a single mirror:

$$\langle E^z(t,z)E^z(t',z')\rangle_R = \frac{1}{\pi^2[(t-t')^2-(z+z')^2]^2}. \qquad (3)$$

Note that in all cases $\langle E^z(t,z)E^z(t',z')\rangle_R > 0$, meaning that the presence of the plates enhances the electric field fluctuations compared to those in empty space. Some physical effects of this enhancement will be the primary topic of this paper.

III. VOLTAGE AND PARTICLE ENERGY FLUCTUATIONS

Consider a particle with electric charge q moving in the z direction, normal to the plates. The work done by the electric field when the particle moves from $z = z_0$ to $z = z_0 + b$ along a spacetime path described by $z = z(t)$, or equivalently $t = t(z)$, is

$$\Delta U = q\int_{z_0}^{z_0+b} E^z(t(z),z)dz. \qquad (4)$$

The corresponding voltage difference is $\Delta V = \Delta U/q$. In the vacuum state, $\langle E^z\rangle = 0$, so the mean work vanishes ($\langle \Delta U\rangle = 0$). However, the variance is nonzero and the contribution to the variance due to the presence of the plates may be written as

$$\langle(\Delta U)^2\rangle = q^2\int_{z_0}^{z_0+b}dz\int_{z_0}^{z_0+b}dz'\langle E^z(t,z)E^z(t',z')\rangle_R. \qquad (5)$$

Assume that the particle moves at an approximately constant speed v, so its worldline may be described by $t = z/v$, or $t' = z'/v$.

A. Single-plate case

First consider the case of one plate, where Eq. (3) applies. The variance of the particle's energy due to the plate may now be written as

$$\langle(\Delta U)^2\rangle = \frac{q^2 v^4}{\pi^2} I(z_0,b,v), \qquad (6)$$

where

$$I(z_0,b,v) = \int_{z_0}^{z_0+b}dz\int_{z_0}^{z_0+b}dz'\frac{1}{[(z-z')^2-v^2(z+z')^2]^2}. \qquad (7)$$

This integral contains a second-order pole at points where $(z-z')^2 = v^2(z+z')^2$. The integral may be defined as a principal value, obtained by writing the integrand as a second derivative:

$$\frac{1}{[(z-z')^2-v^2(z+z')^2]^2} = \frac{\partial}{\partial z}\frac{\partial}{\partial z'}F(z,z'). \qquad (8)$$

An explicit form of $F(z,z')$ is

$$\begin{aligned}
F(z,z') = &\frac{1}{128v^3(zz')^2}[8vzz' + (1-v^2)(z^2-z'^2) \\
&\times(\log\{[(1+v)z' + (v-1)z]^2/\ell^2)\} \\
&-\log\{[(1+v)z + (v-1)z']^2/\ell^2\})], \qquad (9)
\end{aligned}$$

where ℓ is an arbitrary constant with the dimensions of length. Note that $F(z,z')$ is independent of the actual value of ℓ; if we rescale $\ell \to \mu\ell$, then μ cancels. Now $I(z_0,b,v)$ may be expressed as

$$\begin{aligned}
I(z_0,b,v) = &F(z_0+b,z_0+b) - F(z_0+b,z_0) \\
&-F(z_0,z_0+b) + F(z_0,z_0). \qquad (10)
\end{aligned}$$

In the limit in which $v \ll 1$, this result becomes

$$\begin{aligned}
I(z_0,b,v) \sim &\frac{z_0^2+(z_0+b)^2}{8z_0^2(z_0+b)^2v^2} \\
&+\frac{(2z_0+b)^2(2z_0^2+2bz_0-b^2)}{24b^2z_0^2(z_0+b)^2} + O(v^2). \qquad (11)
\end{aligned}$$

The leading term proportional to v^{-2} is independent of b when $b \lesssim z_0$,

$$I(z_0,b,v) \approx \frac{1}{4z_0^2v^2}. \qquad (12)$$

Both the factor of v^{-2} and the lack of dependence upon b in the above result may be traced to the singular nature of the integrand in Eq. (7). In the limit in which $v \to 0$, this integrand approaches $1/(z-z')^4$ and the integral diverges. This leads to the result in which $I(z_0,b,v) \propto 1/v^2$ for small v. If the integrand in Eq. (7) were bounded, we would expect to find $I(z_0,b,v) \propto b^2$ for small b rather than Eq. (12). However, there is a limit to how small b may be for fixed v, as we need to have the $O(v^0)$ term in Eq. (11) be small compared to the $O(1/v^2)$ term. In the case in which $b \ll z_0$, the former term becomes $1/(3b^2)$, which is sufficiently small provided that

$$b \gtrsim \frac{2}{\sqrt{3}} v z_0. \tag{13}$$

The lack of b dependence in Eq. (12) arises because the contribution of the second-order pole in Eq. (7) is independent of the length of the integration interval as long as $vz_0 \lesssim b \lesssim z_0$.

Note that when $v \ll 1$ but $b \gg z_0$, Eq. (11) yields

$$I(z_0, b, v) \approx \frac{1}{8 z_0^2 v^2}, \tag{14}$$

one-half of its value for small b. Furthermore, the decrease in the energy variance as b increases is monotonic. This decrease can be attributed to anticorrelated electric field fluctuations.

We may now combine Eqs. (5) and (12) to write

$$\langle (\Delta U)^2 \rangle \approx \frac{q^2 v^2}{4\pi^2 z_0^2} \tag{15}$$

when $v \ll 1$ and $b \ll z_0$. In this limit, the root-mean-square energy fluctuation is

$$\Delta U_{\rm rms} = \sqrt{\langle (\Delta U)^2 \rangle} \approx \frac{qv}{2\pi z_0}. \tag{16}$$

Note that this energy fluctuation corresponds to a voltage fluctuation of

$$\Delta V_{\rm rms} = \frac{1}{q} \Delta U_{\rm rms} \tag{17}$$

along the worldline of the charged particle. This fluctuation is proportional to the speed v. Remarkably, the fluctuation is independent of the distance traveled, b, as long as

$$\frac{2}{\sqrt{3}} v z_0 \lesssim b \ll z_0. \tag{18}$$

B. Two-plate case

Now we turn to the case of two parallel plates, where the shift in the electric field correlation function is given by Eq. (2). Again we consider a particle moving at constant speed v from z_0 to $z_0 + b$ and write Eq. (5) as

$$\langle (\Delta U)^2 \rangle = \langle (\Delta U)^2 \rangle_{\text{one plate}} + \frac{q^2 v^4}{\pi^2} \sum_{n=-\infty}^{\infty}{}' [I_{2A}(n) + I_{2B}(n)]. \tag{19}$$

Here $\langle (\Delta U)^2 \rangle_{\text{one plate}}$ is the result for a single plate calculated in the previous subsection, and we let

$$I_{2A}(n) = \int_{z_0}^{z_0+b} dz \int_{z_0}^{z_0+b} dz' \frac{1}{[(z-z')^2 - v^2(z+z'-2an)^2]^2} \tag{20}$$

and

$$I_{2B}(n) = \int_{z_0}^{z_0+b} dz \int_{z_0}^{z_0+b} dz' \frac{1}{[(z-z')^2 - v^2(z-z'-2an)^2]^2}. \tag{21}$$

We may evaluate $I_{2A}(n)$ by noting that

$$\frac{1}{[(z-z')^2 - v^2(z+z'-2an)^2]^2} = \frac{\partial}{\partial z}\frac{\partial}{\partial z'} F(z-an, z'-an), \tag{22}$$

so

$$\begin{aligned} I_{2A}(n) = {} & F(z_0+b-an, z_0+b-an) \\ & - F(z_0+b-an, z_0-an) \\ & - F(z_0-an, z_0+b-an) + F(z_0-an, z_0-an). \end{aligned} \tag{23}$$

Here F may be taken to have the form given in Eq. (9). In the limit $v \ll 1$, the quantity $I_{2A}(n)$ is also proportional to $1/v^2$ and takes the form

$$I_{2A}(n) \sim \frac{2z_0^2 - 4anz_0 + 2bz_0 + 2a^2n^2 - 2abn + b^2}{8v^2(an - z_0)^2(an + b - z_0)^2}. \tag{24}$$

As $b \to 0$, this approaches a nonzero value

$$I_{2A}(n) \to \frac{1}{4v^2(an - z_0)^2}. \tag{25}$$

We evaluate $I_{2B}(n)$ by first expressing its integrand as

$$\frac{1}{[(z-z')^2 - v^2(z-z'-2an)^2]^2} = \frac{\partial}{\partial z}\frac{\partial}{\partial z'} G(z, z'), \tag{26}$$

where

$$G(z,z') = \frac{1}{64(nav)^3}[8nav + [(1-v^2)(z-z') + 2nav^2](\log\{[(1+v)(z'-z) + 2nav]^2/\ell^2)\} - \log\{[(1-v)(z-z') + 2nav]^2/\ell^2\})]. \tag{27}$$

This leads to

$$I_{2B}(n) = G(z_0+b, z_0+b) - G(z_0+b, z_0) - G(z_0, z_0+b) + G(z_0, z_0), \tag{28}$$

which has a form independent of b, but proportional to $1/v^2$, when $v \ll 1$,

$$I_{2B}(n) \sim \frac{1}{4a^2v^2n^2}. \tag{29}$$

For the case in which $v \ll 1$ and $b \ll z_0$, we may write Eq. (19) as

$$\langle(\Delta U)^2\rangle = \langle(\Delta U)^2\rangle_{\text{one plate}} + \frac{q^2v^2}{4\pi^2a^2}\sum_{n=-\infty}^{\infty}{}' \left[\frac{1}{n^2} + \frac{1}{(n+z_0/a)^2}\right]. \tag{30}$$

The sums may be evaluated in closed form using

$$\sum_{n=-\infty}^{\infty}{}' \frac{1}{n^2} = 2\zeta(2) = \frac{\pi^2}{3}, \tag{31}$$

where ζ is the Riemann zeta function and [11]

$$\sum_{n=1}^{\infty}\left[\frac{1}{(n+x)^2} + \frac{1}{(n-x)^2}\right] = -\frac{1}{x^2} + \pi^2\csc^2(\pi x). \tag{32}$$

These identities lead to our result for the particle energy variance in the two-plate case

$$\langle(\Delta U)^2\rangle = \frac{q^2v^2}{12a^2}\left[1 + 3\csc^2\left(\frac{\pi z_0}{a}\right)\right]. \tag{33}$$

Note that the contribution of the $1/x^2$ term in Eq. (32) has canceled the $\langle(\Delta U)^2\rangle_{\text{one plate}}$ term in Eq. (30). Furthermore, $\langle(\Delta U)^2\rangle$ is symmetric about the midpoint, $z_0 = a/2$. In the limit in which $z_0 \ll a$, it reduces to the one-plate result, Eq. (15). In the limit in which $a - z_0$ is small, we have

$$\langle(\Delta U)^2\rangle \sim \frac{q^2v^2}{4\pi^2(a-z_0)^2}, \tag{34}$$

which is the one-plate result due to the plate at $z = a$.

Note that here and in the previous subsection, we have assumed sudden switching at $z = z_0$ and $z = z_0 + b$ in expressions such as Eqs. (20) and (21). This still produces finite results, as we are interested in the difference between the Minkowski and Casimir vacua, which is sensitive to modes whose wavelength is of the order of the distance to the plates. Had we been dealing with effects in the Minkowski vacuum alone, sudden switching could produce infinite results, as modes of arbitrarily short wavelength could contribute. Here we are justified in using sudden switching if the timescale for the onset of the coupling of the modified vacuum fluctuations to the charge is short compared to other timescales. If this is not the case, then the switching needs to be modeled using a smooth function, as discussed in Refs. [5,7].

C. Some estimates

Here we wish to make some numerical estimates of the magnitude of the voltage or particle energy fluctuations. Note that in all cases studied above the energy variance is proportional to q^2v^2 and inversely proportional to the square of the distance between the starting point and the nearest plate. Thus, we may use the one-plate result, Eq. (16), for the root-mean-square energy fluctuation as an illustration. Let $K = \frac{1}{2}mv^2$ be the particle's kinetic energy. In the case where the particle is an electron, we may write

$$\Delta U_{\text{rms}} = \frac{e}{\pi z_0}\sqrt{\frac{K}{2m}} \approx 1.9\times10^{-4}\ \text{eV}\sqrt{\frac{K}{1\ \text{eV}}}\left(\frac{100\ \text{nm}}{z_0}\right). \tag{35}$$

Thus, the magnitude of the energy fluctuations increases as the square root of K, but the fractional fluctuation, $\Delta U_{\text{rms}}/K$, is inversely proportional to $\sqrt{K}$. In any case, the energy and hence the velocity fluctuations are relatively small for nonrelativistic particles. This justifies our assumption that v remains approximately constant.

D. Electric field fluctuations and quantum tunneling

It is well known that a quantum particle can tunnel through a potential barrier even when its energy is below the maximum of the barrier. It is also well known that a nonzero temperature can enhance the rate at which particles can pass over the barrier. This process, thermal activation, arises not from quantum tunneling but rather from the fraction of particles in the tail of the Boltzmann distribution that have enough energy to fly over the barrier classically. It is less well known that even at zero temperature, quantum electric field vacuum fluctuations can enhance the tunneling rate compared to that predicted in single-particle quantum mechanics [12,13]. In quantum electrodynamics, this effect arises as a one-loop radiative correction to the tree-level scattering amplitude for an electron to scatter from a potential barrier. The basic physical process may be understood as follows: when the electron is in the vicinity of the barrier, it is equally likely to receive forward and backward kicks from the electric field vacuum fluctuations. However, the net effect of the forward kicks is greater, so the tunneling rate is increased by the field fluctuations. The root-mean-squared energy fluctuation of an electron with initial kinetic energy K while passing a barrier of width a is found in Ref. [13] to be

$$\Delta U_{\rm MV} \approx \frac{e^2 K}{m^2 a^2}. \tag{36}$$

Here the effect is due to vacuum electric field fluctuations in the Minkowski vacuum of empty space. This may be compared to the effect in the Casimir vacuum given by Eq. (35) to find

$$\frac{\Delta U_{\rm rms}}{\Delta U_{\rm MV}} = \frac{a^2 m^{3/2}}{\pi e z_0 \sqrt{K}} \approx 1.9\times 10^4 \left(\frac{a}{1\,{\rm nm}}\right)^2 \left(\frac{100\,{\rm nm}}{z_0}\right)\sqrt{\frac{1\,{\rm eV}}{K}}. \tag{37}$$

For a wide choice of parameters, $\Delta U_{\rm rms} \gg \Delta U_{\rm MV}$, so the effect due to the presence of plates is much larger than the effect in empty space. The actual increase in tunneling rate due to the presence of plates will depend upon the relative magnitudes of $\Delta U_{\rm rms}$ and $|K - V_{\rm max}|$, where $V_{\rm max}$ is the maximum value of the potential. When these two quantities become comparable, the increase can be very large.

E. Effects of finite reflectivity

Thus far, we have assumed perfectly reflecting plates. For a metal, this is a good approximation for electromagnetic waves with angular frequencies below the plasma frequency, ω_p, but not for higher frequency modes. Thus, in a regime where the dominant contribution to a Casimir effect comes from modes where $\omega \lesssim \omega_p$, we can expect the results assuming perfect reflectivity to be a reasonable approximation. This is illustrated in calculations of the mean squared electric field, $\langle E^2(z)\rangle$, at a distance z from a single plate [14]. In the limit in which $\omega_p z \gtrsim 1$, we have

$$\langle E^2(z)\rangle \sim \frac{3}{16\pi^2 z^4}. \tag{38}$$

This is also the result at all values of z for a perfectly reflecting plate, as may be found from the results in Ref. [10]. In the case in which $\omega_p z \lesssim 1$, the mean squared electric field becomes

$$\langle E^2(z)\rangle \sim \frac{\sqrt{2}\omega_p}{32\pi z^3}. \tag{39}$$

Note that finite reflectivity modifies the singular behavior of $\langle E^2(z)\rangle$ as $z \to 0$ but does not remove it. This implies that another physical cutoff is required. One possibility is that the assumption of an exactly smooth plane is too strong, and that surface roughness provides this cutoff. For our purposes, we need not answer this question, but rather confine the use of the perfectly reflecting results to the region where $z > 1/\omega_p$.

Note that results such as Eq. (39) give local expectation values, not correlation functions. A more detailed study of the latter in the presence of boundaries with finite reflectivity is needed. Until such a study has been performed, it is not clear how sensitive results such as Eq. (16) are to finite reflectivity.

IV. A CAVITY-INDUCED CURRENT EXPERIMENT

Here we briefly summarize a recent experiment by Moddel and co-workers [8,9]. This experiment involves electrical current in a metal-insulator-metal interface which is adjacent to a cavity, as illustrated in Fig. 1, which is adapted from Fig. 1 in Refs. [8,9]. A potential difference V_0 is imposed between the palladium and nickel electrodes, which causes a current I to flow through the layer of insulator separating the two electrodes. On the far side of

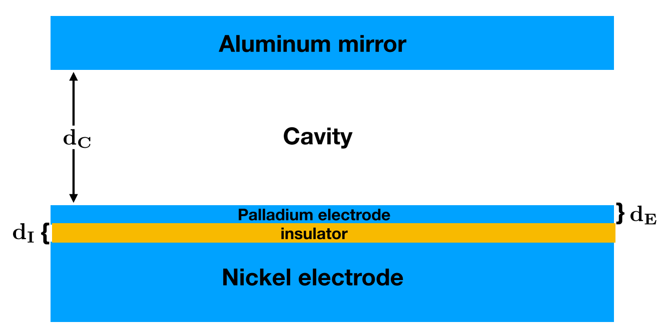

FIG. 1. An optical cavity of thickness d_C is bounded by an aluminum mirror and a M-I-M interface. The latter consists of a nickel electrode, a palladium electrode of thickness d_E, and a layer of insulator of thickness d_I.

the palladium electrode, there is an optical cavity of thickness d_C, and beyond the cavity an aluminum mirror is located. The key result, illustrated in Fig. 3(a) of Ref. [8] and Fig. 4(a) of Ref. [9], is that the magnitude of I for a fixed V_0 is inversely related to the cavity thickness d_C. In effect, the electrical resistance of the insulator layer decreases as d_C decreases.

The cavity is filled with a transparent dielectric, PMMA, and has various thicknesses, $d_C = 33$, 79, 230, and 1100 nm. The insulator layer consists of 1.3 nm aluminum oxide Al_2O_3 and 1 nm of nickel oxide, for a net thickness of $d_I = 2.3$ nm. The palladium electrode has a thickness of $d_E = 8.3$ nm, the aluminum mirror has a thickness of 150 nm, and the nickel electrode has a thickness of either 38 or 50 nm. The plasma frequencies of aluminum, palladium, and nickel are, respectively, $\omega_p(\mathrm{Al}) = 15$ eV, $\omega_p(\mathrm{Pd}) = 7.4$ eV, and $\omega_p(\mathrm{Ni}) = 9.5$ eV. The corresponding length scales are $1/\omega_p(\mathrm{Al}) = 14$ nm, $1/\omega_p(\mathrm{Pd}) = 22$ nm, and $1/\omega_p(\mathrm{Ni}) = 27$ nm.

The measured current flows through the layer of the insulator, possibly by quantum tunneling. The distance of this layer from the aluminum mirror is $z_0 \approx d_C$, and in all cases $\omega_p(\mathrm{Al}) z_0 > 1$. Hence, we may approximate the aluminum mirror as a perfect mirror for the purpose of estimating its effect on the electric field fluctuations at the M-I-M location. The palladium electrode may be viewed as approximately transparent because $\omega_p(\mathrm{Pd}) d_E \ll 1$. The effect of the nickel electrode is difficult to assess. It is too close to the insulator to be treated as a perfect mirror, as $\omega_p(\mathrm{Ni}) d_I \ll 1$, but its effect could be significant.

The applied potential differences between the nickel and palladium electrodes in Ref. [9] are of the order of 0.1 mV, which would produce kinetic energies of the order of $K \approx 10^{-4}$ eV for freely accelerating electrons. We will use this value of K and set $z_0 = 33$ nm in Eq. (35) to estimate the magnitude of the electron energy fluctuations to be of the order of

$$\Delta U_{\mathrm{rms}} \approx 0.06 K \approx 6 \times 10^{-6}\ \mathrm{eV}. \tag{40}$$

The corresponding potential differences in Ref. [8] are of the order of 0.2 V, which leads to the estimate

$$\Delta U_{\mathrm{rms}} \approx 0.0013 K \approx 2.5 \times 10^{-4}\ \mathrm{eV}. \tag{41}$$

It is unclear whether either of these are large enough to explain the results for the current flowing through the insulator discussed in Refs. [8,9]. A more detailed model of the potential barrier involved is needed. It should also be noted that Ref. [9] seems to find a small current even at zero applied voltage. There does not seem to be a plausible explanation of this effect in terms of vacuum electric field fluctuations.

V. FINITE TEMPERATURE EFFECTS

Thus far, we have assumed that the quantized electromagnetic field is in the Casimir vacuum state at zero temperature. Here we wish to estimate the magnitude of the thermal corrections to the electric field fluctuations. We consider a single perfectly reflecting plate here, as in Sec. III A, but now assume a thermal bath of photons at temperature T. We use the well-known result [15] in which a finite temperature two-point function may be constructed by requiring periodicity in imaginary time, with period $\beta = 1/(k_B T)$, where k_B is Boltzmann's constant. For a review, see Ref. [16].

This may be done explicitly as an image sum of the form

$$\langle E^z(t,z) E^z(t',z')\rangle_\beta = \frac{1}{\pi^2} \sum_{n=-\infty}^{\infty} \frac{1}{[(t-t'+in\beta)^2 - (z+z')^2]^2}. \tag{42}$$

Here the $n = 0$ term is the zero temperature function given in Eq. (3), and the $n \neq 0$ terms describe the finite temperature corrections. Again we consider a particle with electric charge q moving perpendicularly to the plate at a speed v from $z = z_0$ to $z = z_0 + b$. The variance of the particle's energy fluctuations is given by an integral of the finite temperature correlation function analogous to that in Eq. (5). We may obtain the contribution to this variance coming from finite temperature effects by omitting the $n = 0$ term in the sum. Denote this contribution by $\langle (\Delta U)^2 \rangle_T$. It may be expressed as

$$\langle (\Delta U)^2 \rangle_T = \frac{q^2 v^4}{\pi^2} {\sum_n}' I_\beta(z_0, b, v, n). \tag{43}$$

Here the prime on the summation indicates a sum on all nonzero integers and

$$I_\beta(z_0, b, v, n) = \int_{z_0}^{z_0+b} dz \int_{z_0}^{z_0+b} dz' \, \frac{1}{[(z - z' + in\beta v)^2 - v^2 (z + z')^2]^2}. \tag{44}$$

Unlike in the zero temperature case, the integrand in Eq. (44) is now free of singularities for $n \neq 0$, and the integration is more straightforward. We may simplify the integration by assuming (1) $v \ll 1$ and (2) $\beta \gg z_0$. In this case, we may drop the $v^2 (z + z')^2$ term in the denominator and write

$$I_\beta(z_0, b, v, n) \approx \int_{z_0}^{z_0+b} dz \int_{z_0}^{z_0+b} dz' \frac{1}{(z - z' + in\beta v)^4} = \frac{1}{6(in\beta v + b)^2} + \frac{1}{6(in\beta v - b)^2} + \frac{1}{3n^2\beta^2 v^2}. \tag{45}$$

If $b \ll \beta v$, we may expand this result to lowest order in b, with the result

$$I_\beta(z_0, b, v, n) \approx \frac{b^2}{(v\beta n)^4} + O(b^4). \tag{46}$$

In contrast to the corresponding result for zero temperature, Eq. (12), $I_\beta(z_0, b, v, n)$ vanishes as $b \to 0$. Next we use the above form in Eq. (43) and employ the identity

$$\sum\nolimits_n{}' \frac{1}{n^4} = 2\zeta(4) = \frac{\pi^4}{45}, \tag{47}$$

where ζ denotes the Riemann zeta function. The result is

$$\langle(\Delta U)^2\rangle_T \approx \frac{\pi^2 q^2 b^2}{45\beta^4}. \tag{48}$$

The ratio of this expression to the zero temperature result, Eq. (15), gives a fractional measure of the magnitude of the thermal effect:

$$r = \frac{\langle(\Delta U)^2\rangle_T}{\langle(\Delta U)^2\rangle_0} = \frac{4\pi^4}{45}\left(\frac{bz_0}{v\beta^2}\right)^2. \tag{49}$$

Let us estimate this ratio for the case of the experiment described in Sec. IV. At room temperature, $T = 300$ K, $\beta \approx 7.6$ μm. Take $b = 2.3$ nm and $z_0 = 33$ nm. If the charged particles are electrons with a kinetic energy of $K = 0.2$ eV, and hence speeds of $v = 8.9 \times 10^{-4}$, then we find that $r \approx 1.9 \times 10^{-5}$. In this case, the thermal effect is very small. In the case in which $K = 10^{-3}$ eV, $v \approx 2.0 \times 10^{-5}$, leading to $r \approx 0.0038$, which is still a relatively small thermal correction. Note that here we are discussing the effects of finite temperature on electric field fluctuations, and not the effect of thermal activation coming from the thermal kinetic energy of the particles. The latter effect could be important at room temperature, depending upon the details of the potential barrier, but seems unlikely to depend upon the distance to a mirror, as was observed in the Moddel and co-workers [8,9] experiment. We have restricted our attention to the case of a single perfectly reflecting plate. The study of the thermal corrections in the cases of two plates and of finite reflectivity is a topic for future work.

VI. SUMMARY

In this paper, we examined the effects of vacuum electric field fluctuations on a charged particle moving perpendicularly to one or two perfectly reflecting plates. This was done by integrating the electric field correlation function in the Casimir vacuum along a segment of the particle's worldline and results in expressions describing fluctuations in the voltage difference along the segment and in the particle's energy. We then considered the possibility that these energy fluctuations could be linked to enhanced quantum tunneling through a potential barrier. The possibility that this effect has already been observed was discussed.

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