

Gauge hierarchy from electroweak vacuum metastability

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We consider the possibility that the gauge hierarchy is a byproduct of the metastability of the electroweak vacuum, i.e., that whatever mechanism is responsible for the latter also sets the running Higgs mass to a value smaller than its natural value by many orders of magnitude. We find that the metastability of the electroweak vacuum, together with the requirement that such a nontrivial vacuum exists, requires the Higgs mass to be smaller than the instability scale by around 1 order of magnitude. While this bound is quite weak in the Standard Model (SM), as the instability scale is $\sim 10^{11}$ GeV, simple and well-motivated extensions of the SM significantly tighten the bound by lowering the instability scale. We first consider the effect of right-handed neutrinos in the ν MSM with approximate $B - \tilde{L}$ symmetry, which allows for masses of order TeV for the right-handed neutrinos and $\mathcal{O}(1)$ Yukawa couplings. We find that right-handed neutrinos cannot by themselves fully explain the gauge hierarchy, as the tightest upper bound compatible with current experimental constraints is $\sim 10^8$ GeV. As we demonstrate on the example of the minimal $SU(4)/Sp(4)$ composite Higgs model, this bound can be lowered significantly through the interplay of the neutrinos and a dimension-six operator. In this scenario, the bound can be brought down considerably, with the smallest value accessible by our perturbative treatment being of order $\simeq 10$ TeV, and consistently several orders of magnitude below its natural value. While this is insufficient to fully solve the gauge hierarchy problem, our results imply that, assuming the SM symmetry-breaking pattern, small running Higgs masses are a universal property of theories giving rise to metastability, suggesting a common origin of the two underlying fine-tunings and providing a strong constraint on any attempt to explain metastability.

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I. INTRODUCTION

The principles of *naturalness* and symmetry have guided the development of fundamental physics over the last 50 years. However, in the light of our Universe's apparent fine-tuning, it appears increasingly likely that particle physics has entered a *post-naturalness* era [1]. One of the most important observations motivating this perspective are the measured values of the Higgs and top quark mass, $M_h = 125.1$ GeV and $M_t = 172.4$ GeV, which imply that the Standard Model (SM) couplings remain perturbative until energies exceeding even the Planck scale. This, together

with the absence of new physics in flavor, precision, and LHC experiments, points to the SM being valid up to very high energies and potentially even the Planck scale.

An obvious drawback of such a *grand desert* above the electroweak scale is that the Higgs mass requires fine-tuning to cancel large radiative corrections, which cannot be explained by (technical) naturalness alone [2], leading to the so-called *hierarchy problem*. Furthermore, it opens up the question why the laws of nature appear to include two fundamental scales, set by the Planck and the Higgs mass, which are separated from one another by 17 orders of magnitude, which is known as the *Higgs naturalness* or *gauge hierarchy problem*.

As we argue throughout this article, this last question might in parts be answered by considering yet another apparent fine-tuning of the Higgs sector. The masses of both the Higgs and the top quark lie in an extraordinarily small window corresponding to a metastable electroweak vacuum [3–17]. When extrapolated to high energies, the Higgs quartic coupling becomes negative at the instability

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scale $\mu_I \sim 10^{11}$ GeV and remains small, which allows our vacuum to decay through bubble nucleation. Using the most recent global averages given in [18,19], its lifetime, defined as the characteristic time to form a bubble in a true vacuum within our past light cone, is found to be at 1σ ,¹

$$\tau_{\text{EW}} \sim 10^{983_{-430}^{+1410}} \text{ years.} \quad (1)$$

This hinges on a delicate cancellation between the exponentially small decay rate per unit volume of the vacuum, $\frac{\Gamma}{V} \sim \exp\left(-\frac{8\pi^2}{3|\lambda(\mu_*)|}\right)$, which in the pure Standard Model is exquisitely sensitive to the quartic coupling at the scale $\mu_* \sim 10^{17}$ GeV, where the latter achieves a minimum, and the exponentially large space-time volume of the observable Universe. In other words, metastability requires a remarkable conspiracy between the cosmological constant and the couplings of the Standard Model.

A way forward is suggested by a common theme underlying many fine-tunings, namely, that they are problems of near criticality. The metastability of the electroweak vacuum requires that the decay rate per space-time volume be relatively close to the critical range for the percolation phase transition of bubble nucleation [20]. The gauge hierarchy problem can be interpreted as the Higgs having a nearly vanishing mass relative to the fundamental scale, close to the phase transition between broken/unbroken electroweak symmetry [21].²

The main point of this article is that two of these examples—the metastability of the electroweak vacuum and the smallness of the (running) Higgs mass compared to the Planck mass—are in fact not entirely independent, as a small Higgs mass is a necessary condition for metastability. In other words, *any* explanation of metastability also offers a path toward a solution to the Higgs naturalness problem.

A. Gauge hierarchy from metastability

Our central result is that the metastability of the electroweak vacuum, together with the very requirement that such a nontrivial vacuum exists, implies an upper bound on the Higgs mass. In other words, metastability necessarily implies a hierarchy between the running Higgs mass and its natural value, whose precise extent depends on the lifetime.

¹We take into account the correlated errors in the top Yukawa, Higgs quartic, and strong gauge coupling as given in [19]. An extensive discussion of the lifetime's sensitivity to other parameters can be found in [13].

²However, even with a positive Higgs mass squared, the electroweak phase would still be spontaneously broken at the QCD scale by the Higgs coupling to the quark condensate. See [22,23] for discussions of this other phase of the SM. Thus, one must not only explain why the Higgs mass is small, but also why the Higgs mass squared is negative.

We show this in three steps:

- (1) Under our assumptions, the running Higgs mass is bounded from above through the instability scale, $m_h^2 < |\beta_\lambda(\mu_I)|(\dots)\mu_I^2$.
- (2) The instability scale lies below the so-called *instanton scale*, $\mu_I^2 \ll \mu_S^2$, which we introduce in the next section.
- (3) The instanton scale lies below the natural value for the running Higgs mass, $\mu_S^2 \lesssim m_{h,\text{nat}}^2$.

We furthermore show that through the effects of well-motivated SM extensions it is possible to obtain a bound just 2 orders of magnitude above the observed value within the limitations of our perturbative treatment and potentially even lower if a more elaborate analysis was to be performed.

The relation between Higgs mass parameter and instability scale was first pointed out in the context of the SM in [11], whose authors obtained the following upper bound³:

$$m_h^2 < |\beta_\lambda(\mu_I)|e^{-3/2}\mu_I^2. \quad (2)$$

Thus, once the instability scale μ_I has been determined, the condition for the existence of a false vacuum constrains the Higgs mass by about 1 more order of magnitude.

Until recently, the usefulness of this bound has widely been dismissed, in particular, by the authors of [11] themselves, as it appears to suffer from two important problems. The first of them is that it simply shifts the question to why the instability scale lies so far below the natural value for the Higgs mass, and more importantly, why it exists in the first place. From our perspective, the existence of the instability scale is given, as it is necessary to allow the vacuum to decay. To achieve metastability with a relatively short lifetime, the quartic coupling must become negative at the instability scale μ_I and then continue to fall off until it reaches a sufficiently negative value $\lambda(\mu_S)$ to obtain the desired lifetime, with μ_S being the *instanton scale* defined in Sec. II. Since λ depends only logarithmically on the scale, this running stretches over several orders of magnitude. Furthermore, as we argue in Sec. II, the properties of the decay rate imply a hierarchy between μ_S and the natural value for m_h^2 .

The second reason why one might question the usefulness of (2) is that it only requires the Higgs mass to lie slightly below the instability scale, which, assuming continued validity of the SM, can be found at roughly 10^{11} GeV, leaving unexplained an additional 8 orders of magnitude to reach the electroweak scale. It is now, however, important to observe that the precise extent of this remaining gap is sensitive to beyond-the-SM physics. In the context of the usefulness of inequality (2), this has recently been discussed in [24], whose authors argue that

³In Sec. III, we review its derivation and derive a more precise version of the bound.

the interplay of a dimension-six operator and additional fermions can significantly lower the instability scale and thus strengthen the bound (2). In Sec. II, we discuss how such SM extensions effect the vacuum's decay rate, before providing explicit numerical results for the SM with gravity as well as two well-motivated Standard Model extensions, namely, the minimal SU(4)/Sp(4) composite Higgs model, which serves as an example for the impact of a dimension-6 term, and the ν MSM with approximate $B - \tilde{L}$ symmetry, whose right-handed neutrinos serve as an example for additional fermion singlets. Besides lowering the instability scale and thus our bound on the Higgs mass, we find that the influence of the additional fermions also shortens the vacuum's lifetime, to the extent that, e.g., the Page time can be achieved. We provide accurate predictions of the vacuum's lifetime as a function of the parameters characterizing the SM extensions of interest. As an important side result, we not only update existing stability bounds by combining for the first time all relevant next-to-leading-order (NLO) corrections, both from gravity and functional determinants, at up to 3-loop accuracy, but also provide a translation of any prediction of the vacuum's lifetime to parameters of the considered SM extensions.

Before discussing in more detail the SM extensions considered here, it is worth emphasizing that our upper bound (2) relies on two assumptions: (i) metastability of the electroweak vacuum and (ii) the existence of such a nontrivial vacuum, i.e., a negative Higgs mass squared.

B. Achieving metastability

One framework in which metastability may naturally arise is the landscape of string theory, together with the mechanism of eternal inflation [25–29] for dynamically populating its vacua. The *principle of mediocrity*, generally assumed in this context (e.g., [30]), entails that we live at asymptotically late times in the unfolding of eternal inflation, when the relative occupational probabilities of different vacua have settled to a near-equilibrium distribution.

However, a more interesting possibility for our purposes is based on the idea that we exist during the approach to equilibrium, i.e., at times much earlier than the exponentially long mixing time for the landscape. In this case, a vacuum like ours should be likely not because it is typical according to a quasistationary distribution, but rather because it has the right properties to be accessed early on in the evolution [31]. This perspective offers a dynamical selection mechanism for vacua based on search optimization [32–35]: vacua that are easily accessed reside in optimal regions where the search algorithm defined by local landscape dynamics is efficient. This idea was formalized with the definition of an accessibility or early-time measure in [33,35]. Importantly, optimal regions of the landscape display nonequilibrium critical phenomena, in the sense that their vacuum dynamics are tuned at *dynamical criticality*. This suggests a deep connection

between the near criticality of our Universe and non-equilibrium phase transitions on the landscape.

A key prediction is that optimal regions are characterized by vacua that are relatively short lived, with lifetimes of order their de Sitter Page time.⁴ For our vacuum, this optimal lifetime is

$$\tau_{\text{Page}} \sim \frac{M_{\text{Pl}}^2}{H_0^3} \simeq 10^{130} \text{ years.} \quad (3)$$

While this lies around 850 orders of magnitude below the central value of the SM, it should be clear that the latter is highly sensitive to beyond-the-SM physics. Indeed, as we show, well-motivated extensions of the SM, such as right-handed neutrinos, can very well shorten the vacuum lifetime down to the Page time.

Importantly, the early-time approach to eternal inflation offers a *raison d'être* for the conspiracy underlying Higgs metastability. That being said, while we are primarily motivated by nonequilibrium eternal inflationary dynamics, it is worth emphasizing once again that our analysis is logically independent of this proposal. The results below pertain more generally to *any* theoretical framework which predicts that our vacuum should be metastable.

One other example for such a framework is described in [24]. These authors propose that the parameters of the Higgs potential are functions of an additional scalar field, the *apeiron*, and provide a dynamical mechanism based on quantum first-order phase transitions in the early Universe favoring values of the field corresponding to a near-critical Higgs potential. For the quartic coupling λ , this translates to the selection of an RG trajectory close to the transition from a stable vacuum to a potential with both a false vacuum at the electroweak scale and a true vacuum at some higher energy, albeit the existence of the latter requires some UV completion to stabilize it. As we argue in Sec. III, these are the necessary conditions for metastability, while the favored RG trajectory of λ amounts to the prediction of an ideal lifetime.

What makes this approach particularly interesting from the perspective of this article is that it allows for a prediction of the Higgs mass parameter and vacuum expectation value. Following the reasoning of [24,37], the probability distributions for these quantities are strongly peaked around values near the instability scale. In the Appendix A 1, we show that this prediction corresponds to the saturation of our bound, making it an important special case of our universally applicable result.

II. ELECTROWEAK VACUUM DECAY

We begin with a brief review of Higgs metastability and how it is effected by the SM extensions of interest for our

⁴It is intriguing that the de Sitter Page time has also emerged recently in a completely different context, as the quantum break time of de Sitter space [36].

later reasoning, in particular, a H^6 operator. The electro-weak vacuum corresponds to the minimum of the Higgs potential, which at tree level is given by

$$V(\mathbf{H}) = -\frac{1}{2}m_h^2|\mathbf{H}|^2 + \lambda|\mathbf{H}|^4, \quad (4)$$

with \mathbf{H} denoting the standard Higgs doublet. The Higgs boson arises from fluctuations around the minimum of this potential, which can in unitary gauge be parametrized as

$$\mathbf{H} = \left(0, \frac{H}{\sqrt{2}}\right); \quad H = v + h, \quad (5)$$

where $v = \frac{m_h}{\sqrt{2\lambda}}$ corresponds to the potential's minimum and $m^2 = 2\lambda v^2 = m_h^2$ to the tree-level mass of the fluctuation h . In a similar manner, the masses of the fermions as well as the massive gauge bosons are proportional to v , as they are generated through the coupling of these particles to the Higgs field. Thus, most particles of the SM can be found at energies somewhere below v . That is, v , or equivalently m_h , sets the electroweak scale.

In a purely classical theory, the configuration $H = v$ represents the theory's unique, stable vacuum up to SU(2) transformations of the full Higgs doublet. This is, however, not necessarily true once the scale dependence of the parameters, due to quantum effects, is taken into account. Doing so gives rise to the RG-improved effective potential, which, for energies well above the electroweak scale, takes the simple form

$$V_{\text{eff}}(h) \simeq \frac{1}{4}\lambda_{\text{eff}}(h)h^4, \quad (6)$$

where $\lambda_{\text{eff}}(h)$ combines the running quartic coupling, its loop corrections, and the Higgs' wave function renormalization factors.

Depending on the precise values of the couplings, it is possible for λ to turn negative at high energies [4]. The data compiled at the LHC indicate that this might indeed be the case, as the measured values of the quartic and Yukawa couplings lie just shy of the critical value beyond which λ would remain positive at all scales. For the most accurate values of the couplings available, λ turns negative at the instability scale $\mu_I \simeq 10^{11}$ GeV, which signals metastability—the electroweak vacuum can decay through nucleation of a bubble of a large Higgs field within a small region of space, which then expands indefinitely.

The lifetime of the electroweak vacuum is determined by the bubble nucleation rate per unit volume, Γ/V . Its dominant contribution arises from the so-called bounce solution, which is the instanton describing the tunneling from the false vacuum into the region of negative λ , as famously shown in [38–40] and built upon in [41,42]. Once the rate per unit volume is known, it is straightforward to obtain the lifetime τ_{EW} of the vacuum, i.e., the time at which the probability that a bubble has formed within an

observer's past light cone becomes unity. Assuming vacuum energy dominance, it is given by

$$\tau_{\text{EW}} = \frac{3H_0^3}{4\pi} \left(\frac{\Gamma}{V}\right)^{-1}, \quad (7)$$

where H_0 is the Hubble constant. It is also common in the literature to define the lifetime as the characteristic time scale $\tau_{\text{EW}} \sim (\frac{\Gamma}{V})^{-1/4}$, as is, e.g., the case in [17].⁵ An additional technical hurdle for a potential of the form (6) arises from its classical scale invariance. As a result, there exists not just one bounce configuration but a one-parameter family of solutions,

$$H_R(r) = \frac{2\sqrt{2}}{\sqrt{-\lambda}} \frac{R}{R^2 + r^2}, \quad (8)$$

whose parameter R is the size of the bubble of true vacuum, and r is the four-dimensional Euclidean radius. This degeneracy manifests itself in the form of a nontrivial zero mode, the dilatation mode, which on its own would result in a divergent decay rate. An extensive discussion of the SM lifetime, including a clear solution to the scale invariance problem, was recently given in [17]. The problem is naturally solved by the RG running of λ , which breaks the scale invariance radiatively, resulting in the decay rate being dominated by the bounce of size $R = \mu_\star^{-1}$, where μ_\star satisfies

$$\beta_\lambda(\mu_\star) = 0. \quad (9)$$

Intuitively, the decay rate is dominated by the configuration associated with the scale at which λ is smallest.

Before performing the integral over the dilatation mode, the decay rate at NLO takes the form

$$\frac{\Gamma}{V} = \int \frac{dR}{R^5} e^{-\frac{8\pi^2}{3|\lambda(R^{-1})|} D(R^{-1})}, \quad (10)$$

where D summarizes all but the gravitational corrections at NLO, which we give in the Appendix. The integral over R can be performed explicitly by resumming loop corrections to the Euclidean action, leading to the result given in [17].⁶

⁵The difference in our lifetime is also partially due to an increase in the world average of the top mass since [17] was published.

⁶Note that we evaluate all couplings at the scale R^{-1} , while in [17] they are evaluated at μ_\star , which is more convenient if one wishes to perform the integral exactly. These authors show that setting the scale equal to μ_\star and demanding scale invariance of the nucleation rate induce a series of quantum corrections, allowing them to perform the integral order by order and finally resumming all terms obtained in this way. In order to obtain (11), all one has to do is perform the sum over all the corrections right away in the exponent.

As the scale μ_* lies quite close to the Planck mass, high-precision predictions of the lifetime require an analysis of gravitational corrections. Derivations of the leading-order contributions arising from gravity can be found in, e.g., [43–45]. This complicates the above prescription as it introduces terms in the Euclidean action that explicitly break scale invariance. The general influence of such terms on the decay rate has been discussed with a special focus on the interplay of gravitational corrections with RG running in [43]. In general, introducing an explicit breaking of scale invariance affects the Euclidean action not only directly via the additional term but also indirectly by changing the scale of the dominant bounce.

One way to understand this is to reconsider (10), supplemented with a gravitational term for concreteness. We assume that the latter is subdominant, so that the saddle point of the path integral can be approximated to leading order by the family of bounces (8). Including the leading-order gravitational correction to the Euclidean action given in, e.g., [45] leads to a modified version of (10),

$$\frac{\Gamma}{V} = \int \frac{dR}{R^5} e^{-\frac{8\pi^2}{3|\lambda(R^{-1})|} - \frac{256\pi^3}{45\lambda^2(R^{-1})(RM_{\text{Pl}})^2}} D(R^{-1}). \quad (11)$$

The technique used in [17] to perform the R integral is now spoiled by the gravitational term, so that we must be content with a saddle-point approximation,⁷

$$\frac{\Gamma}{V} \simeq e^{-S_E(\lambda(\mu_S), \mu_S)} \sqrt{\frac{2\pi}{\frac{d^2}{d\ln\mu^2} S_E(\lambda(\mu_S), \mu_S)}} \mu_S^4 \Lambda(\mu_S), \quad (12)$$

where the Euclidean action is

$$S_E(\lambda(\mu_S), \mu_S) = \frac{8\pi^2}{3|\lambda(\mu_S)|} + \frac{256\pi^3}{45\lambda^2(\mu_S)} \frac{\mu_S^2}{M_{\text{Pl}}^2}. \quad (13)$$

All couplings, as well as the corrections summarized in D , are evaluated at the saddle point $R^{-1} = \mu_S$ minimizing the Euclidean action, which is the solution of

$$\beta_\lambda(\mu_S) \left(\frac{64\pi}{15} \frac{\mu_S^2}{M_{\text{Pl}}^2} - \lambda(\mu_S) \right) = \lambda(\mu_S) \frac{64\pi}{15} \frac{\mu_S^2}{M_{\text{Pl}}^2}. \quad (14)$$

This result agrees up to a factor of $\mathcal{O}(1)$ with the rate given in [45]. It is straightforward to prove that $\mu_S < \mu_*$, in general, with approximate equality $\mu_S \simeq \mu_*$ whenever $\mu_* \ll M_{\text{Pl}}$.

⁷In the following analytical discussions, we treat the R^{-5} prefactor as effectively $\mathcal{O}(1)$ when performing the saddle-point approximation, as its dependence on R is negligible compared to the $e^{\frac{1}{2}}$ of the Euclidean action. We nevertheless included it in our numerical calculations, confirming that this approximation is well justified.

As the left-hand side is suppressed by β_λ , it can be expected that the effect of the gravitational term, which is to lower μ_S relative to μ_* , becomes dominant already for values of μ_S 1 order of magnitude below M_{Pl} , leading to sub-Planckian values of μ_S even for values of μ_* multiple orders of magnitude above M_{Pl} . As this would clearly remain true if one were to replace the Planck mass by any cutoff scale Λ , it can, in general, be expected that any new physics stabilizing the Standard Model below μ_* has a strong impact on the decay rate even if the direct correction to the Euclidean action is suppressed by the naturally small factor $\frac{\mu_S^2}{\Lambda^2}$. Therefore, we are careful in what follows to distinguish between these two scales.

This effect also arises naturally in SM extensions that lead to an extended Higgs potential, most importantly (partially) composite Higgs models and models with a second, heavy Higgs doublet like the (ν) MSSM. After integrating out all heavier degrees of freedom, the most general Higgs potential takes the form

$$V_{\text{full}}(H) = -\frac{m_h^2}{4} H^2 + \frac{\lambda}{4} H^4 + \frac{C_6}{\Lambda^2} H^6 + \dots, \quad (15)$$

where the values of the Wilson coefficients at the matching scale Λ can be obtained by standard techniques. Through power counting and anticipating that μ_S tends to be smaller than Λ , it is clear that for a majority of generic potentials the dimension-six term will be the most important, so that we focus on its effect throughout the remainder of this article.

At lower energy, the coefficients $\{C_{2n}\}_{n>2}$ are obtained by integrating their beta functions. In principle, the dimension-six correction induces additional terms in the SM couplings' beta functions. However, these are suppressed by a factor of $\frac{m_h^2}{\Lambda^2}$ [46], which, as we are about to show, is necessarily $\ll 1$ for metastable vacua. Hence, we can safely neglect these corrections in the beta functions. The main impact of the dimension-six operator will be on the instanton scale μ_S and corresponding decay rate.

We proceed to calculate the vacuum decay rate, including the dimension-six correction to the potential

$$\Delta V_{\text{eff}} = \frac{C_6}{\Lambda^2} H^6. \quad (16)$$

As such, the impact of beyond-the-SM physics is treated perturbatively, neglecting higher-order terms in the full potential (15). This approximation is justified provided that

$$\left| \frac{\Delta V_{\text{eff}}}{V_{\text{eff}}} \right| = \frac{4C_6}{|\lambda|} \cdot \frac{H^2}{\Lambda^2} \ll 1. \quad (17)$$

On the bounce solution, which to leading order is given by (8), clearly $H_R(r)$ is maximal at the origin: $H_R(r) \leq H_R(0) \simeq 2\sqrt{2}\mu_S/\sqrt{|\lambda(\mu_S)|}$, where we have focused on

the dominant bounce with $R = \mu_S^{-1}$. Combining with (17), we are led to define an expansion parameter,

$$\epsilon \equiv \frac{32C_6}{\lambda^2} \cdot \frac{\mu_S^2}{\Lambda^2}. \quad (18)$$

For concreteness, we define the regime of validity of our perturbative treatment as $\epsilon \leq 1$. Given the relation between the lifetime and λ , our perturbative treatment is expected to break down either for small values of Λ or for sufficiently long-lived vacua.

Including the dimension-six operator (16) to the Higgs potential yields a correction to the bounce Euclidean action (13) given by

$$\Delta S_E = \frac{128\pi^2}{5|\lambda|^3} \frac{C_6}{R^2\Lambda^2}. \quad (19)$$

Thus, the decay rate (11) becomes, to leading order,

$$\frac{\Gamma}{V} = \int \frac{dR}{R^5} \exp\left(-\frac{8\pi^2}{3|\lambda(R^{-1})|} - \frac{128\pi^2(R^{-1})}{5|\lambda(R^{-1})|^3} \frac{C_6}{(R\Lambda)^2} - \frac{256\pi^3}{45\lambda^2(R^{-1})} \frac{1}{(RM_{\text{Pl}})^2}\right) \cdot D(R^{-1}). \quad (20)$$

By analogy with the gravitational correction, it is easy to see that the new term has two effects. First, it increases the value of the Euclidean action, thereby stabilizing the vacuum. Clearly, the strength of this effect increases with $\frac{C_6}{\Lambda^2}$; i.e., it is most significant either for large C_6 or small Λ . Second, it shifts the instanton scale μ_S further away from μ_* . For $\Lambda \lesssim \mu_*$, in particular, μ_S is shifted to scales below or close to Λ .

This can be quantified in parallel to the treatment of the gravitational term. As before, we perform the R integral in the saddle-point approximation, with the scale μ_S of the dominant bounce satisfying the modified equation,

$$\begin{aligned} \beta_\lambda(\mu_S) \left(\frac{64\pi}{15} \frac{\mu_S^2}{M_{\text{Pl}}^2} - \lambda(\mu_S) - \frac{144}{5} \frac{C_6}{\lambda(\mu_S)} \frac{\mu_S^2}{\Lambda^2} \right) + \beta_{C_6}(\mu_S) \frac{48}{5} \frac{\mu_S^2}{\Lambda^2} \\ = \frac{64\pi}{15} \lambda(\mu_S) \frac{\mu_S^2}{M_{\text{Pl}}^2} - \frac{96}{5} C_6(\mu_S) \frac{\mu_S^2}{\Lambda^2}. \end{aligned} \quad (21)$$

The decay rate in this approximation is once again given by (12), with the Euclidean action now of the form

$$S_E = \frac{8\pi^2}{3|\lambda(\mu_S)|} + \frac{128\pi^2 C_6(\mu_S)}{5|\lambda(\mu_S)|^3} \frac{\mu_S^2}{\Lambda^2} + \frac{256\pi^3}{45\lambda^2(\mu_S)} \frac{\mu_S^2}{M_{\text{Pl}}^2}. \quad (22)$$

III. UPPER BOUND ON THE RUNNING HIGGS MASS FROM METASTABILITY

A key point of our analysis is the observation that the metastability of the electroweak vacuum, together with the

very requirement that such a nontrivial vacuum exists, implies an upper bound on the Higgs mass. This connection was first pointed out in the context of the SM in [11], although its usefulness was quickly dismissed as the resulting bound lies several orders of magnitude above the observed value.

At first sight, how the vacuum lifetime can translate to a bound on the Higgs mass is somewhat nontrivial. The lifetime is highly sensitive to the quartic coupling, but it has no explicit dependence on the vacuum expectation value or, equivalently, the Higgs mass. The additional necessary ingredient is the demand of a nontrivial electroweak vacuum. This requires the existence of a minimum of the effective potential at field values below the instability scale μ_I , where the quartic coupling becomes negative.

To see this, let us follow [11] and minimize the effective potential, including the logarithmic running of λ but neglecting the running of m_h . Demanding that $\frac{d}{dh} V_{\text{eff}}(h) = 0$, we find

$$m_h^2 = \left(2\lambda(v) + \frac{\beta_\lambda(v)}{2} \right) v^2. \quad (23)$$

At scales somewhat larger than the instability scale, both quadratic and quartic terms appear with a negative sign, so that no extremum can occur. Thus, since we are interested in an upper bound for m_h , we can focus on values of v near μ_I , such that $\lambda(v) \simeq \beta_\lambda(\mu_I) \ln \frac{v}{\mu_I}$. This gives

$$m_h^2 \simeq \left(2 \ln \frac{v}{\mu_I} + \frac{1}{2} \right) \beta_\lambda(\mu_I) v^2. \quad (24)$$

If understood as a function of v , the right-hand side is bounded from above since $\beta_\lambda(\mu_I) < 0$, which implies that a solution can only exist for sufficiently small m_h ,

$$m_h^2 \lesssim |\beta_\lambda(\mu_I)| e^{-3/2} \mu_I^2. \quad (25)$$

Thus, once the instability scale has been determined, the condition for the existence of a false vacuum constrains the Higgs mass by about 1 more order of magnitude.

A more precise version of this bound can be obtained by including the nonlogarithmic 1-loop corrections to λ , following the power counting scheme developed in [47]. As shown in Appendix A 1, doing so replaces (25) by

$$m_h^2 \lesssim |\beta_\lambda(\mu_I)| \exp\left(-\frac{3}{2} - 2 \frac{\lambda_1(\mu_I)}{\beta_\lambda(\mu_I)}\right) \mu_I^2. \quad (26)$$

This of course reduces to (25) if λ_1 is neglected, as assumed in [11]. Unfortunately, (25) on its own is insufficient to explain the large hierarchy between the electroweak scale and the Planck scale, for two reasons. First, it only shifts the question to why the instability scale lies so far below the Planck scale, and why it exists in the first place. Second,

it only requires the Higgs mass to lie slightly below the instability scale, which as mentioned earlier is roughly 10^{11} GeV, leaving unexplained an additional 8 orders of magnitude to reach the electroweak scale.

The goal of this analysis is to address these two points as follows. With regard to the first point, as mentioned earlier, we assume the existence of the instability scale to be the result of some underlying UV physics determining the Higgs' couplings at some high energy. To achieve metastability, the quartic coupling must become negative at the instability scale μ_I and then continue to fall off until it reaches a sufficiently negative value $\lambda(\mu_S)$ to obtain the desired lifetime. Since λ depends only logarithmically on the scale, this running stretches over several orders of magnitude. This, together with relation (25), provides us with an upper bound for the Higgs mass parameter and therefore also the electroweak scale, as sketched in Fig. 1. As to the second point, in Sec. V, we explore beyond-the-SM physics naturally lowering the instability scale and thereby strengthening the bound on the Higgs mass.

We first derive a simple analytical relation between the vacuum's lifetime and the strength of the mass inequality, showing that less stable vacua lead to a stronger bound on the Higgs mass.

For the purpose of an approximate analytical treatment, it suffices to use the leading-order expression for the decay rate,

$$\frac{\Gamma}{V} \propto \mu_S^4 \exp\left(-\frac{8\pi^2}{3|\lambda(\mu_S)|}\right), \quad (27)$$

where μ_S satisfies (14). Note that, although the correction through either gravity or a dimension-6 term can be significant in minimizing S_E , the validity of the semi-classical description of course requires $\mu_S \ll M_{\text{Pl}}$ or $\mu_S \ll \Lambda$, respectively, so that the direct correction to the numerical value of S_E is small and can be safely neglected for the purpose of this estimate.

Thus, at this level, the decay rate is fully determined by the two parameters $\lambda(\mu_S)$ and μ_S . For a given μ_S , it is easy to

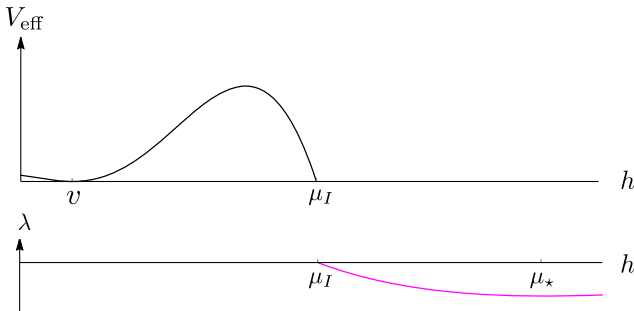


FIG. 1. The effective potential as well as the running of the quartic coupling, not to scale. If the scale μ_* where the quartic coupling reaches a minimum lies significantly below the Planck scale, then $\mu_S \simeq \mu_*$.

see that a larger $\lambda(\mu_S)$ (i.e., smaller $|\lambda(\mu_S)|$) corresponds to a smaller decay rate and hence a longer lifetime. Indeed, combining (7) and (27) gives

$$\begin{aligned} \lambda(\mu_S) &\simeq -\frac{8\pi^2}{223.14 + 3 \ln\left(\frac{\mu_S^4}{H_0^4 \text{ GeV}}\right) + 3 \ln\left(\frac{\tau_{\text{EW}}}{\text{yrs}}\right)} \\ &\simeq -\frac{8\pi^2}{1551.15 + 12 \ln\left(\frac{\mu_S}{4 \times 10^{16} \text{ GeV}}\right) + 3 \ln\left(\frac{\tau_{\text{EW}}}{\text{yrs}}\right)}, \end{aligned} \quad (28)$$

where in the last step we have substituted $H_0 \simeq 10^{-42}$ GeV and normalized μ_S to its SM value. Notice from the first line that, given the smallness of H_0 compared to typical values of μ_S , the Hubble constant plays a significant role in determining the lifetime. This highlights the fact that metastability does not only require a conspiracy of SM couplings but also of the Hubble and instanton scales.

The key point is that, once a lifetime τ_{EW} and an instanton scale μ_S are specified, the quartic coupling $\lambda(\mu_S)$ is determined through (28). Further substitution of μ_S and $\lambda(\mu_S)$ back into (14) then determines $\beta_\lambda(\mu_S)$. This gives us all the necessary data to perform a RG evolution and determine the instability scale μ_I at which λ crosses zero. While we soon solve the RG equations numerically, it is worth plowing ahead with an analytical estimate to get some intuition on how different parameters of the theory ultimately affect the Higgs mass bound.

To proceed, let us Taylor expand the quartic coupling around μ_S and evaluate the result at μ_I ,

$$\begin{aligned} 0 = \lambda(\mu_I) &= \lambda(\mu_S) + \beta_\lambda(\mu_S)(\ln \mu_I - \ln \mu_S) \\ &+ \frac{1}{2} \beta'_\lambda(\mu_S)(\ln \mu_I - \ln \mu_S)^2 + \dots \end{aligned} \quad (29)$$

To leading order, this gives

$$\mu_I \simeq \mu_S \exp\left(-\frac{|\lambda(\mu_S)|}{|\beta_\lambda(\mu_S)|}\right). \quad (30)$$

In the regime where $\mu_S \simeq \mu_*$, which occurs whenever $\mu_* \ll M_{\text{Pl}}$ or $\mu_* \ll \Lambda$, respectively, we must work to subleading order, since $\beta_\lambda(\mu_*) = 0$. The result in this case is an even larger hierarchy,

$$\mu_I \simeq \mu_* \exp\left(-\sqrt{2 \frac{|\lambda(\mu_*)|}{\beta'_\lambda(\mu_*)}}\right). \quad (31)$$

For all parameter values of interest in our discussion, the right-hand sides of (30) and (31) imply the existence of a significant hierarchy between the instability scale μ_I and the instanton scale μ_S . This approximate result can now also be used to understand the dependence of our bound (26) on the Higgs mass on the lifetime. It is straightforward to see

that, for a given value of μ_S , shorter lifetimes imply smaller values of μ_I and thus stronger upper bounds on m_h .

IV. METASTABILITY BOUNDS IN THE STANDARD MODEL WITH GRAVITY

As a first example for the consequences of our previous arguments, we assume that the SM is valid up to the Planck scale and neglect all Yukawa couplings except the top and bottom quarks and that of the tau. Furthermore, following [11], we treat $y_t(M_{\text{Pl}})$ and $\lambda(M_{\text{Pl}})$ as free parameters characterizing the vacuum, while, for simplicity, fixing all other couplings at the Planck scale to their SM-extrapolated values.

We now perform the RG running numerically to accurately evaluate the Higgs mass bound (26). This requires us to determine the instability scale μ_I and all relevant couplings at that scale. We assume that the SM is valid up to the Planck scale and keep track of the most important coupling constants in determining the lifetime, namely, the Higgs quartic, the top, bottom, and tau Yukawa, and the three gauge couplings. We run these couplings using the complete 3-loop beta functions for all but the small bottom and tau Yukawas and taking into account the most important 4-loop contribution for the QCD coupling, given in the Appendix.

Following the reasoning of [11], we assume $y_t(M_{\text{Pl}})$ and $\lambda(M_{\text{Pl}})$ to be subject to the mechanism responsible for picking the vacuum and thus consider them as free parameters probing the landscape. While scanning different values of these parameters, we keep the gauge as well as the bottom and tau Yukawa couplings fixed at the Planck scale. We determine them by integrating the beta functions given Appendix A 3 with the initial values of the relevant couplings at the top mass given in [19],

$$\begin{aligned} \lambda(M_t) &= 0.12607, & y_t(M_t) &= 0.9312, \\ y_b(M_t) &= 0.0155334, & y_\tau(M_t) &= 0.0102566, \\ g_s(M_t) &= 1.1618, & g'(M_t) &= 0.358545, \\ g(M_t) &= 0.64765, \end{aligned} \quad (32)$$

leading to the Planck scale values,

$$\begin{aligned} \lambda(M_{\text{Pl}}) &= -0.00971659, & y_t(M_{\text{Pl}}) &= 0.380196, \\ y_b(M_{\text{Pl}}) &= 0.00456352, & y_\tau(M_{\text{Pl}}) &= 0.00940597, \\ g_s(M_{\text{Pl}}) &= 0.486966, & g'(M_{\text{Pl}}) &= 0.477243, \\ g(M_{\text{Pl}}) &= 0.505385. \end{aligned} \quad (33)$$

Having solved for the RG evolution of these couplings, we determine μ_S by solving (14), which in turn can be used to calculate the decay rate and thus the lifetime of the vacuum. For this purpose, we use the full expression (12) for the decay rate. Repeating this for a sufficiently tight

mesh of vacua, we obtain a collection of data points $(y_t(M_{\text{Pl}}), \lambda(M_{\text{Pl}}), \mu_S, \tau_{\text{EW}}, \bar{m}_h)$, where \bar{m}_h denotes the upper bound on the running Higgs mass for the considered vacuum. When scanning the landscape of potential vacua, we neglect those with a lifetime shorter than 10^{10} years, as well as those whose dominant instanton reaches into a Planckian regime at its center, $H_{R=\mu_S^{-1}}(r=0) \sim M_{\text{Pl}}$, for which quantum gravity effects might be relevant.

We treat gravitational corrections in the way described in Sec. II, taking into account only the leading-order correction. This is justified, as the condition $H_{R_S} \lesssim M_{\text{Pl}}$ ensures that the natural expansion parameter, $\epsilon_{\text{grav}} = \frac{1}{\sqrt{\lambda(\mu_S)}} \frac{\mu_S^2}{M_{\text{Pl}}^2}$ is smaller than $\frac{1}{8}$. Furthermore, as was argued in [48], it can be expected that the higher-order gravitational corrections due to backreactions should have a negligible influence on the decay rate.

Figure 2 shows the numerical dependence of \bar{m}_h on the lifetime, where we replace $\lambda(M_{\text{Pl}})$ by the lifetime and keep $y_t(M_{\text{Pl}})$ as a second parameter. We find that once the lifetime has been imposed, a hierarchy of several orders of magnitude arises naturally, without the need for further fine-tuning of the couplings.

Despite falling short of explaining the full hierarchy, this establishes a strong connection between metastability and the smallness of the Higgs mass. To summarize the logic, in any framework where there is a fundamental reason for vacuum metastability, such as in the early-time approach to eternal inflation, the requirement of an electroweak vacuum requires the running Higgs mass to be sufficiently small, with shorter lifetimes leading to stronger bounds.

As we see throughout the remainder of this article, the insight that shorter lifetimes lead to a stronger bound on the Higgs mass remains valid even for the considered Standard Model extensions and is at least qualitatively independent of the choice of parameter(s) used to achieve different lifetimes. While this universality, which could be expected from the simplicity of (30) and (26), might be nothing but a mathematical peculiarity, we think that it is reasonable to believe that it might be of significance for the understanding of our Universe and, in particular, the Higgs mass.

V. METASTABILITY BOUNDS BEYOND THE STANDARD MODEL

The remaining gap between our bound and the observed value of the Higgs mass is mostly due to the hierarchy between the latter and the instability scale μ_I , which is not constrained by vacuum decay via an instanton. If, however, our vacuum's instability scale could be found at a lower energy, the bound arising from the requirement of metastability would improve significantly. In general, taking the idea seriously that the near criticality of the Higgs mass is a consequence of the vacuum's metastability, this favors SM extensions causing large negative contributions to β_λ at

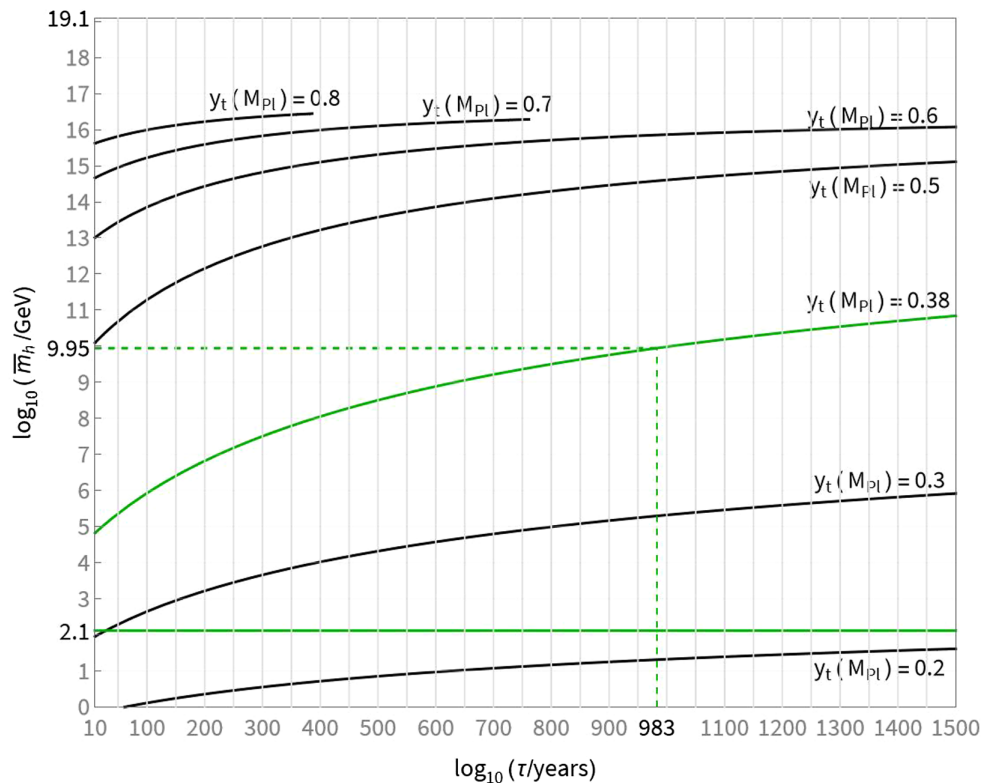


FIG. 2. The upper bound \bar{m}_h on the running mass as a function of the vacuum's lifetime for different values of $y_t(M_{\text{Pl}})$. The dashed green line denotes the central values of parameters inferred from experiments, and the solid green line marks the measured value of the Higgs mass.

energies significantly below the pure SM's instability scale. The simplest way to generate such terms is through the influence of relatively light fermions with a large Yukawa coupling to the Higgs. The currently best-motivated candidate for a particle with these properties is famously the right-handed neutrino.⁸ While we use it for the purpose of concreteness, it should be clear that, at least qualitatively, our results could be recovered from any suitable fermion.

In generic seesaw models, the combination of relatively light right-handed neutrinos and a large Yukawa coupling is obviously in conflict with current experimental constraints and the very idea of the seesaw itself. While it is, in principle, possible for the eigenvalues of the light neutrino mass matrix to be smaller than their pendants for the heavy neutrinos by many orders of magnitude, the cancellations necessary for this would require a significant amount of fine-tuning. An attractive way around this issue is presented by models in which the mass of the light neutrinos is protected by an approximate symmetry. For instance, the $B - \tilde{L}$ symmetric ν MSM [49–55], reviewed in Sec. VA, allows for $\mathcal{O}(1)$ couplings despite a right-handed neutrino mass as small as $\mathcal{O}(1)$ TeV.

⁸The case of a generic singlet neutrino has recently been discussed in [24], whereas we chose to focus on this particular example for concreteness but also due to its potential phenomenological relevance.

In Secs. VB–VD, we demonstrate how adding light right-handed neutrinos to the SM decreases both the instability scale and the vacuum lifetime. However, as we show that the effect on the latter is far more pronounced than that on the instability scale, lowering it to the desired extent would render the vacuum severely unstable.

Again falling back to our initial assumption that the observed value of the Higgs mass is indeed the result of metastability, this suggests considering further SM extensions capable of conspiring with right-handed neutrinos in a similar way as gravity, i.e., by stabilizing the vacuum at scales below the instability scale, thus allowing for stronger Yukawa couplings. The simplest way to achieve this is through new physics that generates a dimension-six term in the Higgs potential, which happens, e.g., in models with a (partially) composite Higgs or two Higgs doublets like the MSSM. For a general dimension-six operator at tree level, this mechanism and its interplay with the influence of fermion singlets has recently been discussed in [24]. For concreteness and to allow for an efficient treatment of the running of the dimension-six Wilson coefficient, for our numerical analysis, we impose the matching condition of the $SU(4)/Sp(4)$ composite Higgs model,

$$C_6(\Lambda_f) = -\frac{\pi^2}{12}\lambda(\Lambda_f), \quad (34)$$

where $\Lambda_f = 4\pi f$, with the technipion decay constant f . It is however crucial to keep in mind that this case is generic, and our results can be easily generalized to a general theory.

Using our results from Sec. II, we show in Secs. V E and V F that through the involvement of such an operator the instability scale can be lowered drastically, and with it the bound on the Higgs mass can be shifted closely above its observed value in agreement with all observational constraints. To be concrete, our perturbative treatment allows for a bound as strong as 10^4 GeV, in agreement with all observational constraints, and potentially even smaller if it were not for the breakdown of approximation for the bounce.

A. Achieving light right-handed neutrinos with a large Yukawa coupling

The ν MSM consists of the SM together with three heavy, right-handed singlet neutrinos N_I , $I = 1, 2, 3$. Their Lagrangian is given by

$$\mathcal{L} = \bar{N}_I i\gamma^\mu \partial_\mu N_I - Y_{\alpha I} \bar{\mathbf{L}}_\alpha N_I (\epsilon \mathbf{H}^*) - \frac{1}{2} M_{IJ} \bar{N}_I^c N_J + \text{H.c.}, \quad (35)$$

where $\bar{\mathbf{L}}_\alpha$ denotes the lepton doublets ($\alpha = e, \mu, \tau$), and $Y_{\alpha I}$ is the matrix of Yukawa couplings and ϵ the totally anti-symmetric SU(2) matrix. The last term is a Majorana mass matrix. This model is usually referred to as the ν MSM or type I seesaw, as the mass matrix of the left-handed neutrinos after symmetry breaking is given by

$$m_\nu = -\frac{v^2}{2} Y M^{-1} Y^T, \quad (36)$$

which is to be understood as a matrix in generation space. This relation implies that, in generic cases, increasing the mass of the right-handed neutrinos decreases the mass of the left-handed ones and vice versa. In other words, the lightness of the left-handed neutrinos is either the result of the large masses of their right-handed counterparts or tiny Yukawa couplings.

However, there is a subtlety to this argument. As the masses of the left-handed neutrinos are the eigenvalues of the mass matrix (36), it is possible that cancellations between the different elements of the latter lead to small eigenvalues despite Yukawa couplings of order ~ 1 and right-handed neutrinos with masses in the TeV range [56]. While the fine-tuning necessary to achieve such cancellations could of course be accidental, it can naturally arise as the result of an additional approximate symmetry protecting the masses of the left-handed neutrinos. One of the most important examples for such a symmetry is related to the preservation of the $B - \tilde{L}$ number, where \tilde{L} denotes an extension of the lepton number by right-handed neutrinos [49] and given by

$$N_3 \rightarrow e^{i\alpha} N_3, \quad N_2 \rightarrow e^{-i\alpha} N_2, \quad N_1 \rightarrow e^{i\beta} N_1. \quad (37)$$

Imposing this as an exact symmetry, and taking into account several observational constraints, the Majorana mass matrix and the matrix of Yukawa couplings are restricted to be of the form

$$M = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & M \\ 0 & M & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} 0 & Y_1 & 0 \\ 0 & Y_2 & 0 \\ 0 & Y_3 & 0 \end{pmatrix}, \quad (38)$$

up to rotations of the right-handed neutrinos in flavor space. Consistency with observations requires this symmetry to be broken, which can be made manifest by introducing symmetry-breaking terms for both the Yukawa coupling as well as the mass term. While these are crucial to obtain everything from neutrino oscillations to the masses of the light mass eigenstates, i.e., the SM neutrinos, their effect on the decay rate is negligible as long as the symmetry is only slightly broken.

The influence of right-handed neutrino singlets on vacuum stability has been discussed at different levels of detail in [57–61]. Considering its strong dependence on the quartic coupling λ , it should be clear that the neutrinos' main influence on the decay rate is via their contribution to β_λ . While their precise effect depends on the detailed form of the Yukawa coupling and Majorana mass matrix, it is generally true that right-handed neutrinos lead to a more negative λ and thus a shorter lifetime.

B. Vacuum decay in symmetry protected seesaw models

Assuming the Yukawa couplings to be of the form (38) allows for the following replacements in the ν MSM's beta functions, which we give in full detail to 2-loop accuracy in Appendix A 3,

$$Y_\nu Y_\nu^\dagger Y_\nu \rightarrow Y_\nu \cdot (Y_1^2 + Y_2^2 + Y_3^2), \\ \text{Tr}(Y_\nu^\dagger Y_\nu) \rightarrow Y_1^2 + Y_2^2 + Y_3^2. \quad (39)$$

As argued above, observations require the addition of symmetry-breaking terms to (38) and thus modifications to this relation. However, under our assumption that $|Y| \sim \mathcal{O}(1)$ while $M \lesssim 10^{11}$ GeV, such additional terms should be strongly suppressed compared to the leading-order terms, allowing us to safely neglect them in the following. The same is true for corrections of the heavy neutrino masses, so that we match the ν MSM with the pure SM at the scale $\mu \simeq M$. This reduces the number of parameters to four: the heavy neutrino mass M and the three couplings $Y_1(M)$, $Y_2(M)$, and $Y_3(M)$.

Furthermore, given that we neglect the terms breaking the $B - \tilde{L}$ symmetry, and taking into account that our analysis is, in general, insensitive to all properties of the

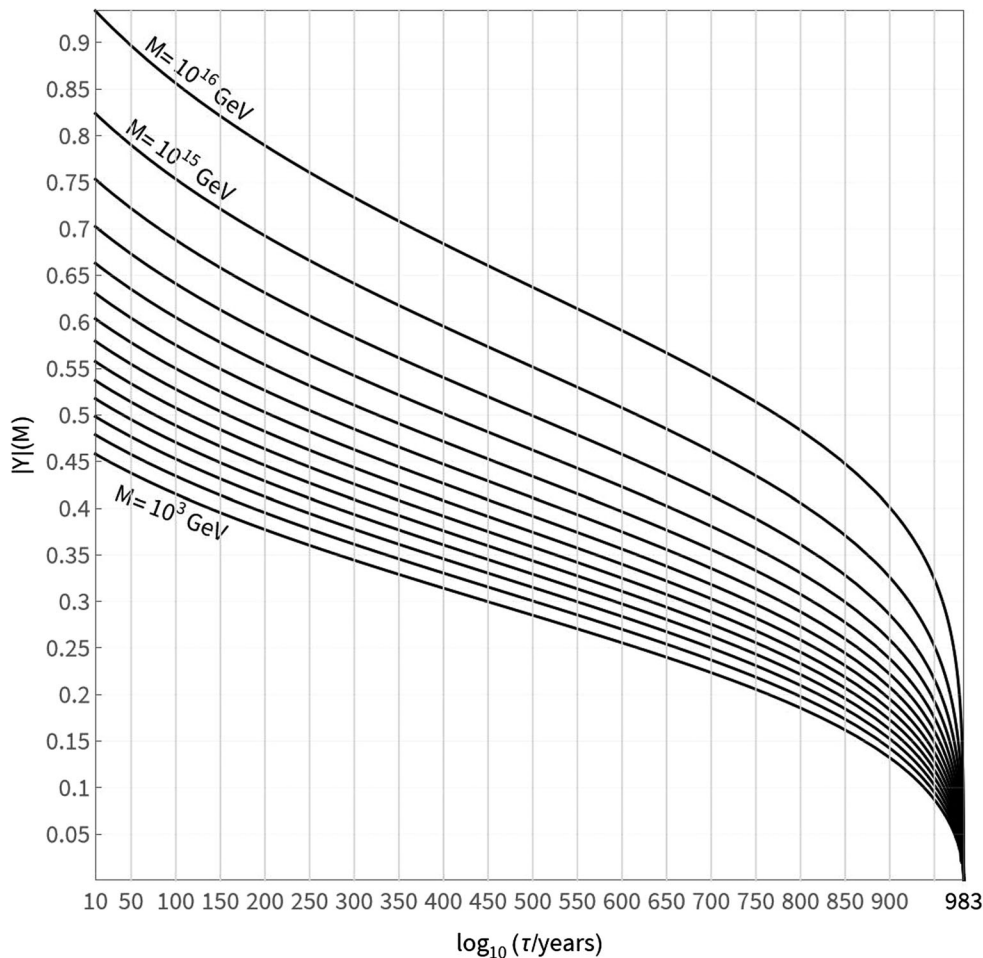


FIG. 3. The value of $|Y(M)|$ necessary to realize a given lifetime, shorter than the central value of 10^{983} years for the pure SM. Each curve corresponds to a different value of M , in increasing order of magnitude.

neutrinos except their Yukawa couplings, we find an additional $SO(3)$ symmetry among Y_1 , Y_2 , and Y_3 .⁹ This symmetry is also respected by the correction to the effective potential required for our upper bound on the Higgs mass, as well as by the threshold corrections at the matching scale of the pure SM's quartic coupling with the one of UV theory, making it evident that the same holds for the NLO corrections to the decay rate from the neutrinos' determinants. Thus, as long as we are only interested in the neutrinos' influences on Higgs vacuum decay, we can further simplify the Yukawa matrix by rotating its components as

$$Y \rightarrow \begin{pmatrix} 0 & 0 & 0 \\ 0 & |Y| & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \text{where } |Y|^2 = Y_1^2 + Y_2^2 + Y_3^2, \quad (40)$$

⁹Note that this symmetry would be broken, e.g., through the neutrino contribution to the tau Yukawa. However, given the smallness of the latter as well as the smallness of the interval over which the neutrino contribution is relevant, we can safely neglect the corresponding terms for simplicity.

leaving us with just two parameters: M and $|Y(M)|$.¹⁰

The neutrino's contribution to the quartic coupling's loop correction, λ_1 , is given in, e.g., [60–62], and the beta functions of the extended SM are known to 2-loop accuracy [63]. The only missing quantity to extend the NLO formula (12) is the neutrinos' kinetic terms' functional determinants. Recalling that we are interested in the regime $M \lesssim \mu_I$, the masses of both light and heavy neutrino states are negligible at scales relevant to the instanton. Making use of this fact, it is straightforward to see that the neutrino fluctuations contribute in the same way as the top quark under the replacement $y_t \rightarrow |Y|$.

C. The numerical relation between neutrinos' Yukawa couplings and the lifetime

We numerically compute the lifetime using (12), supplemented with the neutrino determinant. To do so, we first

¹⁰This is equivalent to the case of a hierarchy between the Yukawa couplings of the neutrinos, which is often used in the literature to simplify results [57–61].

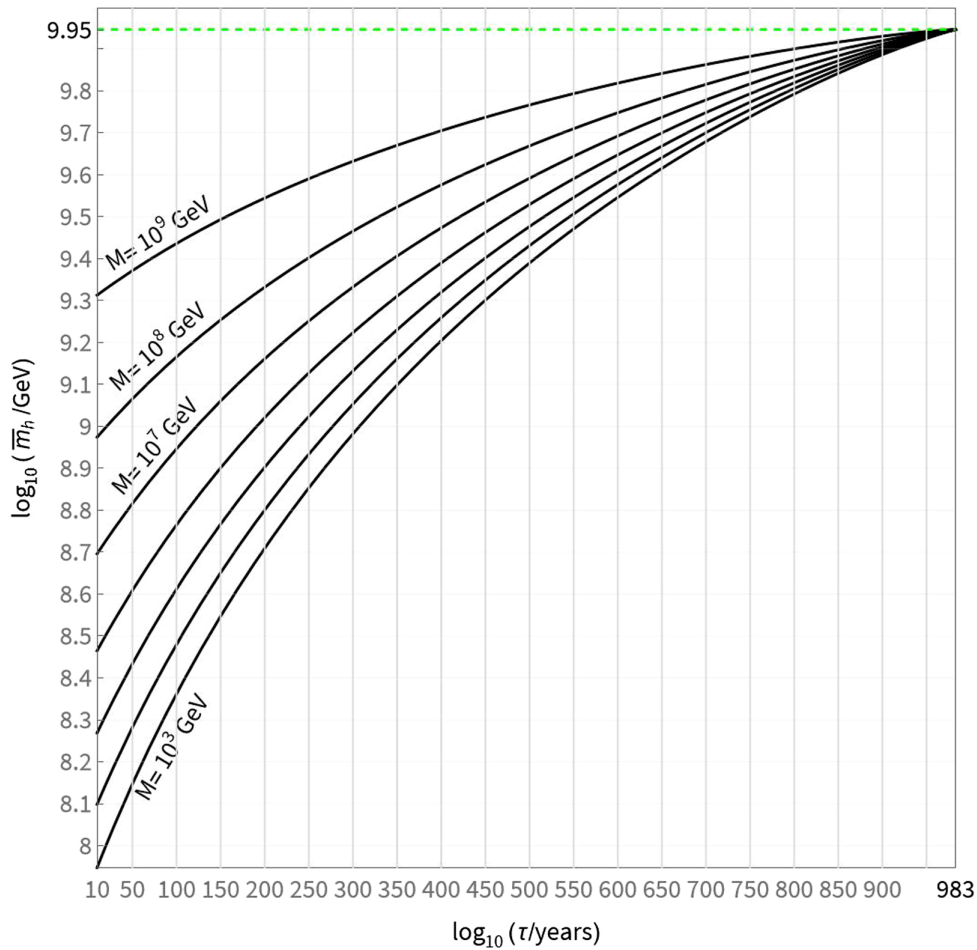


FIG. 4. The upper bound on the running Higgs mass as a function of the lifetime. Different lifetimes are achieved by varying the neutrino coupling parameter $|Y(M)|$ (per Fig. 3), while all other couplings are fixed to their observed values near the electroweak scale. Each curve corresponds to a different value of M . The green dashed line marks the bound obtained for the considered set of parameters in the pure SM.

match the SM couplings with those of the $B - \tilde{L}$ symmetric ν MSM at the scale M , following the effective field theory procedures outlined in [64,65] and using the threshold corrections for λ and y_t given in [57].¹¹ Then, for a given value of $|Y(M)|$, we run the couplings up to the Planck scale using the full ν MSM beta functions at 2-loop accuracy found in [63], which we give in the Appendix in the conventions used in this article.

Having determined the running of the couplings, we obtain the instanton scale μ_S as before by solving (14). It is worth noting that μ_S lies significantly below μ_* (where β_λ vanishes) in this case. Indeed, the right-handed neutrino Yukawa couplings with values relevant for our purposes generally push μ_* beyond the Planck scale. Nevertheless, as explained in Sec. II, gravitational corrections keep μ_S

below the Planck scale due to their nonlogarithmic dependence on the scale.

Figure 3 shows the value of $|Y(M)|$ necessary to achieve a certain lifetime for all values of M of interest. We consider lifetimes ranging from 10^{10} years to the SM central value of 10^{983} years. This plot allows us to convert any prediction of the vacuum's lifetime to a relation of the relevant right-handed neutrino properties, assuming either a strong hierarchy between the Yukawa couplings or the $B - \tilde{L}$ symmetry. As a side result, we provide updated stability bounds on the Yukawa couplings, which can be read off from the left boundary of Fig. 3. For $M = 10^{12} - 10^{15}$ GeV, our results are almost identical to the ones given in [57], with a minor deviation due to our more careful treatment of gravitational corrections and, more importantly, updated experimental input for the SM couplings.

Before proceeding, it is important to stress once again that the inclusion of right-handed neutrino Yukawa couplings with properties relevant for our purpose generally shifts the minimum of λ to scales beyond the Planck mass.

¹¹The authors of [57] kindly informed us about a small typo in their Eq. G(20), where a factor of $\frac{1}{4}$ in the first term has to be replaced by $\frac{1}{2}$. We have taken this into account in our calculation.

While this on its own would render the most important instantons inaccessible to us, we are saved by gravitational corrections. As described in Sec. II, due to their non-logarithmic dependence on the scale, gravitational corrections shift μ_S below the Planck scale, even though the gravitational term is strongly suppressed compared to the Minkowski contribution. This observation is crucial, as it provides a hint at how the small Higgs mass we observe might be achieved via a metastability bound.

D. The neutrino Yukawa couplings' effect on the metastability bound

Having established the possibility to achieve any desired lifetime shorter than in the pure SM, we turn to our second objective, namely, to address the smallness of the electro-weak scale. Unlike in Sec. III, we keep λ and y_t fixed to their SM values at the electro-weak scale and instead scan the lifetime by varying $|Y(M)|$. We further modify our previous procedure by including the neutrino correction to the decay rate, as well as the neutrino contributions to λ_1 and the beta functions. We focus on the range $M \lesssim \mu_I$, for

otherwise the running of λ would be unaffected up to the instability scale, and the Higgs mass bound would be unchanged.

Figure 4 shows the upper bound on the running Higgs mass, \bar{m}_h , as a function of the lifetime for different values of M . Stronger Yukawa couplings lead to a faster decline of λ , and thus a smaller μ_I , which in turn yields a stronger bound on the Higgs mass. Furthermore, it is clear that the larger the neutrino mass is, the higher is the energy at which their Yukawa couplings become relevant, and the smaller is their influence on μ_I .

Our results lie within with experimental bounds on neutrino parameters. While in the ν MSM many of their properties depend on the symmetry-breaking parts of the Yukawa couplings and the mass matrix, the symmetric Lagrangian (35) allows for a mixing of the active neutrinos with the Dirac fermion $\Psi = N_2 + N_3^c$, the strength of which is determined by the combination $\frac{\nu}{M}|Y|$ [56]. Although experiments do not directly constrain this combination, current experimental bounds allow for $|Y| \lesssim 0.5$ for $M = 1$ TeV [66], i.e., the full range of parameters investigated in this section.

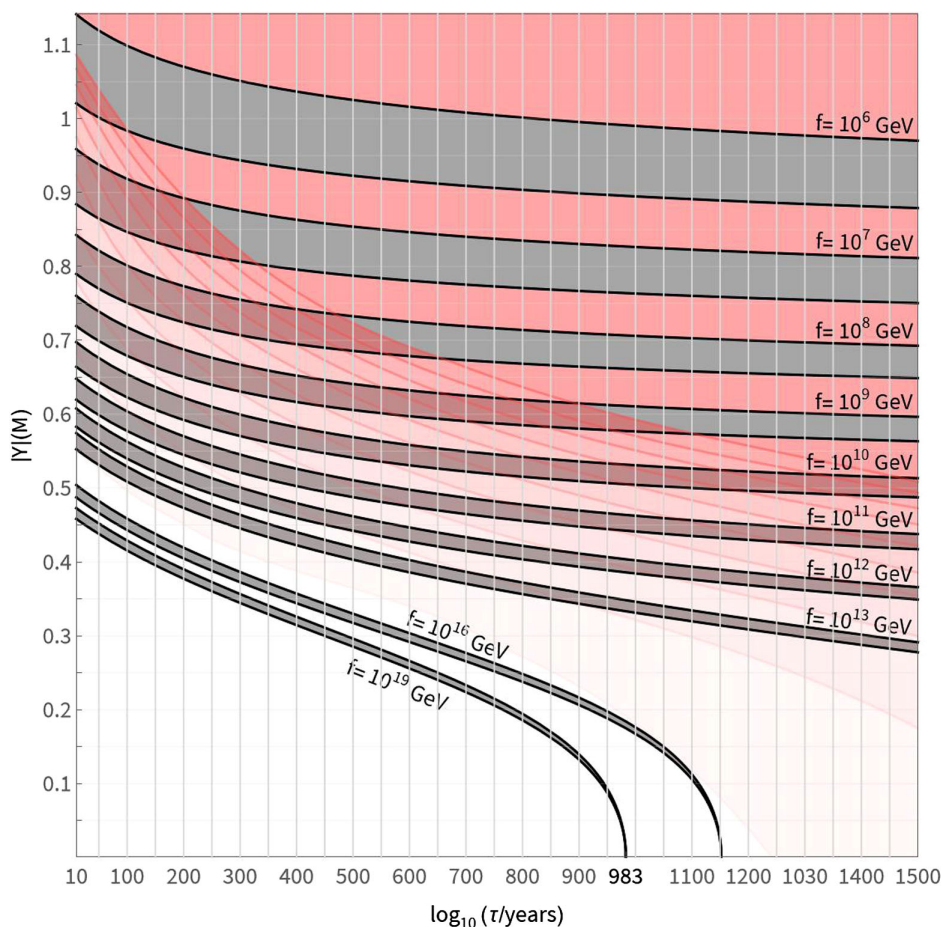


FIG. 5. The value of $|Y(M)|$ necessary to realize a given lifetime for different values of f . The gray bars cover the range $M = 1\text{--}5$ TeV. In the white region, the expansion parameter defined in (18) is smaller than 0.1 for $M = 1$ TeV, in the dark red region larger than 1. The shaded regions indicate the transition in steps of $\Delta\epsilon = 0.1$.

There is an important subtlety arising from the inclusion of neutrinos or additional fermions, in general. As we are mostly interested in scenarios with $M < \mu_I \sim \bar{m}_h$, the restricted Higgs mass is, in general, *not* the SM parameter but that of the ν MSM, which is related to its SM counterpart via a threshold correction $\delta m_h^2 = \frac{M^2 |Y|^2}{(4\pi)^2}$. While this expression is at least 1 order of magnitude smaller than \bar{m}_h^2 for all parameters considered in this article and thus negligible in the context our bound, it is important to note that this is not automatically true for all setups. The smallness of the correction compared to \bar{m}_h^2 can be related to the relative smallness of $|Y|$, which on the one hand leads to a small correction and on the other hand does not allow for a fast enough decline of λ to push μ_I close to M . This last point is further enhanced by the threshold correction in λ . If the matching scale M lies close enough to the would-be instability scale, the threshold correction for λ can be large enough for the latter to jump to a negative value without vanishing exactly, spoiling our analysis. While also this poses no problem for the range of parameters covered in this article, it might have consequences for a refined analysis of the model discussed in the next subsection with parameters currently not accessible by our perturbative treatment.

E. The numerical relation between neutrinos Yukawa couplings, dimension-six operator, and the lifetime

Combining our previous results, it is straightforward to include the effect of a dimension-six operator. We restrict

ourselves to small values of M , for concreteness $M \sim 1\text{--}5$ TeV, which yield the tightest bound on the Higgs mass (see Fig. 4) while being safely compatible with experimental constraints for $M \gtrsim 2$ TeV. With these values fixed, we scan different values of f and $|Y(M)|$.

All SM couplings are matched at the scale M using the same procedure as before, in particular, taking into account threshold corrections for λ and y_t and run using their 3-loop beta functions, neglecting the contributions $\sim m_h^2/\Lambda_f^2$. As in the most interesting cases the instanton scale lies close to the scale of new physics, we take into account the RG running of C_6 at 1-loop only.

In this manner, we can compute the lifetime of the vacuum for each pair of values $|Y(M)|$ and f —or, equivalently, the value of $|Y(M)|$ corresponding to every possible lifetime, given f . This relation is depicted in Fig. 5, for values of f ranging from 10^6 to 10^{19} GeV. In principle, smaller values of f would also be possible but are not accessible by our perturbative analysis. The red shading indicates different values of the expansion parameter ϵ assuming $M = 1$ TeV, defined in (18), ranging from 0.1 to 1. Consistent with the discussion below (18), we see that the perturbative expansion is indeed most reliable for shorter lifetimes and larger f .

F. The neutrino Yukawa couplings and dimension-six term's combined effect on the metastability bound

Having related the lifetime to the neutrino Yukawa couplings, we have gathered everything necessary to

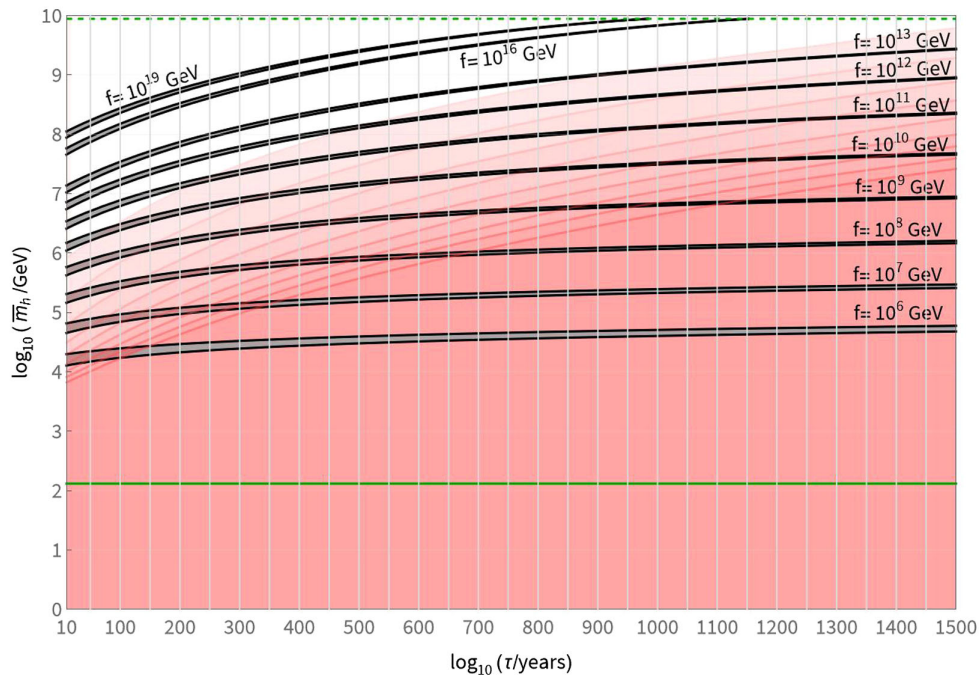


FIG. 6. The upper bound on the running Higgs mass as a function of the lifetime for different values of f , with the range of f being only restricted by the applicability of our perturbative treatment and not physical reasons. The gray bars represent the interval $M = 1\text{--}5$ TeV. The shading in the background again marks areas of different ϵ for $M = 1$ TeV, ranging from 0.1 to 1 in steps of size 0.1.

include the effect of the dimension-six term on the upper bound on the running Higgs mass as a function of the vacuum's lifetime. The modified relation is shown in Fig. 6, again considering $M = 1\text{--}5$ TeV for concreteness. We see that metastability forces m_h lie consistently at least 2 orders of magnitude below f , which is of the same order as the natural value for the mass parameter.

Figure 6 confirms our expectations. By balancing right-handed neutrinos with the stabilizing effect of a dimension-six operator, it is possible to simultaneously achieve a significantly smaller lifetime and decrease the upper bound on the running Higgs mass down to $\simeq 10$ TeV. From there, naively extrapolating our results beyond the applicability of our approximations, it appears as if lowering f just slightly further might allow for an upper bound on the Higgs mass of order 1 TeV, which, together with the necessity of a potential barrier separating electroweak and true vacuum, might be sufficient to explain the observed value of 10^2 GeV as well as the remaining small hierarchy.

VI. CONCLUSIONS

The absence of new physics in flavor, precision, and LHC experiments points to the SM being valid up to very high energy, leaving the lightness of the Higgs unexplained and seemingly fine-tuned. It appears increasingly doubtful that the cherished principle of naturalness, which has guided much of particle model building, can explain the gauge hierarchy problem. A puzzling consequence of extrapolating the SM to a very high energy scale is the metastability of the electroweak vacuum. We believe this numerical conspiracy is no accident. It is striking that Higgs metastability, the gauge hierarchy problem, and the cosmological constant problem can all be interpreted as problems of near criticality.

In this article, we considered an unconventional approach to the hierarchy problem, which rests on the idea that the gauge hierarchy is a consequence of a more primitive property of our vacuum, namely, its metastability. This is motivated, e.g., by the early-time framework for eternal inflation put forth recently, based on search optimization on the string landscape. A key prediction of this framework is that optimal regions of the landscapes are characterized by vacua that are relatively short lived, with lifetimes of order of their de Sitter Page time. As such, this offers a *raison d'être* for the conspiracy underlying Higgs metastability. While we are primarily motivated by the nonequilibrium approach to eternal inflation, our results pertain more generally to any theoretical framework predicting that our vacuum should be metastable.

Central to our analysis is the observation that the metastability of the electroweak vacuum, together with the very requirement that such a nontrivial vacuum exists, implies that the Higgs mass is bounded from above by the instability scale. Simply put, a small Higgs mass is a

necessary condition for a metastable vacuum. This was first pointed out in the context of the SM in [11], though the bound is quite weak in this case, as the instability scale is $\sim 10^{11}$ GeV.

A key point of this article is that simple, well-motivated extensions of the SM can significantly tighten the upper bound on the Higgs mass, as low as ~ 10 TeV and potentially even further through a more elaborate analysis, by lowering the instability scale μ_I . Furthermore, the vacuum lifetime can be shortened, to the extent that the Page time can be achieved. In other words, our viewpoint is that tightening the Higgs mass bound down to a value slightly above the electroweak scale together with achieving a shorter lifetime act as guiding principles in sifting through possible SM extensions. As an important by-product of our analysis, we provided accurate calculations of the vacuum's lifetime as a function of the parameters characterizing the SM extensions of interest, which allowed us to update existing stability bounds by combining for the first time all relevant NLO corrections, both from gravity and functional determinants, at up to 3-loop accuracy.

We first considered the inclusion of right-handed neutrinos, a necessary extension to the SM to explain the mass of their left-handed counterparts. With a right-handed mass of order TeV and $\mathcal{O}(1)$ Yukawa couplings, the lifetime is dramatically shorter while the Higgs mass bound is significantly tighter. An elegant model where this can be naturally realized while satisfying experimental constraints is the ν MSM model with approximate $B - \tilde{L}$ symmetry. However, we found that right-handed neutrinos by themselves cannot fully explain the gauge hierarchy—the tightest upper bound compatible with experimental constraints is $\sim 10^8$ GeV—still a few orders of magnitude away from the electroweak scale. Within the ν MSM, this discrepancy can only be alleviated by further increasing the Yukawa couplings, but this would inevitably render the electroweak vacuum unstable.

This observation led us to consider further SM extensions that have a stabilizing effect on the vacuum, thereby allowing for stronger right-handed neutrino Yukawa couplings. For concreteness, we studied the minimal $SU(4)/Sp(4)$ composite Higgs model as a simple example that can achieve this. The main impact of compositeness is through the perturbative addition of a dimension-six correction to the Higgs potential. As such, while we focus on this particular minimal composite Higgs model, our results apply more generally to any theory that can be well approximated by a dimension-six operator at low energy.

The stabilizing effect of this dimension-six operator allowed for stronger right-handed neutrino Yukawa couplings. This results in shorter lifetimes, of order the de Sitter Page time, while simultaneously lowering the upper bound on the Higgs mass. Within the reach of our perturbative analysis, the bound can be moved down to up to $\simeq 10$ TeV and potentially even further if corrections to

the bounce profile were to be taken into account to increase our analysis' range of applicability.

It is crucial to keep in mind that our result is really a bound rather than an independent solution to the gauge hierarchy problem and is independent of the mechanism setting the lifetime. The existence of our bound provides a strong constraint for any such mechanism, as it implies that its applicability either requires a small Higgs mass or, ideally, should be able to yield the correct Higgs mass. Our result can, however, also be understood as a strong indicator for the shared origin of the different fine-tunings observed in the Higgs sector, as it suggests that a large hierarchy between running Higgs mass and Planck scale is necessary for metastability.

The upper bound on the Higgs mass relies on two assumptions: (i) metastability of the electroweak vacuum and (ii) the existence of such a nontrivial vacuum, i.e., a negative Higgs mass squared. While the former is motivated, e.g., by the nonequilibrium approach to eternal inflation, the latter remains an assumption in our analysis. In future work, we will consider further SM extensions which naturally give rise to a negative Higgs mass squared. The ν MSM model with approximate $B - L$ symmetry, with right-handed neutrino masses at the TeV scale, has the appealing feature of having the electroweak scale as its unique mass scale. A natural extension of the present work would be to consider classically conformally invariant models [67–69], such as theories where scale invariance is radiatively broken in a hidden sector and mediated to the SM via a Higgs-portal coupling [68,70–84]. It will be interesting to see whether such extensions can further tighten the bound closer to the measured Higgs mass.

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APPENDIX: COLLECTED RELEVANT CALCULATIONS

1. Upper bound on the Higgs mass

In this Appendix, we derive the more precise version of the Higgs mass bound, given by (26), by including

nonlogarithmic 1-loop corrections to λ . The full RG-improved effective potential is given by

$$V_{\text{eff}}(H) = -\frac{m_h^2}{4} e^{2\Gamma[H]} H^2 + \frac{1}{4} (\lambda(H) + \lambda_1(H) + \dots) e^{4\Gamma[H]} H^4, \quad (\text{A1})$$

where λ_n denotes nonlogarithmic corrections to λ at n -loop order. The extrema of this potential, $\frac{d}{dH} V_{\text{eff}}(H)|_{H=v} = 0$, are given by

$$\frac{m_h^2}{v^2} = \frac{e^{2\Gamma(v)}}{1 + \gamma(v)} \left(2(\lambda(v) + \lambda_1(v) + \dots) + \frac{1}{2} (\beta_\lambda(v) + \beta_{\lambda_1}(v) + \dots) \right), \quad (\text{A2})$$

where γ denotes the Higgs field's anomalous dimension.

The existence of a metastable vacuum is equivalent to the existence of solutions to this equation. A first restriction on these solutions can be read off directly from (A1): at scales somewhat larger than the instability scale, $\lambda(\mu)$ becomes sufficiently negative to compensate for the positive loop corrections $\{\lambda_n(\mu)\}_n$. Thus, at those scales, both the quadratic and the quartic term appear with a negative sign, and no extremum can occur. This implies that we can focus on values of v near the instability scale, with the immediate consequence that $\lambda(v) \simeq \beta_\lambda(\mu_I) \ln \frac{v}{\mu_I}$. Thus, λ becomes effectively 1-loop [47], and therefore, a consistent leading-order perturbative expansion of (A2) must account for all 1-loop terms. This includes, in particular, λ_1 , which has been neglected in [11].

Restricting ourselves to 1-loop accuracy, (A2) can be brought to the form

$$m_h^2 = v^2 \left(2 \ln \frac{v}{\mu_I} + 2 \frac{\lambda_1(\mu_I)}{\beta_\lambda(\mu_I)} + \frac{1}{2} \right) \beta_\lambda(\mu_I). \quad (\text{A3})$$

Understood as a function of v , the right-hand side is bounded from above since $\beta_\lambda(\mu_I) < 0$. Maximizing over v , we obtain the inequality

$$m_h^2 \lesssim |\beta_\lambda(\mu_I)| \exp \left(-\frac{3}{2} - 2 \frac{\lambda_1(\mu_I)}{\beta_\lambda(\mu_I)} \right) \mu_I^2. \quad (\text{A4})$$

If λ_1 is neglected, as assumed in [11], this bound reduces to (25).

Equation (A3) can furthermore be used to classify the effective potential. Clearly, for values of m_h^2 larger than the bound (A4), it has no extremum. In the case that m_h^2 is strictly smaller than the bound but larger than 0, the right-hand side of Eq. (A3) has two solutions, corresponding to the top of the potential barrier and the vev, which—neglecting the nonlogarithmic term λ_1 for now—ranges

from 0 to $e^{-1/4}\mu_I$. For $v = e^{-3/4}\mu_I$, the inequality (A4) is saturated, corresponding to a saddle point of the effective potential at v , implying that corrections due to the running of λ effectively cancel the tree-level mass parameter m_h^2 in $\frac{d^2}{dH^2} V_{\text{eff}}(H)$. This is now precisely the critical point at the center of the probability distribution derived in [24], confirming its consistency with our bound on the running Higgs mass, which it saturates.

For $m_h^2 < 0$, the potential no longer permits spontaneous symmetry breaking without the influence of further quantum corrections, and (A3) has only one solution, again corresponding to the potential wall, which is now located beyond the instability scale. Since in this limit the potential's only minimum lies at $H = 0$, we find $v < e^{-1/4}\mu_I$ when neglecting λ_1 . The same discussion can be performed after restoring it, leading to the more appropriate bound,

$$v \leq \exp\left(-\frac{1}{4} - \frac{\lambda_1(\mu_I)}{\beta_\lambda(\mu_I)}\right)\mu_I. \quad (\text{A5})$$

In Secs. V E and V F, we add a dimension-six operator to the potential, of the form $\Delta V = \frac{C_6}{\Lambda^2} H^6$, where to leading order $C_6 \sim \lambda$. This term leads to a correction in (A4) of order $\lambda \frac{v^2}{\Lambda^2}$. Recalling that $v^2 \lesssim \mu_I^2 \ll \Lambda^2$, this correction is suppressed compared to the terms originating from the pure SM, so that it could justifiably be neglected in our perturbative discussion.

2. NLO tunneling formula for the Standard Model

For completeness, we include here the NLO formula for the vacuum decay rate derived in [17]. Before performing the integral over the dilatation modes, the decay rate per unit volume is given by

$$\frac{\Gamma}{V} = \int \frac{dR}{R^5} e^{-S_E(\lambda(R^{-1}), R)} D(R^{-1}). \quad (\text{A6})$$

The factor $D(R^{-1})$ is defined as

$$D(R^{-1}) \equiv \frac{72}{\sqrt{6\pi^2}} S_E^4(\lambda(R^{-1}), R) \exp\left[12\zeta'(-1) - \frac{25}{3} + \pi^2 - \gamma_E - \frac{3}{2} \ln 2 - \frac{3}{2} S_{\text{fin}}^+(X) - 3S_{\text{fin}}^+(Y) + \frac{3}{2} S_{\text{fin}}^{\bar{\psi}\psi}(\sqrt{Z_t}) + \frac{3}{2} S_{\text{fin}}^{\bar{\psi}\psi}(\sqrt{Z_b}) - 3S_{\text{loops}}^{\bar{\psi}\psi}(Z_t) - 3S_{\text{loops}}^{\bar{\psi}\psi}(Z_b) - \frac{1}{2} S_{\text{diff}}^{\text{AG}}(X) - S_{\text{diff}}^{\text{AG}}(Y) - S_{\text{loops}}^{\text{AG}}(X) - 2S_{\text{loops}}^{\text{AG}}(Y)\right], \quad (\text{A7})$$

where $X \equiv -\frac{g^2 + g'^2}{12\lambda}$, $Y \equiv -\frac{g^2}{12\lambda}$, and $Z_i \equiv \frac{v_i^2}{\lambda}$. The correction $S_{\text{fin}}^+(x)$ appearing in the exponent is given by

$$S_{\text{fin}}(x) = x^2(6\gamma_E + 51 - 6\pi^2) + 6x + \frac{11}{36} + \ln 2\pi + \frac{3}{4\pi^2} \zeta(3) - 4\zeta'(-1) - \ln\left(\frac{\cos(\frac{\pi}{2}\kappa_x)}{6\pi x}\right) - x\kappa_x \left[\psi^{(-1)}\left(\frac{3+\kappa_x}{2}\right) - \psi^{(-1)}\left(\frac{3-\kappa_x}{2}\right)\right] + \left(6x - \frac{1}{6}\right) \left[\psi^{(-2)}\left(\frac{3+\kappa_x}{2}\right) + \psi^{(-2)}\left(\frac{3-\kappa_x}{2}\right)\right] + \kappa_x \left[\psi^{(-3)}\left(\frac{3+\kappa_x}{2}\right) - \psi^{(-3)}\left(\frac{3-\kappa_x}{2}\right)\right] - 2 \left[\psi^{(-4)}\left(\frac{3+\kappa_x}{2}\right) + \psi^{(-4)}\left(\frac{3-\kappa_x}{2}\right)\right], \quad (\text{A8})$$

where $\kappa_x \equiv \sqrt{1 - 24x}$, and ψ^n is the polygamma function. The other corrections to the action are

$$S_{\text{diff}}^{\text{AG}}(x) = x^2(121 - 12\pi^2) - \frac{45}{2}x^2, \\ S_{\text{loops}}^{\text{AG}}(x) = -\frac{5}{18} - \frac{1}{3}(\gamma_E - \ln 2) - x(7 + 6(\gamma_E - \ln 2)) - 9x^2\left(\frac{1}{2} + \gamma_E - \ln 2\right), \\ S_{\text{loops}}^{\bar{\psi}\psi}(x) = -x\left(\frac{13}{8} + \frac{2}{3}(\gamma_E - \ln 2)\right) + x^2\left(\frac{5}{18} + \frac{1}{3}(\gamma_E - \ln 2)\right), \quad (\text{A9})$$

as well as

$$S_{\text{fin}}^{\bar{\psi}\psi}(x) = 16\psi^{(-1)}(2) - \frac{8}{3}\psi^{(-2)}(2) + \frac{4}{3}x^2(1 - \gamma_E) - \frac{x^4}{3}(1 - 2\gamma_E) - \frac{4}{3}x(1 - x^2)[\psi^{(-1)}(2+x) - \psi^{(-1)}(2-x)] + \frac{4}{3}x(1 - 3x^2)[\psi^{(-2)}(2+x) + \psi^{(-2)}(2-x)] + 8x[\psi^{(-3)}(2+x) - \psi^{(-3)}(2-x)] - 8[\psi^{(-4)}(2+x) + \psi^{(-4)}(2-x)]. \quad (\text{A10})$$

We refer the reader to [17] for further details and the meaning of the different subscripts.

3. Beta functions

We provide the beta functions used in our calculations. The beta function of the quartic coupling λ at 3-loop order is given by

$$\begin{aligned}
\beta_\lambda = & \frac{1}{(4\pi)^2} [24\lambda^2 - 6y_t^4 - 6y_b^4 - 2y_\tau^4 + \frac{3}{8}(2g^4 + (g^2 + g'^2)^2) - \lambda(9g^2 + 3g'^2 - 12y_t^2 - 12y_b^2 - 4y_\tau^2) + |Y|^2(4\lambda - 2|Y|^2)] \\
& + \frac{1}{(4\pi)^4} \left[\frac{1}{48}(915g^6 - 289g^4g'^2 - 559g^2g'^4 - 379g'^6) + 30y_t^6 + 30y_b^6 + 10y_\tau^6 - y_t^4 \left(\frac{8}{3}g^2 + 32g_s^2 + 3\lambda + 6y_b^2 \right) \right. \\
& - y_b^4 \left(\frac{4}{3}g^2 + 32g_s^2 + 3\lambda + 6y_t^2 \right) - y_\tau^4(4g'^2 + 3\lambda) \\
& + \lambda \left(-\frac{73}{8}g^4 + \frac{39}{4}g^2g'^2 + \frac{629}{24}g'^4 + 108g^2\lambda + 36g'^2\lambda - 312\lambda^2 \right) \\
& + y_t^2 \left(-\frac{9}{4}g^4 + \frac{21}{2}g^2g'^2 - \frac{19}{4}g'^4 + \lambda \left(\frac{45}{2}g^2 + \frac{85}{6}g'^2 + 80g_s^2 - 144\lambda - 42y_b^2 \right) \right) \\
& + y_b^2 \left(-\frac{9}{4}g^4 + \frac{9}{2}g^2g'^2 - \frac{5}{4}g'^4 + \lambda \left(\frac{45}{2}g^2 + \frac{25}{6}g'^2 + 80g_s^2 - 144\lambda - 42y_t^2 \right) \right) \\
& + y_\tau^2 \left(-\frac{3}{4}g^4 + \frac{11}{2}g^2g'^2 - \frac{25}{4}g'^4 + \lambda \left(\frac{15}{2}g^2 + \frac{75}{6}g'^2 - 48\lambda \right) \right) \\
& + |Y|^2 \left(-96\lambda^2 + \lambda(5g^2 + 15g'^2 - 2|Y|^2) - \frac{3}{2}g^4 + 20|Y|^4 \right) \Big] \\
& + \frac{1}{(4\pi)^6} [\lambda^3(12022.7\lambda + 1746y_t^2 - 774.904g^2 - 258.3g'^2) \\
& + \lambda y_t^2(3536.52y_t^2 + 321.54g_s^2 - 719.078g^2 - 212.896g'^2) \\
& + \lambda^2(-1580.56g^4 - 1030.734g'^4 - 1055.466g^2g'^2) \\
& + \lambda y_t^4(-446.764y_t^2 - 1325.732g_s^2 - 10.94g^2 - 70.05g'^2) \\
& + \lambda y_t^2(713.936g_s^4 - 639.328g^4 - 415.888g'^4 + 30.288g_s^2g^2 + 58.18g_s^2g'^2 + 18.716g^2g'^2) \\
& + \lambda g^4(-114.288g_s^2 + 1730.966g^2 + 265.46g'^2) + \lambda g'^4(-46.562g_s^2 + 343.072g^2 + 260.814g'^2) \\
& + y_t^6(-486.298y_t^2 + 500.988g_s^2 + 146.276g^2 + 113.1g'^2) \\
& + y_t^4(-100.402g_s^4 + 31.768g^4 + 88.6g'^4 + 26.698g_s^2g^2 + 58.566g_s^2g'^2 - 234.52g^2g'^2) \\
& + y_t^2g_s^2(32.928g^4 + 3.644g'^4 + 37.954g^2g'^2) + y_t^2g^4(125g^2 + 43.470g'^2) \\
& + y_t^2g'^4(58.318g^2 + 102.936g'^2) + g_s^2(15.072g^6 + 7.138g'^6 + 5.024g^4g'^2 + 6.138g^2g'^4) \\
& - 228.182g^8 - 23.272g'^8 - 126.296g^6g'^2 + 36.112g^4g'^4 - 14.288g^2g'^6] + \mathcal{O}\left(\frac{m_h^2}{\Lambda_f^2}\right). \tag{A11}
\end{aligned}$$

For the top Yukawa coupling y_t , we have, also at 3-loop order,

$$\begin{aligned}
\beta_{y_t} = & \frac{y_t}{(4\pi)^2} \left[-\frac{9}{4}g^2 - \frac{17}{12}g'^2 - 8g_s^2 + \frac{9}{2}y_t^2 + |Y|^2 \right] \\
& + \frac{y_t}{(4\pi)^4} \left[-\frac{23}{4}g^4 - \frac{3}{4}g^2g'^2 + \frac{1187}{216}g'^4 + 9g^2g_s^2 + \frac{19}{9}g'^2g_s^2 - 108g_s^4 + y_t^2 \left(\frac{225}{16}g^2 + \frac{131}{16}g'^2 + 36g_s^2 - \frac{11}{4}y_b^2 - \frac{9}{4}y_\tau^2 \right) \right. \\
& + y_b^2 \left(\frac{99}{16}g^2 + \frac{7}{48}g'^2 + 4g_s^2 - \frac{1}{4}y_b^2 + \frac{5}{4}y_\tau^2 \right) + y_\tau^2 \left(\frac{15}{8}g^2 + \frac{25}{8}g'^2 - \frac{9}{4}y_\tau^2 \right) \\
& \left. + 6(\lambda^2 - 2y_t^4 - 2\lambda y_t^2) + |Y|^2 \left(\frac{5}{8}y_t^2 - \frac{9}{8}y_\tau^2 - \frac{9}{4}|Y|^2 + \frac{5}{8}g^2 + \frac{15}{8}g'^2 \right) \right]
\end{aligned}$$

$$\begin{aligned}
& + \frac{y_t}{(4\pi)^6} \left[y_t^4 \left(58.6028 y_t^2 + 198\lambda - 157g_s^2 - \frac{1593}{16}g^2 - \frac{2437}{48}g'^2 \right) + \lambda y_t^2 \left(\frac{15}{4}\lambda + 16g_s^2 - \frac{135}{2}g^2 - \frac{127}{6}g'^2 \right) \right. \\
& + y_t^2 (363.764g_s^4 + 16.990g^4 - 67.839g'^4 + 48.370g_s^2g^2 + 30.123g_s^2g'^2 + 58.048g^2g'^2) \\
& + \lambda^2 (-36\lambda + 45g^2 + 15g'^2) + \lambda \left(-\frac{171}{16}g^4 - \frac{1089}{144}g'^4 + \frac{39}{8}g^2g'^2 \right) \\
& - 619.35g_s^6 + 169.829g^6 + 74.074g'^6 + 73.654g_s^4g^2 - 25.16g_s^4g'^2 \\
& \left. - 21.072g_s^2g^4 - 61.997g_s^2g'^4 - \frac{107}{4}g_s^2g^2g'^2 - 7.905g^4g'^2 - 12.339g^2g'^4 \right] + \mathcal{O}\left(\frac{m_h^2}{\Lambda_f^2}\right).
\end{aligned}$$

Meanwhile, the beta function of the bottom Yukawa is, at 2-loop order,

$$\begin{aligned}
\beta_{y_b} = & \frac{y_b}{(4\pi)^2} \left[\frac{3}{2}y_t^2 + \frac{9}{2}y_b^2 + y_\tau^2 - 8g_s^2 - \frac{9}{4}g^2 - \frac{5}{12}g'^2 + |Y|^2 \right] \\
& + \frac{y_b}{(4\pi)^4} \left[\frac{5}{8}y_t^2|Y|^2 - \frac{9}{8}y_b^2|Y|^2 - \frac{9}{4}|Y|^4 + \frac{15}{8}|Y|^2 \left(\frac{1}{3}g^2 + g'^2 \right) + 6\lambda^2 - 108g_s^4 - \frac{23}{4}g^4 - \frac{127}{216}g'^4 \right. \\
& + 9g_s^2g^2 + \frac{31}{9}g_s^2g'^2 - \frac{9}{4}g^2g'^2 + y_t^2 \left(-\frac{1}{4}y_t^2 - \frac{11}{4}y_b^2 + \frac{5}{4}y_\tau^2 + 4g_s^2 + \frac{99}{16}g^2 + \frac{91}{48}g'^2 \right) \\
& \left. + y_b^2 \left(-12y_b^2 - \frac{9}{4}y_\tau^2 - 12\lambda + 36g_s^2 + \frac{225}{16}g^2 + \frac{79}{16}g'^2 \right) + y_\tau^2 \left(-\frac{9}{4}y_\tau^2 + \frac{15}{8}g^2 + \frac{25}{8}g'^2 \right) \right].
\end{aligned}$$

At the same order, the tau Yukawa's beta function is

$$\begin{aligned}
\beta_{y_\tau} = & \frac{y_\tau}{(4\pi)^2} \left[3y_t^2 + 3y_b^2 + \frac{5}{2}y_\tau^2 - \frac{9}{4}g^2 - \frac{15}{4}g'^2 \right] \\
& + \frac{y_\tau}{(4\pi)^4} \left[6\lambda^2 - \frac{23}{4}g^4 + \frac{457}{24}g'^4 + \frac{9}{4}g^2g'^2 + y_t^2 \left(-\frac{27}{4}y_t^2 + \frac{3}{2}y_b^2 - \frac{27}{4}y_\tau^2 + 20g_s^2 + \frac{45}{8}g^2 + \frac{85}{24}g'^2 \right) \right. \\
& \left. + y_b^2 \left(-\frac{27}{4}y_b^2 - \frac{27}{4}y_\tau^2 + 20g_s^2 + \frac{45}{8}g^2 + \frac{25}{24}g'^2 \right) + y_\tau^2 \left(-3y_\tau^2 - 12\lambda + 165g^2 + \frac{179}{16}g'^2 \right) \right]. \tag{A12}
\end{aligned}$$

The beta functions of the gauge couplings g' , g , and g_s are, again at 3-loop order, respectively, given by

$$\begin{aligned}
\beta_{g'} = & \frac{g'^3}{(4\pi)^2} \frac{41}{6} + \frac{g'^3}{(4\pi)^4} \left[\frac{199}{18}g'^2 + \frac{9}{2}g^2 + \frac{44}{3}g_s^2 - \frac{17}{6}y_t^2 - \frac{1}{2}|Y|^2 \right] \\
& + \frac{g'^3}{(4\pi)^6} \left[y_t^2 \left(\frac{315}{16}y_t^2 - \frac{29}{5}g_s^2 - \frac{785}{32}g^2 - \frac{2827}{288}g'^2 \right) + \lambda \left(-3\lambda + \frac{3}{2}g^2 + \frac{3}{2}g'^2 \right) \right. \\
& \left. + 99g_s^4 + \frac{1315}{64}g^4 - \frac{388613}{5184}g'^4 - \frac{25}{9}g_s^2g^2 - \frac{137}{27}g_s^2g'^2 + \frac{205}{96}g^2g'^2 \right], \tag{A13}
\end{aligned}$$

$$\begin{aligned}
\beta_g = & -\frac{g^3}{(4\pi)^2} \frac{19}{6} + \frac{g^3}{(4\pi)^4} \left[\frac{3}{2}g'^2 + \frac{35}{6}g^2 + 12g_s^2 - \frac{3}{2}y_t^2 - \frac{1}{2}|Y|^2 \right] \\
& + \frac{g^3}{(4\pi)^6} \left[y_t^2 \left(\frac{147}{16}y_t^2 - 7g_s^2 - \frac{729}{32}g^2 - \frac{593}{96}g'^2 \right) + \lambda \left(-3\lambda + \frac{3}{2}g^2 + \frac{1}{2}g'^2 \right) \right. \\
& \left. + 81g_s^4 + \frac{324953}{1728}g^4 - \frac{5597}{576}g'^4 + 39g_s^2g^2 - \frac{1}{3}g_s^2g'^2 + \frac{291}{32}g^2g'^2 \right], \tag{A14}
\end{aligned}$$

and

$$\beta_{g_s} = -\frac{g_s^3}{(4\pi)^2}7 + \frac{g_s^3}{(4\pi)^4} \left[\frac{11}{6}g_s^2 + \frac{9}{2}g^2 - 26g_s^2 - 2y_t^2 \right] + \frac{g_s^3}{(4\pi)^6} \left[y_t^2(15y_t^2 - 40g_s^2 - 93/8g^2 - 101/24g^2) + \frac{65}{2}g_s^4 + \frac{109}{8}g^4 - \frac{2615}{216}g^4 + 21g_s^2g^2 + \frac{77}{9}g_s^2g^2 - \frac{1}{8}g^2g^2 \right]. \quad (\text{A15})$$

These are at 3-loop order, except for β_{g_s} which includes the dominant 4-loop term. For the neutrinos' Yukawa couplings, the beta function at 2-loop order is

$$\beta_{Y_i} = \frac{Y_i}{(4\pi)^2} \left[\frac{5}{2}|Y|^2 + 3y_t^2 - \frac{3}{4}g^2 - \frac{9}{4}g^2 \right] + \frac{Y_i}{(4\pi)^4} \left[\frac{3}{2}|Y|^4 - \frac{9}{4}|Y|^2(3y_t^2 + |Y|^2) - \frac{9}{4}(3y_t^4 + |Y|^4) + \frac{3}{2}\lambda^2 - 64\lambda|Y|^2 + \frac{|Y|^2}{16}(93g^2 + 135g^2) \right. \\ \left. + \frac{5}{2} \left(y_t^2 \left(\frac{17}{12}g^2 + \frac{9}{4}g^2 + 8g_s^2 \right) + \frac{3}{4}|Y|^2 \left(\frac{1}{3}g^2 + g^2 \right) \right) + \frac{7}{48}g^4 - \frac{9}{4}g^2g^2 - \frac{23}{4}g^4 \right] + \mathcal{O}\left(\frac{m_h^2}{\Lambda_f^2}\right). \quad (\text{A16})$$

The running of the Wilson coefficient C_6 at 1-loop order is determined by

$$\beta_{C_6} = \frac{C_6}{(4\pi)^2} \left[-\frac{9}{2}(3g^2 + g^2) + 108\lambda + 18y_t^2 \right]. \quad (\text{A17})$$

Lastly, the running mass m_h^2 satisfies

$$\beta_{m_h^2} = \frac{3m_h^2}{8\pi^2} \left[2\lambda + y_t^2 - \frac{3}{4}g^2 - \frac{3}{20}g^2 + \mathcal{O}\left(\frac{m_h^2}{\Lambda_f^2}\right) \right]. \quad (\text{A18})$$

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