

Polarized vector meson production in semi-inclusive DIS

Xue-shuai Jiao and Kai-bao Chen^{*}

School of Science, Shandong Jianzhu University, Jinan, Shandong 250101, China



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We make a systematic calculation for polarized vector meson production in semi-inclusive lepton-nucleon deep inelastic scattering $e^-N \rightarrow e^-VX$. We consider the general case of neutral current electroweak interactions at high energies which give rise to parity-violating effects. We present a general kinematic analysis for the process and show that the cross sections are expressed by 81 structure functions. We further give a parton model calculation for the process and show the results for the structure functions in terms of the transverse momentum dependent parton distribution functions and fragmentation functions at the leading order and leading twist of perturbative quantum chromodynamics. The results show that there are 27 nonzero structure functions at this order, among which 15 are related to the tensor polarization of the vector meson. Thirteen structure functions are generated by parity-violating effects. We also present the result and a rough numerical estimate for the spin alignment of the vector meson.

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I. INTRODUCTION

Parton distribution functions (PDFs) and fragmentation functions (FFs) are related to the hadron structure and hadronization mechanism, respectively. They are two important quantities in describing high energy reactions (see, e.g., [1–7] for recent reviews). In semi-inclusive reaction processes, e.g., lepton-nucleon semi-inclusive deep inelastic scattering (SIDIS) with hadron production at small transverse momentum, the transverse momentum dependent (TMD) factorization applies [8–12]. The sensitive observables in experiments are often different azimuthal asymmetries that are theoretically expressed by the convolutions of TMD PDFs and FFs in general. Therefore, processes such as SIDIS, Drell-Yan, or semi-inclusive hadron production in electron-positron annihilation give us opportunities for accessing the three-dimensional hadron structure and hadronization mechanism [13–39].

One can access the spin dependent TMD FFs in semi-inclusive processes by measuring the polarizations or spin dependent azimuthal asymmetries of the produced hadron. For example, for Λ hyperon production, the polarizations of Λ can be measured by its self-analyzing weak decay. Very similar to hyperon polarizations, the polarizations of vector mesons can also be determined by the angular distribution of their decay products through the strong decay into two

spin zero hadrons. Since the vector mesons are spin-1, there will be both vector and tensor polarizations for them. Measurements have been carried out, e.g., for the e^+e^- annihilation process at the Large Electron-Positron Collider more than two decades ago for the spin alignment of vector mesons [40–42], which has attracted much attention (see, e.g., [43] for a recent phenomenological study and references therein). One can also study hadron polarizations in the SIDIS process; e.g., the Λ hyperon polarizations can be studied in $e^-N \rightarrow e^-\Lambda X$. In the QCD parton model and TMD factorization, the Λ polarizations in SIDIS are expressed by the convolution of the TMD PDFs with the corresponding spin dependent TMD FFs (see [44] for a recent phenomenological study). When we consider the final state hadron to be a vector meson V with spin-1, i.e., $e^-N \rightarrow e^-VX$, we can access not only the vector polarization dependent FFs but also the tensor polarization dependent FFs [36].

The Electron-Ion Collider (EIC) has been proposed to be built as the next generation collider on deep inelastic scattering with high energy and high luminosity [45,46]. It gives us new opportunities for exploring physics on quantum chromodynamics (QCD) and nucleon structure. Since the beam energy of the EIC is relatively high, it has a wider kinematic coverage. The momentum transfer Q^2 between the incident lepton and the nucleon has a chance to be comparable with M_Z^2 , i.e., the mass square of the Z boson. Therefore, parity-violating effects can arise through the interference of the electromagnetic (EM) and weak interactions [47]. It will also generate new structure functions that are complementary to those from the pure EM interaction. Experimentally, parity-violating asymmetries in deep inelastic scattering (DIS) experiments were

^{*}chenkaibao19@sdjzu.edu.cn

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first observed at SLAC [48,49] and have been measured widely [50–60]. Proposals for precise measurements are also available [61–63]. On the theoretical side, electroweak inclusive and semi-inclusive DIS processes have been studied extensively [23,39,47,64–69]. However, systematic studies are still lacking. These include a full kinematic analysis and QCD parton model calculations for the cross section, and the systematic treatment for the hadron polarization effects.

The rest of the paper is organized as follows. In Sec. II, we present the general form of the cross section for $e^-N \rightarrow e^-VX$ in terms of structure functions by carrying out a kinematic analysis. In Sec. III, we calculate the hadronic tensor in the QCD parton model and give the results expressed by the convolution of the TMD PDFs and FFs. In Sec. IV, we give the results for the structure functions as well as the spin alignment of the vector meson in terms of TMD PDFs and FFs. Section V is a summary.

II. THE GENERAL FORM OF THE CROSS SECTION AND THE STRUCTURE FUNCTIONS

A. The process and notations

We consider the SIDIS process at high energies with unpolarized electron and nucleon beams, i.e.,

$$e^-(l) + N(p_N) \rightarrow e^-(l') + V(p_h, S) + X, \quad (1)$$

where V is a vector meson with spin-1. The momenta of the incident and outgoing particles are shown in the brackets. S denotes the polarizations of the vector meson. The coordinate system for $e^-N \rightarrow e^-VX$ in the photon-nucleon collinear frame is shown in Fig. 1, where the incoming proton and the virtual photon move along the $\pm z$ axis and the x axis is determined by the transverse momenta of the leptons. The azimuthal angle ϕ is spanned by the transverse momentum of the vector meson with respect to the transverse momentum of the outgoing electron.

The standard variables for SIDIS are defined as $Q^2 = -q^2$, $x = \frac{Q^2}{2p_N \cdot q}$, $y = \frac{p_N \cdot q}{p_N \cdot l}$, $z_h = \frac{p_N \cdot p_h}{p_N \cdot q}$, and $s = (p_N + l)^2$. We will use the light-cone coordinate system, in which a four vector a^μ is expressed as $a^\mu = (a^+, a^-, \vec{a}_\perp)$ with

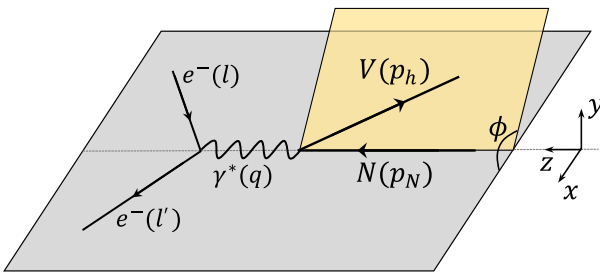


FIG. 1. The coordinate system for $e^-N \rightarrow e^-VX$. The symbols in the brackets denote the four momenta of the particles.

$a^\pm = (a^0 \pm a^3)/\sqrt{2}$ and $\vec{a}_\perp = (a^1, a^2)$. The momenta of the particles take the following forms:

$$p_N^\mu = (p_N^+, 0, \vec{0}_\perp), \quad (2)$$

$$l^\mu = \left(\frac{1-y}{y} x p_N^+, \frac{Q^2}{2xy p_N^+}, \frac{Q\sqrt{1-y}}{y}, 0 \right), \quad (3)$$

$$q^\mu = \left(-x p_N^+, \frac{Q^2}{2x p_N^+}, \vec{0}_\perp \right), \quad (4)$$

$$p_h^\mu = \left(\frac{x p_N^+ \vec{p}_{h\perp}^2}{z_h Q^2}, \frac{z_h Q^2}{2x p_N^+}, \vec{p}_{h\perp} \right), \quad (5)$$

$$\vec{p}_{h\perp} = |p_{h\perp}|(\cos \phi, \sin \phi). \quad (6)$$

We also define two unit light-cone vectors $n^\mu = (0, 1, \vec{0}_\perp)$ and $\bar{n}^\mu = (1, 0, \vec{0}_\perp)$. With these notations, the transverse metric is given by $g_\perp^{\mu\nu} = g^{\mu\nu} - \bar{n}^\mu n^\nu - n^\mu \bar{n}^\nu$. We will also use the transverse antisymmetric tensor defined as $\varepsilon_\perp^{\mu\nu} = \varepsilon^{\mu\nu\alpha\beta} \bar{n}_\alpha n_\beta$ with $\varepsilon_\perp^{12} = 1$.

We consider the leading order approximation for QED with the exchange of a single virtual photon γ^* or a Z boson with momentum $q = l - l'$ between the electron and the nucleon. The differential cross section is given by

$$d\sigma = \frac{2\alpha_{\text{em}}^2}{sQ^4} A_r L_r^{\mu\nu}(l, l') W_{r,\mu\nu}(q, p_N, p_h, S) \frac{d^3 l' d^3 p_h}{2E_{l'} 2E_h}. \quad (7)$$

The symbol r can be $\gamma\gamma$, ZZ , and γZ . They correspond to the electromagnetic, the weak, and the interference contributions, respectively. A summation over r is implicit in Eq. (7), i.e.,

$$d\sigma = d\sigma_{\gamma\gamma} + d\sigma_{ZZ} + d\sigma_{\gamma Z}. \quad (8)$$

The A_r factors and the leptonic tensors are determined in perturbative theory. The A_r factors can be calculated as

$$A_{\gamma\gamma} = e_q^2, \quad (9)$$

$$A_{ZZ} = \frac{Q^4}{[(Q^2 + M_Z^2)^2 + \Gamma_Z^2 M_Z^2] \sin^4 2\theta_W} \equiv \chi, \quad (10)$$

$$A_{\gamma Z} = \frac{2e_q Q^2 (Q^2 + M_Z^2)}{[(Q^2 + M_Z^2)^2 + \Gamma_Z^2 M_Z^2] \sin^2 2\theta_W} \equiv \chi_{\text{int}}, \quad (11)$$

where Γ_Z is the width of the Z boson and θ_W is the Weinberg angle. The leptonic tensors are given by

$$L_{\gamma\gamma}^{\mu\nu}(l, l') = 2(l^\mu l'^\nu + l'^\mu l^\mu - g^{\mu\nu} l \cdot l'), \quad (12)$$

$$L_{ZZ}^{\mu\nu}(l, l') = c_1^e L_{\gamma\gamma}^{\mu\nu}(l, l') - 2ic_3^e \varepsilon^{\mu\nu l l'}, \quad (13)$$

$$L_{\gamma Z}^{\mu\nu}(l, l') = c_V^e L_{\gamma\gamma}^{\mu\nu}(l, l') - 2ic_A^e \epsilon^{\mu\nu l l'}, \quad (14)$$

where c_V^e and c_A^e are from the weak interaction vertex defined by $\Gamma_\mu^e = \gamma_\mu(c_V^e - c_A^e \gamma_5)$, $c_1^e = (c_V^e)^2 + (c_A^e)^2$, and $c_3^e = 2c_V^e c_A^e$. We see that $L_{\gamma\gamma}^{\mu\nu}$ and $L_{\gamma Z}^{\mu\nu}$ can be obtained from $L_{ZZ}^{\mu\nu}$ by the replacements of $(c_1^e, c_3^e) \rightarrow (1, 0)$ and $(c_1^e, c_3^e) \rightarrow (c_V^e, c_A^e)$, respectively. The corresponding hadronic tensors are defined as

$$\begin{aligned} W_{\gamma\gamma}^{\mu\nu}(q, p_N, p_h, S) &= \sum_X \delta^4(p_N + q - p_h - p_X) \\ &\times \langle p_N | J_{\gamma\gamma}^\mu(0) | p_h, S; X \rangle \\ &\times \langle p_h, S; X | J_{\gamma\gamma}^\nu(0) | p_N \rangle, \end{aligned} \quad (15)$$

$$\begin{aligned} W_{ZZ}^{\mu\nu}(q, p_N, p_h, S) &= \sum_X \delta^4(p_N + q - p_h - p_X) \\ &\times \langle p_N | J_{ZZ}^\mu(0) | p_h, S; X \rangle \\ &\times \langle p_h, S; X | J_{ZZ}^\nu(0) | p_N \rangle, \end{aligned} \quad (16)$$

$$\begin{aligned} W_{\gamma Z}^{\mu\nu}(q, p_N, p_h, S) &= \sum_X \delta^4(p_N + q - p_h - p_X) \\ &\times \langle p_N | J_{ZZ}^\mu(0) | p_h, S; X \rangle \\ &\times \langle p_h, S; X | J_{\gamma\gamma}^\nu(0) | p_N \rangle, \end{aligned} \quad (17)$$

where the current operators are given by $J_{\gamma\gamma}^\mu(0) = \bar{\psi}(0)\gamma^\mu\psi(0)$ and $J_{ZZ}^\mu(0) = \bar{\psi}(0)\Gamma_\mu^q\psi(0)$ with $\Gamma_\mu^q = \gamma_\mu(c_V^q - c_A^q\gamma_5)$.

For the phase space factors, in terms of the standard SIDIS variables, we have

$$\frac{d^3 l'}{2E_{l'}} = \frac{ys}{4} dx dy d\psi, \quad \frac{d^3 p_h}{2E_h} = \frac{dz_h}{2z_h} d^2 p_{h\perp}, \quad (18)$$

where ψ is the azimuthal angle of \vec{l}' around \vec{l} . Therefore the cross section in Eq. (7) can be written as

$$\frac{d\sigma}{dx dy dz_h d\psi d^2 p_{h\perp}} = \frac{y\alpha_{\text{em}}^2}{4z_h Q^4} A_r L_r^{\mu\nu}(l, l') W_{r,\mu\nu}(q, p_N, p_h, S). \quad (19)$$

B. Cross section in terms of structure functions

In this subsection, we make a kinematic analysis for the process. There is no restriction from parity since we consider both the EM and the weak interaction contributions. The most general form of the cross section is obtained by adding the contributions from the EM, the weak, and the interference terms together. Formally, the pure weak interaction contribution will generate all the possible structure functions. The EM and the interference contributions will not introduce new types of structure

functions, so we first concentrate on the analysis of the pure weak interaction contribution.

We give the decomposition of the hadronic tensors in terms of basic Lorentz tensors (BLTs). The general form of the hadronic tensor satisfies the constraints of Hermiticity and current conservation. The hadronic tensor is divided into a symmetric part and an antisymmetric part, i.e., $W_{ZZ}^{\mu\nu} = W_{ZZ}^{S\mu\nu} + W_{ZZ}^{A\mu\nu}$. More explicitly, we have

$$W_{ZZ}^{S\mu\nu} = \sum_{\sigma,i} W_{\sigma i}^S h_{\sigma i}^{S\mu\nu} + \sum_{\sigma,i} \tilde{W}_{\sigma i}^S \tilde{h}_{\sigma i}^{S\mu\nu}, \quad (20)$$

$$W_{ZZ}^{A\mu\nu} = \sum_{\sigma,i} W_{\sigma i}^A h_{\sigma i}^{A\mu\nu} + \sum_{\sigma,i} \tilde{W}_{\sigma i}^A \tilde{h}_{\sigma i}^{A\mu\nu}, \quad (21)$$

where $h_{\sigma i}^{\mu\nu}$'s and $\tilde{h}_{\sigma i}^{\mu\nu}$'s represent the space reflection even and odd BLTs, respectively. They are constructed from available kinematical variables in the process, e.g., p_N , p_h , q , and the polarization vector or tensor. $W_{\sigma i}$'s are scalar coefficients. The subscript σ specifies the polarizations of the vector meson.

There are nine BLTs for the unpolarized part [36], i.e.,

$$h_{Ui}^{S\mu\nu} = \left\{ g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}, p_{Nq}^\mu p_{Nq}^\nu, p_{hq}^\mu p_{hq}^\nu, p_{Nq}^{\{\mu} p_{hq}^{\nu\}} \right\}, \quad (22)$$

$$\tilde{h}_{Ui}^{S\mu\nu} = \{ \epsilon^{\{\mu q p_N p_h\}} p_{Nq}^{\nu\}}, \epsilon^{\{\mu q p_N p_h\}} p_{hq}^{\nu\} \}, \quad (23)$$

$$h_U^{A\mu\nu} = \{ p_{Nq}^{\{\mu} p_{hq}^{\nu\}} \}, \quad (24)$$

$$\tilde{h}_{Ui}^{A\mu\nu} = \{ \epsilon^{\mu\nu q p_N}, \epsilon^{\mu\nu q p_h} \}. \quad (25)$$

The subscript U denotes the unpolarized part. We have defined the four vectors such as $p_{Nq}^\mu \equiv p_N^\mu - q^\mu(p_N \cdot q)/q^2$ satisfying $q \cdot p_{Nq} = 0$, and similar for p_{hq}^μ . We have also used the notations $A^{\{\mu} B^{\nu\}} \equiv A^\mu B^\nu + A^\nu B^\mu$ and $A^{[\mu} B^{\nu]} \equiv A^\mu B^\nu - A^\nu B^\mu$ for simplicity.

The vector polarization of the vector meson, similar to spin-1/2 hadrons, is described by the helicity λ and the transverse polarization vector S_T^μ . It has been shown in [36] that the polarization dependent BLTs can be constructed from the unpolarized BLTs by multiplying the corresponding polarization dependent scalars or pseudoscalars. Therefore, we have

$$h_{Vi}^{S\mu\nu} = \{ [\lambda, (p_{h\perp} \cdot S_T)] \tilde{h}_{Ui}^{S\mu\nu}, \epsilon_{\perp}^{p_{h\perp} S_T} h_{Uj}^{S\mu\nu} \}, \quad (26)$$

$$\tilde{h}_{Vi}^{S\mu\nu} = \{ [\lambda, (p_{h\perp} \cdot S_T)] h_{Ui}^{S\mu\nu}, \epsilon_{\perp}^{p_{h\perp} S_T} \tilde{h}_{Uj}^{S\mu\nu} \}, \quad (27)$$

$$h_{Vi}^{A\mu\nu} = \{ [\lambda, (p_{h\perp} \cdot S_T)] \tilde{h}_{Ui}^{A\mu\nu}, \epsilon_{\perp}^{p_{h\perp} S_T} h_{Uj}^{A\mu\nu} \}, \quad (28)$$

$$\tilde{h}_{Vi}^{A\mu\nu} = \{ [\lambda, (p_{h\perp} \cdot S_T)] h_{Ui}^{A\mu\nu}, \epsilon_{\perp}^{p_{h\perp} S_T} \tilde{h}_{Uj}^{A\mu\nu} \}. \quad (29)$$

There are in total 27 BLTs for the vector polarization dependent part.

The tensor polarization of the vector meson is described by five independent components. They are given by a Lorentz scalar S_{LL} , a transverse Lorentz vector S_{LT}^μ with two independent components S_{LT}^x and S_{LT}^y , and a transverse Lorentz tensor $S_{TT}^{\mu\nu}$ with two independent components $S_{TT}^{xx} = -S_{TT}^{yy}$ and $S_{TT}^{xy} = S_{TT}^{yx}$ [22,36]. Since S_{LL} is a Lorentz scalar, there are nine S_{LL} -dependent BLTs in analog with the unpolarized part. They are given by

$$h_{LLi}^{S\mu\nu} = S_{LL} h_{Ui}^{S\mu\nu}, \quad (30)$$

$$\tilde{h}_{LLi}^{S\mu\nu} = S_{LL} \tilde{h}_{Ui}^{S\mu\nu}, \quad (31)$$

$$h_{LL}^{A\mu\nu} = S_{LL} h_U^{A\mu\nu}, \quad (32)$$

$$\tilde{h}_{LLi}^{A\mu\nu} = S_{LL} \tilde{h}_{Ui}^{A\mu\nu}. \quad (33)$$

The S_{LT}^μ dependent BLTs are given by

$$h_{LTi}^{S\mu\nu} = \{(p_{h\perp} \cdot S_{LT}) h_{Ui}^{S\mu\nu}, \epsilon_{\perp}^{p_{h\perp} S_{LT}} \tilde{h}_{Uj}^{S\mu\nu}\}, \quad (34)$$

$$\tilde{h}_{LTi}^{S\mu\nu} = \{(p_{h\perp} \cdot S_{LT}) \tilde{h}_{Ui}^{S\mu\nu}, \epsilon_{\perp}^{p_{h\perp} S_{LT}} h_{Uj}^{S\mu\nu}\}, \quad (35)$$

$$h_{LTi}^{A\mu\nu} = \{(p_{h\perp} \cdot S_{LT}) h_U^{A\mu\nu}, \epsilon_{\perp}^{p_{h\perp} S_{LT}} \tilde{h}_{Uj}^{A\mu\nu}\}, \quad (36)$$

$$\tilde{h}_{LTi}^{A\mu\nu} = \{(p_{h\perp} \cdot S_{LT}) \tilde{h}_{Ui}^{A\mu\nu}, \epsilon_{\perp}^{p_{h\perp} S_{LT}} h_U^{A\mu\nu}\}. \quad (37)$$

For the $S_{TT}^{\mu\nu}$ dependent part, we have

$$h_{TTi}^{S\mu\nu} = \{S_{TT}^{p_h p_h} h_{Ui}^{S\mu\nu}, \tilde{S}_{TT}^{p_h p_h} \tilde{h}_{Uj}^{S\mu\nu}\}, \quad (38)$$

$$\tilde{h}_{TTi}^{S\mu\nu} = \{S_{TT}^{p_h p_h} \tilde{h}_{Ui}^{S\mu\nu}, \tilde{S}_{TT}^{p_h p_h} h_{Uj}^{S\mu\nu}\}, \quad (39)$$

$$h_{TTi}^{A\mu\nu} = \{S_{TT}^{p_h p_h} h_U^{A\mu\nu}, \tilde{S}_{TT}^{p_h p_h} \tilde{h}_{Uj}^{A\mu\nu}\}, \quad (40)$$

$$\tilde{h}_{TTi}^{A\mu\nu} = \{S_{TT}^{p_h p_h} \tilde{h}_{Ui}^{A\mu\nu}, \tilde{S}_{TT}^{p_h p_h} h_U^{A\mu\nu}\}, \quad (41)$$

where $S_{TT}^{p_h p_h} = p_{h\perp\alpha} p_{h\perp\beta} S_{TT}^{\alpha\beta}$ and $\tilde{S}_{TT}^{p_h p_h} = -\tilde{p}_{h\perp\alpha} p_{h\perp\beta} S_{TT}^{\alpha\beta}$ with $\tilde{p}_{h\perp}^\alpha = \epsilon_{\perp}^{\alpha\mu} p_{h\perp\mu}$.

Substitute the hadronic tensors in Eqs. (20) and (21) into Eq. (19), and after making Lorentz contractions with the leptonic tensor, we will obtain the general form for the cross section. To be explicit, we first parametrize the components of the transverse polarization vector and tensor as

$$S_T^\mu = |S_T| (0, 0, \cos \phi_S, \sin \phi_S), \quad (42)$$

$$S_{LT}^x = |S_{LT}| \cos \phi_{LT}, \quad (43)$$

$$S_{LT}^y = |S_{LT}| \sin \phi_{LT}, \quad (44)$$

$$S_{TT}^{xx} = |S_{TT}| \cos 2\phi_{TT}, \quad (45)$$

$$S_{TT}^{xy} = |S_{TT}| \sin 2\phi_{TT}. \quad (46)$$

Then, the cross section can be divided into six parts according to the polarization states of the vector meson. It is given by

$$\frac{d\sigma}{dx dy dz_h d\mu d^2 p_{h\perp}} = \frac{\alpha_{em}^2}{xy Q^2} (\mathcal{W}_U + \lambda \mathcal{W}_L + |S_T| \mathcal{W}_T + S_{LL} \mathcal{W}_{LL} + |S_{LT}| \mathcal{W}_{LT} + |S_{TT}| \mathcal{W}_{TT}), \quad (47)$$

where the subscripts of \mathcal{W} denote the polarization states of the vector meson. The explicit expressions for each part are calculated to be

$$\begin{aligned} \mathcal{W}_U &= A(y) W_U^T + E(y) W_U^L + B(y) (\sin \phi \tilde{W}_{U1}^{\sin \phi} + \cos \phi \tilde{W}_{U1}^{\cos \phi}) + E(y) (\sin 2\phi \tilde{W}_U^{\sin 2\phi} + \cos 2\phi \tilde{W}_U^{\cos 2\phi}) \\ &\quad + C(y) W_U + D(y) (\sin \phi \tilde{W}_{U2}^{\sin \phi} + \cos \phi \tilde{W}_{U2}^{\cos \phi}), \end{aligned} \quad (48)$$

$$\begin{aligned} \mathcal{W}_L &= A(y) \tilde{W}_L^T + E(y) \tilde{W}_L^L + B(y) (\sin \phi \tilde{W}_{L1}^{\sin \phi} + \cos \phi \tilde{W}_{L1}^{\cos \phi}) + E(y) (\sin 2\phi \tilde{W}_L^{\sin 2\phi} + \cos 2\phi \tilde{W}_L^{\cos 2\phi}) \\ &\quad + C(y) \tilde{W}_L + D(y) (\sin \phi \tilde{W}_{L2}^{\sin \phi} + \cos \phi \tilde{W}_{L2}^{\cos \phi}), \end{aligned} \quad (49)$$

$$\begin{aligned} \mathcal{W}_T &= \sin \phi_S [B(y) W_{T1}^{\sin \phi_S} + D(y) W_{T2}^{\sin \phi_S}] + \sin(\phi + \phi_S) E(y) W_T^{\sin(\phi + \phi_S)} \\ &\quad + \sin(\phi - \phi_S) [A(y) W_T^{T, \sin(\phi - \phi_S)} + E(y) W_T^{L, \sin(\phi - \phi_S)} + C(y) W_T^{\sin(\phi - \phi_S)}] \\ &\quad + \sin(2\phi - \phi_S) [B(y) W_{T1}^{\sin(2\phi - \phi_S)} + D(y) W_{T2}^{\sin(2\phi - \phi_S)}] + \sin(3\phi - \phi_S) E(y) W_T^{\sin(3\phi - \phi_S)} + \cos \phi_S [B(y) \tilde{W}_{T1}^{\cos \phi_S} + D(y) \tilde{W}_{T2}^{\cos \phi_S}] \\ &\quad + \cos(\phi + \phi_S) E(y) \tilde{W}_T^{\cos(\phi + \phi_S)} + \cos(\phi - \phi_S) [A(y) \tilde{W}_T^{T, \cos(\phi - \phi_S)} + E(y) \tilde{W}_T^{L, \cos(\phi - \phi_S)} + C(y) \tilde{W}_T^{\cos(\phi - \phi_S)}] \\ &\quad + \cos(2\phi - \phi_S) [B(y) \tilde{W}_{T1}^{\cos(2\phi - \phi_S)} + D(y) \tilde{W}_{T2}^{\cos(2\phi - \phi_S)}] + \cos(3\phi - \phi_S) E(y) \tilde{W}_T^{\cos(3\phi - \phi_S)}, \end{aligned} \quad (50)$$

$$\begin{aligned} \mathcal{W}_{LL} = & A(y)W_{LL}^T + E(y)W_{LL}^L + B(y)(\sin\phi\tilde{W}_{LL1}^{\sin\phi} + \cos\phi W_{LL1}^{\cos\phi}) + E(y)(\sin 2\phi\tilde{W}_{LL}^{\sin 2\phi} + \cos 2\phi W_{LL}^{\cos 2\phi}) \\ & + C(y)W_{LL} + D(y)(\sin\phi\tilde{W}_{LL2}^{\sin\phi} + \cos\phi W_{LL2}^{\cos\phi}), \end{aligned} \quad (51)$$

$$\begin{aligned} \mathcal{W}_{LT} = & \sin\phi_{LT}[B(y)\tilde{W}_{LT1}^{\sin\phi_{LT}} + D(y)\tilde{W}_{LT2}^{\sin\phi_{LT}}] + \sin(\phi + \phi_{LT})E(y)\tilde{W}_{LT}^{\sin(\phi+\phi_{LT})} \\ & + \sin(\phi - \phi_{LT})[A(y)\tilde{W}_{LT}^{T,\sin(\phi-\phi_{LT})} + E(y)\tilde{W}_{LT}^{L,\sin(\phi-\phi_{LT})} + C(y)\tilde{W}_{LT}^{\sin(\phi-\phi_{LT})}] \\ & + \sin(2\phi - \phi_{LT})[B(y)\tilde{W}_{LT1}^{\sin(2\phi-\phi_{LT})} + D(y)\tilde{W}_{LT2}^{\sin(2\phi-\phi_{LT})}] + \sin(3\phi - \phi_{LT})E(y)\tilde{W}_{LT}^{\sin(3\phi-\phi_{LT})} \\ & + \cos\phi_{LT}[B(y)W_{LT1}^{\cos\phi_{LT}} + D(y)W_{LT2}^{\cos\phi_{LT}}] + \cos(\phi + \phi_{LT})E(y)W_{LT}^{\cos(\phi+\phi_{LT})} \\ & + \cos(\phi - \phi_{LT})[A(y)W_{LT}^{T,\cos(\phi-\phi_{LT})} + E(y)W_{LT}^{L,\cos(\phi-\phi_{LT})} + C(y)W_{LT}^{\cos(\phi-\phi_{LT})}] \\ & + \cos(2\phi - \phi_{LT})[B(y)W_{LT1}^{\cos(2\phi-\phi_{LT})} + D(y)W_{LT2}^{\cos(2\phi-\phi_{LT})}] + \cos(3\phi - \phi_{LT})E(y)W_{LT}^{\cos(3\phi-\phi_{LT})}, \end{aligned} \quad (52)$$

$$\begin{aligned} \mathcal{W}_{TT} = & \sin(\phi - 2\phi_{TT})[B(y)\tilde{W}_{TT1}^{\sin(\phi-2\phi_{TT})} + D(y)\tilde{W}_{TT2}^{\sin(\phi-2\phi_{TT})}] + \sin 2\phi_{TT}E(y)\tilde{W}_{TT}^{\sin 2\phi_{TT}} \\ & + \sin(2\phi - 2\phi_{TT})[A(y)\tilde{W}_{TT}^{T,\sin(2\phi-2\phi_{TT})} + E(y)\tilde{W}_{TT}^{L,\sin(2\phi-2\phi_{TT})} + C(y)\tilde{W}_{TT}^{\sin(2\phi-2\phi_{TT})}] \\ & + \sin(3\phi - 2\phi_{TT})[B(y)\tilde{W}_{TT1}^{\sin(3\phi-2\phi_{TT})} + D(y)\tilde{W}_{TT2}^{\sin(3\phi-2\phi_{TT})}] + \sin(4\phi - 2\phi_{TT})E(y)\tilde{W}_{TT}^{\sin(4\phi-2\phi_{TT})} \\ & + \cos(\phi - 2\phi_{TT})[B(y)W_{TT1}^{\cos(\phi-2\phi_{TT})} + D(y)W_{TT2}^{\cos(\phi-2\phi_{TT})}] + \cos 2\phi_{TT}E(y)W_{TT}^{\cos 2\phi_{TT}} \\ & + \cos(2\phi - 2\phi_{TT})[A(y)W_{TT}^{T,\cos(2\phi-2\phi_{TT})} + E(y)W_{TT}^{L,\cos(2\phi-2\phi_{TT})} + C(y)W_{TT}^{\cos(2\phi-2\phi_{TT})}] \\ & + \cos(3\phi - 2\phi_{TT})[B(y)W_{TT1}^{\cos(3\phi-2\phi_{TT})} + D(y)W_{TT2}^{\cos(3\phi-2\phi_{TT})}] + \cos(4\phi - 2\phi_{TT})E(y)W_{TT}^{\cos(4\phi-2\phi_{TT})}. \end{aligned} \quad (53)$$

We have defined the following functions of y for simplicity:

$$\begin{aligned} A(y) &= y^2 - 2y + 2, \\ B(y) &= 2(2 - y)\sqrt{1 - y}, \\ C(y) &= y(2 - y), \\ D(y) &= 2y\sqrt{1 - y}, \\ E(y) &= 2(1 - y). \end{aligned} \quad (54)$$

There are 81 structure functions in total, which is exactly the number of the corresponding 81 independent BLTs for the hadronic tensor. Among all the structure functions, 39 of them, i.e., the \tilde{W} 's, correspond to the space reflection odd part of the cross section and the remaining 42 correspond to the space reflection even part. We also note that there are 45 structure functions that depend on the tensor polarizations of the vector meson. It can be checked that the cross section for $e^-N \rightarrow e^-VX$ given in Eqs. (47)–(53) has the same form in terms of the azimuthal angle dependence as that for $e^+e^- \rightarrow Z^0 \rightarrow V\pi X$ given in [36]. This is because these two processes share the same set of BLTs when constructing the general form of the hadronic tensors.

III. PARTON MODEL CALCULATIONS

A. The hadronic tensor results in terms of TMD PDFs and FFs

We now calculate the hadronic tensor using the QCD parton model. At the leading order of perturbative quantum

chromodynamics and the leading twist, the hadronic tensor is given by

$$\begin{aligned} W_{ZZ}^{\mu\nu}(q, p_N, p_h, S) = & \int d^4k_i d^4k_f \delta^4(q + k_i - k_f) \\ & \times \text{Tr} [\hat{\Phi}(k_i, p_N) \Gamma^\mu \hat{\Xi}(k_f, p_h, S_h) \Gamma^\nu], \end{aligned} \quad (55)$$

where $\hat{\Phi}$ and $\hat{\Xi}$ are the parton correlators related to the parton distribution and the fragmentation process. They are defined as [25]

$$\hat{\Phi}_{ij}(k_i, p_N) = \int \frac{d^4\xi}{(2\pi)^4} e^{i\xi \cdot k_i} \langle p_N | \bar{\psi}_j(0) \psi_i(\xi) | p_N \rangle, \quad (56)$$

$$\begin{aligned} \hat{\Xi}_{ij}(k_f, p_h, S) = & \sum_X \int \frac{d^4\eta}{(2\pi)^4} e^{-i\eta \cdot k_f} \langle 0 | \psi_i(0) | p_h, S; X \rangle \\ & \times \langle p_h, S; X | \bar{\psi}_j(\eta) | 0 \rangle. \end{aligned} \quad (57)$$

We have suppressed the gauge links for short notations. Taking approximations for the momenta in the δ function of the hadronic tensor, i.e., $k_i^- \approx 0$ and $k_f^+ \approx 0$, and after integration, we get

$$W_{ZZ}^{\mu\nu} = 2z_h \int d^2k_{iT} d^2k_{fT} \delta^2(\vec{k}_{iT} - \vec{k}_{fT} - \vec{p}_{h\perp}/z_h) \times \text{Tr}[\hat{\Phi}(x, k_{iT}) \Gamma^\mu \hat{\Xi}(z_h, k_{fT}, S) \Gamma^\nu], \quad (58)$$

where the TMD parton correlators are defined by

$$\hat{\Phi}_{ij}(x, k_{iT}) = \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3} e^{ixp_N^+ \xi^- + ik_{iT} \cdot \xi_T} \times \langle p_N | \bar{\psi}_i(0) \psi_j(\xi^-, \xi_T) | p_N \rangle, \quad (59)$$

$$\hat{\Xi}_{ij}(z_h, k_{fT}, S) = \sum_X \int \frac{d\eta^+ d^2\eta_T}{2z_h(2\pi)^3} e^{-i\eta^+ p_h^- / z_h - i\eta_T \cdot k_{fT}} \times \langle 0 | \psi_i(0) | p_h, S; X \rangle \times \langle p_h, S; X | \bar{\psi}_j(\eta^+, \eta_T) | 0 \rangle. \quad (60)$$

The parton correlators are 4×4 matrices in Dirac space. They can be expanded under the basis of Gamma matrices, i.e.,

$$\hat{\Phi} = \frac{1}{2} (\Phi_\alpha \gamma^\alpha - \tilde{\Phi}_\alpha \gamma_5 \gamma^\alpha + \Phi_{\alpha\beta} i\sigma^{\alpha\beta} \gamma_5) + \dots, \quad (61)$$

$$\hat{\Xi} = \frac{1}{2} (\Xi_\alpha \gamma^\alpha + \tilde{\Xi}_\alpha \gamma_5 \gamma^\alpha + \Xi_{\alpha\beta} i\sigma^{\alpha\beta} \gamma_5) + \dots, \quad (62)$$

where \dots denotes terms irrelevant for the leading twist. The Φ_α 's, etc., are expanding coefficients or are called correlation functions. We have omitted the arguments of the correlators and correlation functions for short notations. For an unpolarized nucleon, the Lorentz decomposition of the correlation functions only generate two TMD PDFs at the leading twist [70]. We have

$$\Phi^\alpha = \bar{n}^\alpha f_1(x, k_{iT}) + \dots, \quad (63)$$

$$\Phi^{\alpha\beta} = -\frac{1}{M_N} \bar{n}^\alpha \tilde{k}_{iT}^\beta h_1^\perp(x, k_{iT}) + \dots \quad (64)$$

The first term defines the number density distribution, and h_1^\perp is the Boer-Mulders function which is chiral-odd.

Similarly, for the fragmentation part, the correlation functions are decomposed as [36]

$$\Xi^\alpha = n^\alpha \left(D_1 + \frac{\tilde{k}_{fT} \cdot S_T}{M_h} D_{1T}^\perp + S_{LL} D_{1LL} + \frac{k_{fT} \cdot S_{LT}}{M_h} D_{1LT}^\perp + \frac{S_{TT}^{k_f k_f}}{M_h^2} D_{1TT}^\perp \right) + \dots, \quad (65)$$

$$\tilde{\Xi}^\alpha = n^\alpha \left(\lambda_h G_{1L} + \frac{k_{fT} \cdot S_T}{M_h} G_{1T}^\perp + \frac{\tilde{k}_{fT} \cdot S_{LT}}{M_h} G_{1LT}^\perp - \frac{\tilde{S}_{TT}^{k_f k_f}}{M_h^2} G_{1TT}^\perp \right) + \dots, \quad (66)$$

$$\Xi^{\alpha\beta} = -\frac{n^{[\alpha} \tilde{k}_{fT}^{\beta]}}{M_h} (H_1^\perp + S_{LL} H_{1LL}^\perp) + n^{[\alpha} S_T^{\beta]} H_{1T}^\perp + \frac{n^{[\alpha} k_{fT}^{\beta]}}{M_h} \left(\lambda_h H_{1L}^\perp + \frac{k_{fT} \cdot S_T}{M_h} H_{1T}^\perp \right) - n^{[\alpha} \tilde{S}_{LT}^{\beta]} H_{1LT}^\perp - \frac{n^{[\alpha} \tilde{S}_{TT}^{k_f k_f}]}{M_h} H_{1TT}^\perp - \frac{n^{[\alpha} \tilde{k}_{fT}^{\beta]}}{M_h} \left(\frac{k_{fT} \cdot S_{LT}}{M_h} H_{1LT}^\perp + \frac{S_{TT}^{k_f k_f}}{M_h^2} H_{1TT}^\perp \right) + \dots \quad (67)$$

We have omitted the arguments of z_h and k_{fT} for the TMD FFs for short notations. There are 18 TMD FFs at the leading twist, ten of them are tensor polarization dependent.

Substituting the decomposition of the parton correlators of Eqs. (61)–(67) into the hadronic tensor in Eq. (58), by carrying out the traces we can obtain the results for the hadronic tensor. The relevant traces we need are

$$\text{Tr}[\bar{n} \Gamma^\mu n \Gamma^\nu] = \text{Tr}[\gamma_5 \bar{n} \Gamma^\mu \gamma_5 n \Gamma^\nu] = -4c_1^q g_\perp^{\mu\nu} - 4ic_3^q \epsilon_\perp^{\mu\nu}, \quad (68)$$

$$\text{Tr}[\bar{n} \Gamma^\mu \gamma_5 n \Gamma^\nu] = \text{Tr}[\gamma_5 \bar{n} \Gamma^\mu n \Gamma^\nu] = 4c_3^q g_\perp^{\mu\nu} + 4ic_1^q \epsilon_\perp^{\mu\nu}, \quad (69)$$

$$\text{Tr}[i\sigma^{-\alpha} \gamma_5 \Gamma^\mu i\sigma^{+\beta} \gamma_5 \Gamma^\nu] = 4c_2^q (g_\perp^{\mu\nu} g_\perp^{\alpha\beta} - g^{\mu\alpha} g^{\nu\beta} - g^{\nu\alpha} g^{\mu\beta}), \quad (70)$$

where $c_2^q = (c_V^q)^2 - (c_A^q)^2$ and the indices α and β are both transverse. To further simplify the expressions, we define a symmetric tensor $\alpha_T^{\mu\nu}(a_T, b_T) = a_T^\mu b_T^\nu - (a_T \cdot b_T) g_\perp^{\mu\nu}$, and we also use the following notations for the combinations of FFs:

$$\mathcal{D}_1 \equiv D_1 + \frac{\tilde{k}_{fT} \cdot S_T}{M_h} D_{1T}^\perp + S_{LL} D_{1LL} + \frac{k_{fT} \cdot S_{LT}}{M_h} D_{1LT}^\perp + \frac{S_{TT}^{k_f k_f}}{M_h^2} D_{1TT}^\perp, \quad (71)$$

$$\mathcal{G}_1 \equiv \lambda_h G_{1L} + \frac{k_{fT} \cdot S_T}{M_h} G_{1T}^\perp + \frac{\tilde{k}_{fT} \cdot S_{LT}}{M_h} G_{1LT}^\perp - \frac{\tilde{S}_{TT}^{k_f k_f}}{M_h^2} G_{1TT}^\perp, \quad (72)$$

$$\mathcal{H} \equiv H_1^\perp + S_{LL} H_{1LL}^\perp + \frac{k_{fT} \cdot S_{LT}}{M_h} H_{1LT}^\perp + \frac{S_{TT}^{k_f k_f}}{M_h^2} H_{1TT}^\perp. \quad (73)$$

Using these notations, we obtain the hadronic tensor given by

$$\begin{aligned}
W_{ZZ}^{\mu\nu} = & \frac{2z_h}{x} \mathcal{C} \left\{ (c_3^q g_{\perp}^{\mu\nu} + i c_1^q \varepsilon_{\perp}^{\mu\nu}) f_1 \mathcal{G}_1 - (c_1^q g_{\perp}^{\mu\nu} + i c_3^q \varepsilon_{\perp}^{\mu\nu}) f_1 \mathcal{D}_1 \right. \\
& + 2c_2^q h_1^{\perp} \left[\frac{\alpha_T^{\mu\nu}(\tilde{k}_{iT}, S_T)}{M_N} H_{1T} \right. \\
& + \frac{\alpha_T^{\mu\nu}(\tilde{k}_{iT}, k_{fT})}{M_N M_h} \left(\lambda_h H_{1L}^{\perp} + \frac{k_{fT} \cdot S_T}{M_h} H_{1T}^{\perp} \right) \\
& + \frac{\alpha_T^{\mu\nu}(k_{iT}, S_{LT})}{M_N} H_{1LT} + \frac{\alpha_T^{\mu\nu}(k_{iT}, S_{TT}^{k_f})}{M_N M_h} H_{1TT}^{\perp} \\
& \left. \left. + \frac{\alpha_T^{\mu\nu}(k_{iT}, k_{fT})}{M_N M_h} \mathcal{H} \right] \right\}. \quad (74)
\end{aligned}$$

The convolution $\mathcal{C}[\dots]$ is defined as

$$\begin{aligned}
\mathcal{C}[wfD] = & x \int d^2 k_{iT} d^2 k_{fT} \delta^2(\vec{k}_{iT} - \vec{k}_{fT} - \vec{p}_{h\perp}/z_h) \\
& \times wf(x, k_{iT}) D(z_h, k_{fT}). \quad (75)
\end{aligned}$$

We see that the hadronic tensor is given by the convolution of the TMD PDFs and FFs. There are 18 different convolution modules, in which half of them are related to the chiral-odd TMD PDFs and FFs.

B. The cross section in the parton model

Substituting the hadronic tensor Eq. (74) into Eq. (19), and making Lorentz contractions with the leptonic tensor, we will obtain the cross section expressed by the TMD PDFs and FFs. However, we notice that the indices in the hadronic tensor are taken by the convolution variables k_{iT} and k_{fT} . In order to compare with the structure function results, we need to do a transformation to rewrite these terms using physical observables, so that the azimuthal angle modulations will be explicit. To this end, we define a unit perpendicular vector $\hat{h}^\mu = p_{h\perp}^\mu / |\vec{p}_{h\perp}|$ representing the direction of the transverse momentum of the vector meson. Under the convolution, we have, from Lorentz covariance, e.g.,

$$\mathcal{C}[k_{iT}^\mu F(k_{iT}, k_{fT}, \hat{h})] = A \hat{h}^\mu, \quad (76)$$

where $F(k_{iT}, k_{fT}, \hat{h})$ is an arbitrary scalar function of k_{iT} , k_{fT} , and \hat{h} . The coefficient A can be obtained by contraction on both sides with the unit vector \hat{h}_μ , i.e.,

$$A = -\mathcal{C}[(\hat{h} \cdot k_{iT}) F(k_{iT}, k_{fT}, \hat{h})]. \quad (77)$$

This means that, under the convolution, one can equivalently replace k_{iT}^μ with $-(\hat{h} \cdot k_{iT}) \hat{h}^\mu$. For terms with two or more indices, we give the detailed derivations and expressions in the Appendix. These algebras can also be found with a compact form in Ref. [16].

When contracting the hadronic tensor with the leptonic tensor, we need the following basic contraction results, i.e.,

$$L_{ZZ}^{\mu\nu} \cdot (c_1^q g_{\perp\mu\nu} + i c_3^q \varepsilon_{\perp\mu\nu}) = -\frac{2Q^2}{y^2} T_0^q(y), \quad (78)$$

$$L_{ZZ}^{\mu\nu} \cdot (c_3^q g_{\perp\mu\nu} + i c_1^q \varepsilon_{\perp\mu\nu}) = -\frac{2Q^2}{y^2} \tilde{T}_1^q(y), \quad (79)$$

$$L_{ZZ}^{\mu\nu} \cdot \alpha_{T\mu\nu}(a_T, b_T) = c_1^e \frac{2Q^2 E(y)}{y^2} |\vec{a}_T| |\vec{b}_T| \cos(\phi_a + \phi_b), \quad (80)$$

$$L_{ZZ}^{\mu\nu} \cdot \alpha_{T\mu\nu}(\tilde{a}_T, b_T) = -c_1^e \frac{2Q^2 E(y)}{y^2} |\vec{a}_T| |\vec{b}_T| \sin(\phi_a + \phi_b), \quad (81)$$

where we have defined

$$T_0^q(y) = c_1^e c_1^q A(y) + c_3^e c_3^q C(y), \quad (82)$$

$$\tilde{T}_1^q(y) = c_1^e c_3^q A(y) + c_3^e c_1^q C(y). \quad (83)$$

We will also define and use the following dimensionless coefficients to simplify the results:

$$\bar{w}_0 = -\frac{k_{fT}^2}{M_h^2}, \quad (84)$$

$$w'_0 = -\frac{k_{iT} \cdot k_{fT}}{M_N M_h}, \quad (85)$$

$$w_1 = -\frac{\hat{h} \cdot k_{iT}}{M_N}, \quad (86)$$

$$\bar{w}_1 = -\frac{\hat{h} \cdot k_{fT}}{M_h}, \quad (87)$$

$$w_2 = \frac{2(\hat{h} \cdot k_{iT})(\hat{h} \cdot k_{fT}) + k_{iT} \cdot k_{fT}}{M_N M_h}, \quad (88)$$

$$w'_2 = \frac{(\hat{h} \cdot k_{iT})(\hat{h} \cdot k_{fT}) + k_{iT} \cdot k_{fT}}{M_N M_h}, \quad (89)$$

$$\bar{w}_2 = \frac{2(\hat{h} \cdot k_{fT})^2 + k_{fT}^2}{M_h^2}, \quad (90)$$

$$\begin{aligned}
w_3 = & \frac{-1}{M_N M_h^2} [4(\hat{h} \cdot k_{fT})^2 (\hat{h} \cdot k_{iT}) + k_{fT}^2 (\hat{h} \cdot k_{iT}) \\
& + 2(\hat{h} \cdot k_{fT})(k_{iT} \cdot k_{fT})], \quad (91)
\end{aligned}$$

$$w_4 = \frac{1}{M_N M_h^3} \{k_{fT}^2 [k_{iT} \cdot k_{fT} + 4(\hat{h} \cdot k_{fT})(\hat{h} \cdot k_{iT})] + 4(\hat{h} \cdot k_{fT})^2 [k_{iT} \cdot k_{fT} + 2(\hat{h} \cdot k_{fT})(\hat{h} \cdot k_{iT})]\}. \quad (92)$$

We divide the cross section into two parts, according to the chirality of the TMD PDFs or FFs involved, i.e.,

$$d\sigma_{ZZ} = d\sigma_{ZZ}|_{\chi\text{-even}} + d\sigma_{ZZ}|_{\chi\text{-odd}}. \quad (93)$$

The chiral-even part is calculated as

$$\begin{aligned} \frac{d\sigma_{ZZ}|_{\chi\text{-even}}}{dx dy dz_h d\psi d^2 p_{h\perp}} &= \frac{\alpha_{\text{em}}^2 \chi}{xy Q^2} \{T_0^q(y) \mathcal{C}[f_1(D_1 + S_{LL} D_{1LL}) + |S_T| \sin(\phi - \phi_S) \bar{w}_1 f_1 D_{1T}^\perp - |S_{LT}| \cos(\phi - \phi_{LT}) \bar{w}_1 f_1 D_{1LT}^\perp \\ &\quad + |S_{TT}| \cos(2\phi - 2\phi_{TT}) \bar{w}_2 f_1 D_{1TT}^\perp] - \tilde{T}_1^q(y) \mathcal{C}[\lambda f_1 G_{1L} - |S_T| \cos(\phi - \phi_S) \bar{w}_1 f_1 G_{1T}^\perp \\ &\quad + |S_{LT}| \sin(\phi - \phi_{LT}) \bar{w}_1 f_1 G_{1LT}^\perp - |S_{TT}| \sin(2\phi - 2\phi_{TT}) \bar{w}_2 f_1 G_{1TT}^\perp]\}. \end{aligned} \quad (94)$$

The chiral-odd part is calculated as

$$\begin{aligned} \frac{d\sigma_{ZZ}|_{\chi\text{-odd}}}{dx dy dz_h d\psi d^2 p_{h\perp}} &= \frac{\alpha_{\text{em}}^2 \chi}{xy Q^2} c_1^e c_2^q E(y) \mathcal{C}\{\cos 2\phi 2w_2 h_1^\perp (H_1^\perp + S_{LL} H_{1LL}^\perp) - \lambda \sin 2\phi 2w_2 h_1^\perp H_{1L}^\perp - |S_T| \sin(\phi + \phi_S) 2w_1 h_1^\perp H_{1T}^\perp \\ &\quad + |S_T| [\bar{w}_0 w_1 \sin(\phi + \phi_S) + w_3 \sin(3\phi - \phi_S)] h_1^\perp H_{1T}^\perp + |S_{LT}| \cos(\phi + \phi_{LT}) 2w_1 h_1^\perp H_{1LT}^\perp \\ &\quad - |S_{LT}| [w_3 \cos(3\phi - \phi_{LT}) + \bar{w}_0 w_1 \cos(\phi + \phi_{LT})] h_1^\perp H_{1LT}^\perp + |S_{TT}| [w_4 \cos(4\phi - 2\phi_{TT}) \\ &\quad + \bar{w}_0 w'_0 \cos 2\phi_{TT}] h_1^\perp H_{1TT}^\perp - |S_{TT}| \cos 2\phi_{TT} 2w'_0 h_1^\perp H_{1TT}^\perp\}. \end{aligned} \quad (95)$$

It is straightforward to obtain the EM and the interference contributions by doing replacements for the electroweak coefficients, e.g., $c_2^q \rightarrow 1$ and c_V^q for the EM and the interference parts, respectively. To further unify the notations, we define $T_{0,r}^q(y)$'s and $\tilde{T}_{1,r}^q(y)$'s with $r = ZZ, \gamma Z$, and $\gamma\gamma$. For the weak interaction part, we have $T_{0,ZZ}^q(y) = T_0^q(y)$ and $\tilde{T}_{1,ZZ}^q(y) = \tilde{T}_1^q(y)$. For γZ and $\gamma\gamma$ parts, we have

$$T_{0,\gamma Z}^q(y) = c_V^e c_V^q A(y) + c_A^e c_A^q C(y), \quad (96)$$

$$\tilde{T}_{1,\gamma Z}^q(y) = c_V^e c_A^q A(y) + c_A^e c_V^q C(y), \quad (97)$$

$$T_{0,\gamma\gamma}^q(y) = A(y), \quad (98)$$

$$\tilde{T}_{1,\gamma\gamma}^q(y) = 0. \quad (99)$$

For simplicity, we will not show the total cross section explicitly. Instead, we give the explicit parton model results for the structure functions in the next section.

IV. THE STRUCTURE FUNCTIONS RESULTS AND THE SPIN ALIGNMENT

A. Parton model results of the structure functions

By comparing the cross section given by the structure functions in Eq. (47) and the parton model results in Eqs. (94) and (95), we get the structure functions results in terms of the convolution of the TMD PDFs and FFs. We include all the contributions from $\gamma\gamma$, ZZ , and γZ channels,

i.e., Eq. (8). To simplify the expressions, we define the following electroweak coefficients:

$$c_{11}^{\text{ew}} = c_1^e c_1^q \chi + c_V^e c_V^q \chi_{\text{int}} + e_q^2, \quad (100)$$

$$c_{12}^{\text{ew}} = c_1^e c_2^q \chi + c_V^e c_V^q \chi_{\text{int}} + e_q^2, \quad (101)$$

$$c_{13}^{\text{ew}} = c_1^e c_3^q \chi + c_V^e c_A^q \chi_{\text{int}}, \quad (102)$$

$$c_{31}^{\text{ew}} = c_3^e c_1^q \chi + c_A^e c_V^q \chi_{\text{int}}, \quad (103)$$

$$c_{33}^{\text{ew}} = c_3^e c_3^q \chi + c_A^e c_A^q \chi_{\text{int}}. \quad (104)$$

We give the structure functions results in the following:

$$W_U^T = c_{11}^{\text{ew}} \mathcal{C}[f_1 D_1], \quad (105)$$

$$W_U = c_{33}^{\text{ew}} \mathcal{C}[f_1 D_1], \quad (106)$$

$$W_U^{\cos 2\phi} = 2c_{12}^{\text{ew}} \mathcal{C}[w_2 h_1^\perp H_1^\perp], \quad (107)$$

$$\tilde{W}_L^T = -c_{13}^{\text{ew}} \mathcal{C}[f_1 G_{1L}], \quad (108)$$

$$\tilde{W}_L = -c_{31}^{\text{ew}} \mathcal{C}[f_1 G_{1L}], \quad (109)$$

$$W_L^{\sin 2\phi} = -2c_{12}^{\text{ew}} \mathcal{C}[w_2 h_1^\perp H_{1L}^\perp], \quad (110)$$

$$W_T^{T, \sin(\phi - \phi_S)} = c_{11}^{\text{ew}} \mathcal{C}[\bar{w}_1 f_1 D_{1T}^\perp], \quad (111)$$

$$W_T^{\sin(\phi - \phi_S)} = c_{33}^{\text{ew}} \mathcal{C}[\bar{w}_1 f_1 D_{1T}^\perp], \quad (112)$$

$$\tilde{W}_T^{T,\cos(\phi-\phi_S)} = c_{13}^{\text{ew}} \mathcal{C}[\bar{w}_1 f_1 G_{1T}^\perp], \quad (113)$$

$$\tilde{W}_T^{\cos(\phi-\phi_S)} = c_{31}^{\text{ew}} \mathcal{C}[\bar{w}_1 f_1 G_{1T}^\perp], \quad (114)$$

$$W_T^{\sin(\phi+\phi_S)} = c_{12}^{\text{ew}} \mathcal{C}[w_1 h_1^\perp (\bar{w}_0 H_{1T}^\perp - 2H_{1T})], \quad (115)$$

$$W_T^{\sin(3\phi-\phi_S)} = c_{12}^{\text{ew}} \mathcal{C}[w_3 h_1^\perp H_{1T}^\perp], \quad (116)$$

$$W_{LL}^T = c_{11}^{\text{ew}} \mathcal{C}[f_1 D_{1LL}], \quad (117)$$

$$W_{LL} = c_{33}^{\text{ew}} \mathcal{C}[f_1 D_{1LL}], \quad (118)$$

$$W_{LL}^{\cos 2\phi} = 2c_{12}^{\text{ew}} \mathcal{C}[w_2 h_1^\perp H_{1LL}^\perp], \quad (119)$$

$$W_{LT}^{T,\cos(\phi-\phi_{LT})} = -c_{11}^{\text{ew}} \mathcal{C}[\bar{w}_1 f_1 D_{1LT}^\perp], \quad (120)$$

$$W_{LT}^{\cos(\phi-\phi_{LT})} = -c_{33}^{\text{ew}} \mathcal{C}[\bar{w}_1 f_1 D_{1LT}^\perp], \quad (121)$$

$$\tilde{W}_{LT}^{T,\sin(\phi-\phi_{LT})} = -c_{13}^{\text{ew}} \mathcal{C}[\bar{w}_1 f_1 G_{1LT}^\perp], \quad (122)$$

$$\tilde{W}_{LT}^{\sin(\phi-\phi_{LT})} = -c_{31}^{\text{ew}} \mathcal{C}[\bar{w}_1 f_1 G_{1LT}^\perp], \quad (123)$$

$$W_{LT}^{\cos(\phi+\phi_{LT})} = c_{12}^{\text{ew}} \mathcal{C}[w_1 h_1^\perp (2H_{1LT} - \bar{w}_0 H_{1LT}^\perp)], \quad (124)$$

$$W_{LT}^{\cos(3\phi-\phi_{LT})} = -c_{12}^{\text{ew}} \mathcal{C}[w_3 h_1^\perp H_{1LT}^\perp], \quad (125)$$

$$W_{TT}^{T,\cos(2\phi-2\phi_{TT})} = c_{11}^{\text{ew}} \mathcal{C}[\bar{w}_2 f_1 D_{1TT}^\perp], \quad (126)$$

$$W_{TT}^{\cos(2\phi-2\phi_{TT})} = c_{33}^{\text{ew}} \mathcal{C}[\bar{w}_2 f_1 D_{1TT}^\perp], \quad (127)$$

$$\tilde{W}_{TT}^{T,\sin(2\phi-2\phi_{TT})} = c_{13}^{\text{ew}} \mathcal{C}[\bar{w}_2 f_1 G_{1TT}^\perp], \quad (128)$$

$$\tilde{W}_{TT}^{\sin(2\phi-2\phi_{TT})} = c_{31}^{\text{ew}} \mathcal{C}[\bar{w}_2 f_1 G_{1TT}^\perp], \quad (129)$$

$$W_{TT}^{\cos 2\phi_{TT}} = c_{12}^{\text{ew}} \mathcal{C}[w_0' h_1^\perp (\bar{w}_0 H_{1TT}^\perp - 2H_{1TT}')], \quad (130)$$

$$W_{TT}^{\cos(4\phi-2\phi_{TT})} = c_{12}^{\text{ew}} \mathcal{C}[w_4 h_1^\perp H_{1TT}^\perp]. \quad (131)$$

We see that there are 27 nonzero structure functions at the leading twist. Among these structure functions, 15 are related to the tensor polarizations of the vector meson. The structure functions denoted by \tilde{W} 's are related to c_{13}^{ew} and c_{31}^{ew} , which are labels of parity odd structure. If we only consider the EM interaction, 14 terms that are associated with c_{11}^{ew} and c_{12}^{ew} will survive. After reducing to EM interaction, it can be checked that the unpolarized and vector polarization dependent parts are consistent with the results given in, e.g., [25,28]. The other 13 structure functions (those related to c_{13}^{ew} , c_{31}^{ew} , and c_{33}^{ew}) will vanish. These 13 structure functions are generated by the weak

interaction and the interference between the EM and weak interactions.

B. The spin alignment of the vector meson

Compared with the hyperon production, the tensor polarizations are unique for polarized vector meson production. The tensor polarizations of the vector meson can be measured through the angular distribution of their decay products [41]. Among different components of the tensor polarizations, the spin alignment is perhaps the most interesting one that has been studied a lot. The spin alignment ρ_{00}^V is defined by the 00 component of the spin density matrix in the helicity basis. In terms of the differential cross section, it is given by [34]

$$\rho_{00}^V = \frac{d\sigma(\lambda=0)}{\sum_{\lambda=\pm 1,0} d\sigma(\lambda)}. \quad (132)$$

For the helicity $\lambda = \pm 1$ states, $S_{LL} = 1/2$, while $S_{LL} = -1$ for the $\lambda = 0$ state, and all the other polarization components are zero. Therefore, from the general form of the cross section in Eq. (47), we get

$$\rho_{00}^V = \frac{1}{3} - \frac{\mathcal{W}_{LL}}{3\mathcal{W}_U}. \quad (133)$$

Substituting the parton model results of the structure functions, we obtain

$$\begin{aligned} \rho_{00}^V = \frac{1}{3} \{ & 1 - [(c_{11}^{\text{ew}} A(y) + c_{33}^{\text{ew}} C(y)) \mathcal{C}[f_1 D_{1LL}] + 2c_{12}^{\text{ew}} E(y) \\ & \times \mathcal{C}[w_2 h_1^\perp H_{1LL}^\perp] \cos 2\phi] / [(c_{11}^{\text{ew}} A(y) + c_{33}^{\text{ew}} C(y)) \mathcal{C}[f_1 D_1] \\ & + 2c_{12}^{\text{ew}} E(y) \mathcal{C}[w_2 h_1^\perp H_1^\perp] \cos 2\phi] \}. \end{aligned} \quad (134)$$

We see that the spin alignment depends on both the chiral-even FF D_{1LL} and the chiral-odd FF H_{1LL}^\perp . It also depends on the azimuthal angle ϕ in general. However, experimentally, it is much easier to measure the ϕ integrated spin alignment $\langle \rho_{00}^V \rangle$. In this case, we have

$$\langle \rho_{00}^V \rangle = \frac{1}{3} - \frac{(c_{11}^{\text{ew}} A(y) + c_{33}^{\text{ew}} C(y)) \mathcal{C}[f_1 D_{1LL}]}{3(c_{11}^{\text{ew}} A(y) + c_{33}^{\text{ew}} C(y)) \mathcal{C}[f_1 D_1]}. \quad (135)$$

If we only consider the EM interaction, we can easily get the expression reduced from Eq. (135) given by

$$\langle \rho_{00}^V \rangle|_{\text{em}} = \frac{1}{3} - \frac{e_q^2 \mathcal{C}[f_1 D_{1LL}]}{3e_q^2 \mathcal{C}[f_1 D_1]} \quad (136)$$

that a summation over different quark flavors is implicit both in Eqs. (135) and (136). We see that $\langle \rho_{00}^V \rangle|_{\text{em}}$ is much simpler than $\langle \rho_{00}^V \rangle$ and independent of y . It is clear that the spin alignment of the vector meson is independent of the

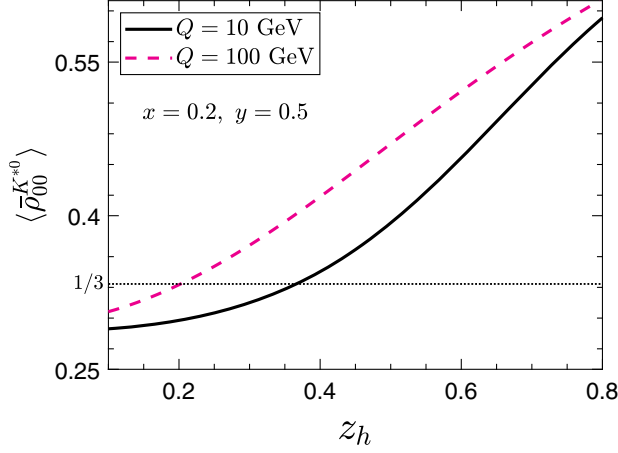


FIG. 2. A rough numerical estimate of the spin alignment for K^*0 production.

quark polarization and will deviate from $1/3$ if the D_{1LL} FF is nonzero.

We take the production of the K^*0 vector meson as an example to give a rough numerical estimate of the spin alignment. For the corresponding TMD PDFs and FFs, we consider only light flavors and take the Gaussian ansatz, i.e.,

$$f_1(x, k_{iT}) = f_{1q}(x) \frac{1}{\pi \Delta_f^2} e^{-\vec{k}_{iT}^2 / \Delta_f^2}, \quad (137)$$

$$D_1(z_h, k_{fT}) = D_{1q}^{K^*0}(z_h) \frac{1}{\pi \Delta_D^2} e^{-\vec{k}_{fT}^2 / \Delta_D^2}, \quad (138)$$

$$D_{1LL}(z_h, k_{fT}) = D_{1LLq}^{K^*0}(z_h) \frac{1}{\pi \Delta_{LL}^2} e^{-\vec{k}_{fT}^2 / \Delta_{LL}^2}. \quad (139)$$

They are factorized into a collinear distribution part and a Gaussian distribution part for the transverse momentum dependence. It should be noted that the Gaussian widths, i.e., Δ_f , Δ_D , and Δ_{LL} can depend on different flavors in principle. However, if we substitute Eqs. (137)–(139) into Eq. (135), carry out the convolution integrals and further integrate over the transverse momentum, $p_{h\perp}$, of the produced vector meson, the Gaussian widths will cancel. More explicitly, we get the $p_{h\perp}$ integrated (or averaged) spin alignment given by the collinear part of the TMD PDFs and FFs, i.e.,

$$\langle \bar{\rho}_{00}^{K^*0} \rangle = \frac{1}{3} - \frac{[c_{11}^{\text{ew}} A(y) + c_{33}^{\text{ew}} C(y)] f_{1q}(x) D_{1LLq}^{K^*0}(z_h)}{3[c_{11}^{\text{ew}} A(y) + c_{33}^{\text{ew}} C(y)] f_{1q}(x) D_{1q}^{K^*0}(z_h)}, \quad (140)$$

where a summation over quark flavors is also implicit in the numerator and the denominator of Eq. (140).

We choose $x = 0.2$ and $y = 0.5$ as a typical value and show the rough numerical estimate of the spin alignment as a function of z_h in Fig. 2 for $Q = 10$ GeV and 100 GeV, respectively. In the numerical calculation, we have taken

the CT14 next-to-leading order PDFs [71] for $f_{1q}(x)$. For the FFs $D_{1q}^{K^*0}(z_h)$ and $D_{1LLq}^{K^*0}(z_h)$, we use the parametrization results given in [72]. The factorization scales for the PDFs and FFs are set to $\mu_f = Q$. It is clear to see that the spin alignment is deviated from $1/3$ at both low and high Q values. We also note that the spin alignment increases monotonically with z_h . These properties may be checked in the SIDIS experiments such as JLab or EIC in the future.

V. SUMMARY

Semi-inclusive deep inelastic scattering is an important process for accessing the three-dimensional partonic structure of the nucleon and the hadronization mechanism. We present a systematic calculation for $e^-N \rightarrow e^-VX$ with unpolarized electron and nucleon beams and polarized vector meson production at high energies. We give a full kinematic analysis for this process by considering both the electromagnetic and the weak interactions that introduce the parity-violating effects. The results show that the cross sections are expressed by 81 structure functions. Among all the structure functions, 39 correspond to the space reflection odd part of the cross section and 42 correspond to the space reflection even part. We also carry out a parton model calculation for the process and show that there are 27 nonzero structure functions at the leading twist, in which 15 are related to the tensor polarizations of the vector meson and 13 are generated by the parity-violating effects. The structure functions are given by the convolution of the corresponding TMD PDFs and FFs. We also present the result for the spin alignment of the vector meson. A rough numerical estimate is made, as an example, for the K^*0 spin alignment. It gives us ways for accessing the tensor polarization dependent FFs by measuring the polarization of the vector meson. Future experimental studies such as the EIC will provide us with better opportunities to study the nucleon structure and the hadronization mechanism as well as the polarization effects in detail.

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APPENDIX: TRANSFORMATIONS FOR THE CONVOLUTION

For terms with two indices, according to Lorentz covariance, we have

$$\mathcal{C}[k_{iT}^\mu k_{fT}^\nu F(k_{iT}, k_{fT}, \hat{h})] = B_1 \hat{h}^\mu \hat{h}^\nu + B_2 g_{\perp}^{\mu\nu}. \quad (\text{A1})$$

The coefficients B_1 and B_2 can be obtained by contracting with $\hat{h}_\mu \hat{h}_\nu$ and $g_{\perp\mu\nu}$ on both sides, i.e.,

$$\mathcal{C}[(\hat{h} \cdot k_{iT})(\hat{h} \cdot k_{fT})F(k_{iT}, k_{fT}, \hat{h})] = B_1 - B_2, \quad (\text{A2})$$

$$\mathcal{C}[(k_{iT} \cdot k_{fT})F(k_{iT}, k_{fT}, \hat{h})] = -B_1 + 2B_2. \quad (\text{A3})$$

One can solve

$$B_1 = \mathcal{C}\{[2(\hat{h} \cdot k_{iT})(\hat{h} \cdot k_{fT}) + k_{iT} \cdot k_{fT}]F(k_{iT}, k_{fT}, \hat{h})\}, \quad (\text{A4})$$

$$B_2 = \mathcal{C}\{[(\hat{h} \cdot k_{iT})(\hat{h} \cdot k_{fT}) + k_{iT} \cdot k_{fT}]F(k_{iT}, k_{fT}, \hat{h})\}. \quad (\text{A5})$$

For terms with three indices, we only need to consider the form of $\mathcal{C}[k_{iT}^\rho k_{fT}^\mu k_{fT}^\nu F(k_{iT}, k_{fT}, \hat{h})]$ in the calculation. Note that the $\mu\nu$ indices are symmetric in this case and can be expressed by three bases. We have

$$\mathcal{C}[k_{iT}^\rho k_{fT}^\mu k_{fT}^\nu F(k_{iT}, k_{fT}, \hat{h})] = C_1 \hat{h}^\rho \hat{h}^\mu \hat{h}^\nu + C_2 \hat{h}^\rho g_\perp^{\mu\nu} + C_3 g_\perp^{\rho\{\mu} \hat{h}^{\nu\}}. \quad (\text{A6})$$

By contractions on both side with the three bases, one can also solve

$$C_1 = \mathcal{C}\{[-4(\hat{h} \cdot k_{fT})^2(\hat{h} \cdot k_{iT}) - 2(\hat{h} \cdot k_{fT})(k_{iT} \cdot k_{fT}) - k_{fT}^2(\hat{h} \cdot k_{iT})]F(k_{iT}, k_{fT}, \hat{h})\}, \quad (\text{A7})$$

$$C_2 = \mathcal{C}\{[-(\hat{h} \cdot k_{fT})^2(\hat{h} \cdot k_{iT}) - k_{fT}^2(\hat{h} \cdot k_{iT})]F(k_{iT}, k_{fT}, \hat{h})\}, \quad (\text{A8})$$

$$C_3 = \mathcal{C}\{[-(\hat{h} \cdot k_{fT})^2(\hat{h} \cdot k_{iT}) - (\hat{h} \cdot k_{fT})(k_{iT} \cdot k_{fT})]F(k_{iT}, k_{fT}, \hat{h})\}. \quad (\text{A9})$$

For terms with four k_T 's, we need to consider the convolution with the form of $\mathcal{C}[k_{iT}^{\{\mu} k_{fT}^{\nu\}} k_{fT}^\rho k_{fT}^\sigma F(k_{iT}, k_{fT}, \hat{h})]$. In this case, the $\mu\nu$ and $\rho\sigma$ indices are symmetric, respectively. Thus, we have

$$\mathcal{C}[k_{iT}^{\{\mu} k_{fT}^{\nu\}} k_{fT}^\rho k_{fT}^\sigma F(k_{iT}, k_{fT}, \hat{h})] = D_1 \hat{h}^\mu \hat{h}^\nu \hat{h}^\rho \hat{h}^\sigma + D_2 \hat{h}^\mu \hat{h}^\nu g_\perp^{\rho\sigma} + D_3 \hat{h}^\rho \hat{h}^\sigma g_\perp^{\mu\nu} + D_4 \frac{1}{2} (\hat{h}^\rho \hat{h}^{\{\mu} g_\perp^{\nu\}\sigma} + \hat{h}^\sigma \hat{h}^{\{\mu} g_\perp^{\nu\}\rho}) + D_5 g_\perp^{\mu\nu} g_\perp^{\rho\sigma}. \quad (\text{A10})$$

Also by contractions on both sides with the five bases, one can solve and get

$$D_1 = \mathcal{C}\{[2k_{fT}^2[k_{iT} \cdot k_{fT} + 4(\hat{h} \cdot k_{fT})(k_{iT} \cdot k_{fT})] + 8(\hat{h} \cdot k_{fT})^2[k_{iT} \cdot k_{fT} + 2(\hat{h} \cdot k_{fT})(\hat{h} \cdot k_{iT})]F(k_{iT}, k_{fT}, \hat{h})\}, \quad (\text{A11})$$

$$D_2 = 2\mathcal{C}\{[k_{fT}^2 + (\hat{h} \cdot k_{fT})^2][k_{iT} \cdot k_{fT} + 2(\hat{h} \cdot k_{fT})(\hat{h} \cdot k_{iT})]F(k_{iT}, k_{fT}, \hat{h})\}, \quad (\text{A12})$$

$$D_3 = 2\mathcal{C}\{[k_{fT}^2 + 2(\hat{h} \cdot k_{fT})^2][k_{iT} \cdot k_{fT} + (\hat{h} \cdot k_{fT})(\hat{h} \cdot k_{iT})]F(k_{iT}, k_{fT}, \hat{h})\}, \quad (\text{A13})$$

$$D_4 = 2\mathcal{C}\{(\hat{h} \cdot k_{fT})[(k_{iT} \cdot k_{fT})(\hat{h} \cdot k_{fT}) + (\hat{h} \cdot k_{iT})(k_{fT}^2 + 2(\hat{h} \cdot k_{fT})^2)]F(k_{iT}, k_{fT}, \hat{h})\}, \quad (\text{A14})$$

$$D_5 = 2\mathcal{C}\{[k_{fT}^2 + (\hat{h} \cdot k_{fT})^2][k_{iT} \cdot k_{fT} + (\hat{h} \cdot k_{fT})(\hat{h} \cdot k_{iT})]F(k_{iT}, k_{fT}, \hat{h})\}. \quad (\text{A15})$$

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