Exploring the universal relations with the correlation analysis of neutron star properties

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Motivated by finding a way to construct the universal relations between the neutron star properties and obtaining the application scope of the universal relations, the linear correlation properties between several properties of neutron stars are analyzed. Based on this, several existing universal relations are tested and several new universal relations are predicted. The results show that in the whole typical central density range of neutron stars, the better the linear correlation properties between the combinations of physical quantities are, the better the universality of the universal relations will be. It is shown that the quantities possessing desired linear correlation properties are the compactness β, moment of inertia I, gravitational redshift z and gravitational binding energy E_g, etc. Based on the linear correlation analysis of the above acquired quantities, several existing universal relations about the dimensionless gravitational binding energy and several new universal relations about the dimensionless moment of inertia are tested and predicted respectively. Moreover, the moment of inertia of PSR J0030 + 0451 can be constrained by the newly discovered universal relation. In conclusion, the linear correlation analysis of the neutron star properties is an effective and feasible method to explore the universal relations.

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I. INTRODUCTION

Neutron star is a kind of star with the ultrahigh density, extreme pressure and temperature, and the super strong magnetic field in the universe [1]. Such an environment cannot be reproduced in the terrestrial laboratory at present, so it naturally becomes an ideal laboratory for the research of nuclear physics and astrophysics. In recent years, with the discovery of the gravitational radiation event of binary neutron stars merger (GW170817) [2–4], the study on neutron stars has officially entered a new golden age. At present, the research on neutron stars and their matter states is a hotspot in the world and has attracted great attention [1,5–9].

In the study on neutron stars, many neutron star properties significantly depend on the selection of the equation of state (EOS) with an obvious uncertainty, which makes it difficult to further understand the properties of neutron stars and the constraint on the EOS [6–8]. One of the current solutions is to construct a general relation between different neutron star properties, i.e., the universal relation (independent of the specific EOS), to indirectly determine the specific properties and EOS of neutron stars [7,10–13]. For some unobservable or hard to be observed quantities of neutron stars, the universal relations provide a way to constrain these quantities, and can be used to further determine the EOS [7,9–15].

During the past decades, people have established a mount of universal relations, for example, the universal relations between different quasioscillation modes of neutron stars [16–21], and the famous I-Love-Q relations between the moment of inertia I, the love number and the normalized quadrupole moment Q of neutron stars established by Yagi *et al.*. in recent years [10,11]. Currently, the universal relations have been widely used in many research fields of neutron stars, such as the general isolated neutron star systems, including the static and rotating cases [12,13,22–27]; the binary neutron star system, including the general binary star system and the binary neutron star merger [28,29]; the protoneutron star and supernova explosion [30–33] and some physical processes related to the internal structure of neutron stars, such as the glitch phenomena [34]. The universal relation can be applied to the fundamental physics, such as, the I-Love-Q relations, which can be used to test the validity of general relativity and other gravitational theories [10,11].

How to find and establish the universal relations is a challenging work at present. The general practice way is to use the experience to establish the universal relations through repeated attempts, which has a large uncertainty [7,12,13]. It is necessary to develop a reasonable method to

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predict and construct the universal relations. As the universal relations generally reflect the relations between two quantity combinations of neutron stars, actually there exists a notable linear relation or strong correlation between these two combinations [12,13,35]. Thus, it is conceivable that analyzing the correlation properties between the relevant quantity combinations may be of great help to establish the universal relations. As we know, the linear correlation coefficient (Pearson coefficient) is a very effective method that can quantitatively reflect the strength of the linear correlation between different quantities [35,36]. Therefore, in this work, we try to start from analysing the correlation properties between the fundamental global quantities of neutron stars, to find the right quantity candidates, and further test and predict the universal relations.

The paper is organized as follows. In Sec. II, the selected EOSs and the corresponding neutron star properties are briefly reviewed. In Sec. III, we will use the linear correlation analysis of neutron star properties to test and predict the universal relations, specifically including the linear correlation properties of the universal relations in Sec. III A, the linear correlation properties of the neutron star properties in Sec. III B, testing the universal relations with the linear correlation analysis of neutron star properties in Sec. III C, and predicting new universal relations with the linear correlation analysis of neutron star properties in Sec. III D. Finally, a brief summary is given in Sec. IV. In this work, we use the geometric unit ($G = c = 1$).

II. THE SELECTED EOS AND NEUTRON STAR PROPERTIES

In this work, the main purpose is to use the correlation properties of neutron star global quantities to explore the universal relations. Therefore, for achieving this goal, we need to choose a certain amount of representative EOSs of neutron stars to calculate the linear correlation properties of neutron star quantities. In this work, we selected 15 sets of EOS from the microscopic and phenomenological nuclear many-body theories, including one hybrid stars EOS, i.e., ALF2 (nuclear + quark matter) [37]; 12 nucleonic EOSs, i.e., APR3 [38], APR4 [38], ENG [39], MPA1 [40], SLy [41], DD-F [42], DD2 [43], DD-LZ1 [44], DD-ME1 [45], DD-ME2 [46], PKA1 [47], PKO3 [48], as well as the Soft-EOS and Stiff-EOS [49], which are the possible stiffest and softest EOSs in the selected models of Hebeler *et al.*. under the constraint of causality, respectively. Also, for the crust region of neutron star at low density, the BPS EOS [50] and the NV EOS [51] are adopted to describe the outer crust and the inner crust respectively.

Figure 1 shows the curves of P-ρ and M-R relations of neutron stars described by the above selected 15 EOSs. In the left panel, the curves of P-ρ of different EOS display distinctly different behaviors. Among the curves, the Stiff-EOS gives the stiffest pressure curve and the ALF2 gives the softest one. Correspondingly, for the curves of M-R relations in the right panel attained by solving the TOV equations [52,53] with the input of above P-ρ relations, the results show significant difference. The Stiff-EOS gives the highest maximum mass and the ALF2 EOS gives the lowest one, which is consistent with the results of P-ρ curves. Also, for verifying the rationality of the selected EOSs, we have added the recent astronomical observational constraints for neutron stars in the right panel of Fig. 1, which are shown by different color areas, i.e., the M-R constraints from the observations of GW170817 [4], NICER [54] and PSR J0740 + 6620 ($M = 2.14^{+0.20}_{-0.18}\ M_\odot$, 95% credibility level) [55]. It is shown that in the selected 15 EOSs, the curves of M-R relations given by most EOSs can be in good agreement with the recent astronomical observation constraints on neutron stars.

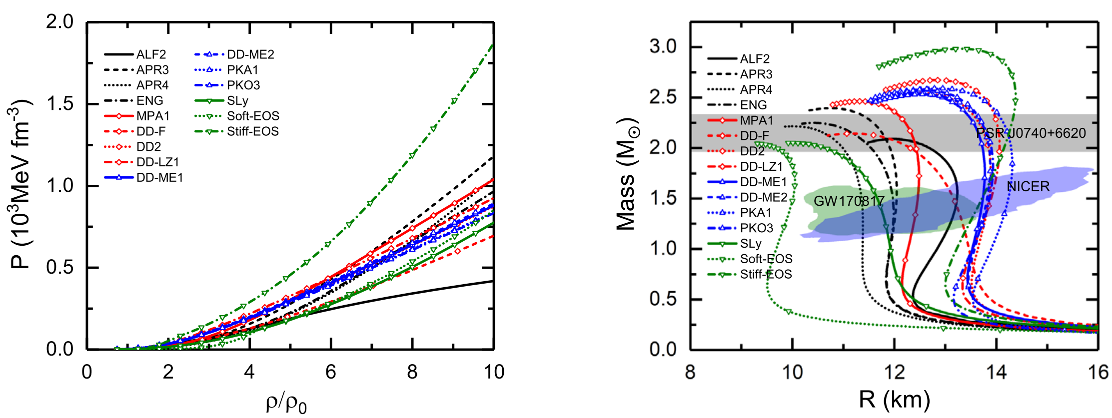

FIG. 1. Left panel: the pressure P of neutron stars as functions of the baryonic density in unit ρ/ρ_0, where ρ_0 means the nuclear saturation density; right panel: the corresponding M-R relation of neutron stars. All of the results are given by the 15 selected EOSs. In the right panel, the green, blue and gray areas represent the constraints from the GW170817 [4] with 90% credibility level, NICER [54] and PSR J0740 + 6620 [55] with 95% credibility level respectively.

III. TESTING AND PREDICTING THE UNIVERSAL RELATIONS WITH THE LINEAR CORRELATION ANALYSIS OF NEUTRON STAR PROPERTIES

The universal relation is an effective method to study the relations of global quantities of neutron stars [7,12,13]. It's easy to understand that the universal relations are associated with the correlation properties between the global quantities of neutron stars [12,13,35]. Therefore, using the correlation properties between different global quantities to explore the new universal relations is a possible way. The linear correlation analysis is an effective method to study the correlation properties between two quantities. Its coefficient r, which reflects the strength of linear correlation between two quantities, can be expressed as

$$r(X,Y) = \frac{\mathrm{Cov}(X,Y)}{\sqrt{D(X)}\sqrt{D(Y)}}, \tag{3.1}$$

where X, Y represent the object variables, $\mathrm{Cov}(X,Y)$ is the covariance, and $D(X)$, $D(Y)$ represent the variance of X, Y respectively [35,36]. The closer the absolute value of the correlation coefficient $|r|$ is to 1, the greater the correlation strength between the two quantities will be and the closer it is to linearity; conversely, the closer $|r|$ is to 0, the smaller the correlation strength between the two quantities will be. Based on this, the linear correlation analysis of the neutron stars global quantities will be used to explore the universal relations in this section.

A. Linear correlation properties of the universal relations

In this subsection, we will analyze the linear correlation properties between different physical quantities or combinations in universal relations of neutron stars. First, we need to select some appropriate universal relations. At present, there are a large number of universal relations. For universal relations describing the static and spherical symmetric neutron stars, the main property quantities involved are the binding energy E_b, compactness parameter β, moment of inertia I, tidal deformability Λ, gravitational redshift z and so on. In this work we mainly focus on the dimensionless forms of these quantities. For binding energy E_b, it can be divided into three forms: the total binding energy E_t, the gravitational binding energy E_g and the nuclear binding energy E_n. It has been shown that only E_g can provide information on the internal mass distribution of neutron stars and has preferable universal relations with other global properties [22,30]. Here we mainly concern E_t and E_g, and adopt their dimensionless form, i.e., E_t/M, E_g/M, where M represents the gravitational mass of neutron stars [12]. For compactness parameter $\beta = M/R$ and moment of inertia I, they can also provide information about the global mass distribution of neutron stars [12,56,57]. Here we use the dimensionless form of I, such as I/MR^2, I/M^3, I/R^3. For tidal deformability Λ, which reflects the deformation of neutron stars under the gravitational field of companion stars, it can be simply expressed as $\Lambda = \frac{2}{3}k_2\beta^{-5}$, where k_2 is the tidal deformation polarization factor which reflects the difficulty of neutron star deformation [56]. For gravitational redshift z, it can reflect the magnitude of frequency shift toward the red end of electromagnetic wave on the surface of neutron star caused by the strong gravitational field of neutron stars [13,58].

Figure 2 shows the selected four universal relations, which were first established in Ref. [12] as $\beta - |E_g|/M$, $(|E_g|/M)^{-2} - I/M^3$, $(|E_g|/M)^{-5} - \Lambda$ and $\beta - |E_t|/M$ from plots (a)-(d). It should be pointed out that the EOSs adopted here is different from the previous work [12]. In Fig. 2, it can be seen that according to the relative errors between the respective fitting curves and numerical results, the first three universal relations, namely $\beta - |E_g|/M$, $(|E_g|/M)^{-2} - I/M^3$ and $(|E_g|/M)^{-5} - \Lambda$, show a notable universal behavior. But for the fourth relation $\beta - |E_t|/M$, the results show that its universal behavior is poor. The same results are also found in Ref. [12] for the above four universal relations.

The left panel of Fig. 3 shows the linear correlation properties between the different physical quantity combinations of the above four universal relations in Fig. 2, which are respectively named A, B, C and D for simplicity. The results of famous I-Love-C relations [59] are also introduced for comparison. It is shown that among the above selected universal relations, A, B, C, I-Love and I-C all show a good linear correlation property in the whole central density range of typical neutron stars ($\rho_0 \sim 5\rho_0$), while D has a very poor correlation. At very high central density ($\rho_C \geq 5\rho_0$), there is a significant change for the linear correlation behavior which shows that the correlations of B, I-Love and I-C decrease obviously, A and C remain unchanged. Actually, this is beyond the general density range of typical neutron star, as shown in the right panel of Fig. 3. When $\rho_C > 5\rho_0$, the result of $M - \rho_C$ relation given by the Stiff-EOS has exceeded the maximum mass point, and then the neutron star will enter the unstable region. So we do not concern about it in this work. As a result, it is found that the linear correlation behaviors of A, B and C are very similar to those of I-Love and I-C, except D. Therefore, the above analysis can show that the better the universal relations of neutron stars are, the better the linear correlation properties between the corresponding physical quantity combinations will be and vice versa. We also tested other universal relations in this work, such as the universal relations established firstly in Ref. [13]. The results also support their conclusion.

B. Linear correlation properties of the neutron star properties

In this subsection, we will try to study the linear correlation properties between the fundamental quantities

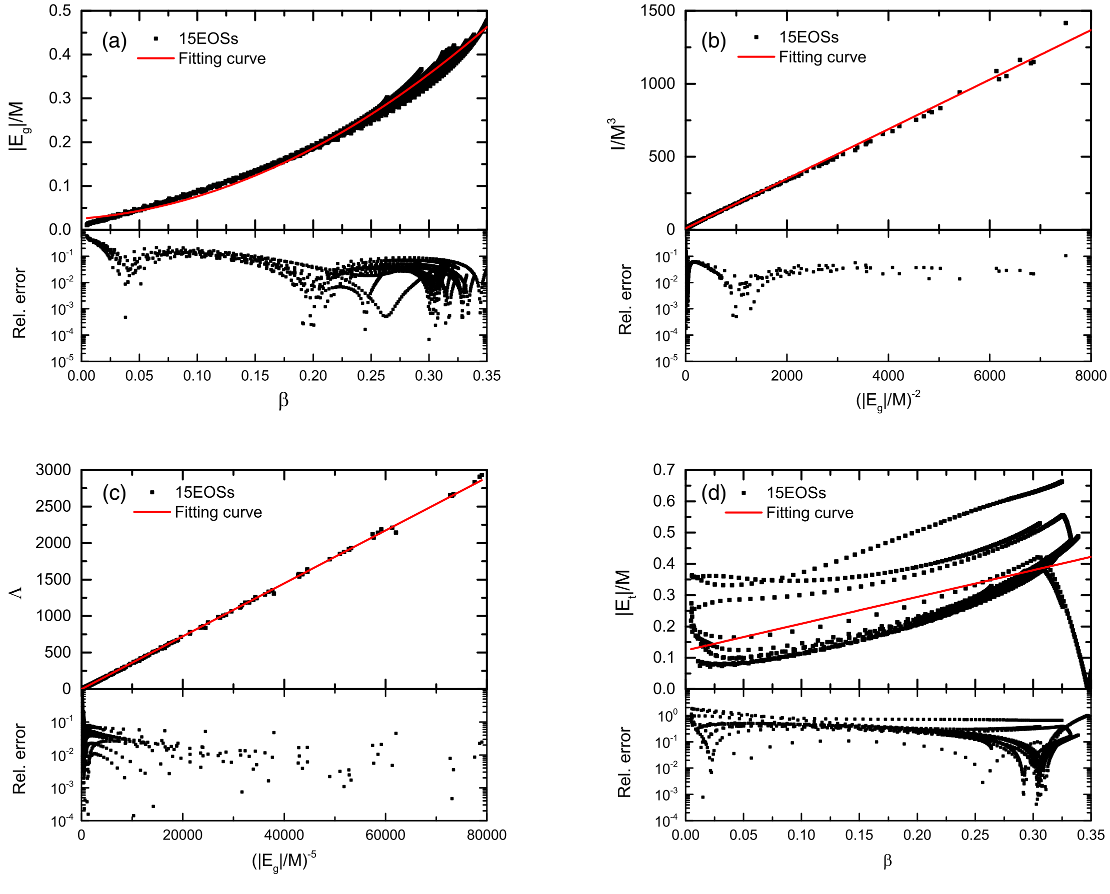

FIG. 2. The universal relations of neutron stars and their relative error between the fitting curve and the numerical results, i.e., $|Q_{\rm fit} - Q|/|Q_{\rm fit}|$, where Q means the physical quantities in universal relations. Plot (a): relation between $|E_g|/M$ and β; plot (b): relation between $(|E_g|/M)^{-2}$ and I/M^3; plot (c): relation between $(|E_g|/M)^{-5}$ and Λ; plot (d): relation between $|E_t|/M$ and β. All the above four relations are extracted from Ref. [12].

of neutron stars to find the quantities with good correlation properties and explore the universal relations. Here, we mainly concern some important global quantities of neutron stars, such as E_t, E_g, β, I, Λ, z.

Figure 4 shows the linear correlation properties between several global quantities and other global quantities of neutron stars in a wide density range. From the results in Fig. 4, it can be seen that the correlation properties of

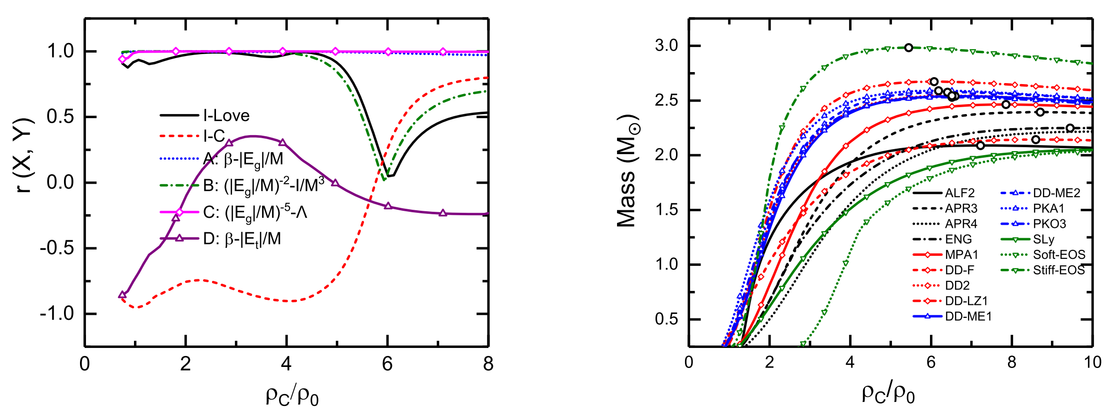

FIG. 3. Left panel: the linear correlation coefficients $r(X, Y)$ between the different global quantities in the four known universal relations in Fig. 2 (named A, B, C and D) as functions of the central density of neutron stars in unit (ρ_C/ρ_0). The results of relations of I-Love and I-C [59] are also shown for comparison. Right panel: neutron star mass as functions of its central density. The results are given by the 15 selected EOSs same as Fig. 1, the hollow black circles on each curve of $M - \rho_C$ represent the maximum mass points of neutron stars.

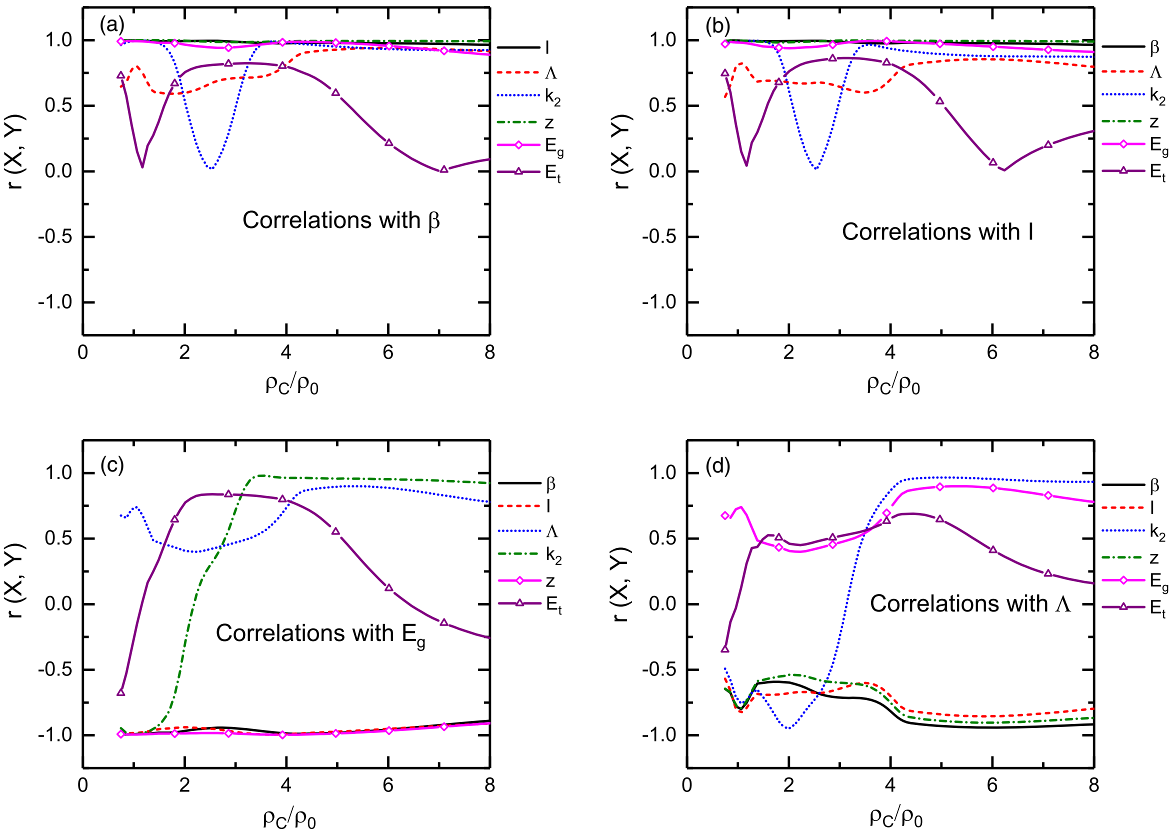

FIG. 4. The linear correlation coefficients $r(X, Y)$ between different neutron star global quantities as functions of the central density of neutron stars. Plot (a): correlations with the compactness β; plot (b): correlations with the dimensionless moment of inertia I/MR^2; plot (c): correlations with the gravitational binding energy E_g; plot (d): correlations with the tidal deformability Λ.

different quantities are significantly different. Figure 4(a) shows that in the density range of typical neutron stars ($\rho_0 \sim 5\rho_0$), the correlations between β and I, z, E_g are preferable, but it is poor with other quantities, such as E_t. In Fig. 4(d), it is shown that the correlations of the tidal deformability Λ with almost all other quantities are not very notable. We also calculated the linear correlation properties of other quantities one by one. Finally, it is found that in the whole central density range of typical neutron stars, the quantities which have good linear correlation properties with each other are mainly β, I, z and E_g, while the rest are generally poor. These quantities with good linear correlation properties can be used to explore the universal relations according to the conclusion in Sec. III A.

C. Testing the universal relations with the linear correlation analysis of neutron star properties

In this subsection, we will use the neutron star global quantities with preferable linear correlation properties to test the existing universal relations and verify the validity of this method. As the universal relations usually reflect the relations between two quantity combinations, we first need to construct some common combinations, such as the dimensionless quantities E_g/M, I/M^3, I/R^3, etc. Here we choose the dimensionless gravitational binding energy E_g/M as the research object to study its correlation properties with other quantities, and then test the universal relations. It is worth noting that in general universal relations, the exponent is generally introduced into the quantity combinations for universality, such as Q^n (Q represents the neutron star quantity, n is the exponent). As the exponent n generally needs to be determined separately, it will bring additional difficulties to our next correlation calculations. Therefore, in order to eliminate the influence of exponent on linear correlation calculation, we will utilize the logarithmic forms for the neutron star quantity combinations. For example, the universal relation $(|E_g|/M)^{-5} - \Lambda$ [12], as mentioned earlier, can be expressed as

$$\Lambda \propto \left(\frac{E_g}{M}\right)^{-5}. \tag{3.2}$$

In logarithmic form, the Eq. (3.2) will be simplified as

$$\log \Lambda \propto -5 \log\left(\frac{E_g}{M}\right), \tag{3.3}$$

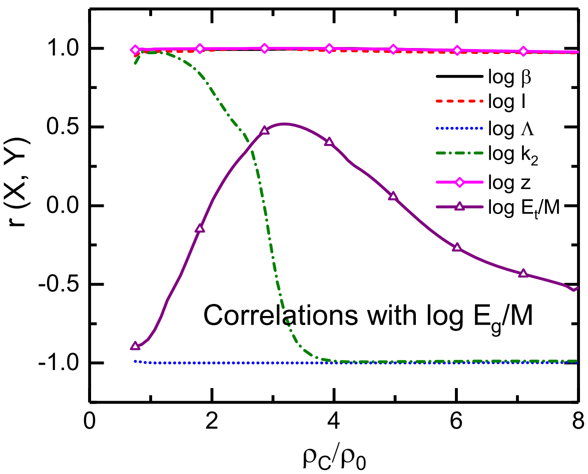

FIG. 5. The linear correlation coefficients $r(X, Y)$ of the dimensionless gravitational binding energy in logarithmic form $\log(E_g/M)$ with other neutron stars properties in logarithmic form $\log Q$ as functions of the central density of neutron stars.

where the coefficient $'' - 5''$ in Eq. (3.3) does not affect the strength of linear correlation between these two quantity combinations, so it can be omitted. That is to say, in order to study the correlation between $(|E_g|/M)^{-5}$ and Λ, we only need to study the correlation between $\log(E_g/M)$ and $\log\Lambda$.

Figure 5 shows the correlation properties between $\log(E_g/M)$ and other property combinations in logarithmic form $\log Q$. Similar to the analysis of Fig. 4, it can be seen from Fig. 5 that in the whole density range of typical neutron stars, $\log(E_g/M)$ has a remarkable correlation with $\log\beta$, $\log I$, $\log\Lambda$ and $\log z$; for other quantities, i.e., $\log k_2$ and $\log(E_t/M)$, they are poorly correlated. According to the conclusion of Fig. 3, the possible universal relations with $\log(E_g/M)$ are mainly the above four quantities with the notable linear correlations. Therefore, we will list these relations with good correlations. To simplify, we only display some of them. Figure 6 shows the relations $\log(E_g/M) - \log\Lambda$ and $\log(E_g/M) - \log\beta$ and the relative errors between their corresponding fitting curves and numerical results. For $\log(E_g/M) - \log\Lambda$, it can be seen that there is an almost strict linear relation between $\log(E_g/M)$ and $\log\Lambda$, and the relative error is within a reasonable range. This is an ideal universal relation. For $\log(E_g/M) - \log\beta$, there is an approximate quadratic curve between two quantities, and the relative error is also within the reasonable range. It is also a desired universal relation. In fact, the above two universal relations were first established in Ref. [12]. Our results in Fig. 6 are consistent with their conclusion. According to our calculation, the other two relations $\log(E_g/M) - \log(I/MR^2)$ and $\log(E_g/M) - \log z$ are also the notable universal relations, which is consistent with the results in Refs. [12,13]. So it is an effective method to use the linear correlation analysis of neutron stars properties to test the universal relations. Next, we will use it to predict new universal relations of neutron stars in the next subsection.

D. Predicting the new universal relations with the linear correlation analysis of neutron star properties

In this subsection, we will try to use the linear correlation analysis to predict the new universal relations. Since the universal relations about E_g/M have been tested in Sec. III C, in this subsection, we mainly focus on the universal relations about I/M^3 and I/R^3. Figure 7 shows the linear correlation properties between $\log(I/M^3)$ and other property quantities $\log Q$ in logarithmic form. From the results, it can be seen that in the whole typical central density range of neutron stars ($\rho_0 \sim 5\rho_0$), the quantities which have good linear correlation properties with $\log(I/M^3)$ mainly include $\log\beta$, $\log\Lambda$, $\log z$ and $\log(E_g/M)$, while for $\log k_2$, $\log(E_t/M)$, their correlations are very poor. In fact, according to previous knowledge, for the relation between I/M^3 and Λ, it is the familiar I-Love relation [10,11]; for relations $I/M^3 - z$ and $I/M^3 - E_g/M$, these two universal relations have also been confirmed, see

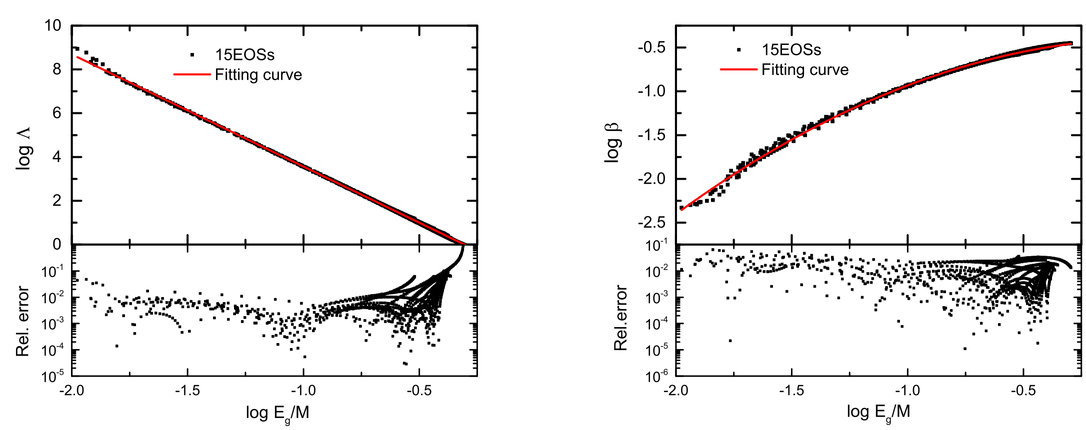

FIG. 6. The universal relations and their corresponding relative error between the fitting curve and the numerical results. Left panel: relation between $\log(E_g/M)$ and $\log\Lambda$; right panel: relation between $\log(E_g/M)$ and $\log\beta$.

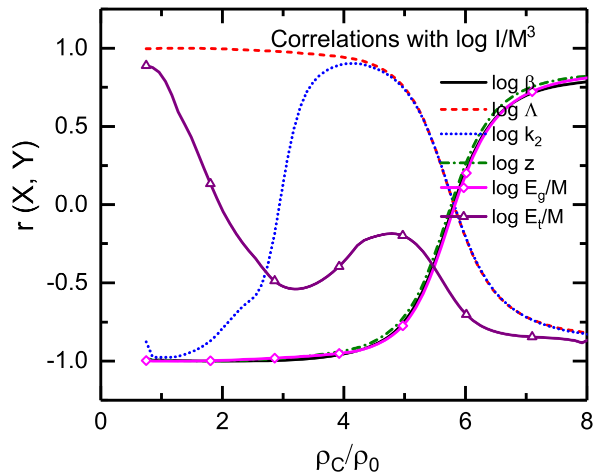

FIG. 7. Similar as Fig. 5, but for the results of dimensionless moment of inertia $\log(I/M^3)$.

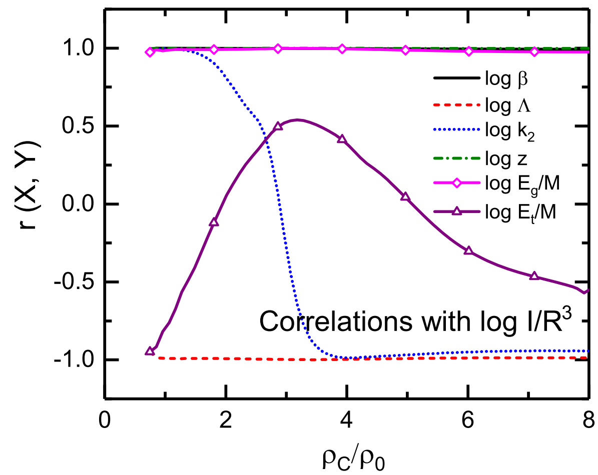

FIG. 9. Similar as Fig. 5, but for the results of dimensionless moment of inertia $\log(I/R^3)$.

Refs. [12,13] for details. For the relation between I/M^3 and β, there is no work to discuss it so far. In Fig. 8, we show the results of $\log(I/M^3) - \log\beta$ and the relative error between their corresponding fitting curve and numerical results. As shown in Fig. 8, there is a notable universal relation between $\log(I/M^3)$ and $\log\beta$.

Similarly, Fig. 9 shows the linear correlation properties between the dimensionless moment of inertia $\log(I/R^3)$ and other quantities in logarithmic form. From Fig. 9, it can be seen that in the typical central density region of neutron stars, the quantities that are notably correlated to $\log(I/R^3)$ mainly include $\log\beta$, $\log\Lambda$, $\log z$ and $\log(E_g/M)$. So we will mainly focus on the relations with these four quantities next. In fact, the universal relations between I/R^3 and β, z have been confirmed, see Refs. [13,60] for details. In Fig. 10, we show the results of $\log(I/R^3) - \log(E_g/M)$. The results show that there is a notable universal relation between $\log(I/R^3)$ and $\log(E_g/M)$. Similarly, we also test the relation of $\log(I/R^3) - \log\Lambda$, and it is also a desired universal relation.

The newly discovered universal relations mentioned above are all about the moment of inertia of neutron stars, i.e., $I/M^3 - \beta$, $I/R^3 - \Lambda$ and $I/R^3 - E_g/M$. We can use these relations to constrain the moment of inertia of some observable neutron stars. The moment of inertia of PSR J0030 + 0451 has been studied extensively [60–62]. For example, Jiang *et al.* calculated the moment of inertia for 1.4 $M_\odot$ neutron stars $I_{1.4}$ with the radius $R_{1.4}$ of PSR J0030 + 0451. Their results shown that when $R_{1.4} = 12.1^{+1.2}_{-0.8}$ km, the corresponding $I_{1.4} = 1.43^{+0.30}_{-0.13} \times 10^{45}$ g cm^2 [61]. Similarly, Li *et al.* used the bayesian

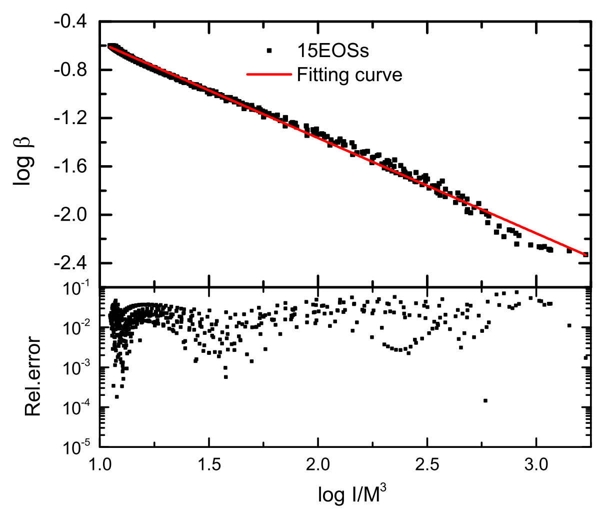

FIG. 8. Similar as Fig. 6, but for the relation between $\log(I/M^3)$ and $\log\beta$.

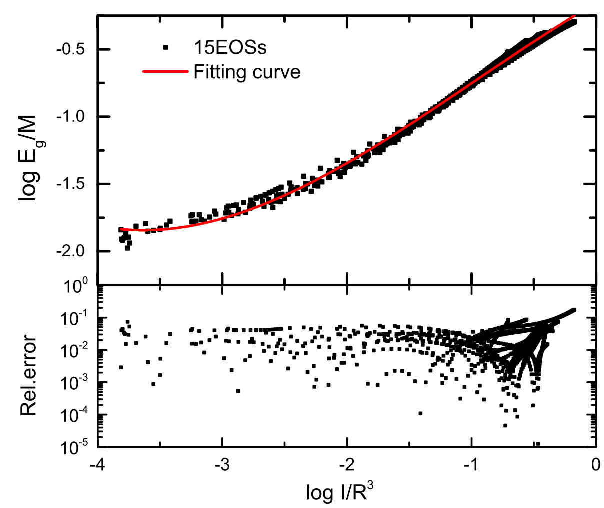

FIG. 10. Similar as Fig. 6, but for the relation between $\log(I/R^3)$ and $\log(E_g/M)$.

analysis to predict the moment of inertia of PSR J0030 + 0451 is $I = 1.95^{+0.70}_{-0.50} \times 10^{45}$ g cm^2 at 68% credible level with NICER constraints [54,60,63]. Here we also take the pulsar PSR J0030 + 0451 as an example to constrain its moment of inertia. Considering the $R_{1.4}$ or $\beta_{1.4}$ of PSR J0030 + 0451 can be measured by NICER observation, here we only employ the relation $I/M^3 - \beta$ to obtain the constraint. According to the results shown in Fig. 8, the relation $I/M^3 - \beta$ can be expressed as

$$\log I/M^3 = -1.261 \log\beta + 0.277. \quad (3.4)$$

If $R_{1.4} = 12.1^{+1.2}_{-0.8}$ km [61], i.e., $\beta_{1.4} = 0.171^{+0.012}_{-0.015}$, the corresponding $I_{1.4} = 2.09^{+0.27}_{-0.17} \times 10^{45}$ g cm^2. Similarly, if $\beta_{1.4} = 0.159^{+0.025}_{-0.022}$ [62], namely $R_{1.4} = 13.00^{+2.09}_{-1.77}$ km, the corresponding $I_{1.4} = 2.29^{+0.47}_{-0.39} \times 10^{45}$ g cm^2. It is shown that our constraint on $I_{1.4}$ for PSR J0030 + 0451 is relatively higher than that in the Ref. [62].

In conclusion, combing the analysis of the above results in Sec. III C and Sec. III D, it can be seen that it is an effective and feasible method to use the linear correlation analysis of neutron star properties to test and predict the universal relations.

IV. SUMMARY

Universal relation is an effective method to study the relations between the neutron star properties, but at present there is no complete method to predict and establish it. It is well known that the universal relations can be naturally associated with the correlation properties between different neutron star properties. So in this work, we use 15 EOSs derived from different nuclear many-body theories to calculate the linear correlation properties between different neutron star global quantities, and screened out the appropriate quantities with preferable correlation properties to explore the universal relations.

Firstly, we analyzed the linear correlation properties between two quantity combinations in four existing universal relations. It is found that in the typical central density range of neutron stars ($\rho_0 \sim 5\rho_0$), the better the linear correlation properties between the two quantity combinations are, the better the universality of the universal relations will be; otherwise, the worse. So we analyzed the linear correlation properties between different neutron star properties. The results show that the quantities with preferable correlation properties are mainly the compactness parameter β, moment of inertia I, gravitational redshift z and gravitational binding energy E_g, which can be used to explore the universal relations.

Then we take the selected neutron star properties as objects to test and predict the universal relations. First we take the dimensionless gravitational binding energy E_g/M as an example to test the existing universal relations. It is shown that our universal relations about E_g/M can be in good agreement with the previous results. Next, we take the dimensionless moment of inertia I/M^3 and I/R^3 as objects to predict the new universal relations. It is found that the quantities that are well correlated with I/M^3 and I/R^3 mainly are β, Λ, z and E_g/M. Finally, in addition to testing several existing universal relations, we also found several new universal relations, such as $I/M^3 - \beta$ and $I/R^3 - E_g/M$. Further, we use the relation $I/M^3 - \beta$ as an application, to constrain the moment of inertia of PSR J0030 + 0451. The results show that $I_{1.4} = 2.29^{+0.47}_{-0.39} \times 10^{45}$ g cm^2 when $R_{1.4} = 13.00^{+2.09}_{-1.77}$ km for PSR J0030+0451. The above results show that it is an effective and feasible method to use the linear correlation analysis of neutron star properties to explore the universal relations.

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