# Transverse polarization of hyperons produced in semi-inclusive deep inelastic scattering

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We study the transverse polarization of hyperons produced in semi-inclusive deep inelastic scattering, $ep \to e\Lambda^\uparrow X$, in the framework of the collinear twist-3 factorization. The cross section from the twist-3 distribution functions and the twist-3 quark fragmentation functions is computed in the leading order with respect to the QCD coupling constant. The constraint relations among the twist-3 FFs are taken into account to simplify the formula. The formula is relevant to large-$P_T$ hyperon production in the future Electron Ion Collider experiment.

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## I. INTRODUCTION

Transverse polarization of hyperons produced in unpolarized collisions have been observed in many high-energy inclusive processes such as $pp \to \Lambda^\uparrow X$, $e^+e^- \to \Lambda^\uparrow X$, and $ep \to e\Lambda^\uparrow X$.[1] This phenomenon is an example of transverse single spin asymmetries (SSAs) in which only one particle appearing in the process is transversely polarized. The SSAs associated with the initial state spin such as $p^\uparrow p \to hX$ and $ep^\uparrow \to ehX$ ($h = \pi, K, \eta, \ldots$) have also been observed. The parton model or perturbative QCD at twist-2 level fails to produce large SSAs [1], and its description in terms of QCD has been a challenge in QCD spin physics. In the collinear factorization of perturbative QCD, the SSAs occur as a twist-3 observable, which reflects multiparton correlations either in the initial nucleon or in the fragmentation processes [2–4]. Derivation of the twist-3 cross sections for SSAs required lots of technical development. By now the collinear twist-3 formalism for all kinds of the twist-3 distribution functions (DFs) and the fragmentation functions (FFs) has been well established in the leading order (LO) with respect to the QCD coupling constant [5–20]. Those functions are, in general, not independent from each other but obey some constraint relations based on the operator identities. The complete set of those relations have been also derived [21,22], which are crucial to obtain the frame-independent expressions for the twist-3 cross sections [21–23]. There have been also some attempts to extend the twist-3 calculation to the next-to-leading order level [24–26].

In this paper, we study the transverse polarization of hyperons produced in semi-inclusive deep inelastic scattering (SIDIS), $ep \to e\Lambda^\uparrow X$, in the LO collinear twist-3 factorization. As for the case of $pp \to \Lambda^\uparrow X$ [20,27–30], two kinds of the twist-3 cross sections contribute. One is from the twist-3 DF in unpolarized proton combined with the "transversity" FF for the polarized $\Lambda$. The other one is from the twist-3 polarized FFs for $\Lambda$ combined with the unpolarized parton DFs in the proton. In the latter contribution, both twist-3 quark FFs [30] and twist-3 gluon FFs [20] contribute due to the chiral-even nature of the FFs and DFs. Here we report the LO twist-3 cross section formula for $ep \to e\Lambda^\uparrow X$ from the twist-3 DFs and the twist-3 quark FFs. This is relevant for the large-$P_T$ polarized hyperon production in the future Electron-Ion-Collider (EIC) experiment. The contribution from twist-3 purely gluonic FFs will be reported in a separate publication [31].

[1]Throughout this paper we collectively denote spin-1/2 hyperons as $\Lambda$.

The remainder of this paper is organized as follows: In Sec. II, we summarize the twist-3 DFs and FFs relevant to the present study. In Sec. III, after summarizing the kinematics of $ep \to e\Lambda^{\uparrow}X$ (Sec III A), we present the cross section from the twist-3 DFs (Sec. III B) and from the twist-3 quark FFs (Sec. III C). Section IV is devoted to a brief summary of the present study.

## II. TWIST-3 QUARK DFS AND FFS

In this section, we summarize the twist-3 distribution function (DF) and the fragmentation functions (FF), which are necessary to calculate the polarized cross section for $ep \to e\Lambda^{\uparrow}X$. For the twist-3 distribution, we need only one function $E_F(x_1, x_2)$ in an unpolarized proton, which is defined from the quark-gluon correlation function as [32]

$$M_F^{\alpha}(x_1,x_2) = \int \frac{d\lambda}{2\pi}\int\frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2-x_1)}\langle p|\bar{\psi}_j(0) \times [0,\mu n] gF^{\alpha n}(\mu n)[\mu n, \lambda n]\psi_i(\lambda n)|p\rangle = -\frac{M_N}{4}\epsilon^{\alpha\beta n p}(\gamma_5\gamma_\beta \not{p})_{ij} E_F(x_1,x_2) + \cdots, \quad (1)$$

where $M_N$ is the nucleon mass, $|p\rangle$ is the nucleon state with momentum $p$ which can be regarded as lightlike, $n$ is another lightlike vector satisfying $p\cdot n = 1$, and $i$, $j$ denotes the spinor indices. We use the convention for the $\epsilon$-tensor as $\epsilon^{0123} = 1$ and the notation $\epsilon^{\alpha\beta n p} \equiv \epsilon^{\alpha\beta\mu\nu} n_\mu p_\nu$ is used. $[\mu n, \lambda n] = \mathrm{P}\exp[ig\int_\lambda^\mu d\tau\, n\cdot A(\tau n)]$ is the gauge link operator, which makes the correlation function color gauge invariant. From $P$, $T$-invariance, one has $E_F(x_1,x_2) = E_F(x_2,x_1)$. The support of $E_F(x_1,x_2)$ is $|x_{1,2}| < 1$ and $|x_1 - x_2| < 1$. The antiquark or "charge-conjugated" distribution $\bar{E}_F(x_1,x_2)$ for $E_F$ is defined by $\psi \to C\bar{\psi}^T$, $\bar{\psi} \to -\psi^T C^{-1}$ ($C$ is the charge conjugation matrix) and $F_{\alpha\beta} \to -F_{\alpha\beta}^T$ in (1) and it satisfies the relation $\bar{E}_F(x_1,x_2) = E_F(-x_2,-x_1)$.

We need several kinds of FFs, which are summarized below using the notation in [21]. The simplest ones are defined from the light cone correlation functions of quark fields:

$$\Delta_{ij}(z) = \frac{1}{N}\sum_X \int\frac{d\lambda}{2\pi} e^{-i\frac{\lambda}{z}}\langle 0|[\infty w, 0]\psi_i(0)|h(P_h,S_\perp)X\rangle \times \langle h(P_h,S_\perp)X|\bar{\psi}_j(\lambda w)[\lambda w, \infty w]|0\rangle = \left(\gamma_5 \not{S}_\perp \frac{\not{P}_h}{z}\right)_{ij} H_1(z) + M_h \epsilon^{\alpha S_\perp w P_h}(\gamma_\alpha)_{ij} \times \frac{D_T(z)}{z} + M_h(\gamma_5\not{S}_\perp)_{ij}\frac{G_T(z)}{z} + \cdots. \quad (2)$$

where $|h(P_h,S_\perp)\rangle$ denotes the hyperon state with mass $M_h$, momentum $P_h$ and the transverse spin vector $S_\perp$ normalized as $S_\perp^2 = -1$. Since we are interested in the twist-3 cross section, we treat $P_h$ as lightlike, and $w$ is another lightlike vector satisfying $P_h \cdot w = 1$. $H_1(z)$ is the twist-2 transversity FF, $D_T(z)$ and $G_T(z)$ are twist-3 and are called *intrinsic* twist-3 FFs. $D_T(z)$ is naively $T$-odd, while $G_T(z)$ is naively $T$-even. In (2), gauge link operator $[\lambda w, \infty w] \equiv \mathrm{P}\exp[ig\int_\infty^\lambda d\tau\, w\cdot A(\tau w)]$, is inserted, which makes the correlation function gauge invariant.

The next one is the twist-3 *kinematical* FFs which are defined as

$$\Delta_{\partial ij}^{\alpha}(z) = \frac{1}{N}\sum_X \int\frac{d\lambda}{2\pi} e^{-i\frac{\lambda}{z}}\langle 0|[\infty w, 0]\psi_i(0)|h(P_h,S_\perp)X\rangle \times \langle h(P_h,S_\perp)X|\bar{\psi}_j(\lambda w)[\lambda w, \infty w]|0\rangle\overleftarrow{\partial}^{\alpha} = -iM_h\epsilon^{\alpha S_\perp w P_h}(\not{P}_h)_{ij}\frac{D_{1T}^{\perp(1)}(z)}{z} + iM_h S_\perp^{\alpha}(\gamma_5\not{P}_h)_{ij}\frac{G_{1T}^{\perp(1)}(z)}{z} + \cdots, \quad (3)$$

where each FF is defined to be real, and they are related to $k_T^2/M_h^2$-moment of the transverse-momentum-dependent (TMD) FFs [33]. $G_{1T}^{\perp(1)}(z)$ is naively $T$-even, while $D_{1T}^{\perp(1)}(z)$ is naively $T$-odd and contributes to the hyperon polarization.

Next we introduce the twist-3 *dynamical* FFs which are defined from the three parton correlation function as:

$$\Delta_{Fij}^{\alpha}(z,z_1) = \frac{1}{N}\sum_X \int\frac{d\lambda}{2\pi}\int\frac{d\mu}{2\pi} e^{-i\frac{\lambda}{z_1}} e^{-i\mu(\frac{1}{z}-\frac{1}{z_1})}\langle 0|\psi_i(0)|h(P_h,S_\perp)X\rangle\langle h(P_h,S_\perp)X|\bar{\psi}_j(\lambda w) gF^{\alpha w}(\mu w)|0\rangle = M_h\epsilon^{\alpha S_\perp w P_h}(\not{P}_h)_{ij}\frac{\hat{D}_{\mathrm{FT}}^{*}(z,z_1)}{z} - iM_h S_\perp^{\alpha}(\gamma_5\not{P}_h)_{ij}\frac{\hat{G}_{\mathrm{FT}}^{*}(z,z_1)}{z} + \cdots, \quad (4)$$

where the gauge link is suppressed for simplicity. The dynamical FFs $\hat{D}_{\mathrm{FT}}(z,z_1)$ and $\hat{G}_{\mathrm{FT}}(z,z_1)$ are complex functions and their complex conjugates are defined in (4). The real parts of these functions are naively $T$-even, while the imaginary parts are naively $T$-odd and contribute to the hyperon polarization. Replacing $gF^{w\alpha}(\mu w)$ by the covariant derivative $D^{\alpha}(\mu w) = \partial^{\alpha} - igA^{\alpha}(\mu w)$, one can define another set of the twist-3 FFs, $\hat{D}_{\mathrm{DT}}(z,z_1)$ and $\hat{G}_{\mathrm{DT}}(z,z_1)$, by the same tensor decomposition as above. But they can be related to the above functions [17,19]:

$$\Im\hat{D}_{\rm DT}(z,z_1) = P\frac{1}{1/z-1/z_1}\Im\hat{D}_{\rm FT}(z,z_1) - \delta\left(\frac{1}{z}-\frac{1}{z_1}\right)D_{1T}^{\perp(1)}(z), \tag{5}$$

$$\Im\hat{G}_{\rm DT}(z,z_1) = P\frac{1}{1/z-1/z_1}\Im\hat{G}_{\rm FT}(z,z_1). \tag{6}$$

Three kinds of twist-3 FFs (2)–(4) are not independent, but are subject to the EOM relation and the Lorentz invariance relations (LIRs). Here we quote those relations from [21]. The EOM relation involving the naively $T$-odd FFs is given by

$$\int_z^\infty \frac{dz_1}{z_1^2}\frac{1}{1/z-1/z_1}(\Im\hat{D}_{\rm FT}(z,z_1)-\Im\hat{G}_{\rm FT}(z,z_1)) = \frac{D_T(z)}{z}+D_{1T}^{\perp(1)}(z), \tag{7}$$

and the LIR reads

$$-\frac{2}{z}\int_z^\infty \frac{dz_1}{z_1^2}\frac{\Im\hat{D}_{\rm FT}(z,z_1)}{(1/z_1-1/z)^2} = \frac{D_T(z)}{z}+\frac{d(D_{1T}^{\perp(1)}(z)/z)}{d(1/z)}. \tag{8}$$

It has been shown that these relations are crucial to guarantee the gauge invariance and the frame independence of twist-3 cross sections for various processes [21,23,30].

In addition to those in (4), there is another type of dynamical FFs defined from the matrix elements like $\sim\langle 0|gF_a^{\alpha w}|hX\rangle\langle hX|\bar{\psi}t^a\psi|0\rangle$ with $t^a$ the generator of color SU(3). They are, however, related to the purely gluonic twist-3 FFs by the EOM relations and the LIRs as was shown in [22]. It has been also shown that the combination of the contributions from the twist-3 purely gluonic FFs and these dynamical FFs gives the gauge and frame independent cross section for $pp\to\Lambda^\uparrow X$ thanks to the LIR s and the EOM relations [20]. Therefore we will discuss those contributions together in a separate publication [31].

## III. TWIST-3 CROSS SECTION FOR $ep\to e\Lambda^\uparrow X$

### A. Kinematics

Here we summarize the kinematics for

$$e(\ell)+p(p)\to e(\ell')+\Lambda^\uparrow(P_h,S_\perp)+X, \tag{9}$$

where $\ell$, $\ell'$, $p$ and $P_h$ are the momenta of each particle, and $S_\perp$ is the transverse spin vector of the produced $\Lambda^\uparrow$. To derive the cross section for this process, we define the following five Lorentz invariants:

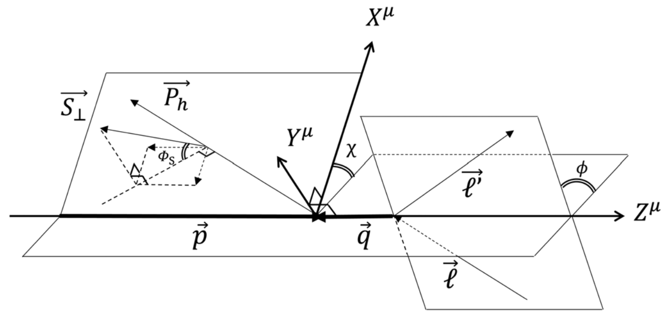


FIG. 1. Hadron frame in which $\vec{q}$ and $\vec{p}$ are collinear. The angles $\phi$ and $\chi$ are, respectively, the azimuthal angles for the lepton plane and the hadron plane (measured from a certain reference plane), and $\Phi_S$ is the azimuthal angle of the transverse spin vector of $\Lambda^\uparrow$, $\vec{S}_\perp$, measured from the hadron plane.

$$S_{ep}=(p+\ell)^2,\quad x_{bj}=\frac{Q^2}{2p\cdot q},\quad Q^2=-q^2=-(\ell-\ell')^2,\quad z_f=\frac{p\cdot P_h}{p\cdot q},\quad q_T=\sqrt{-q_t^2}, \tag{10}$$

where the spacelike four momentum $q_t$ is defined by

$$q_t^\mu = q^\mu - \frac{P_h\cdot q}{p\cdot P_h}p^\mu - \frac{p\cdot q}{p\cdot P_h}P_h^\mu, \tag{11}$$

which satisfies $q_t\cdot p = q_t\cdot P_h = 0$.

We work in the hadron frame [34], in which the momenta of the virtual photon and the initial proton are collinear as shown in Fig. 1. In this frame those momenta take

$$q^\mu=(q^0,\vec{q})=(0,0,0,-Q), \tag{12}$$

$$p^\mu=\left(\frac{Q}{2x_{bj}},0,0,\frac{Q}{2x_{bj}}\right). \tag{13}$$

We define the azimuthal angles of the hadron plane and the lepton plane as $\chi$ and $\phi$, respectively. With these angles, $P_h^\mu$ can be written as

$$P_h^\mu=\frac{z_fQ}{2}\left(1+\frac{q_T^2}{Q^2},\frac{2q_T}{Q}\cos\chi,\frac{2q_T}{Q}\sin\chi,-1+\frac{q_T^2}{Q^2}\right), \tag{14}$$

which implies the transverse momentum of $\Lambda^\uparrow$ is $P_{hT}=z_fq_T$. For this $P_h^\mu$, $w^\mu$ in (2) takes the following form:

$$w^\mu=\frac{1}{z_fQ(1+q_T^2/Q^2)^2}\left(1+\frac{q_T^2}{Q^2},-\frac{2q_T}{Q}\cos\chi, -\frac{2q_T}{Q}\sin\chi,1-\frac{q_T^2}{Q^2}\right). \tag{15}$$

The initial and the scattered lepton momenta are, respectively, given by

$$\ell^\mu = \frac{Q}{2}(\cosh\psi, \sinh\psi\cos\phi, \sinh\psi\sin\phi, -1), \quad (16)$$

$$\ell'^\mu = \frac{Q}{2}(\cosh\psi, \sinh\psi\cos\phi, \sinh\psi\sin\phi, 1), \quad (17)$$

with

$$\cosh\psi = \frac{2x_{bj}S_{ep}}{Q^2} - 1. \quad (18)$$

In order to calculate the cross section, we introduce the following four vectors orthogonal to each other [34],

$$T^\mu = \frac{1}{Q}(q^\mu + 2x_{bj}p^\mu) = (1,0,0,0), \quad (19)$$

$$X^\mu = \frac{1}{q_T}\left\{\frac{P_h^\mu}{z_f} - q^\mu - \left(1 + \frac{q_T^2}{Q^2}\right)x_{bj}p^\mu\right\} = (0, \cos\chi, \sin\chi, 0), \quad (20)$$

$$Z^\mu = -\frac{q^\mu}{Q} = (0,0,0,1), \quad (21)$$

$$Y^\mu = \epsilon^{\mu\nu\rho\sigma}T_\nu X_\rho Z_\sigma = (0, -\sin\chi, \cos\chi, 0). \quad (22)$$

The polar angle $\theta$ of $\vec{P}_h$ measured from the $Z$-axis can be written as

$$\cos\theta = \frac{P_{hz}}{|\vec{P}_h|} = \frac{q_T^2 - Q^2}{q_T^2 + Q^2}, \quad (23)$$

$$\sin\theta = \frac{P_{hT}}{|\vec{P}_h|} = \frac{2q_T Q}{q_T^2 + Q^2}, \quad (24)$$

where $P_{hT} = \sqrt{P_{hx}^2 + P_{hy}^2}$. The transverse spin vector $\vec{S}_\perp$ of the hyperon resides in the plane which is orthogonal to $\vec{P}_h$, and we define the azimuthal angle of $\vec{S}_\perp$ measured from the hadron plane around $\vec{P}_h$ as $\Phi_s$ (See Fig. 1). Then $S_\perp^\mu$ can be written as

$$S_\perp^\mu = \cos\theta\cos\Phi_S X^\mu + \sin\Phi_S Y^\mu - \sin\theta\cos\Phi_S Z^\mu. \quad (25)$$

The polarized cross section for (9) can be written as

$$d\Delta\sigma = \frac{1}{2S_{ep}}\frac{d^3\vec{P}_h}{(2\pi)^3 2P_h^0}\frac{d^3\vec{\ell}'}{(2\pi)^3 2\ell'0}\frac{e^4}{q^4}L^{\mu\nu}(l,l')W_{\mu\nu}(p,q,P_h), \quad (26)$$

where $W_{\mu\nu}$ is the hadronic tensor and $L^{\mu\nu}$ is the unpolarized leptonic tensor defined by

$$L^{\mu\nu}(\ell,\ell') = 2(\ell^\mu\ell'^\nu + \ell^\nu\ell'^\mu) - Q^2 g^{\mu\nu}. \quad (27)$$

Using the kinematic variables introduced in (10), the differential cross section can be written as

$$\frac{d^6\Delta\sigma}{dx_{bj}dQ^2dz_f dq_T^2 d\phi d\chi} = \frac{\alpha_{\rm em}^2 z_f}{128\pi^4 S_{ep}^2 x_{bj}^2 Q^2} \times L^{\mu\nu}(\ell,\ell')W_{\mu\nu}(p,q,P_h), \quad (28)$$

where $\alpha_{\rm em} = e^2/(4\pi)$ is the fine structure constant in QED. The hadronic tensor $W_{\mu\nu}$ satisfies the current conservation $q^\mu W_{\mu\nu} = q^\nu W_{\mu\nu} = 0$ and can be expanded by the following six tensors, $\mathcal{V}_k^{\mu\nu}$ [34]:

$$\begin{aligned}
\mathcal{V}_1^{\mu\nu} &= X^\mu X^\nu + Y^\mu Y^\nu, \\
\mathcal{V}_2^{\mu\nu} &= g^{\mu\nu} + Z^\mu Z^\nu, \\
\mathcal{V}_3^{\mu\nu} &= T^\mu X^\nu + X^\mu T^\nu, \\
\mathcal{V}_4^{\mu\nu} &= X^\mu X^\nu - Y^\mu Y^\nu, \\
\mathcal{V}_8^{\mu\nu} &= T^\mu Y^\nu + Y^\mu T^\nu, \\
\mathcal{V}_9^{\mu\nu} &= X^\mu Y^\nu + Y^\mu X^\nu. \quad (29)
\end{aligned}$$

In order to calculate $L^{\mu\nu}W_{\mu\nu}$ in (26), we introduce the inverse tensors $\tilde{\mathcal{V}}_k^{\mu\nu}$ for $\mathcal{V}_k^{\mu\nu}$:

$$\begin{aligned}
\tilde{\mathcal{V}}_1^{\mu\nu} &= \frac{1}{2}(2T^\mu T^\nu + X^\mu X^\nu + Y^\mu Y^\nu), \\
\tilde{\mathcal{V}}_2^{\mu\nu} &= T^\mu T^\nu, \\
\tilde{\mathcal{V}}_3^{\mu\nu} &= -\frac{1}{2}(T^\mu X^\nu + X^\mu T^\nu), \\
\tilde{\mathcal{V}}_4^{\mu\nu} &= \frac{1}{2}(X^\mu X^\nu - Y^\mu Y^\nu), \\
\tilde{\mathcal{V}}_8^{\mu\nu} &= -\frac{1}{2}(T^\mu Y^\nu + Y^\mu T^\nu), \\
\tilde{\mathcal{V}}_9^{\mu\nu} &= \frac{1}{2}(X^\mu Y^\nu + Y^\mu X^\nu). \quad (30)
\end{aligned}$$

With these $\mathcal{V}_k^{\mu\nu}$ and $\tilde{\mathcal{V}}_k^{\mu\nu}$, one obtains $L^{\mu\nu}W_{\mu\nu}$ as

$$\begin{aligned}
L^{\mu\nu}W_{\mu\nu} &= \sum_{k=1,\ldots,9}[L_{\mu\nu}\mathcal{V}_k^{\mu\nu}][W_{\rho\sigma}\tilde{\mathcal{V}}_k^{\rho\sigma}] \\
&= Q^2\sum_{k=1,\ldots,9}\mathcal{A}_k(\phi-\chi)[W_{\rho\sigma}\tilde{\mathcal{V}}_k^{\rho\sigma}], \quad (31)
\end{aligned}$$

where $\mathcal{A}_k(\varphi)$ ($\varphi \equiv \phi - \chi$, $k = 1, \ldots, 4, 8, 9$) is defined by

$$\mathcal{A}_k(\varphi) = L_{\mu\nu}\mathcal{V}_k^{\mu\nu}/Q^2, \quad (32)$$

and they are calculated to be

$$
\begin{aligned}
\mathcal{A}_1(\varphi) &= 1+\cosh^2\psi, \\
\mathcal{A}_2(\varphi) &= -2, \\
\mathcal{A}_3(\varphi) &= -\cos\varphi\sinh 2\psi, \\
\mathcal{A}_4(\varphi) &= \cos 2\varphi \sinh^2\psi, \\
\mathcal{A}_8(\varphi) &= -\sin\varphi\sinh 2\psi, \\
\mathcal{A}_9(\varphi) &= \sin 2\varphi \sinh^2\psi.
\end{aligned} \tag{33}
$$

Corresponding to $\mathcal{A}_k(\phi-\chi)$ $(k=1,2...9)$, the cross section for (9) consists of five components with different dependences on the azimuthal angle $\phi-\chi$.

### B. Contribution from unpolarized twist-3 distribution and the transversity fragmentation function

In this section we calculate the twist-3 cross section for $ep\to e\Lambda^\uparrow X$ which arises from the twist-3 DF in the nucleon. The method of the calculation is described in detail in [6], which developed the formalism for a similar twist-3 process $ep^\uparrow\to e\pi X$. At leading order with respect to the QCD coupling constant, only the dynamical twist-3 DF in the nucleon $E_F(x_1,x_2)$ defined in (1) contributes together with the transversity FF $H_1(z)$ for $\Lambda^\uparrow$, which is schematically shown in Fig. 2. Since the original calculation in [6] overlooked some of the diagrams for the hard part [11,35], we shall also include those new diagrams below. The cross section occurs as pole contributions from the hard part, which are classified into the hard pole (HP), soft-gluon-pole (SGP) and soft-fermion-pole (SFP). For the SFP contribution, the new type of diagrams found in [11] cancel the contribution from the quark's SFP function $E_F(0,x)$ paired with the quark's transversity FF arising from the SFP diagrams considered in [6]. Accordingly, in the present case, the SFP contribution occurs from the left four diagrams in Fig. 1 of [35] in which the lower quark line (for anti-quark) crossing the final state cut fragments into $\Lambda^\uparrow$, together with the diagrams obtained by reversing the arrows on the quark lines. Therefore the SFP contribution survives only for the antiquark's transversity FF paired with the quark's SFP DF or the quark's transversity FF paired with the *antiquark's* SFP DF [35]. We expect these contributions involving the FF or SFP functions for an antiquark should be much smaller compared with those from the SGP and HP ones. Hence we will not consider them below. It was also found in [35] that there are some other HP diagrams not considered in [6]. We will also include those new types of HP contributions below.

Using (28) and (31), and factorizing the twist-2 transversity FF $H_1(z)$ from the hadronic tensor, one can write this cross section as

$$
\frac{d^6\Delta\sigma^{\rm tw3-dist}}{dx_{bj}dQ^2dz_f dq_T^2 d\phi d\chi} = \frac{z_f\alpha_{\rm em}^2}{128\pi^4 S_{ep}^2 x_{bj}^2}\sum_{k=1}^{9}\mathcal{A}_k(\phi-\chi) \times\int\frac{dz}{z^2}H_1(z)[w_{\mu\nu}(p,q,P_h/z)\tilde{\mathcal{V}}_k^{\mu\nu}], \tag{34}
$$

where the summation over quark flavors as well as the factor associated with the quark's fractional electric charge is omitted. Applying the formalism described in [6], we obtain $w_{\mu\nu}$ in (34) in terms of the gauge-invariant correlation function $M_F^\alpha(x_1,x_2)$ in (1) as

$$
w_{\mu\nu}(p,q,P_h/z) = \int dx_1\int dx_2 {\rm Tr}\left[i\omega^\alpha{}_\beta M_F^\beta(x_1,x_2) \times\frac{\partial S_{\sigma,\mu\nu}^{\rm HP/SGP}(k_1,k_2,q,P_h/z)p^\sigma}{\partial k_2^\alpha}\bigg|_{k_i=x_ip}\right], \tag{35}
$$

where $\omega^\alpha{}_\beta = g^\alpha{}_\beta - p^\alpha n_\beta$ and the summation over color indices is implicit. $S_{\sigma,\mu\nu}^{\rm HP/SGP}$ is the partonic hard part corresponding to the original nucleon matrix element $M^\sigma(k_1,k_2)\sim\mathcal{FT}\langle p|\overline{\psi}gA^\sigma\psi|p\rangle$. Although we start from $\int d^4k_1\int d^4k_2 M^\sigma(k_1,k_2)S_\sigma(k_1,k_2)$ for the cross section, reorganization of the collinear expansion using the Ward identity allows us to convert the gauge field into the field strength in the correlation function and leads to the gauge invariant expression for the twist-3 cross section as shown in (35) (see [6] for the details).

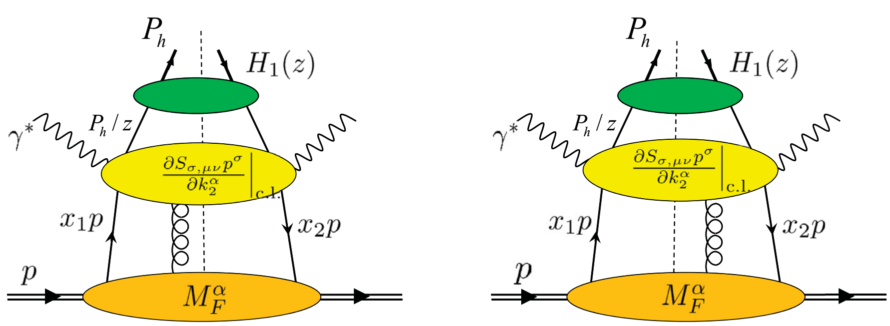


FIG. 2. Twist-3 distribution function contribution to $ep\to e\Lambda^\uparrow X$. The top blob indicates the fragmentation correlator for $H_1(z)$, the bottom one represents the correlation function for $E_F(x_1,x_2)$ and the middle one corresponds to the partonic hard part.

The LO diagrams for $S^{\rm HP}_{\sigma,\mu\nu}$ are given in Fig. 2 of [6] and the left two diagrams of Fig. 2 of [35].[2] The LO diagrams for $S^{\rm SGP}_{\sigma,\mu\nu}$ are given in Fig. 8 of [6]. We remind the reader that the hard part for the HP contribution satisfies the following relation owing to the Ward identity [6],

$$\left.\frac{\partial S^{\rm HP}_{\sigma,\mu\nu}(k_1,k_2,q,P_h/z)p^\sigma}{\partial k_2^\alpha}\right|_{k_i=x_ip} = \frac{1}{x_1-x_2}S^{\rm HP}_{\alpha,\mu\nu}(x_1p,x_2p,q,P_h/z), \tag{36}$$

and thus one can obtain the HP contribution without calculating the derivative in (35). The hard part contains the $\delta$-function corresponding to the on-shell condition for the final unobserved parton, which takes

$$\delta((xp+q-P_h/z)^2) = \frac{1}{Q^2\hat{z}}\delta\left(\frac{q_T^2}{Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)\right), \tag{37}$$

where the variables $\hat{x}$ and $\hat{z}$ are defined as

$$\hat{x}=\frac{x_{bj}}{x}, \qquad \hat{z}=\frac{z_f}{z}. \tag{38}$$

Calculating the LO diagrams for the hard part, we have obtained the HP contribution as

$$\begin{aligned}\frac{d^6\Delta\sigma^{\rm HP}}{dx_{bj}dQ^2dz_fdq_T^2d\phi d\chi} &= \frac{\alpha_{\rm em}^2\alpha_s}{16\pi^2S_{ep}^2x_{bj}^2Q^2}\left(\frac{-\pi M_N}{4}\right)\int\frac{dz}{z}H_1(z)\int\frac{dx}{x}\left[\frac{2}{1-\hat{x}}E_F(x_{bj},x)\left(\frac{4}{Nq_T}-\frac{4NQ^2(\hat{x}-1)}{q_T^3\hat{x}}\right)\right.\\ &\quad\times\sinh^2\psi\sin\{\Phi_S+2(\phi-\chi)\}+E_F(x_{bj},x_{bj}-x)\left(\frac{-1}{N}\right)\left\{\frac{8\hat{x}}{\hat{z}q_T}(1+\cosh^2\psi)\sin\Phi_S\right.\\ &\quad\left.\left.+\frac{8(1-\hat{x})Q}{\hat{z}q_T^2}\sinh 2\psi\sin(\Phi_s+\phi-\chi)+\frac{8(1-\hat{x})^2Q^2}{\hat{x}\,\hat{z}\,q_T^3}\sinh^2\psi\sin\{\Phi_S+2(\phi-\chi)\}\right\}\right]\\ &\quad\times\delta\left(\frac{q_T^2}{Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)\right),\end{aligned} \tag{39}$$

where $\alpha_s=g^2/(4\pi)$ is the strong coupling constant. The $E_F(x_{bj},x)$ contribution occurs from the diagrams in Fig. 2 of [6] and the $E_F(x_{bj},x_{bj}-x)$ contribution is from the left two diagrams of Fig. 2 of [35]. We note that $k=3$, 8 and $k=4$, 9 terms can be, respectively, combined into the single sin forms $\sim\sin(\Phi_s+\phi-\chi)$ and $\sim\sin\{\Phi_s+2(\phi-\chi)\}$.

For the LO calculation of the SGP contribution we found the method using the master formula [10] is convenient. The result reads

$$\begin{aligned}\frac{d^6\Delta\sigma^{\rm SGP}}{dx_{bj}dQ^2dz_fdq_T^2d\phi d\chi} &= \frac{\alpha_{\rm em}^2\alpha_S}{16\pi^2S_{ep}^2x_{bj}^2Q^2}\left(\frac{-\pi M_N}{4}\right)\frac{q_T}{Q^2}\int\frac{dz}{z}H_1(z)\int\frac{dx}{x}\left(\frac{-1}{2N}\right)\\ &\quad\times\left[-8(1+\cosh^2\psi)\sin\Phi_S\left\{\frac{2\hat{x}}{1-\hat{z}}\left(x\frac{dE_F(x,x)}{dx}-E_F(x,x)\right)+\left(\frac{(1+\hat{z})Q^2}{\hat{z}q_T^2}+\frac{2\hat{x}}{1-\hat{x}}\right)E_F(x,x)\right\}\right.\\ &\quad+8\sinh 2\psi\sin(\Phi_S+\phi-\chi)\left\{\frac{2\hat{x}Q}{(1-\hat{z})q_T}\left(x\frac{dE_F(x,x)}{dx}-E_F(x,x)\right)-\frac{Q}{2q_T}\left(\frac{Q^2}{q_T^2}+1\right)E_F(x,x)\right\}\\ &\quad-8\sinh^2\psi\sin\{\Phi_S+2(\phi-\chi)\}\left\{\frac{2\hat{x}Q^2}{(1-\hat{z})q_T^2}\left(x\frac{dE_F(x,x)}{dx}-E_F(x,x)\right)\right.\\ &\quad\left.\left.-\left(\frac{3Q^4}{\hat{z}q_T^4}+\frac{Q^2}{q_T^2}\right)E_F(x,x)\right\}\right]\delta\left(\frac{q_T^2}{Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)\right).\end{aligned} \tag{40}$$

It turned out that the LO SGP cross sections for $k=3$, 8 and $k=4$, 9 can be also transformed into the single sin forms as in the HP contribution.

To summarize this subsection, the cross section in (34) is given by the sum of (39) and (40).

[2] Since both $E_F$ and $H_1$ are chiral-odd, right two diagrams in the same figure vanish.

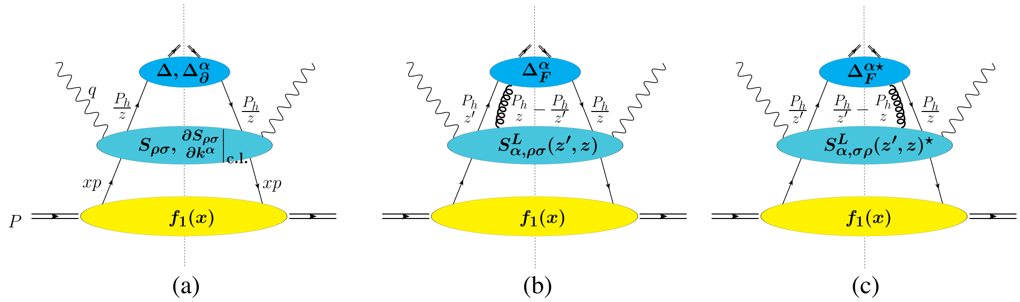


FIG. 3. Twist-3 quark fragmentation function contribution to $ep \to e\Lambda^\uparrow X$. The top blob represents the fragmentation correlator, the bottom one indicates the unpolarized quark DF, and the middle one corresponds to the partonic hard part. The contribution from the unpolarized gluon DF should also be included.

### C. Contribution from twist-3 quark fragmentation function

The formalism for calculating the twist-3 quark FF contribution has been developed in [17] for the process $ep^\uparrow \to e\pi X$. This contribution is diagrammatically shown in Fig. 3. From Eq. (54) of [17], one can write the twist-3 quark FF contribution to (9) as

$$\frac{d^6\Delta\sigma^{\text{tw3-frag}}}{dx_{bj}dQ^2dz_f dq_T^2 d\phi d\chi} = \frac{\alpha_{\text{em}}^2}{128\pi^4 x_{bj}^2 S_{ep}^2 Q^2} z_f Q^2 \sum_{k=1,\dots,4,8,9} \mathcal{A}_k(\phi-\chi) \int \frac{dx}{x} f_1(x) \\ \times \left[ \int \frac{dz}{z^2} \text{Tr}[\Delta(z) S_{\rho\sigma}(z)] \tilde{\mathcal{V}}_k^{\rho\sigma} + \Omega^\alpha{}_\beta \int \frac{dz}{z^2} \text{Tr}\left[ \Delta_\partial^\beta(z) \frac{\partial S_{\rho\sigma}(k)}{\partial k^\alpha}\bigg|_{\text{c.l.}} \right] \tilde{\mathcal{V}}_k^{\rho\sigma} \right. \\ \left. + \Omega^\alpha{}_\beta \int \frac{dz}{z^2} \int \frac{dz'}{z'^2} \text{P}\left( \frac{1}{1/z' - 1/z} \right) \{ \Im \text{Tr}[\Delta_F^\beta(z,z') S^L_{\alpha,\rho\sigma}(z',z)] + (\rho \leftrightarrow \sigma) \} \tilde{\mathcal{V}}_k^{\rho\sigma} \right], \qquad (41)$$

where $\Omega^\alpha{}_\beta = g^\alpha{}_\beta - P_h^\alpha w_\beta$, $f_1(x)$ denotes the twist-2 unpolarized quark or gluon distribution function, and the summation over quark flavors as well as the factor associated with the quark's fractional electric charge is omitted. In (41), $S_{\rho\sigma}(k)$ and $S^L_{\alpha,\rho\sigma}(z',z)$ are the partonic hard parts originally associated, respectively, with the fragmentation matrix elements $\sum_X \langle 0|\psi|hX\rangle\langle hX|\bar\psi|0\rangle$ and $\sum_X \langle 0|\psi|hX\rangle\langle hX|\bar\psi g A^\alpha|0\rangle$. Inserting (2)–(4) into (41), one can obtain the twist-3 FF contribution to the cross section as[3]

$$\frac{d^6\Delta\sigma^{\text{tw3-frag}}}{dx_{bj}dQ^2dz_f dq_T^2 d\phi d\chi} = \frac{\alpha_{\text{em}}^2 \alpha_s(-M_h)}{16\pi^2 x_{bj}^2 S_{ep}^2 Q^2} \sum_{k=1}^{9} \mathcal{A}_k(\phi-\chi) \mathcal{S}_k \int \frac{dx}{x} f_1(x) \int \frac{dz}{z} \delta\left( \frac{q_T^2}{Q^2} - \left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right) \right) \\ \times \left[ \frac{D_T(z)}{z} \hat\sigma_{\text{int}}^k - \left\{ \frac{d}{d(1/z)} \frac{D_{1T}^{\perp(1)}(z)}{z} \right\} \hat\sigma_{kin1}^k - D_{1T}^{\perp(1)}(z) \hat\sigma_{kin2}^k + \int \frac{dz'}{z'^2} \text{P}\left( \frac{1}{1/z - 1/z'} \right) \right. \\ \times \left\{ \Im \hat{D}_{\text{FT}}(z,z') \left[ \hat\sigma_{\text{DF3}}^k - \frac{2}{z}\left( \frac{1}{1/z-1/z'} \right) \hat\sigma_{\text{DF4}}^k + \frac{z'}{z} \hat\sigma_{\text{DF1}}^k + \frac{1}{z}\left( \frac{1}{z'} - \frac{1}{(1-q_T^2/Q^2)z_f} \right)^{-1} \hat\sigma_{\text{DF2}}^k \right] \right. \\ \left. \left. + \Im \hat{G}_{\text{FT}}(z,z') \left[ \hat\sigma_{GF3}^k - \frac{2}{z}\left( \frac{1}{1/z-1/z'} \right) \hat\sigma_{GF4}^k + \frac{z'}{z} \hat\sigma_{GF1}^k + \frac{1}{z}\left( \frac{1}{z'} - \frac{1}{(1-q_T^2/Q^2)z_f} \right)^{-1} \hat\sigma_{GF2}^k \right] \right\} \right], \qquad (42)$$

where $\mathcal{S}_{1,2,3,4} = \sin\Phi_S$ and $\mathcal{S}_{8,9} = \cos\Phi_S$, and $\hat\sigma_{\text{int}}^k, \hat\sigma_{kin1}^k, \cdots$ etc. represent the partonic hard cross sections. In (42) we have explicitly separated the $z'$-dependence of the cross section for the dynamical FFs, and introduced the $z'$-independent partonic hard cross section, $\hat\sigma_{\text{DF1}}^k$, $\hat\sigma_{\text{DF2}}^k$, $\hat\sigma_{\text{DF3}}^k$, $\hat\sigma_{\text{DF4}}^k$ for $\Im\hat{D}_{\text{FT}}(z,z')$, and likewise for $\Im\hat{G}_{\text{FT}}(z,z')$.

From actual calculation, it is easy to find

$$\hat\sigma_{\text{DF3}}^k = -\hat\sigma_{GF3}^k, \qquad \hat\sigma_{GF4}^k = 0. \qquad (43)$$

[3]The $T$-even functions $G_T(z)$ in (2) and $G_{1T}^{(1)\perp}(z)$ in (3) do not contribute to the symmetric part of the hadronic tensor.

Thanks to these relations, the EOM relation (7) and the LIR (8) allow one to absorb the contribution from the dynamical FFs with, $\hat{\sigma}^k_{\rm DF3}$, $\hat{\sigma}^k_{\rm DF4}$, $\hat{\sigma}^k_{GF3}$ and $\hat{\sigma}^k_{GF4}$, into those from the intrinsic and the kinematical FFs. Defining the new partonic hard cross sections,

$$\hat{\sigma}^k_T \equiv \hat{\sigma}^k_{\rm int} + \hat{\sigma}^k_{\rm DF3} + \hat{\sigma}^k_{\rm DF4}, \tag{44}$$

$$\hat{\sigma}^k_{\perp D} \equiv \hat{\sigma}^k_{kin1} - \hat{\sigma}^k_{\rm DF4}, \tag{45}$$

$$\hat{\sigma}^k_{\perp} \equiv \hat{\sigma}^k_{kin2} - \hat{\sigma}^k_{\rm DF3}, \tag{46}$$

one can rewrite (42) in the following form:

$$\begin{aligned}\frac{d^6\Delta\sigma^{\rm tw3-frag}}{dx_{bj}dQ^2dz_f dq_T^2 d\phi d\chi} &= \frac{\alpha_{\rm em}^2\alpha_s(-M_h)}{16\pi^2x_{bj}^2S_{ep}^2Q^2}\sum_{k=1}^{9}\mathcal{A}_k(\phi-\chi)\mathcal{S}_k\int\frac{dx}{x}f_1(x)\int\frac{dz}{z}\delta\left(\frac{q_T^2}{Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)\right)\\ &\times\left[\frac{D_T(z)}{z}\hat{\sigma}^k_T - \left\{\frac{d}{d(1/z)}\frac{D_{1T}^{\perp(1)}(z)}{z}\right\}\hat{\sigma}^k_{\perp D} - D_{1T}^{\perp(1)}(z)\hat{\sigma}^k_{\perp}\right.\\ &+\int\frac{dz'}{z'^2}\mathrm{P}\left(\frac{1}{1/z-1/z'}\right)\left\{\Im\hat{D}_{\rm FT}(z,z')\left[\frac{z'}{z}\hat{\sigma}^k_{\rm DF1}+\frac{1}{z}\left(\frac{1}{z'}-\frac{1}{(1-q_T^2/Q^2)z_f}\right)^{-1}\hat{\sigma}^k_{\rm DF2}\right]\right.\\ &\left.\left.+\Im\hat{G}_{\rm FT}(z,z')\left[\frac{z'}{z}\hat{\sigma}^k_{GF1}+\frac{1}{z}\left(\frac{1}{z'}-\frac{1}{(1-q_T^2/Q^2)z_f}\right)^{-1}\hat{\sigma}^k_{GF2}\right]\right\}\right].\end{aligned} \tag{47}$$

This is the final form we use to present the twist-3 quark FF contribution to $ep\to e\Lambda^\uparrow X$ shown in Fig. 3.

In order to present the partonic hard cross sections in (47), it is convenient to introduce the quark FF contribution to the twist-2 unpolarized cross section for $ep\to e\Lambda X$. It reads

$$\frac{d^6\sigma^{\rm unpol}}{dx_{bj}dQ^2dz_f dq_T^2 d\phi d\chi} = \frac{\alpha_{\rm em}^2\alpha_s}{16\pi^2x_{bj}^2S_{ep}^2Q^2}\sum_{k=1}^{4}\mathcal{A}_k(\phi-\chi)\int\frac{dx}{x}f_1(x)\int\frac{dz}{z}D_q(z)\delta\left(\frac{q_T^2}{Q^2}-\left(1-\frac{1}{\hat{x}}\right)\left(1-\frac{1}{\hat{z}}\right)\right)C_F\hat{\sigma}^k_U, \tag{48}$$

where $D_q(z)$ denotes the twist-2 unpolarized quark FF, and $C_F\hat{\sigma}^k_U$ represents the partonic hard cross sections which depend on whether $f_1(x)=q(x)$ (quark DF) or $f_1(x)=G(x)$ (gluon DF). We will see below that some of the hard cross sections in (47) are related to $\hat{\sigma}^k_U$s.

Below we give LO Feynman diagrams for the hard part and the results for the hard cross sections $\hat{\sigma}_{T,\perp D,\perp}$ and $\hat{\sigma}_{{\rm DF1},GF1,DF2,GF2}$, which are the functions of $Q$, $q_T$, $\hat{x}=x_{bj}/x$ and $\hat{z}=z_f/z$.

(1) $\gamma q\to qg$ channel:

The diagrams for the hard part in this channel are shown in Fig. 4. It is convenient to present some of the hard cross sections in (47) in the following form:

$$\hat{\sigma}^k_T = C_F\hat{\sigma}^k_1 - \frac{1}{N}\frac{1-\hat{z}}{q_T\hat{z}}\hat{\sigma}^k_U, \tag{49}$$

$$\hat{\sigma}^k_{\perp D} = \frac{1}{N}\frac{1-\hat{z}}{q_T\hat{z}}\hat{\sigma}^k_U, \tag{50}$$

$$\hat{\sigma}^k_{\perp} = C_F\hat{\sigma}^k_2, \tag{51}$$

$$\hat{\sigma}^k_{\rm DF2} = (1-\hat{z})\left(\frac{C_F}{1-\hat{x}-\hat{z}}-\frac{1}{2N\hat{z}}\right)\hat{\sigma}^k_2, \tag{52}$$

$$\hat{\sigma}^k_{GF2} = (1-\hat{z})\left(\frac{C_F}{1-\hat{x}-\hat{z}}-\frac{1}{2N\hat{z}}\right)\hat{\sigma}^k_3, \tag{53}$$

where $\hat{\sigma}^k_U$s are the unpolarized partonic cross sections introduced in (48) for the unpolarized quark

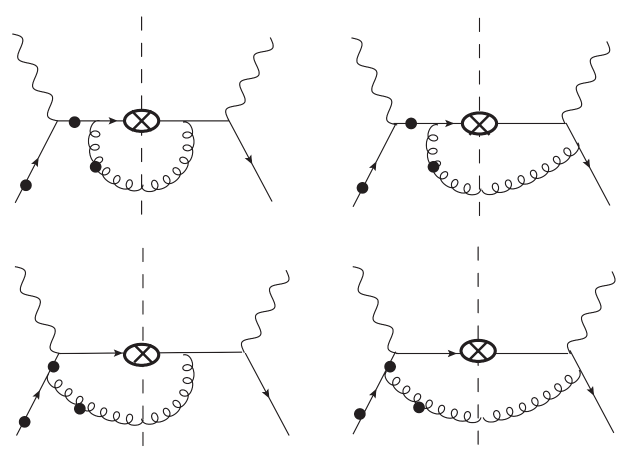

FIG. 4. LO diagrams in the $\gamma q\to qg$ channel contributing to the hard parts $S_{\rho\sigma}(k)$ and $S^L_{\alpha,\rho\sigma}(z',z)$ in (41). A cross in an oval in each diagram represents the fragmentation insertion. For $S^L_{\alpha,\rho\sigma}(z',z)$, an extra gluon line connecting the cross in the oval and each dot in the diagram should be added.

distribution $f_1(x) = q(x)$ and $\hat{\sigma}^k_{1,2,3}$ are newly introduced partonic cross sections, and hence specification of $\hat{\sigma}^k_U$, $\hat{\sigma}^k_{1,2,3}$, $\hat{\sigma}_{\rm DF1}$ and $\hat{\sigma}_{GF1}$ determines the cross sections in (47). We shall now give those in the $\gamma q \to qg$ channel.
(a) Twist-2 unpolarized cross section:

$$\begin{aligned}
\hat{\sigma}^1_U &= \frac{2(2-2\hat{x}+\hat{x}^2+\hat{z}(-2+8\hat{x}-6\hat{x}^2)+\hat{z}^2(1-6\hat{x}+6\hat{x}^2))}{(1-\hat{z})(1-\hat{x})},\\
\hat{\sigma}^2_U &= 8\hat{x}\,\hat{z},\\
\hat{\sigma}^3_U &= \frac{4Q(1-\hat{x}+\hat{z}(-1+2\hat{x}))}{q_T},\\
\hat{\sigma}^4_U &= 4\hat{x}\,\hat{z}\,.
\end{aligned} \tag{54}$$

$\hat{\sigma}^{8,9}_U = 0$ is understood in (49) and (50).
(b) $k=1$:

$$\begin{aligned}
\hat{\sigma}^1_1 &= \frac{4(1-9\hat{x}+14\hat{x}^2-6\hat{x}^3+2\hat{z}^2(1-6\hat{x}+6\hat{x}^2)+\hat{z}(-4+23\hat{x}-30\hat{x}^2+12\hat{x}^3))}{q_T(-1+\hat{x})(-1+\hat{z}+\hat{x})},\\
\hat{\sigma}^1_2 &= -\frac{4((-1+\hat{x})^3-3\hat{z}(-1+\hat{x})^2(-1+2\hat{x})+\hat{z}^3(1-6\hat{x}+6\hat{x}^2)+\hat{z}^2(-3+16\hat{x}-18\hat{x}^2+6\hat{x}^3))}{q_T(-1+\hat{z})(-1+\hat{x})(-1+\hat{z}+\hat{x})},\\
\hat{\sigma}^1_3 &= -\frac{4((-1+\hat{x})^3-3\hat{z}(-1+\hat{x})^2(-1+2\hat{x})+\hat{z}^3(1-6\hat{x}+6\hat{x}^2)+\hat{z}^2(-3+14\hat{x}-18\hat{x}^2+6\hat{x}^3))}{q_T(-1+\hat{x})(-1+\hat{z})(-1+\hat{z}+\hat{x})},\\
\hat{\sigma}^1_{\rm DF1} &= \frac{1}{N}\frac{2(-1+(-1+\hat{z})\hat{x})}{q_T\hat{z}(-1+\hat{x})},\\
\hat{\sigma}^1_{GF1} &= -\frac{1}{N}\frac{2(1+(-1+\hat{z})\hat{x})}{q_T\hat{z}(-1+\hat{x})},
\end{aligned} \tag{55}$$

(c) $k=2$:

$$\begin{aligned}
\hat{\sigma}^2_1 &= \frac{16(-1+2\hat{z})\hat{x}}{q_T},\\
\hat{\sigma}^2_2 &= -\frac{16\hat{x}\,\hat{z}}{q_T},\\
\hat{\sigma}^2_3 &= -\frac{16\hat{z}\,\hat{x}}{q_T},\\
\hat{\sigma}^2_{\rm DF1} &= 0,\\
\hat{\sigma}^2_{GF1} &= 0.
\end{aligned} \tag{56}$$

(d) $k=3$:

$$\begin{aligned}
\hat{\sigma}^3_1 &= \frac{4\hat{x}((-1+\hat{x})^2+\hat{z}^3(-4+8\hat{x})+2\hat{z}^2(5-10\hat{x}+4\hat{x}^2)-2\hat{z}(3-7\hat{x}+4\hat{x}^2))}{Q(-1+\hat{z})(-1+\hat{x})(-1+\hat{z}+\hat{x})},\\
\hat{\sigma}^3_2 &= -\frac{4\hat{x}\,\hat{z}(-2(-1+\hat{x})^2+\hat{z}^2(-2+4\hat{x})+\hat{z}(5-8\hat{x}+4\hat{x}^2))}{Q(-1+\hat{z})(-1+\hat{x})(-1+\hat{z}+\hat{x})},\\
\hat{\sigma}^3_3 &= -\frac{4Q(-2(-1+\hat{x})^2+\hat{z}^2(-2+4\hat{x})+\hat{z}(3-8\hat{x}+4\hat{x}^2))}{q_T^2(-1+\hat{z}+\hat{x})},\\
\hat{\sigma}^3_{\rm DF1} &= \frac{1}{N}\frac{2\hat{x}}{Q(-1+\hat{x})},\\
\hat{\sigma}^3_{GF1} &= -\frac{1}{N}\frac{2\hat{x}}{Q(-1+\hat{x})}.
\end{aligned} \tag{57}$$

(e) $k=4$:

$$\hat{\sigma}_1^4=4\frac{(4\hat{x}\hat{z}^2-2(-1+\hat{x})x+\hat{z}(1-6\hat{x}+4\hat{x}^2))}{q_T(-1+\hat{z}+\hat{x})},$$
$$\hat{\sigma}_2^4=-\frac{4\hat{z}(1+2(-1+\hat{z})\hat{x}+2\hat{x}^2)}{q_T(-1+\hat{z}+\hat{x})},$$
$$\hat{\sigma}_3^4=-\frac{4\hat{z}(-1+2(-1+\hat{z})\hat{x}+2\hat{x}^2)}{q_T(-1+\hat{z}+\hat{x})},$$
$$\hat{\sigma}_{\mathrm{DF1}}^4=\frac{1}{N}\frac{2}{q_T},$$
$$\hat{\sigma}_{GF1}^4=-\frac{1}{N}\frac{2}{q_T}. \tag{58}$$

(f) $k=8$:

$$\hat{\sigma}_1^8=\frac{4\hat{x}(-(-1+\hat{x})^2+2\hat{z}(-1+\hat{x})^2+2\hat{x}\hat{z}^2)}{Q(-1+\hat{z})(-1+\hat{x})(-1+\hat{z}+\hat{x})},$$
$$\hat{\sigma}_2^8=-\frac{4\hat{x}\hat{z}^2}{Q(-1+\hat{z})(-1+\hat{x})(-1+\hat{z}+\hat{x})},$$
$$\hat{\sigma}_3^8=\frac{4\hat{z}Q}{q_T^2(-1+\hat{z}+\hat{x})},$$
$$\hat{\sigma}_{\mathrm{DF1}}^8=-\frac{1}{N}\frac{2\hat{x}}{Q(1-\hat{x})},$$
$$\hat{\sigma}_{GF1}^8=\frac{1}{N}\frac{2\hat{x}}{Q(1-\hat{x})}. \tag{59}$$

(g) $k=9$:

$$\sigma_1^9=\frac{4\{\hat{z}+2\hat{x}\,\hat{z}+2(-1+\hat{x})\hat{x}\}}{q_T(-1+\hat{z}+\hat{x})},$$
$$\sigma_2^9=-\frac{4\hat{z}}{q_T(-1+\hat{z}+\hat{x})},$$
$$\sigma_3^9=\frac{4\hat{z}}{q_T(-1+\hat{z}+\hat{x})},$$
$$\sigma_{\mathrm{DF1}}^9=\frac{1}{N}\frac{2}{q_T},$$
$$\sigma_{GF1}^9=-\frac{1}{N}\frac{2}{q_T}. \tag{60}$$

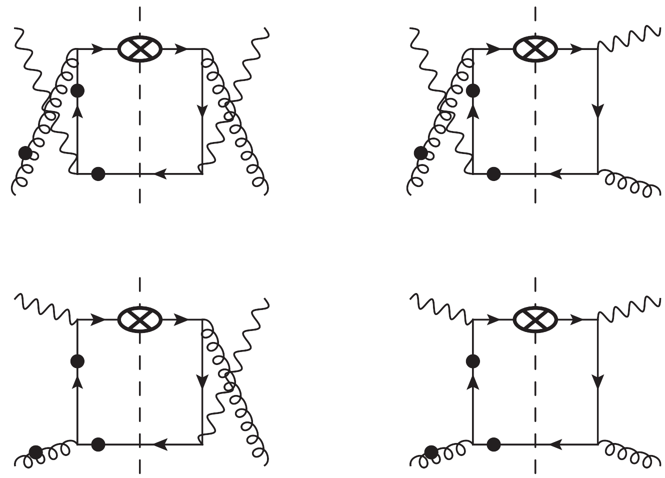

FIG. 5. LO diagrams in the $\gamma g\to q\bar{q}$ channel contributing to the hard parts $S_{\rho\sigma}(k)$ and $S^L_{\alpha,\rho\sigma}(z',z)$ in (41). A cross in an oval in each diagram represents the fragmentation insertion. For $S^L_{\alpha,\rho\sigma}(z',z)$, an extra gluon line connecting the cross in the oval and each dot in the diagram should be added.

(2) $\gamma g\to q\bar{q}$ channel:
Diagrams for the hard part in this channel are shown in Fig. 5. It is convenient to present some of the hard cross sections in (47) in the following form:

$$\hat{\sigma}_T^k=\hat{\sigma}_1^k-\frac{1}{N^2-1}\frac{1-\hat{z}}{q_T\hat{z}}\hat{\sigma}_U^k, \tag{61}$$

$$\hat{\sigma}_{\perp D}^k=\frac{N}{2C_F}\frac{1-\hat{z}}{q_T\hat{z}}\hat{\sigma}_U^k, \tag{62}$$

$$\hat{\sigma}_{\mathrm{DF2}}^k=\frac{2\hat{x}}{1-\hat{x}-\hat{z}}\left(\frac{1-\hat{x}}{1-\hat{x}-\hat{z}}+\frac{1}{N^2-1}\right)\hat{\sigma}_2^k, \tag{63}$$

$$\hat{\sigma}_{GF2}^k=\frac{2\hat{x}}{1-\hat{x}-\hat{z}}\left(\frac{1-\hat{x}}{1-\hat{x}-\hat{z}}+\frac{1}{N^2-1}\right)\hat{\sigma}_3^k, \tag{64}$$

where $\hat{\sigma}_U^k$'s are the unpolarized partonic cross sections in (48) for the unpolarized gluon distribution $f_1(x)=G(x)$, and $\hat{\sigma}_{1,2,3}^k$ are newly introduced here. We will give $\hat{\sigma}_U^k$, $\hat{\sigma}_{1,2,3}^k$, $\hat{\sigma}_\perp^k$, $\hat{\sigma}_{\mathrm{DF1}}^k$, and $\hat{\sigma}_{GF1}^k$ below.

(a) Twist-2 unpolarized cross sections:

$$\hat{\sigma}_U^1=2\frac{1-2\hat{x}+2\hat{x}^2-2\hat{z}(1-6\hat{x}+6\hat{x}^2)+2\hat{z}^2(1-6\hat{x}+6\hat{x}^2)}{\hat{z}(1-\hat{z})},$$
$$\hat{\sigma}_U^2=16\hat{x}(1-\hat{x}),$$
$$\hat{\sigma}_U^3=\frac{8\hat{x}q_T(1-2\hat{z})(1-2\hat{x})}{Q(1-\hat{z})},$$
$$\hat{\sigma}_U^4=8\hat{x}(1-\hat{x}), \tag{65}$$

where $\hat{\sigma}_U^{8,9}=0$ is understood in (61) and (62).

(b) $k = 1$:

$$
\begin{aligned}
\hat{\sigma}_1^1 &= -\frac{2(-1+\hat{x})\{-2+5\hat{x}-3\hat{x}^2+\hat{z}^2(-1+6\hat{x}^2)+\hat{z}(4-11\hat{x}+6\hat{x}^2)\}}{q_T\hat{z}^2(-1+\hat{z}+\hat{x})},\\
\hat{\sigma}_\perp^1 &= \frac{2}{q_T(-1+\hat{z})\hat{z}^2(-1+\hat{z}+\hat{x})}\{\hat{x}-\hat{x}^3+2\hat{z}^4(1-6\hat{x}+6\hat{x}^2)+\hat{z}^2(4-30\hat{x}+55\hat{x}^2-24\hat{x}^3)\\
&\quad +\hat{z}(-1+5\hat{x}-15\hat{x}^2+9\hat{x}^3)+\hat{z}^3(-5+37\hat{x}-54\hat{x}^2+18\hat{x}^3)\},\\
\hat{\sigma}_{\mathrm{DF1}}^1 &= -\frac{1}{N^2-1}\frac{2(3\hat{z}(1-2\hat{x})\hat{x}+\hat{x}(1+\hat{x})+\hat{z}^2(1-6\hat{x}+6\hat{x}^2))}{q_T\hat{z}^2},\\
\hat{\sigma}_{GF1}^1 &= -\frac{1}{N^2-1}\frac{2(\hat{z}(5-6\hat{x})\hat{x}+(-1+\hat{x})\hat{x}+\hat{z}^2(1-6\hat{x}+6\hat{x}^2))}{q_T\hat{z}^2},\\
\hat{\sigma}_2^1 &= \frac{1}{q_T\hat{x}\hat{z}^2}\{(-1+\hat{x})^2(-1+2\hat{x})-3\hat{z}(-1+\hat{x})^2(-1+4\hat{x})+\hat{z}^3(1-6\hat{x}+6\hat{x}^2)+3\hat{z}^2(-1+7\hat{x}-10\hat{x}^2+4\hat{x}^3)\},\\
\hat{\sigma}_3^1 &= \frac{-(-1+\hat{x})^2+3\hat{z}(-1+\hat{x})^2+\hat{z}^2(-3+9\hat{x}-6\hat{x}^2)+\hat{z}^3(1-6\hat{x}+6\hat{x}^2)}{q_T\hat{x}\hat{z}^2}.
\end{aligned}
\tag{66}
$$

(c) $k = 2$:

$$
\begin{aligned}
\hat{\sigma}_1^2 &= -\frac{8(-1+\hat{x})\hat{x}(-1+\hat{x}+\hat{z}\,\hat{x})}{q_T\hat{z}(-1+\hat{z}+\hat{x})},\\
\hat{\sigma}_\perp^2 &= \frac{8(-1+\hat{x})\hat{x}(1+2\hat{z}^2-\hat{x}+\hat{z}(-4+3\hat{x}))}{q_T\hat{z}(-1+\hat{z}+\hat{x})},\\
\hat{\sigma}_{\mathrm{DF1}}^2 &= -\frac{1}{N^2-1}\frac{8q_T\hat{x}^2}{Q^2},\\
\hat{\sigma}_{GF1}^2 &= -\frac{1}{N^2-1}\frac{8q_T\hat{x}^2}{Q^2},\\
\hat{\sigma}_2^2 &= \frac{4q_T\hat{x}(-2+\hat{z}+2\hat{x})}{Q^2},\\
\hat{\sigma}_3^2 &= \frac{4q_T\hat{x}\,\hat{z}}{Q^2}.
\end{aligned}
\tag{67}
$$

(d) $k = 3$:

$$
\begin{aligned}
\hat{\sigma}_1^3 &= -\frac{2\hat{x}(-4(-1+\hat{x})^2+2\hat{z}(3-4\hat{x}+\hat{x}^2)+\hat{z}^2(-1-2\hat{x}+4\hat{x}^2))}{Q(-1+\hat{z})\hat{z}(-1+\hat{z}+\hat{x})},\\
\hat{\sigma}_\perp^3 &= \frac{2\hat{x}(-1-\hat{x}+2\hat{x}^2+\hat{z}^3(-4+8\hat{x})+\hat{z}(-3+16\hat{x}-10\hat{x}^2)+\hat{z}^2(9-26\hat{x}+12\hat{x}^2))}{Q(-1+\hat{z})\hat{z}(-1+\hat{z}+\hat{x})},\\
\hat{\sigma}_{\mathrm{DF1}}^3 &= -\frac{1}{N^2-1}\frac{2\hat{x}(-1-2\hat{x}+\hat{z}(-2+4\hat{x}))}{Q\hat{z}},\\
\hat{\sigma}_{GF1}^3 &= -\frac{1}{N^2-1}\frac{2(-1+2\hat{z})\hat{x}(-1+2\hat{x})}{Q\hat{z}},\\
\hat{\sigma}_2^3 &= \frac{-3+7\hat{x}-4\hat{x}^2+\hat{z}^2(-2+4\hat{x})+2\hat{z}(3-7\hat{x}+4\hat{x}^2)}{Q\hat{z}},\\
\hat{\sigma}_3^3 &= \frac{-1-2\hat{z}(-1+\hat{x})+\hat{x}+\hat{z}^2(-2+4\hat{x})}{Q\hat{z}}.
\end{aligned}
\tag{68}
$$

(e) $k=4$:

$$\begin{aligned}
\hat{\sigma}_1^4 &= -\frac{2(-1+\hat{x})(1-3\hat{x}+2(1+\hat{z})\hat{x}^2)}{q_T\hat{z}(-1+\hat{z}+\hat{x})},\\
\hat{\sigma}_\perp^4 &= \frac{2(-1+\hat{x})(1+(1-8\hat{z}+4\hat{z}^2)x+(-2+6\hat{z})\hat{x}^2)}{q_T\hat{z}(-1+\hat{z}+\hat{x})},\\
\hat{\sigma}_{\mathrm{DF1}}^4 &= -\frac{1}{N^2-1}\frac{2(-1+\hat{x})(-1+2(-1+\hat{z})\hat{x})}{q_T\hat{z}},\\
\hat{\sigma}_{GF1}^4 &= -\frac{1}{N^2-1}\frac{2(-1+\hat{x})(1+2(-1+\hat{z})\hat{x})}{q_T\hat{z}},\\
\hat{\sigma}_2^4 &= \frac{q_T(1+2(-2+\hat{z})\hat{x}+4\hat{x}^2)}{Q^2},\\
\hat{\sigma}_3^4 &= \frac{q_T(-1+2\hat{x}\,\hat{z})}{Q^2}.
\end{aligned} \tag{69}$$

(f) $k=8$:

$$\begin{aligned}
\hat{\sigma}_1^8 &= -\frac{2\hat{x}(-2(-1+\hat{x})^2+4\hat{z}(-1+\hat{x})^2+\hat{z}^2(-1+2\hat{x}))}{Q(-1+\hat{z})\hat{z}(-1+\hat{z}+\hat{x})},\\
\hat{\sigma}_\perp^8 &= -\frac{2\hat{x}(1-\hat{z}-\hat{x}+\hat{z}^2(-1+2\hat{x}))}{Q(-1+\hat{z})\hat{z}(-1+\hat{z}+\hat{x})},\\
\hat{\sigma}_{\mathrm{DF1}}^8 &= \frac{1}{N^2-1}\frac{\hat{x}}{2Q\hat{z}},\\
\hat{\sigma}_{GF1}^8 &= -\frac{1}{N^2-1}\frac{\hat{x}}{2Q\hat{z}},\\
\hat{\sigma}_2^8 &= -\frac{(-1+2\hat{z})(-1+\hat{x})^2}{Q\hat{z}},\\
\hat{\sigma}_3^8 &= \frac{(-1+2\hat{z})(-1+\hat{x})}{Q\hat{z}}.
\end{aligned} \tag{70}$$

(g) $k=9$:

$$\begin{aligned}
\hat{\sigma}_1^9 &= -\frac{2(-1+\hat{x})(1+(-5+2\hat{z})\hat{x}+4\hat{x}^2)}{q_T\hat{z}(-1+\hat{z}+\hat{x})},\\
\hat{\sigma}_\perp^9 &= -\frac{2(-1+\hat{x})(-1+\hat{x}+2\hat{z}\,\hat{x})}{q_T\hat{z}(-1+\hat{z}+\hat{x})},\\
\hat{\sigma}_{\mathrm{DF1}}^9 &= \frac{1}{N^2-1}\frac{q_T\hat{x}}{2Q^2(-1+\hat{z})},\\
\hat{\sigma}_{GF1}^9 &= -\frac{1}{N^2-1}\frac{q_T\hat{x}}{2Q^2(-1+\hat{z})},\\
\hat{\sigma}_2^9 &= -\frac{q_T(-1+2\hat{x})}{Q^2},\\
\hat{\sigma}_3^9 &= \frac{q_T(-1+2\hat{x})}{Q^2}.
\end{aligned} \tag{71}$$

This completes the specification of all partonic hard cross sections in (47) and (48).

## IV. SUMMARY

In this paper, we have studied the twist-3 cross section for the transversely polarized hyperon production in SIDIS $ep \to e\Lambda^{\uparrow}X$ in the framework of collinear factorization. The cross section consists of five structure functions with different dependences on the azimuthal angles. We have presented the LO cross section which occurs from the twist-3 DF in the initial proton combined with the transversity FF for $\Lambda^{\uparrow}$ and the twist-3 quark FFs for $\Lambda^{\uparrow}$ combined with the unpolarized DF in the proton for all five structure functions. The derived cross section is relevant for the large-$P_T$ $\Lambda^{\uparrow}$ production in the future EIC experiment. For completeness the contribution from the twist-3 purely gluon FFs as well as another $q\bar{q}g$-FF of the type $\sim\langle 0|gF^{\alpha w}|hX\rangle\langle hX|\bar{\psi}\psi|0\rangle$ needs to be included (as discussed at the end of Sec. II), which will be reported in a separate paper [31].

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