Asymptotic behavior of null geodesics near future null infinity. II. Curvatures, photon surface, and dynamically transversely trapping surface

Masaya Amo,[1] Tetsuya Shiromizu,[2,3] Keisuke Izumi,[3,2] Hirotaka Yoshino,[4,5] and Yoshimune Tomikawa[6]

[1]*Center for Gravitational Physics, Yukawa Institute for Theoretical Physics, Kyoto University, 606-8502 Kyoto, Japan*
[2]*Department of Mathematics, Nagoya University, Nagoya 464-8602, Japan*
[3]*Kobayashi-Maskawa Institute, Nagoya University, Nagoya 464-8602, Japan*
[4]*Advanced Mathematical Institute, Osaka City University, Osaka 558-8585, Japan*
[5]*Department of Physics, Kobe University, Kobe 657-8501, Japan*
[6]*Faculty of Economics, Matsuyama University, Matsuyama 790-8578, Japan*

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Bearing in mind our previous study on asymptotic behavior of null geodesics near future null infinity, we analyze the behavior of geometrical quantities such as a certain extrinsic curvature and Riemann tensor in the Bondi coordinates. In the sense of asymptotics, the condition for an r-constant hypersurface to be a photon surface is shown to be controlled by a key quantity that determines the fate of photons initially emitted in angular directions. In four dimensions, such a nonexpanding photon surface can be realized even near future null infinity in the presence of enormous energy flux for a short period of time. By contrast, in higher-dimensional cases, no such a photon surface can exist. This result also implies that the dynamically transversely trapping surface, which is proposed as an extension of a photon surface, can have an arbitrarily large radius in four dimensions.

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I. INTRODUCTION

In recent years, there have been unprecedented reports of various observations on black holes: the gravitational-wave observations (see [1] for the first detection) and the shadow imaging [2]. In the discussion of systems far away from massive objects, as in black hole observations, ideal observers are considered to stay at future null infinity in asymptotically flat spacetimes, which are formulated precisely in Refs. [3–11]. Thus, it is important to understand the properties of the asymptotic structure near future null infinity. Although the spacetime asymptotes to the Minkowski spacetime near infinity, there are nontrivial features of the asymptotics. One of them is the supertranslation, which gives an infinite number of independent generators for symmetries [3,4]. By contrast with the cases in four dimensions, however, supertranslations are absent in higher dimensions if one supposes the finiteness of global charges such as mass [11,12]. Recently, supertranslations attract much attention in several topics including the memory effect [13–15] and the soft theorem [16].

In our previous paper [17], meanwhile, the authors examined null geodesics that correspond to worldlines of photons emitted in angular directions of the Bondi coordinates near future null infinity. Surprisingly, there exists a nontrivial difference between four and higher dimensions (see Ref. [18] for the extension for Brans-Dicke theory). In higher dimensions, any of these null geodesics always reaches future null infinity. In four dimensions, by contrast, it is not guaranteed: Gravitational waves and the flow of matter energy could affect the fate of the null geodesics (see Sec. II for a brief review and/or Ref. [17] for detail).

The behavior of geodesics is imprinted in the geometrical quantities, namely curvatures. One of the examples is the photon sphere, which describes the unstable circular orbits of photons in the Schwarzschild black hole spacetime. That is to say, although the photon sphere indicates a collection of specific null geodesics, equivalent conditions can be given with the geometrical quantities as well [19,20]. The concept of the photon sphere has been extended to general spacetimes [20], called the photon surface, which is the collection of certain (not necessarily circular) photon orbits. The generalization of the photon surface has been widely discussed [21–25] (see also Ref. [26] for the stability for the photon surface). In this paper, such an existing idea for photon spheres (or photon surfaces) is applied to the cases for the asymptotic behavior of null geodesics analyzed in our previous work [17]; that is, we shall show the properties of the asymptotic null geodesics emitted in the angular direction near future null infinity in terms of the extrinsic curvature and the Riemann tensor. Our previous work [17] suggests that there is an essential difference between the cases of four and higher dimensions. We will see similar differences in the geometrical quantities; i.e., their nontrivial asymptotic features can be seen only in four-dimensional cases.

It is known that a hyperboloid in the Minkowski spacetime is a photon surface, and thus, an arbitrarily large expanding photon surface can be introduced in general [20]. In this paper, in contrast to such an expanding photon surface, we focus on the condition for an r-constant hypersurface to be a photon surface (say, a nonexpanding photon surface). The formation of such a photon surface is fairly nontrivial, implying the existence of strong gravity. We will see in four dimensions that nonexpanding photon surfaces can exist (locally) in the asymptotic region if enormous outgoing energy flux is present. In addition, we will show that the nonexpanding photon surface described above is, at the same time, the dynamically transversely trapping surface (DTTS) proposed by four of us in order to describe the strong gravity region in terms of behavior of photons [24]. This indicates that a DTTS with an arbitrarily large radius can form near future null infinity. Most of these studies will be done in an approximate way by adopting the leading-order terms in the powers of $1/r$, but an exact analysis using the Vaidya spacetime will also be briefly reported.

The rest of this paper is organized as follows. In Sec. II, we give a brief review of asymptotically flat spacetimes in terms of the Bondi coordinates and our previous work [17]. In Sec. III, we show the extrinsic curvature of the surface given by a constant radial coordinate near future null infinity and discuss the presence of an approximate photon surface. In Sec. IV, we see the fact that the result of Sec. III implies that an arbitrarily large DTTS can form near future null infinity. In Sec. V, we show asymptotic behavior of the Riemann tensor. Section VI is devoted to a summary and discussion. In Appendix, we present the calculations for a Vaidya spacetime.

II. ASYMPTOTIC BEHAVIOR OF SPACETIME AND NULL GEODESICS

In this section, we briefly review the spacetime behavior near future null infinity following Refs. [3,4,11] (see also [8,9,27]) and our previous results on null geodesics [17].

Let $n(\geq 4)$ be the dimension of spacetimes. For asymptotically flat spacetimes, the metric near future null infinity is written in the Bondi coordinates as

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -Ae^B du^2 - 2e^B du dr + h_{IJ}r^2(dx^I + C^I du)(dx^J + C^J du), \quad (1)$$

where the Greek indices denote the spacetime components. A, B, C^I, and h_{IJ} are functions of u, r, and x^I. Here, x^I stands for the angular coordinates. Future null infinity is supposed to be in the limit of $r \to \infty$, while u is finite. We impose the gauge condition as

$$\sqrt{\det h_{IJ}} = \omega_{n-2}, \quad (2)$$

where ω_{n-2} is the volume element of the unit $(n-2)$-dimensional sphere S^{n-2}. The functions A, B, C^I, and h_{IJ} can be expanded with respect to power of $1/r$, whose explicit formulas are shown in Ref. [11]. The vacuum Einstein equation shows us that the nonzero components of the metric behave as[1]

$$\begin{aligned} g_{uu} &= -Ae^B + h_{IJ}C^I C^J r^2 = -1 - A^{(1)} r^{-(n/2-1)} + m r^{-(n-3)} + \mathcal{O}(r^{-(n-1)/2}), \\ g_{ur} &= -e^B = -1 - B^{(1)} r^{-(n-2)} + \mathcal{O}(r^{-(n-3/2)}), \\ g_{IJ} &= h_{IJ} r^2 = \omega_{IJ} r^2 + h^{(1)}_{IJ} r^{-(n/2-3)} + \mathcal{O}(r^{-(n-5)/2}), \\ g_{uI} &= h_{IJ} C^J r^2 = C^{(1)}{}_I r^{-(n/2-2)} + \mathcal{O}(r^{-(n-3)/2}), \end{aligned} \quad (3)$$

where ω_{IJ} denotes the metric of the unit $(n-2)$-dimensional sphere. In general relativity, the integration of $m(u, x^I)$ over the solid angle gives the Bondi mass,

$$M(u) := \frac{n-2}{16\pi} \int_{S^{n-2}} m d\Omega. \quad (4)$$

We can also apply the falloff behavior of Eq. (3) to nonvacuum spacetimes if the falloff behavior of the stress-energy tensor is sufficiently fast such that the lowest order of the Einstein tensor behaves as $G_{\mu\nu} = \mathcal{O}(r^{-n/2})$ in the coordinate system, which asymptotes to the Cartesian coordinate system near null infinity. For example, the stress-energy tensor of the Maxwell field in even dimensions behaves as $T_{\mu\nu} = \mathcal{O}(r^{-(n-2)})$ in the same coordinate, and then the current

[1] In even dimensions, each exponent in the Landau symbol of Eq. (3) actually has the higher order by $r^{1/2}$, but we write it in the same way as in odd dimensions for unification. Note that quantities below, such as the Christoffel symbols, do not have half-integer powers with respect to $1/r$ in even dimensions.

setup works. The example in Appendix also satisfies the falloff behavior of the stress-energy tensor for the $n = 4$ case. We list some components of the inverse metric and the Christoffel symbols, which we will use later:

$$g^{rr} = 1 + A^{(1)} r^{-(n/2-1)} - m r^{-(n-3)} + \mathcal{O}(r^{-(n-1)/2}), \tag{5}$$

and

$$\begin{aligned}
\Gamma^r_{uu} &= \frac{1}{2}\dot{A}^{(1)} r^{-(n/2-1)} - \frac{1}{2}\dot{m} r^{-(n-3)} + \mathcal{O}(r^{-(n-1)/2}), \\
\Gamma^r_{ur} &= -\frac{n-2}{4} A^{(1)} r^{-n/2} + \frac{n-3}{2} m r^{-(n-2)} + \mathcal{O}(r^{-(n+1)/2}), \\
\Gamma^r_{rr} &= -(n-2) B^{(1)} r^{-(n-1)} + \mathcal{O}(r^{-(n-1/2)}), \\
\Gamma^r_{uI} &= \left(\frac{n-4}{4} C^{(1)}_I + \frac{1}{2} A^{(1)}_{,I}\right) r^{-(n/2-1)} + \left(-\frac{1}{2} m_{,I} + \frac{1}{2} C^{(1)J} \dot{h}^{(1)}_{IJ}\right) r^{-(n-3)} + \mathcal{O}(r^{-(n-1)/2}), \\
\Gamma^r_{rI} &= \frac{n}{4} C^{(1)}_I r^{-(n/2-1)} + \mathcal{O}(r^{-(n-1)/2}), \\
\Gamma^r_{IJ} &= -\omega_{IJ} r + \frac{1}{2} \dot{h}^{(1)}_{IJ} r^{-(n/2-3)} + \mathcal{O}(r^{-(n-5)/2}),
\end{aligned} \tag{6}$$

where $C^{(1)}_I \coloneqq C^{(1)J}\omega_{IJ}$, and $A^{(1)}$ is set to be zero for $n = 4$ because it is absorbed into m. The variables with dots, such as $\dot{A}^{(1)}$, denote their derivatives with respect to u.

We will look at the asymptotic behavior of null geodesics near future null infinity (see our previous paper [17] for the details). Let us focus on null geodesics that correspond to worldlines of photons emitted in the tangential directions to the r-constant surfaces near future null infinity, i.e., the ones with $r' = 0$, where $'$ denotes the derivative with respect to the affine parameter. At the emission point, the r-component of the geodesic equation is calculated as

$$\begin{aligned}
r''|_{r'=0} &= -\Gamma^r_{\mu\nu}(x^\mu)'(x^\nu)' \\
&= \left[\omega_{IJ} r - \frac{1}{2}\dot{h}^{(1)}_{IJ} r^{-(n/2-3)} + \frac{1}{2}\dot{m}\omega_{IJ} r^{-(n-5)} \right. \\
&\quad \left. -\frac{1}{2}\dot{A}^{(1)}\omega_{IJ} r^{-(n/2-3)} + \mathcal{O}(r^{-(n-5)/2})\right](x^I)'(x^J)', \end{aligned} \tag{7}$$

by using the null condition $g_{\mu\nu}(x^\mu)'(x^\nu)' = 0$, the future directed condition $u' \geq 0$, and Eq. (6). In four dimensions, Eq. (7) becomes

$$r''|_{r'=0} = [\Omega_{IJ} r + \mathcal{O}(r^0)](x^I)'(x^J)', \tag{8}$$

where Ω_{IJ} is defined as

$$\Omega_{IJ} \coloneqq \omega_{IJ} - \frac{1}{2}\dot{h}^{(1)}_{IJ} + \frac{1}{2}\dot{m}\omega_{IJ}. \tag{9}$$

Therefore, even at sufficiently large r, the trajectory of a photon is not approximated by that in the flat spacetime but determined by Ω_{IJ}. Moreover, the sign of r'' is determined by that of the eigenvalues of Ω_{IJ}, and thus, r'' can be negative. In Ref. [17], we have proved that the null geodesics will reach future null infinity provided that Ω_{IJ} is positive definite and $\dot{m} \leq 0$ in four dimensions. Unless these conditions are satisfied, the null geodesics with the same initial conditions may not be able to reach future null infinity. One may expect that $\dot{h}^{(1)}_{IJ}$ and $\dot{m}$ would be sufficiently small and Ω_{IJ} would be positive definite for almost all situations. However, the possibility that photons do not reach future null infinity would be fairly surprising.

In the case $n \geq 5$, Eq. (7) becomes

$$r''|_{r'=0} = [\omega_{IJ} r + \mathcal{O}(r^{-(n/2-3)})](x^I)'(x^J)'. \tag{10}$$

This implies that the trajectory of photons is approximated by that in the flat spacetime, and any null geodesic always reach future null infinity [17].

III. EXTRINSIC CURVATURE AND PHOTON SURFACE

The behavior of geodesics depends on the geometry of a spacetime. This gives us an expectation that the properties corresponding to the nontrivial asymptotic behavior of null geodesics described in the previous section can be seen in the geometric quantities. In the discussion of the photon sphere (or the photon surface), such a relation has been shown [20]. Photon surfaces are defined to be nowhere-spacelike codimension-one hypersurface S such that, for every point $p \in S$ and every null vector $k^a \in T_pS$ (small latin indices run the coordinate u and angular coordinate indices I), a null geodesic γ tangent to k^a at p is included in S at least for a finite section of the geodesics around p. Alternatively, an equivalent condition of the photon surface is given in terms of the geometric quantities; i.e., S is a photon surface if and only if $\chi_{ab}k^a k^b = 0$ holds for all null tangent vectors k^a to S, where χ_{ab} is the extrinsic curvature.

This condition is also equivalent to the condition that the surface is umbilical, i.e., vanishing of the traceless part of the extrinsic curvature, σ_{ab}, of S, i.e., $\sigma_{ab} = 0$ [20,28]. Similarly, the nontrivial behavior of asymptotic null geodesics found in four dimensions would be reflected in the asymptotic behavior of the geometric quantities, and it is natural to expect that the r-constant hypersurface with vanishing Ω_{IJ} defined by Eq. (9) becomes a photon surface.

We begin this section with the calculation of the extrinsic curvature χ_{ab} of the r-constant hypersurface S_r with sufficiently large r. The induced metric on S_r is given by

$$P_{ab}dx^a dx^b = -Ae^B du^2 + h_{IJ}r^2(dx^I + C^I du)(dx^J + C^J du); \tag{11}$$

that is, each component of the metric is written in

$$\begin{aligned} P_{uu} &= -Ae^B + h_{IJ}C^I C^J r^2 \\ &= -1 - A^{(1)}r^{-(n/2-1)} + mr^{-(n-3)} + \mathcal{O}(r^{-(n-1)/2}), \end{aligned} \tag{12}$$

$$P_{uI} = h_{IJ}C^J r^2 = C_I^{(1)} r^{-(n/2-2)} + \mathcal{O}(r^{-(n-3)/2}), \tag{13}$$

$$P_{IJ} = h_{IJ}r^2 = \omega_{IJ}r^2 + h_{IJ}^{(1)} r^{-(n/2-3)} + \mathcal{O}(r^{-(n-5)/2}). \tag{14}$$

For later convenience, we also write down the inverse of P_{ab},

$$\begin{aligned} P^{uu} &= -A^{-1}e^{-B} = -1 + A^{(1)}r^{-(n/2-1)} - mr^{-(n-3)} \\ &\quad + \mathcal{O}(r^{-(n-1)/2}), \end{aligned} \tag{15}$$

$$P^{uI} = A^{-1}e^{-B}C^I = C^{(1)I} r^{-n/2} + \mathcal{O}(r^{-(n+1)/2}), \tag{16}$$

$$\begin{aligned} P^{IJ} &= h^{IJ}r^{-2} - A^{-1}e^{-B}C^I C^J \\ &= [\omega^{IJ} - h^{(1)IJ}r^{-(n/2-1)} + \mathcal{O}(r^{-(n-1)/2})]r^{-2}. \end{aligned} \tag{17}$$

The extrinsic curvature of the r-constant hypersurface is

$$\chi_{ab} \coloneqq \nabla_a r_b = -\frac{1}{\sqrt{g^{rr}}}\Gamma^r_{ab}, \tag{18}$$

where we used the fact that the outward unit normal vector r_a is written as $r_a = (g^{rr})^{-1/2}(dr)_a$ in the Bondi coordinates. Using Eqs. (5) and (6), the components of χ_{ab} are calculated as

$$\begin{aligned} \chi_{uu} &= -[1 + \mathcal{O}(r^{-1})]\Gamma^r_{uu} \\ &= -\frac{1}{2}\dot{A}^{(1)}r^{-(n/2-1)} + \frac{1}{2}\dot{m}r^{-(n-3)} + \mathcal{O}(r^{-(n-1)/2}), \end{aligned} \tag{19}$$

$$\begin{aligned} \chi_{uI} &= -[1 + \mathcal{O}(r^{-1})]\Gamma^r_{uI} = -\left(\frac{n-4}{4}C_I^{(1)} + \frac{1}{2}A_{,I}^{(1)}\right)r^{-(n/2-1)} \\ &\quad + \left(\frac{1}{2}m_{,I} - \frac{1}{2}C^{(1)J}\dot{h}_{IJ}^{(1)}\right)r^{-(n-3)} + \mathcal{O}(r^{-(n-1)/2}), \end{aligned} \tag{20}$$

$$\begin{aligned} \chi_{IJ} &= -[1 + \mathcal{O}(r^{-1})]\Gamma^r_{IJ} \\ &= \omega_{IJ}r - \frac{1}{2}\dot{h}_{IJ}^{(1)}r^{-(n/2-3)} + \mathcal{O}(r^{-(n-5)/2}). \end{aligned} \tag{21}$$

In the current setup, it is easy to see

$$r''|_{r'=0} = -\Gamma^r_{\mu\nu}k^\mu k^\nu = [1 + \mathcal{O}(r^{-1})]\chi_{ab}k^a k^b \tag{22}$$

on S_r. We can see the relation between the extrinsic curvature χ_{ab} and the behavior of null geodesic momentarily tangent to S_r. Equation (8) shows that, in four dimensions, the behavior of null geodesics cannot be approximated by that in the flat spacetime because of the existence of $\dot{m}$ and $\dot{h}_{IJ}^{(1)}$ in Ω_{IJ}. We can see in Eqs. (18)–(21) that the nontrivial asymptotic properties are imprinted in the extrinsic curvature χ_{ab} in four dimensions: In the leading order of $\chi_{ab}k^a k^b$, $\dot{m}$ and $\dot{h}_{IJ}^{(1)}$ appear together with $\chi_{ab}k^a k^b$ of the flat spacetime. We have seen, moreover, that r'' becomes negative if and only if $\Omega_{IJ}k^I k^J$ is negative, and then r decreases at least for a short period of time. Here, k^I is composed of the angular components of a null vector k^a tangent to S_r. Equations (8) and (22) show that the condition for the negativity of $\Omega_{IJ}k^I k^J$ corresponds to that of $\chi_{ab}k^a k^b$. By contrast, in higher dimensions, $\chi_{ab}k^a k^b$ is always positive, which is consistent with the fact that any null geodesics always reach future null infinity. The leading contributions are the same as those in the flat spacetime, and thus the extrinsic curvature can be approximated by that of the flat spacetime.

Let us see the condition where the r-constant hypersurface becomes a photon surface. It is equivalent to the traceless part of the extrinsic curvature vanishes ($\sigma_{ab} = 0$) or $\chi_{ab}k^a k^b = 0$ for any null vector k^a tangent to S_r. The traceless part of the extrinsic curvature is written in

$$\sigma_{ab} \coloneqq \chi_{ab} - \frac{1}{n-1}\chi P_{ab}, \tag{23}$$

where χ denotes the trace part of χ_{ab}. Each components of σ_{ab} is evaluated as

$$\begin{aligned} \sigma_{uu} &= \frac{n-2}{n-1}r^{-1} - \frac{n-2}{2(n-1)}\dot{A}^{(1)}r^{-(n/2-1)} \\ &\quad + \frac{n-2}{2(n-1)}\dot{m}r^{-(n-3)} + \mathcal{O}(r^{-(n-1)/2}), \end{aligned} \tag{24}$$

$$\sigma_{uI} = -\left(\frac{n^2-n-4}{4(n-1)}C_I^{(1)} + \frac{1}{2}A_{,I}^{(1)}\right)r^{-(n/2-1)} + \left(\frac{1}{2}m_{,I} - \frac{1}{2}C^{(1)J}\dot{h}_{IJ}^{(1)} - \frac{1}{2(n-1)}\dot{A}^{(1)}C_I^{(1)}\right)r^{-(n-3)} + \frac{1}{2(n-1)}\dot{m}C_I^{(1)}r^{-(3n/2-5)} + \mathcal{O}(r^{-(n-1)/2}), \quad (25)$$

$$\sigma_{IJ} = \frac{1}{n-1}\omega_{IJ}r - \frac{1}{2}\dot{h}_{IJ}^{(1)}r^{-(n/2-3)} - \frac{1}{2(n-1)}\dot{A}^{(1)}\omega_{IJ}r^{-(n/2-3)} + \frac{1}{2(n-1)}\dot{m}\omega_{IJ}r^{-(n-5)} + \mathcal{O}(r^{-(n-5)/2}). \quad (26)$$

Motivated by the property of the photon surface that $\sigma_{ab} = 0$, we call a retarded time interval $u_i < u < u_f$ of the r-constant hypersurface S_r near future null infinity an "approximate photon surface" when $\sigma_{ab} = 0$ at the leading order in r^{-1} expansion on this retarded time interval of S_r. One can see that the leading-order contributions of σ_{ab} are generically nonzero, which means that an approximate photon surface does not exist in general near future null infinity. This is quite reasonable. However, only in four dimensions, there is a possibility that the leading-order contributions are canceled with each other, and the results in the previous section suggest that the photon surface would appear if Ω_{IJ} vanishes.

In four dimensions, σ_{ab} becomes

$$\sigma_{uu} = \left(\frac{2}{3} + \frac{1}{3}\dot{m}\right)r^{-1} + \mathcal{O}(r^{-2}), \quad (27)$$

$$\sigma_{uI} = \sigma_{Iu} = \mathcal{O}(r^{-1}), \quad (28)$$

$$\sigma_{IJ} = \left(\frac{1}{3}\omega_{IJ} - \frac{1}{2}\dot{h}_{IJ}^{(1)} + \frac{1}{6}\dot{m}\omega_{IJ}\right)r + \mathcal{O}(r^0). \quad (29)$$

The condition for the approximate photon surface, i.e., for $\sigma_{ab} = 0$ at the leading order, is that

$$\dot{m} = -2 \quad \text{and} \quad \dot{h}_{IJ}^{(1)} = 0 \quad (30)$$

are satisfied.[2] Note that these conditions are not satisfied simultaneously for vacuum spacetimes in general relativity. $\dot{m} = -2$ describes the existence of outgoing matter flux, which determines the rate of change of the Bondi mass. $\dot{h}_{IJ}^{(1)} = 0$ describes the absence of gravitational wave radiation. From the definition of $\Omega_{IJ} := \omega_{IJ} - \frac{1}{2}\dot{h}_{IJ}^{(1)} + \frac{1}{2}\dot{m}\omega_{IJ}$, we see that it is equivalent to

$$\Omega_{IJ} = 0. \quad (31)$$

Here, we used $\omega^{IJ}h_{IJ}^{(1)} = 0$, which is a consequence of the gauge condition Eq. (2). As seen in the previous section, it indicates that r'' vanishes if $r' = 0$; that is, r does not change and thus geodesics stay on S_r.

By contrast, in dimensions higher than four, the traceless part of the extrinsic curvature σ_{ab} becomes

$$\sigma_{uu} = \frac{n-2}{n-1}r^{-1} + \mathcal{O}(r^{-(n/2-1)}), \quad (32)$$

$$\sigma_{uI} = \sigma_{Iu} = \mathcal{O}(r^{-(n/2-1)}), \quad (33)$$

$$\sigma_{IJ} = \frac{1}{n-1}\omega_{IJ}r + \mathcal{O}(r^{-(n/2-3)}). \quad (34)$$

All of the leading terms of σ_{ab} do not vanish, and they are the same as the components of σ_{ab} in the flat spacetime. Thus, there is no approximate photon surface for spacetimes whose dimensions are higher than four.

Note that, since the above study only took account of the leading order terms, the photon surface is an approximate one: The r-constant surface satisfies the condition for the photon surface with the error of $\mathcal{O}(1/r)$. As discussed in Appendix, for an outgoing Vaidya solution, which is an exact solution representing the spherically symmetric with the null outgoing matter, it is possible to make a situation where an r-constant hypersurface becomes a photon surface exactly. This requires a fine-tuning of the behavior of the Bondi mass $M(u) = m(u)/2$.

We emphasize again that the difference of the behavior of the null geodesics near null infinity between four dimensions and higher dimensions arises even when $|\dot{m}|$ and $\dot{h}_{IJ}^{(1)}$ are not so large as $\dot{m} \sim -2$. The trajectory is affected even by small $\dot{m}$ and $\dot{h}_{IJ}^{(1)}$ in four dimensions but is not in higher dimensions. This is similar to the memory effect in the sense that both of them lead to critical differences between four dimensions and higher dimensions due to the asymptotic behavior. We have shown here that this effect is understood in terms of the extrinsic curvature.

IV. DTTS FORMATION NEAR FUTURE NULL INFINITY

We have seen in Sec. III that in a four-dimensional spacetime, the behavior of null geodesics can be drastically different from that in the Minkowski spacetime due to the difference of the extrinsic curvature, and the r-constant hypersurface becomes an approximate photon surface if the conditions of Eq. (30) are satisfied. This immediately implies that the formation of a dynamically transversely trapping surface (DTTS) near future null infinity is possible as well in four dimensions. The DTTS has been defined as an extension of a photon surface by four of us in Ref. [24]. The DTTS has an analogy with an apparent horizon, and

[2] In the normalized basis, $\sigma_{\hat{u}\hat{u}} = \mathcal{O}(r^{-1})$, $\sigma_{\hat{u}\hat{I}} = \mathcal{O}(r^{-2})$, and $\sigma_{\hat{I}\hat{J}} = \mathcal{O}(r^{-1})$, where the hatted indices denote the components with respect to the normalized basis. Since the order of $\sigma_{\hat{u}\hat{I}}$ is higher compared the others, there is no requirement from σ_{uI}.

thus, it is calculable on a spacelike hypersurface in generic spacetimes. To be specific, an $(n-2)$-dimensional closed spacelike surface σ_0 is called a DTTS if there is an $(n-1)$-dimensional timelike surface S that contains σ_0 and satisfies the following three conditions on σ_0:

$$\bar{k} = 0, \tag{35}$$

$$\max(\bar{K}_{ab}k^a k^b) = 0, \tag{36}$$

$$^{(3)}\bar{\pounds}_{\bar{n}}\bar{k} \leq 0. \tag{37}$$

Here, we span the time coordinate t starting from σ_0 in S so that its lapse function is constant on each t-constant surface, and $\bar{k}$ denotes the trace of the extrinsic curvature of t-constant surfaces in S. $\bar{K}_{ab}$ is the extrinsic curvature of S, and k^a is an arbitrary null vector tangent to S. $^{(3)}\bar{\pounds}_{\bar{n}}$ denotes the Lie derivative with respect to the unit normal $\bar{n}$ to σ_0 in S. If the equality in the inequality of Eq. (37) holds, σ_0 is called a marginally DTTS.

The physical meaning of this definition is as follows. From σ_0, we emit photons in tangential directions to S. The condition of Eq. (35) means that S is chosen so that σ_0 is an extremal surface in S. The condition of Eq. (36) determines how the surface S bends in the neighborhood of σ_0: The emitted photons must propagate inside of S or on S, and for each point on σ_0, at least one photon must propagate on S. Then, we consider the time slice of S, and the condition of Eq. (37) implies that σ_0 is a DTTS if σ_0 is a maximal surface.

We call a section of a u-constant surface $S_{u,r}$ in S_r near future null infinity an "approximate marginally DTTS" when Eqs. (35) and (36) and the equality in the inequality of Eq. (37) are satisfied at the leading order in r^{-1} expansion. Note that the DTTS, σ_0, is supposed to be a closed surface in the definition to exclude a trivial one such as a plane in flat spacetime, but we do not explicitly impose this condition for the approximate DTTS because by ignoring higher-order contributions, $S_{u,r}$ is almost closed with a coarse-grained sense although it may not be necessarily closed. For a spherically symmetric spacetime discussed in Appendix, one can construct a closed one.

Let us confirm that the u-constant surface $S_{u,r}$ in the approximate photon surface S_r constructed above in the four-dimensional case is an approximate marginally DTTS.[3] Since the photon surface S_r satisfies $\bar{K}_{ab}k^a k^b = 0$ (here, $\bar{K}_{ab} = \chi_{ab}$), the u-constant surface satisfies the condition of Eq. (36). For the family of u-constant hypersurfaces, the trace $\bar{k}$ of the extrinsic curvature in S_r is

$$\bar{k} = {}^{(3)}\bar{\pounds}_{\bar{n}} \log\sqrt{\det(r^2 h_{IJ})} = \mathcal{O}(r^{-2}) \tag{38}$$

from the gauge condition of Eq. (2). Here, we used $\bar{n} = (-P^{uu})^{-1/2}\partial_u - P^{Iu}(-P^{uu})^{1/2}\partial_I \simeq [1+\mathcal{O}(r^{-1})]\partial_u - C^{(1)I} r^{-2}\partial_I$. Then, the condition of Eq. (35) is approximately satisfied. Although the lapse function of the u coordinate on S_r is not constant on each u-constant surface in general, the change in its value is $\mathcal{O}(1/r)$, and the condition of Eq. (37) is also approximately satisfied. Therefore, the section of u-constant and S_r is an approximate marginally DTTS.

The above approximate study can be performed exactly in the spherically symmetric Vaidya spacetime. Due to the spherical symmetry, the lapse function of the coordinate u is constant on each u-constant hypersurface. In this case, Eq. (38) becomes $\bar{k} = 0$. From the calculation of Appendix, the section of an r-constant hypersurface and a u-constant hypersurface becomes a DTTS if

$$\frac{dM}{du} \leq -\left(1 - \frac{2M}{r}\right)\left(1 - \frac{3M}{r}\right), \tag{39}$$

taking account of all orders with respect to the powers of $1/r$, where M is the Bondi mass defined in Eq. (4). This means that in four dimensions, an arbitrarily large DTTS can form. Although in Ref. [24], four of us have proved the Penrose-like inequality,

$$A \leq 4\pi(3GM_0)^2, \tag{40}$$

for the area A of any DTTS on a time-symmetric spacelike hypersurface with a certain condition such as negativity of the radial pressure, where M_0 is the Arnowitt-Deser-Misner (ADM) mass, this areal inequality does not hold in general. Enormously large energy flux, whose radial pressure is positive, can create a strong gravity field even near future null infinity in the sense of the effect on the motion of transversely emitted photons.

V. RIEMANN CURVATURE OF S_r

In this section, we shall examine the inner geometry of the r-constant surface S_r looking at the Riemann tensor for sufficiently large r. The calculations are done for general situation, i.e., in general dimensions with $\dot{h}^{(1)}_{IJ}$ being generically nonzero. The case where S_r becomes an approximate photon surface in four dimensions is briefly commented on.

After short calculations, we have the $(n-1)$-dimensional Riemann tensor $^{(n-1)}R^a{}_{bcd}$ as

[3] In Ref. [21], four of us defined a loosely trapped surface (LTS), which is another concept to characterize a strong gravity region. The LTS is defined as an $(n-2)$-dimensional surface on some spacelike hypersurface whose mean curvature k satisfies $dk/dr \geq 0$. If we introduce the spacelike hypersurface by $t = \text{constant}$ with $t = u + r$ in the Bondi coordinates, the mean curvature k of the r-constant surface is evaluated as $k \simeq 2/r - m/r^2$ and $dk/dr \simeq (-2 + \dot{m})/r^2$ cannot be non-negative.

$$ {}^{(n-1)}R^{u}{}_{IuJ} \simeq \partial_u^{(n-1)}\Gamma^{u}{}_{IJ} = \frac{1}{2}\ddot{h}^{(1)}_{IJ} r^{-(n/2-3)} + \mathcal{O}(r^{-(n-5)/2}), \tag{41} $$

$$ \begin{aligned} {}^{(n-1)}R^{u}{}_{IJK} &\simeq D_J^{(n-1)}\Gamma^{u}{}_{IK} - D_K^{(n-1)}\Gamma^{u}{}_{IJ} \\ &= \frac{1}{2}(D_J\dot{h}^{(1)}_{IK} - D_K\dot{h}^{(1)}_{IJ})r^{-(n/2-3)} + \mathcal{O}(r^{-(n-5)/2}), \end{aligned} \tag{42} $$

$$ \begin{aligned} {}^{(n-1)}R^{I}{}_{JKL} &= {}^{(\omega)}R^{I}{}_{JKL} + \frac{1}{4}(\dot{h}^{(1)I}{}_K\dot{h}^{(1)}_{JL} - \dot{h}^{(1)I}{}_L\dot{h}^{(1)}_{JK})r^{-(n-4)} \\ &\quad + \mathcal{O}(r^{-(n/2-1)}), \end{aligned} \tag{43} $$

where ${}^{(\omega)}R^{I}{}_{JKL}$ is the Riemann tensor of the $(n-2)$-dimensional round sphere; that is, ${}^{(\omega)}R_{IJKL} = \omega_{IK}\omega_{LJ} - \omega_{IL}\omega_{KJ}$. In the derivation of the above, the Christoffel symbol for the induced metric P_{ab} is required to be calculated,

$$ \begin{aligned} {}^{(n-1)}\Gamma^{u}_{uu} &\simeq \frac{1}{2}P^{uu}\partial_u P_{uu} \\ &= \frac{1}{2}\dot{A}^{(1)} r^{-(n/2-1)} - \frac{1}{2}\dot{m} r^{-(n-3)} + \mathcal{O}(r^{-(n-1)/2}), \end{aligned} \tag{44} $$

$$ \begin{aligned} {}^{(n-1)}\Gamma^{u}_{uI} &\simeq \frac{1}{2}P^{uu}\partial_I P_{uu} + \frac{1}{2}P^{uJ}\partial_u P_{IJ} \\ &= \frac{1}{2}A^{(1)}_{,I} r^{-(n/2-1)} - \frac{1}{2}m_{,I} r^{-(n-3)} \\ &\quad + \frac{1}{2}C^{(1)J}\dot{h}^{(1)}_{IJ} r^{-(n-3)} + \mathcal{O}(r^{-(n-1)/2}), \end{aligned} \tag{45} $$

$$ {}^{(n-1)}\Gamma^{u}_{IJ} \simeq -\frac{1}{2}P^{uu}\partial_u P_{IJ} = \frac{1}{2}\dot{h}^{(1)}_{IJ} r^{-(n/2-3)} + \mathcal{O}(r^{-(n-5)/2}), \tag{46} $$

$$ {}^{(n-1)}\Gamma^{I}_{uu} \simeq P^{IJ}\partial_u P_{Ju} = \dot{C}^{(1)I} r^{-n/2} + \mathcal{O}(r^{-(n+1)/2}), \tag{47} $$

$$ {}^{(n-1)}\Gamma^{I}_{uJ} \simeq \frac{1}{2}P^{IK}\partial_u P_{KJ} = \frac{1}{2}\dot{h}^{(1)I}{}_J r^{-(n/2-1)} + \mathcal{O}(r^{-(n-1)/2}), \tag{48} $$

$$ \begin{aligned} {}^{(n-1)}\Gamma^{I}_{JK} &\simeq \frac{1}{2}P^{IL}(\partial_J P_{LK} + \partial_K P_{LJ} - \partial_L P_{JK}) - \frac{1}{2}P^{Iu}\partial_u P_{JK} \\ &= {}^{(\omega)}\Gamma^{I}_{JK} + \Gamma^{(1)I}{}_{JK} r^{-(n/2-1)} - \frac{1}{2}C^{(1)I}\dot{h}^{(1)}_{JK} r^{-(n-3)} \\ &\quad + \mathcal{O}(r^{-(n-1)/2}), \end{aligned} \tag{49} $$

where ${}^{(\omega)}\Gamma^{I}_{JK}$ denotes the Christoffel symbol with respect to ω_{IJ}, $\Gamma^{(1)I}{}_{JK}$ is defined as

$$ \Gamma^{(1)I}{}_{JK} := \frac{1}{2}(D_J h^{(1)I}{}_K + D_K h^{(1)I}{}_J - D^I h^{(1)}_{JK}), \tag{50} $$

D_I is the covariant derivative with respect to ω_{IJ}, and D^I denotes $\omega^{IJ}D_J$.

In the normalized basis, we see that the orders of the first two components of the Riemann tensor become ${}^{(n-1)}R^{\hat{u}}{}_{\hat{I}\hat{u}\hat{J}} = \mathcal{O}(r^{-(n/2-1)})$ and ${}^{(n-1)}R^{\hat{u}}{}_{\hat{I}\hat{J}\hat{K}} = \mathcal{O}(r^{-n/2})$, where the hatted indices denote the components with respect to the normalized basis. For the third component, we have ${}^{(n-1)}R^{\hat{I}}{}_{\hat{J}\hat{K}\hat{L}} = \mathcal{O}(r^{-2}) + \mathcal{O}(r^{-(n-2)})$.

For four dimensions, Eq. (43) becomes

$$ {}^{(3)}R^{I}{}_{JKL} = {}^{(\omega)}R^{I}{}_{JKL} + \frac{1}{4}(\dot{h}^{(1)I}{}_K\dot{h}^{(1)}_{JL} - \dot{h}^{(1)I}{}_L\dot{h}^{(1)}_{JK}) + \mathcal{O}(r^{-1}). \tag{51} $$

This is a quite impressive result because, as seen soon, the term such as the second one in the right-hand side of Eq. (51) does not appear in dimensions higher than four. The reason of the appearance in four dimensions is that the Ricci scalar of a round sphere with the radius r is $\mathcal{O}(1/r^2)$, while the amplitude of gravitational waves decay as $\mathcal{O}(1/r)$. This behavior is related to the memory effect originated from supertranslations.

One can show that the leading-order term of ${}^{(3)}R^{I}{}_{JKL}$ is identical to the Riemann tensor of the round sphere if and only if $\dot{h}^{(1)}_{IJ} = 0$. Under the condition $\dot{h}^{(1)}_{IJ} = 0$, it is trivial to show that the leading term of ${}^{(3)}R^{I}{}_{JKL}$ coincides with the Riemann tensor of the round sphere. Conversely, when the leading term of ${}^{(3)}R^{I}{}_{JKL}$ is identical to the Riemann tensor of the round sphere at the leading order,

$$ \dot{h}^{(1)}_{IK}\dot{h}^{(1)}_{JL} - \dot{h}^{(1)}_{IL}\dot{h}^{(1)}_{JK} = 0 \tag{52} $$

holds. With Eq. (2) and four-dimensional speciality, it is easy to see that Eq. (52) implies $\dot{h}^{(1)}_{IJ} = 0$. See Ref. [29] for similar discussion. To sum up, $\dot{h}^{(1)}_{IJ} = 0$ is the necessary and sufficient condition for the leading part of ${}^{(3)}R^{I}{}_{JKL}$ to be identical to the Riemann tensor of the round sphere. In addition, we can see that the other components of the Riemann tensor fall off as ${}^{(3)}R^{\hat{u}}{}_{\hat{I}\hat{u}\hat{J}} = \mathcal{O}(r^{-1})$ and ${}^{(3)}R^{\hat{u}}{}_{\hat{I}\hat{J}\hat{K}} = \mathcal{O}(r^{-2})$. Here, we recall the case where S_r is the photon surface at the leading order in four dimensions examined in Sec. III. In that case, Ω_{IJ} vanishes, which gives $\dot{h}^{(1)}_{IJ} = 0$ on S_r. Thus, one can see that ${}^{(3)}R^{I}{}_{JKL}$ is identical to the Riemann tensor of the round sphere at the leading order.

We now discuss the higher-dimensional cases. In these cases, Eq. (43) becomes

$$ {}^{(n-1)}R^{I}{}_{JKL} = {}^{(\omega)}R^{I}{}_{JKL} + \mathcal{O}(r^{-(n-4)}) + \mathcal{O}(r^{-(n/2-1)}), \tag{53} $$

and thus, the leading term of ${}^{(n-1)}R^{I}{}_{JKL}$ is identical to the Riemann tensor of the sphere. The other components of ${}^{(n-1)}R^{a}{}_{bcd}$ rapidly decay for $n \geq 7$. For five dimensions,

${}^{(4)}R^{\hat{u}}{}_{\hat{I}\hat{u}\hat{J}} = \mathcal{O}(r^{-3/2})$ is larger than ${}^{(4)}R^{\hat{I}}{}_{\hat{J}\hat{K}\hat{L}} = \mathcal{O}(r^{-2})$, and, for six dimensions, ${}^{(5)}R^{\hat{u}}{}_{\hat{I}\hat{u}\hat{J}} = \mathcal{O}(r^{-2})$ is comparable to ${}^{(5)}R^{\hat{I}}{}_{\hat{J}\hat{K}\hat{L}} = \mathcal{O}(r^{-2})$.

It is merely trivial from the setup that, at the leading order in both four dimensions and higher dimensions, the Riemann tensor $\mathcal{R}^I{}_{JKL}$ with respect to the induced metric on the u-constant surface in S_r is identical to that of the round sphere. It is easily obtained as

$$\mathcal{R}^I{}_{JKL} \simeq {}^{(\omega)}R^I{}_{JKL} + (D_K\Gamma^{(1)I}{}_{JL} - D_L\Gamma^{(1)I}{}_{JK})r^{-(n/2-1)}. \tag{54}$$

VI. SUMMARY AND DISCUSSION

In this paper, adopting the Bondi coordinates, we have analyzed the asymptotic behavior of the extrinsic curvature and the Riemann tensor of the r-constant surface near future null infinity. In particular, we explored the relations to our previous study, that is, asymptotic behavior of null geodesics. Therein, in four dimensions, one could see that the tensor Ω_{IJ} defined by Eq. (9) determines the fate of photons emitted in angular directions. As a consequence, the nontrivial properties of asymptotic behavior of null geodesics have been shown in terms of the geometric quantities, and we have confirmed the direct relation between vanishing of Ω_{IJ} and the umbilical feature of the nonexpanding photon surface. This result also implies that an arbitrarily large DTTS can be formed if enormously large energy flux is present. These analyses have been done approximately adopting the leading order in the powers of $1/r$, but we briefly commented on the exact condition for the formation of a nonexpanding photon surface and a DTTS near future null infinity for a Vaidya spacetime. The behavior of null geodesics near null infinity in four and higher dimensions differs even when $\dot{m}$ is not so large. Equivalent properties have been also seen in the extrinsic curvature of an r-constant hypersurface. A similarity between our results and the memory effect can be recognized since the memory effect also gives nontrivial difference between four dimensions and higher dimensions due to asymptotic behavior of the spacetimes. We have also seen the contributions from gravitational waves at the leading order of the Riemann tensor on an r-constant hypersurface only in four dimensions.

Finally, we evaluate the order of the energy flux required for the formation of nonexpanding photon surface or the DTTS near future null infinity. For simplicity, we focus on the spherically symmetric case, in which the condition of Eq. (30) is reduced to $\dot{m}(u) = -2$. Using Eq. (4), we find $\dot{M}(u)c^2 = -c^5/G \sim -4 \times 10^{59}$ erg/s, which is the Planck luminosity [30]. Since this is conjectured to be the maximum luminosity in the Universe [31], its realization near future null infinity would be rather difficult. Therefore, it is expected that the photon emitted in the angular directions near future null infinity will reach future null infinity for almost all cases in four dimensions. However, it is mathematically interesting to explore the possibility for the construction of some concrete but unfamiliar examples, and this topic will be addressed in near future.

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APPENDIX: EXPLICIT CONSTRUCTION IN A VAIDYA SPACETIME

In this Appendix, we perform exact calculations for finding the condition for an r-constant hypersurface to be a photon surface in a four-dimensional Vaidya spacetime. The metric of the Vaidya spacetime is

$$ds^2 = -f(u,r)du^2 - 2dudr + r^2\omega_{IJ}dx^Idx^J, \tag{A1}$$

where $f(u,r) = 1 - 2M(u)/r$ and $M(u)$ is the Bondi mass. We span the standard spherical-polar coordinates (θ, ϕ) on the unit sphere. In this spacetime, the nonzero component of the stress-energy tensor is

$$T_{uu} = -\frac{1}{4\pi}\frac{\dot{M}}{r^2}, \tag{A2}$$

to which we can apply the falloff behavior of Eq. (3).

We study the behavior of a photon in this spacetime. Due to the spherical symmetry, it is sufficient to study on the equatorial plane, $\theta = \pi/2$. Since the system is axially symmetric, the angular momentum is conserved:

$$L = r^2\phi'. \tag{A3}$$

The null condition gives

$$-fu'^2 - 2u'r' + \frac{L^2}{r^2} = 0, \tag{A4}$$

and the r-component of the geodesic equations is

$$r'' + \frac{1}{2}(\partial_u f + f\partial_r f)u'^2 + \partial_r f u' r' - fr\left(\frac{L}{r^2}\right)^2 = 0. \quad (A5)$$

Combining these equations, we obtain

$$r'' + \frac{2\dot{M}}{fr}u'r' - \frac{L^2}{fr^3}\left[\left(1-\frac{2M}{r}\right)\left(1-\frac{3M}{r}\right)+\dot{M}\right] = 0. \quad (A6)$$

We now examine the condition for an r-constant hypersurface to be a photon surface. This condition is derived by requiring $r' = 0$ and $r'' = 0$. Then, we have

$$\dot{M} = -\left(1-\frac{2M}{r}\right)\left(1-\frac{3M}{r}\right). \quad (A7)$$

Since the Bondi mass $M(u)$ and the quantity $m(u, x^I)$ in the Bondi coordinates are related as $m = 2M$ in four dimensions, we have the approximate condition of Eq. (30) by ignoring the terms of $\mathcal{O}(M/r)$.

So far, we have considered the condition for S_r to be an appropriate photon surface by ignoring higher-order terms in the r^{-1} expansion. It is also possible for the portion of S_r to be an exact photon surface with $M(u)$ satisfying Eq. (A7). Note that Eq. (A7) cannot be satisfied eternally because the total mass is finite. Here, we consider the finite time interval $0 < u < u_f$ in which Eq. (A7) is exactly satisfied. Let M_0 be the Bondi mass at $u = 0$. Consider the situation in which the Bondi mass decreases intensively so that Eq. (A7) is exactly satisfied in the interval $0 < u < u_f$. The dependence of M on the u coordinate must be fine-tuned as

$$M(u) = M_0 - \frac{r(e^{u/r}-1)(1-2M_0/r)(1-3M_0/r)}{3(1-2M_0/r)-2(1-3M_0/r)e^{u/r}}. \quad (A8)$$

For this expression, the portion of $0 < u < u_f$ of S_r becomes the photon surface. Here, the end of the interval is $u_f = r\log[(1-2M_0/r)/(1-3M_0/r)]$, and then the Bondi mass becomes zero; i.e., $M(u_f) = 0$. After $u = u_f$, $M(u)$ must be zero due to non-negativity of the Bondi mass.

For a two-dimensional section of a u-constant hypersurface and an r-constant hypersurface to be an exact DTTS, $r' = 0$ and $r'' \leq 0$ in Eq. (A6) is the necessary and sufficient condition. This leads to the condition of Eq. (39).

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