Constituent quark axial current couplings to light vector mesons in the vacuum and with a weak magnetic field

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Unusual constituent quark axial current couplings to light vector mesons, ρ and ω, are derived in the vacuum and under weak magnetic field by considering a quark-antiquark interaction mediated by a nonperturbative gluon exchange. Similarly, light axial mesons are found to couple anomalously with the constituent quark vector current. These interactions are of the type of the Wess-Zumino-Witten terms, being strongly anisotropic and dependent on the vector (or axial) meson polarization. They also provide axial (vector) form factors for the vector (axial) mesons and are quite small, suppressed nearly by $1/M^{*2}$ with respect to the vector mesons minimal coupling to the quark vector current. Some three-leg meson vertices are also presented: $\pi - \rho - A_1$ and $V_1 V_2 A$ (where V_1, V_2 are vector mesons and A an axial meson). A vector and axial-vector mesons mixing is identified at nonzero magnetic field which however can contribute only in the presence of a third particle or in a medium. Numerical results are presented for different effective gluon propagators.

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I. INTRODUCTION

Pions, neutrinos and eventually the light axial mesons can be considered to probe the axial content of the nucleon, or their corresponding constituent quarks, and of other hadrons. The nucleon's axial form factor provides a measure of its spin content and the interplay of strong and weak charges and interactions. It has been extensively investigated theoretically and experimentally, for few works [1–12]. The decomposition of the nucleon, or correspondingly constituent quark, axial vertex involves eight tensor structures [10,11]. The physical processes and particles that can probe them can be attached to each of these structures. Meson form factors, however, are far more difficult to be measured. Some calculations for spacelike or timelike rho-meson form factors can be found, for example, in [13–16]. At the same time, the understanding of hadron dynamics depends on different types of interactions. For instance, it becomes important to understand how these mesons not only couple to the nucleon but also how interactions among mesons can manifest in experimental situations. Besides that, the way these couplings might be related to each other might be of relevance for establishing connections to QCD, eventually as a manifestation of fundamental symmetries. It is expected that eventual breakdowns of fundamental symmetries should be observed with higher precision experiments.

Low energies (Strong) couplings of the constituent quark (or nucleon) axial current involve either the pion or the light axial mesons. Among these axial mesons the A_1 and the f_1 are usually considered to be chiral partners to the ρ and ω [17,18]. There are however difficulties to determine axial meson properties, in particular because they are highly unstable. Therefore there are uncertainties about their structures [19,20]. In Ref. [20] several couplings and decays of the A_1 were analyzed with possible mixing angles. To understand further these mesons it becomes important to take into account how they interact with other hadrons before decaying. Light axial meson couplings and decays, including its decay to the pion and a vector meson, have also been found to be relevant to describe the τ-decay by the axial current [21–24]. It has also been considered for the μ $(g-2)$ problem [25] wherein the vector-vector-axial (VVA) correlators are important. Three-leg meson vertices, many times representing decays, can also be searched experimentally and this is planned in different facilities as for example at FAIR/GSI [26]. Eventually, associated information from finite energy density medium should be compatible with their dynamics in the vacuum. As an example, the $\rho - \omega - A_1$ and $\rho - \pi - A_1$ couplings give rise to the in medium mixing $\rho - A_1$ [23,27–29]. Eventually this mixing contributes for the dilepton spectrum in hot and/or dense matter [27,30]. The role of vector and axial mesons at finite energy density up to the chiral phase

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transition has been investigated in different approaches [31,32]. Besides that, further meson interactions might lead to modifications in the nuclear potential [33]. From the strict theoretical point of view, three-leg couplings, such as the $\rho - \pi - A_1$ or $\rho - \omega - \pi$, are suppressed in the $1/N_c$ counting. Nevertheless, they have been found to arise in different approaches such as in the massive Yang Mills gauge theory, or even as Wess-Zumino-Witten (WZW) terms [34,35], in [23,36–40]. Facilities currently working and being built are expected to probe several aspects of vector and axial mesons and their interactions. High density investigations on the restoration of chiral symmetry and meson spectroscopy are planned in FAIR/GSI, NICA and J-PARC. Vector meson photoproduction and production mostly in ultraperipheral collisions have been investigated in HERMES, JLAB, FAIR/GSI and SLAC and currently in LHC besides plans for LHC and EIC. Polarized vector mesons are currently produced and investigated in JLAB for example in [41,42]. Besides that axial meson production in central collisions has also been envisaged [43].

The constituent quark model (CQM) remains one of the leading pictures for global properties of hadron structure around which other contributions can be evaluated, for example in [44–46]. Several nucleon properties are described directly, and mostly, in terms of constituent quarks that carry masses and charges of hadrons and they are directly responsible for the nucleon's couplings to mesons and other particles. This association might somewhat manifest in baryon form factors. A more precise relation between these two levels of description can be helpful to the detailed understanding of the hadron structure and dynamics. The lack of understanding of the confinement mechanism does not prevent a good theoretical description of hadron properties and interactions. It becomes then interesting to understand further predictions of these CQM. Global color (GCM) [47,48] and Nambu–Jona-Lasinio (NJL) [49,50] type models go along the CQM description in terms of constituent quarks. They have been shown to be suitable for the description of many aspects of light meson spectrum and properties. In these models, dynamical chiral symmetry breakdown (DChSB) leads to the formation of chiral condensates, directly associated as sea quark states [51]. DChSB endows quarks with a large mass providing an important link between fundamental symmetries at the QCD and the hadron levels. In the CQM, the pion, as a Goldstone boson, has the axial coupling to (constituent) quarks whose coupling constant g_A is usually given by $g_A = 3/4$, $g_A = 5/3$ or $g_A = 1$ [50,52,53]. Light meson couplings to constituent quarks and form factors have been derived dynamically and analytically [16,54] along with the same spirit of the CQM. By starting with a leading term of QCD quark-effective action, based in one nonperturbative gluon exchange (GCM), standard techniques have been applied and they will be considered in the present work. Background quarks, dressed by a sort of gluon cloud, give rise to constituent quarks. Meson interactions with constituent quarks arise and provide, for example, a pion cloud. This is seen explicitly in the Weinberg's large N_c effective field theory (EFT), makes possible the constituent quark picture to cope with the large N_c expansion [53]. This EFT has also been derived with symmetry breaking corrections with the same analytical techniques considered in the present work in the vacuum and under weak magnetic fields [55,56]. Furthermore, this approach might lead to a straightforward extension to finite baryon densities or finite temperatures. Moreover, the approach considered directly provides relative strength (ratios) of different coupling constants or form factors. These ratios lead to reasonable estimates of their relative importance. This is usually important also for planning experiments or for the interpretation of results.

Besides the interest in calculating hadron couplings in the vacuum, strong magnetic fields have been estimated to appear in noncentral relativistic heavy ion collisions and magnetars and they can produce many different effects [57,58]. Although these magnetic fields can be as large as 10^{15} T, they are not so large if compared to a hadron mass scale such as an effective (constituent) quark mass—or even the pion mass. One has typically $(eB_0) \sim 10^{-2}M^{*2}$–$10M^{*2}$, where M^* is a quark effective mass typical from the CQM. It becomes interesting to identify changes in the quark and hadron dynamics due to the external magnetic fields. Under (relatively) weak magnetic fields, pions, vector and axial mesons develop additional couplings to (constituent) quark currents [59,60]. These couplings may have reduced strengths as compared to the couplings to quark vector and axial currents. At energies in which magnetic fields can be expected to show up in noncentral heavy ion collisions, vector and axial mesons can also be more copiously produced. Therefore the investigation of the effects of magnetic fields in the meson dynamics and couplings become of further interest.

In the present work, light vector meson (rho and omega) anomalous couplings to the constituent quark axial current will be derived in the vacuum and under a constant weak magnetic field by considering the same framework used in [16,54,59–61]. These anomalous couplings can be considered as corrections to the rho/omega (A_1/f_1) form factors that correspond to a very small axial (vector) component. As such, this small axial component should be at the origin of the rho-A_1 mixing expected to occur in the medium. The same can be expected for a $\omega - f_1$ mixing. These couplings can be seen as Wess-Zumino-Witten type terms. These coupling functions are considerably smaller than the known rho vector meson coupling to the nucleon (constituent quark) vector current. Besides that, anomalous three-leg meson couplings that might correspond or contribute for vector and/or axial mesons mixings will also be presented.

The work is organized as follows. In the next section, the main steps of the approach are briefly reminded and the quark determinant will be exhibited. A large quark effective mass expansion is performed and the (next leading order) *anomalous* terms will be presented. The next leading terms, and form factors, are presented in Secs. II and III. These couplings are shortly compared to leading couplings of the pion, light vector and axial mesons previously derived. The action term for the vector meson coupling to the axial current, as a WZW type term, is shown to be topologically conserved. Numerical results are presented in Sec. IV for the low momentum regime, $Q^2 < 1\ \mathrm{GeV}^2$, by considering three different effective gluon propagators. On-shell values for some of the coupling constants are provided and estimations for the axial averaged quadratic radius (AQR) of the vector mesons are calculated. In the last section there is a summary and final remarks.

II. QUARK DETERMINANT AND NEXT LEADING ANOMALOUS COUPLINGS

A QCD-based model for quark-antiquark interaction mediated by a gluon is the GCM, which takes into account nonperturbative/non-Abelian gluon effects by means of an effective gluon propagator. It is given by

$$Z[\eta,\bar\eta] = N\int \mathcal{D}[\bar\psi,\psi]\exp i\int_x\left[\bar\psi(i\not{D}-m)\psi - \frac{g^2}{2}\int_y j^\beta_\mu(x)\tilde{R}^{\mu\nu}_{\beta\alpha}(x-y)j^\alpha_\nu(y)+\bar\psi\eta+\bar\eta\psi\right], \tag{1}$$

where the color quark current is $j^\mu_\alpha = \bar\psi\lambda_\alpha\gamma^\mu\psi$, $\int_x$ stands for $\int d^4x$, $i,j,k = 0,\ldots(N_f^2-1)$ will be used for, $SU(N_f=2)$, isospin indices and $\alpha,\beta\ldots = 1,\ldots(N_c^2-1)$ stands for color in the adjoint representation. The sums in color, flavor and Dirac indices are implicit and $\eta,\bar\eta$ are the quark sources. $D_\mu = \partial_\mu - ieQA_\mu$ is the covariant quark derivative with the minimal coupling to a background electromagnetic field, with the diagonal matrix $\hat{Q} = \mathrm{diag}(2/3,-1/3)$ for up and down electromagnetic charges. To account for the non-Abelian structure of the gluon sector the gluon propagator $\tilde{R}^{\mu\nu}_{\alpha\beta}(x-y)$ must be nonperturbative. As an external input for the model, it will be required to have enough strength to yield dynamical chiral symmetry breaking (DChSB) with a given strength of the (running) quark-gluon coupling constant. DChSB has been found in several works with different approaches, for a few examples [62–64]. It is one of the mechanisms that endows hadrons with large masses with respect to the (measured) quark masses [65]. Other terms from QCD effective action, such as genuine three- and four-quark interactions, due the non-Abelian gluon structure, are not considered. In several gauges the gluon propagator $\tilde{R}^{\mu\nu}_{\alpha\beta}(k)$ can be written in momentum space as $\tilde{R}^{\mu\nu}_{\alpha\beta}(k) = \delta_{\alpha\beta}[(g^{\mu\nu}-\frac{k^\mu k^\nu}{k^2})R_T(k) + \frac{k^\mu k^\nu}{k^2}R_L(k)]$, where $R_T(k), R_L(k)$ are transversal and longitudinal components. Contributions from extra terms due to confinement and gauge boson dynamics proportional to $\delta(p^2)$ [66] can be shown to be smaller or even vanishing [67] for the observables calculated below.

By means of a Fierz transformation, Dirac and isospin structures can be suitably introduced such as to provide the correct quantum numbers for leading low energies quark-antiquark states corresponding to mesons. The background field method and the auxiliary field method are then used to calculate the quark determinant in the presence of DChSB by considering the auxiliary fields associated with quark-antiquark meson states. Besides that, background quark currents give rise to constituent quark currents. In particular, the auxiliary field method (AFM) was employed explicitly in Refs. [16,54,55,61] for bilocal auxiliary fields. These auxiliary fields can be expanded in a complete basis of structureless meson fields. Field renormalization constants can be introduced explicitly with which the unit integral of the AFM can be written as

$$\begin{aligned}1 = N\int D[V^i_\mu,\bar{A}^i_\mu,V_\mu,\bar{A}_\mu]&e^{-\frac{i\alpha}{4}\int_{x,y}(\bar{R}^{\mu\nu})^{-1}((Z_V^{\frac12}V^i_\mu - gZ_gZ_\psi\bar{R}_{\mu\nu}j^{V,\nu^i})(Z_V^{\frac12}V^\mu_i - gZ_gZ_\psi\bar{R}_{\mu\nu}j_{V,\nu}{}^i)}e^{-\frac{i\alpha}{4}\int_{x,y}(\bar{R}^{\mu\nu})^{-1}(Z_A^{\frac12}\bar{A}^i_\mu - gZ_gZ_\psi\bar{R}_{\mu\nu}j^{A,\nu^i})(Z_A^{\frac12}\bar{A}^\mu_i - gZ_gZ_\psi\bar{R}_{\mu\nu}j_{A,\nu}{}^i)}\\
&\times e^{-\frac{i\alpha}{4}\int_{x,y}(\bar{R}^{\mu\nu})^{-1}((Z_V^{\frac12}V_\mu - gZ_gZ_\psi\bar{R}_{\mu\nu}j^{V,\nu})(Z_V^{\frac12}V^\mu - gZ_gZ_\psi\bar{R}_{\mu\nu}j_{V,\nu})}e^{-\frac{i\alpha}{4}\int_{x,y}(\bar{R}^{\mu\nu})^{-1}(Z_A^{\frac12}\bar{A}_\mu - gZ_gZ_\psi\bar{R}_{\mu\nu}j^{A,\nu})(Z_A^{\frac12}\bar{A}^\mu - gZ_gZ_\psi\bar{R}_{\mu\nu}j_{A,\nu})}\\
&\times\int D[S_i]D[P_i]e^{-\frac{i}{2}\int_{x,y}R(x-y)\alpha[(Z_S^{\frac12}S - gZ_gZ_\psi j^S_{(2)})^2 + (Z_P^{\frac12}P_i - gZ_gZ_\psi j^P_{i,(2)})^2]},\end{aligned} \tag{2}$$

where

$$\bar{R}^{\mu\nu} \equiv \bar{R}^{\mu\nu}(x-y) = g^{\mu\nu}(R_T(x-y)+R_L(x-y)) + 2\frac{\partial^\mu\partial^\nu}{\partial^2}(R_T(x-y)-R_L(x-y)), \tag{3}$$

$$R \equiv R(x-y) = 3R_T(z-y) + R_L(x-y). \tag{4}$$

The scalar and pseudoscalar fields give rise to a nonlinear representation in terms of the (Goldstone boson) pion field by a usual chiral rotation. The renormalization constants will not be carried on in the calculations below, being therefore omitted. However, they allow for a systematic elimination of the ultraviolet divergences in the resulting couplings of the effective action.

The Gaussian integration of the quark field can now be performed in the presence of background quark currents and meson fields that are arranged in a chiral invariant way. By making use of the identity $\det A = \exp \mathrm{Tr} \ln(A)$, it yields

$$S_{eff} = -i \,\mathrm{Tr}\, \ln \left\{ i \left[S_0{}^{-1}(x-y) + \Xi_v(x-y) + \Xi_s(x-y) + \sum_q a_q \Gamma_q j_q(x,y) \right] \right\}, \tag{5}$$

where Tr stands for traces of all discrete internal indices and integration of space-time coordinates. The quantity $\Xi_v(x-y)$ encodes the vector and axial meson contributions and $\Xi_s(x-y)$ encodes the pion field contribution that arises from the scalar and pseudoscalar auxiliary fields. In the last term of Eq. (5) there is a sum of background quark currents with Dirac, flavor and color structures among which only the (isosinglet and isotriplet) axial currents will be kept:

$$\sum_q a_q \Gamma_q j_q(x,y) \to -\alpha g^2 \bar{R}^{\mu\nu}(x-y)\gamma_\mu[\sigma_i \gamma_5 \bar{\psi}(y)\gamma_5\gamma_\nu \sigma_i \psi(x) + \gamma_5 \bar{\psi}(y)\gamma_5\gamma_\nu \psi(x)], \tag{6}$$

where σ_i are the Pauli matrices, $\bar{R}^{\mu\nu} = 2[3R_T(x-y) + R_L(x-y)]$ and $\alpha = 4/9$. The remaining terms can be written as

$$\Xi_v(x-y) = -\frac{\gamma^\mu}{2}[F_v \sigma_i (V^i_\mu(x) + \gamma_5 \bar{A}^i_\mu(x) + V_\mu(x) + \gamma_5 \bar{A}_\mu(x))]\delta(x-y), \tag{7}$$

$$\Xi_s(x-y) = F(P_R U + P_L U^\dagger)\delta(x-y), \tag{8}$$

where $U = e^{i\vec{\pi}\cdot\tau/2}$, $P_{R/L} = (1 \pm \gamma_5)/2$ are the chirality right-/left-hand projectors, F_v and F provide the canonical field definition meson fields, for both isotriplet (rho and A_1) and isosinglet (ω and f_1) [18,68], and for the pion. F_v will be incorporated into the vector and axial meson fields to provide their canonical dimensions. The other quark-antiquark mesons, their auxiliary fields, will be neglected. The gap equations for the auxiliary fields are derived as saddle point equations and solved as is usually done for the model (1) and NJL-type models. The scalar field equation will be the only one with a nontrivial nonzero solution and the resulting quark-antiquark scalar condensate ($\bar{S}$) becomes responsible for the increased quark mass $M^* = m + \bar{S}$. The quark propagator can then be written as $S_0^{-1}(x-y) = (i\not{D} - M^*)\delta(x-y)$.

A large quark mass expansion [69] with a zero order derivative expansion [70] of the determinant (5) will be implemented now. It is suitable for the long wavelength limit of the model and they have been performed in previous works of the author mentioned above. The leading terms of the expansion with vector/axial currents and the light mesons have been collected and investigated previously by the author in the vacuum and under weak magnetic fields. Next leading couplings of (isotriplet ρ and isosinglet ω) light vector mesons to constituent quark axial current also appear as nonlocal interactions. With explicit momentum dependencies they can be written as

$$\begin{aligned}\mathcal{L}_{\mathrm{vja}} &= i\delta_{ij}\epsilon^{\sigma\rho\mu\nu} F^{\mathrm{vja}}(K,Q) K_\sigma \mathcal{F}^i_{\rho\mu}(Q) j^{A,j}_\nu(K, K+Q) \\ &\quad + i\epsilon^{\sigma\rho\mu\nu} F^{\mathrm{vja}}(K,Q) K_\sigma \mathcal{F}_{\rho\mu}(Q) j^A_\nu(K, K+Q),\end{aligned} \tag{9}$$

where Q is the vector meson 4-momentum, K is the incoming quark momentum. The isotriplet and isosinglet axial currents were defined as $j^{A,i}_\mu(K, K+Q) = \bar{\psi}(K+Q)\gamma_\mu\gamma_5\sigma^i\psi(K)$ and $j^A_\mu(K, K+Q) = \bar{\psi}(K+Q)\gamma_\mu\gamma_5\psi(K)$. The (Abelian limit of the) stress tensors were defined for the isotriplet and isosinglet states:

$$\begin{aligned}\mathcal{F}^i_{\rho\mu}(Q) &= Q_\rho V^i_\mu(Q) - Q_\mu V^i_\rho(Q), \\ \mathcal{F}_{\rho\mu}(Q) &= Q_\rho V_\mu(Q) - Q_\mu V_\rho(Q).\end{aligned} \tag{10}$$

The non-Abelian contribution was neglected although it can be incorporated and it leads to three- and four-vector or axial meson-quark vertices.

The above form factor is given by

$$\begin{aligned}F_{\mathrm{vja}}(K,Q) &= 4d_2 N_c(\alpha g^2)\int_k ((\tilde{S}_0(k+K)\tilde{S}_0(k+K+Q)\bar{\bar{R}}(-k))),\end{aligned} \tag{11}$$

where $\int_k = \int \frac{d^4k}{(2\pi)^4}$ in the Euclidean momentum space, $\bar{\bar{R}}(-k) = 2R(-k)$, $d_n = \frac{(-1)^{n+1}}{2n}$. The double parentheses was used for denoting the order of the original kernels and it implicitly contains the momentum structure resulting from the trace in Dirac indices. This integral, for infrared (IR) regular gluon propagators, is finite. The following function was defined: $\tilde{S}_0(k) = \frac{1}{k^2+M^{*2}}$. These interactions (9) emerge due to the following *anomalous* trace of Dirac matrices:

$$\mathrm{Tr}_D(\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\gamma_5) = 4i\epsilon_{\mu\nu\rho\sigma}, \tag{12}$$

where $\epsilon^{\rho\mu\sigma\nu}$ is the Levi-Civita tensor. Along this work, the momentum dependence of the form factors is written as $F(K,Q)$ that is a shorthanded notation for $F(K^2,Q^2,K\cdot Q)$. The momentum dependent form factors in Eq. (9) have dimensions of mass^{-2}. Both vector mesons, rho and omega, couple to the corresponding axial current with the same strength $F_{\mathrm{vja}}(K,Q)$.

The above interactions (9) have chiral counterparts of the light axial mesons A_1 and f_1, denoted by $\bar{A}^i_\mu$ and $\bar{A}_\mu$ respectively, interacting with the vector quark currents. These couplings are given by

$$\begin{aligned}\mathcal{L}_{\mathrm{vja}-A} &= i\epsilon^{\sigma\rho\mu\nu}F^{vja}(K,Q)K_\sigma\mathcal{G}^i_{\rho\mu}(Q)j^{V,i}_\nu(K,K+Q)\\ &\quad + i\epsilon^{\sigma\rho\mu\nu}F^{vja}(K,Q)K_\sigma\mathcal{G}_{\rho\mu}(Q)j^V_\nu(K,K+Q),\end{aligned} \tag{13}$$

where $j^{V,i}_\mu(K,K+Q)=\bar{\psi}(K+Q)\gamma_\mu\sigma^i\psi(K)$ and $j^V_\mu(K,K+Q)=\bar{\psi}(K+Q)\gamma_\mu\psi(K)$. The (isotriplet and isosinglet) axial mesons (Abelian) stress tensors in coordinate space are the following:

$$\mathcal{G}^i_{\mu\nu} = \partial_\mu\bar{A}^i_\nu - \partial_\nu\bar{A}^i_\mu, \qquad \mathcal{G}_{\mu\nu} = \partial_\mu\bar{A}_\nu - \partial_\nu\bar{A}_\mu. \tag{14}$$

Non-Abelian terms can be incorporated [37,71].

The above anomalous interactions (9) and (13) are strongly anisotropic with respect to the quark and vector meson momenta, and the vector meson polarization. They arise in the same way as Wess-Zumino-Witten–type terms that were also obtained for example in the bosonized version of the NJL model, sigma models and other approaches [36,39,40,47]. The antisymmetric tensor $\epsilon_{\sigma\rho\mu\nu}$ obviously prevents the contributions of many components of quark and vector meson momenta. Moreover, it makes important to select a particular vector (or axial) meson polarization. As a whole, the vertices $Q_\mu K_\nu F_{\mathrm{vja}}(K,Q)$ probe an anisotropy of the spatial variation of the axial (vector) current and charge distribution in Eq. (9) [Eq. (13)]. These couplings correspond to a small anomalous axial (vector) component of the vector (axial) meson structure. As such, it may be responsible for vector-axial meson mixings. This type of mixing could only occur in the presence of a third particle or in a medium to conserve momenta.

A. Comparison with other couplings of the axial current

The leading light mesons [pion, omega, rho and axial $A_1(1260)$] interactions with the axial isotriplet quark current had been obtained by the same method used above [16,54,55]. By adding to the anomalous coupling above, they can be written as

$$\begin{aligned}\mathcal{L}_{j_A} &= [G_A(Q,K)Q_\mu\pi^i(Q) + G_{\bar{A}}(Q,K)\bar{A}^i_\mu(Q)\\ &\quad + iF_{\mathrm{vja}}(Q,K)\epsilon_{\mu\nu\rho\sigma}K^\nu Q^\rho V^\sigma_i(Q)]j^\mu_{A,i}(K,Q),\end{aligned} \tag{15}$$

where a dimensionless pion field was written in the first term. The analogous terms for the isosinglet axial current were omitted. Other couplings with different—higher order—momentum dependence might arise being outside of the scope of this work. The axial meson coupling to the axial current has been considered recently in different approaches [72]. The following form factors were used in Eq. (15):

$$G_A(K,Q) = G_V(K,Q) = 4FC_{A,V}\int_k((M^*\tilde{S}_0(k+K)\tilde{S}_0(k+Q+K)\bar{\bar{R}}(-k))), \tag{16}$$

$$G_{\bar{A}}(K,Q) = C_{A,V}\int_k \frac{(k+K)\cdot(k+K+Q)+M^{*2}}{((k+K)^2+M^{*2})((k+K+Q)^2+M^{*2})}\bar{\bar{R}}(-k), \tag{17}$$

where $C_{A,V} = 4N_c d_2(\alpha g^2)$ and F is the pion decay constant. The form factor $F_{\mathrm{vja}}(K,Q)$, Eq. (11), is suppressed by $\sim 1/M^*$ or $\sim 1/M^{*2}$ with respect to G_A and $G_{\bar{A}}$ and also with respect to the rho or omega coupling to the quark vector current. Besides that, it has a tensor structure dependent on KQ and it presents an intrinsic anisotropy in momentum space that contributes to make him smaller. Note that $G_{\bar{A}}(K,Q)$ (as well as the vector meson coupling) is dimensionless for the canonical normalization of axial-vector fields.

It is also interesting to note that, for constant quark effective masses M^*, the pion axial coupling, $G_A(K,Q)$, is directly proportional to the anomalous coupling constant or form factor $F_{\mathrm{vja}}(K,Q)$. They can be related by

$$\frac{F_{\mathrm{vja}}(K,Q)}{G_A(K,Q)} = \frac{1}{4M^*F}. \tag{18}$$

Therefore the anomalous coupling constant $F_{\rm vja}$ can be considered to be suppressed with respect to $G_A \sim 1$.

B. Some three-leg meson couplings

The meson sector of the quark determinant (5) has been investigated extensively by many groups, for example in [37,39]. Next, the leading coupling terms that involve a vector meson and an axial meson are exhibited. Although not all of them are nonzero in the vacuum they are all displayed and their possible contributions discussed below. The leading vector-axial mesons couplings with a pion can be written as

$$\begin{aligned}\mathcal{L}_{\rm mix} = {} & G_{v-a-\pi}(K,Q)(i\epsilon_{ijk}\pi_i(Q+K)V^j_\mu(Q)A^\mu_k(K) + \pi_i(Q+K)V^i_\mu(Q)A^\mu(K)) \\ & + G_{v-a-\pi}(K,Q)(\pi_i(Q+K)V_\mu(Q)A^\mu_i(K)) \\ & + G_{kv-a-\pi}(K,Q)i\epsilon_{ijk}(\partial^\nu\pi_i(Q+K)Q_\mu V^j_\nu K A^\mu_k(K) + \partial_\nu\pi_i(Q+K)V^i_\mu(Q)K^\mu A^\nu(K)) \\ & + G_{kv-a-\pi}(K,Q)i\epsilon_{ijk}\pi_i(Q+K)Q_\mu V^j_\nu(Q)K^\mu A^\nu_k(K) + \mathcal{O}_k,\end{aligned} \tag{19}$$

where $\mathcal{O}_k$ contains further couplings with higher order momentum dependence, Q, K are the momenta carried by the vector and axial mesons, respectively. In this equation the pion field is canonically normalized. The coupling functions are given by

$$G_{v-a-\pi}(K,Q) = d_2N_c 6\int_k((M^* T(K,Q)\tilde{S}_0(k)\tilde{S}_0(k+K)\tilde{S}_0(k+K+Q))), \tag{20}$$

$$G_{kv-a-\pi}(K,Q) = d_2N_c 6\int_k((M^*\tilde{S}_0(k)\tilde{S}_0(k+K)\tilde{S}_0(k+K+Q))), \tag{21}$$

where $T(K,Q)\equiv(2k\cdot(k+K)+K\cdot(K+Q)-M^{*2})/2$. Similar couplings to the $\rho-\pi-A_1$ have been proposed in other works within different or similar approaches, for example in [21–23,37,40,73–75] and references therein. The integral of the coupling function $G_{v-a-\pi}$ is ultraviolet (UV) logarithmic divergent and $G_{kv-a-\pi}$ is UV finite. They have respectively dimensions of mass and mass^{-1}.

This UV divergence in Eq. (20) is the same as the momentum dependent free vector and axial meson terms. The free and self-interacting light vector and axial mesons sector have been investigated in very similar approaches, see for example in [37,71,76]. This divergence is directly eliminated by the vector and axial field renormalization constants in the same way some couplings were made finite in [61]. The Abelian contributions were found to be given by

$$\mathcal{L}_{free} = -\frac{g_f^{(0)}}{4}(\mathcal{F}_i^{\mu\nu}\mathcal{F}^i_{\mu\nu} + \mathcal{G}_i^{\mu\nu}\mathcal{G}^i_{\mu\nu} + \mathcal{F}^{\mu\nu}\mathcal{F}_{\mu\nu} + \mathcal{G}^{\mu\nu}\mathcal{G}_{\mu\nu}),$$

where the following effective parameter has been defined in the long wavelength and zero momentum limit considered before: $g_f^{(0)} = d_1 4N_c \mathrm{Tr}'((\tilde{S}_0^2(k)))$. This parameter can be set finite as a renormalization condition, the one for Z_V and Z_A, and this makes all the coupling constants and coupling functions in Eq. (19) finite. The mass terms for the vector and axial mesons can also be found from this approach with complementary contributions in similar developments [77,78]. However the masses do not provide important information for the coupling constants and form factors addressed in the present work. To obtain terms consistent with the massive Yang Mills approach, one must impose as renormalization point: $g_f^{(0)} = 1$. It follows that

$$G_{v-a-\pi}(0,0) \sim \frac{g_f^{(0)}M^*}{2} + \frac{1}{e}g_{F\rho\omega}, \tag{22}$$

where $g_{F\rho\omega}$ is the (finite) coupling constant of the neutral rho meson coupling to a photon and to the omega meson: $\mathcal{L}_{F\rho\omega} = -g_{F\rho\omega}F_{\mu\nu}\mathcal{F}_3^{\nu\rho}\mathcal{F}^\mu_\rho$ for $F_{\mu\nu}$ the background photon stress tensor.

Finally, there are other three-leg couplings that can also be associated to mixing of ρ and other vector and axial mesons, in particular the $\rho-\omega-A_1$ vertex [27,79]. Besides that, there are also couplings that yield fusion of vector mesons into an axial meson g_{VVA}. They can be written as

$$\begin{aligned}\mathcal{L}_{3-v} = {} & g_{\omega\rho A_1}(K,Q)2\epsilon^{\mu\nu\alpha\beta}V_\mu(K+Q)(\mathcal{F}^i_{\nu\alpha}(K)A^i_\beta(Q) + \mathcal{G}^i_{\nu\alpha}(Q)V^i_\beta(K)) + \mathcal{O}_\mathcal{F} \\ & + g_{\omega\omega f_1}(K,Q)2\epsilon^{\mu\nu\alpha\beta}V_\mu(K+Q)(\mathcal{F}_{\nu\alpha}(K)A_\beta(Q) + \mathcal{G}_{\nu\alpha}(Q)V_\beta(K)) + \mathcal{O}_\mathcal{F} \\ & + g_{\rho\rho A_1}(K,Q)2\epsilon^{\mu\nu\alpha\beta}i\epsilon_{ijk}V^i_\mu(K+Q)(\mathcal{F}^j_{\nu\alpha}(K)A^k_\beta(Q) + \mathcal{G}^j_{\nu\alpha}(Q)V^k_\beta(K)) + \mathcal{O}_\mathcal{F},\end{aligned} \tag{23}$$

where $\mathcal{O}_{\mathcal{F}}$ contains different momentum structures. These other structures are obtained by exchanging the roles of the rho and the omega, or the A_1 meson, i.e., $\mathcal{F} \leftrightarrow \mathcal{F}_i \leftrightarrow \mathcal{G}_i$ and correspondingly $V^i_\mu \leftrightarrow V_\mu \leftrightarrow A^i_\mu$. The coupling function $g_{\omega\rho A_1}$, which is symmetrized due to the possible different order of external lines, was defined as

$$g_{\omega\rho A_1}(K,Q) = C_3 \int_k ((S_0(k)S_0(k+K)\tilde{S}_0(k+K+Q) + \tilde{S}_0(k)S_0(k+K)S_0(k+K+Q))), \quad (24)$$

$$g_{\omega\rho A_1}(K,Q) = g_{\omega\omega f_1}(K,Q) = g_{\rho\rho A_1}(K,Q), \quad (25)$$

where $C_3 = 6N_c d_2$, the momentum integral has the same UV divergence as Eq. (20) and they can be renormalized exactly in the same way as discussed above. The right-hand side of Eq. (25) is also obtained from the naive quark model relation with a vector meson dominance (VMD) hypothesis [26]. For constant quark effective mass, it can be written

$$g_{\omega\rho A_1}(Q,K) = \frac{1}{M^*} G_{v-a-\pi}(Q,K). \quad (26)$$

These coupling constants are also directly proportional to coupling constants found in the framework of the Skyrme model or massive Yang Mills model [37,38,80].

C. Possible quantization of meson couplings to axial and vector couplings

The coupling $F_{\mathrm{vja}}(K,Q)$ in the action can be written in the coordinate space and, with the same construction of Witten [35,81], it can be expressed as a five dimensional closed surface of a total divergence by means of the Stoke's theorem. In this case a quantization condition emerges. By writing it back in the momentum space, Eq. (9) can be written as

$$n\Gamma = -\frac{i}{240\pi^2} \int d^4K d^4Q \epsilon_{\sigma\rho\mu\nu} F^{\mathrm{vja}}(K,Q) \times K_\sigma \mathcal{F}^i_{\rho\mu}(Q) j^{A,i}_\nu(K,K+Q), \quad (27)$$

where n is an integer. This integral, as a term in the action, corresponds therefore to a topologically conserved quantity assuming integer multiple values of Γ. The axial current however has specific properties. For instance, the axial charge is only partially conserved due to both quark masses and the Adler-Bell-Jackiw (ABJ) anomaly. So the following question arises: how can a topologically conserved coupling involving the axial current be related to the partial conservation of the axial current? Seemingly the vector meson anomalous coupling selects a component of $j^{A,i}_\nu$ that would be quantized and conserved. By decomposing the above integral into the different Lorentz components, i.e., $n\Gamma = \epsilon^{\sigma\rho\mu\nu}\Gamma_{(\sigma\rho\mu\nu)}$, one can write explicitly one of these terms as

$$\Gamma_{(xyz0)} = -\frac{i}{240\pi^2} \int d^4K d^4Q F^{\mathrm{vja}}(K,Q) K_x Q_y [\rho^-_z(Q)\bar{u}(K+Q)\gamma_0\gamma_5 d(K) + \rho^+_z(Q)\bar{d}(K+Q)\gamma_0\gamma_5 u(K)], \quad (28)$$

where $\rho^\pm_z(Q)$ are the z-polarization component profile of the charged rho field. The quantization condition however involves the sum of all the couplings of all the components $\Gamma_{(xyz0)}$.

Besides that, for constant effective mass, these equations can be rewritten in terms of the pion axial coupling to constituent quarks, Eq. (16). From Eq. (18) it can be written that

$$n\Gamma = -\frac{i}{240\pi^2 \times 4M^* F} \int d^4K d^4Q \epsilon^{\sigma\rho\mu\nu} G_A(K,Q) K_\sigma \mathcal{F}^i_{\rho\mu}(Q) j^{A,j}_\nu(K,K+Q). \quad (29)$$

Note however, that the momenta K and Q are orthogonal to each other, differently from the pion axial coupling to the constituent quark current. This equation (29) has a higher order dependence on the constituent quark momentum K_σ but it is of the same order of the meson momentum (pion or vector meson) Q_ρ.

By applying the same reasoning to the axial meson coupling to the constituent quark vector current (13) the following quantization conditions are obtained:

$$m_t\bar{\Gamma} = \int d^4x_1 d^4x_2 i\epsilon^{\sigma\rho\mu\nu} F^{\mathrm{vja}}(x_1,x_2) \mathcal{G}^i_{\rho\mu}(x_1) \partial_\sigma j^{V,i}_\nu(x_2,x_1), \quad (30)$$

$$m_s\tilde{\Gamma} = \int d^4x_1 d^4x_2 i\epsilon^{\sigma\rho\mu\nu} F^{\mathrm{vja}}(x_1,x_2) \mathcal{G}_{\rho\mu}(x_1) \partial_\sigma j^V_\nu(x_2,x_1), \quad (31)$$

where m_s, m_t are integers for the isosinglet and isotriplet axial meson interactions. Notwithstanding the vector current is conserved only for degenerate quark masses these conditions should not disappear in the case of nondegeneracy of quark masses. A similar question to the one raised above arises: how can a topologically conserved quantity, involving the constituent quark vector current, be related to the nonconservation of the (global) vector current that is due to the nondegenerate quark masses? Similarly to the axial current coupling to the vector meson, the axial meson profile and coupling might "select" a topologically preserved component of the vector current. The effect of quark mass nondegeneracy will be investigated in another work.

III. ANOMALOUS VECTOR MESON COUPLINGS TO THE QUARK AXIAL CURRENT UNDER WEAK MAGNETIC FIELD

The effect of a magnetic field, weak with respect to the constituent quark mass M^*, will be presented due to two different mechanisms described in detail in Ref. [60]. The validity of the (semi)classical approximation for the magnetic field to describe observables in heavy ion collisions has been an object of attention in the past years [82,83]. The dependence of the quark propagator on the weak magnetic field is considered and, second, the overall photon couplings to the legs of the meson-current coupling. For the degenerate quark effective mass M^*, the quark propagator, with the leading contribution from the weak magnetic field aligned in the z direction, can be written for degenerate quark masses as [84,85]

$$G(k) = S_0(k) + S_1(k)(eB_0) = \frac{\not{k} + M^*}{k^2 - M^{*2} + i\epsilon} + i\gamma_1\gamma_2 \frac{(\gamma_0 k^0 - \gamma_3 k^3 + M^*)}{(k^2 - M^{*2} + i\epsilon)^2} \hat{Q}(eB_0). \tag{32}$$

The anomalous light vector meson couplings to the axial current in the presence of this constant weak magnetic field provide similar expressions in both mechanisms mentioned above that can be added. Below some of resulting corrections for the couplings, with the form factors, of the previous section are shown for the isotriplet and isosinglet vector and axial mesons. By writing them with a constant small multiplicative factor $(eB_0)/M^{*2}$, they can be written as

$$\begin{aligned}\mathcal{L}_{\mathrm{vjaB}} = {}& \frac{(eB_0)}{M^{*2}} \epsilon_{ij3} \frac{F^B_{\mathrm{vja}}(K,Q)}{M^{*2}} [\epsilon^{12\rho\mu} K_\rho Q_\nu \cdot V^i_\nu(Q) + 2\epsilon_{12\rho\nu} K^\rho \mathcal{F}^{\mu\nu}_i(Q)] j^{A,j}_\mu(K, K+Q) \\ & + \frac{(eB_0)}{M^{*2}} \frac{F^B_{\mathrm{vja}}(K,Q)}{3M^{*2}} [\epsilon^{12\rho\mu} K_\rho Q_\nu \cdot V_\nu(Q) + 2\epsilon_{12\rho\nu} K^\rho \mathcal{F}^{\mu\nu}(Q)] j^{A,3}_\mu(K, K+Q),\end{aligned} \tag{33}$$

$$\begin{aligned}\mathcal{L}_{\mathrm{mix},B} = {}& \frac{(eB_0)}{M^{*2}} G^{B,1}_{v-a}(K,Q) i\epsilon_{12\mu\nu} M^{*2} i\epsilon_{ij3} V^\mu_i(Q) \bar{A}^\nu_j(K) \delta(Q+K) \\ & + \frac{(eB_0)}{M^{*2}} G^{B,1}_{v-a-\pi}(Q,K) i\epsilon_{12\mu\nu} M^* (\pi_3(Q) V^\mu(K) \bar{A}^\nu(Q+K) + T_{ijk}\pi_i(Q) V^\mu_j(K) \bar{A}^\nu_k(K+Q)),\end{aligned} \tag{34}$$

where $T_{ijk} = tr_F(Q\sigma_i\sigma_j\sigma_k)$. The coupling functions can be written in the Euclidean momentum space as

$$F^B_{\mathrm{vja}}(K,Q) = 4d_2 N_c (\alpha g^2) M^{*4} \int_k ((\tilde{S}_0(k+K)\tilde{S}_0(k+K)\tilde{S}_0(k+K+Q)\bar{R}(-k))), \tag{35}$$

$$G^{B,1}_{v-a}(K,Q)\delta(K+Q) = 2d_2 N_c \int_k ((M^{*2}\tilde{S}_0(k+Q)\tilde{S}_0(k+Q)\tilde{S}_0(k))), \tag{36}$$

$$G^{B,1}_{v-a-\pi}(Q,K) = 3d_2 N_c M^{*2} \int_k (((k\cdot(k+Q) - M^{*2})\tilde{S}_0(k+Q)\tilde{S}_0(k+Q)\tilde{S}_0(k+K+Q)\tilde{S}_0(k))), \tag{37}$$

where the vector mesons are the canonically normalized ones. These coupling functions (coupling constants) are all finite and they are dimensionless. The couplings $F^B_{\mathrm{vja}}(K,Q)$ might be seen as magnetic field corrections to the anomalous vector meson form factor.

The mixing $G^{B,1}_{v-a}(K,Q)$ disappears in the absence of the magnetic field. Note that the similar term for isosinglet vector and axial fields does not show up. This term, however, is trivially zero in the absence of other particles due to conservation of momentum, and this is made explicit in the delta function $\delta(K+Q)$. In the presence of another particle, or in a finite density medium, the conservation of momentum can be satisfied and this anomalous mixing can contribute because vector and axial mesons propagate in orthogonal directions. This coupling should contribute for in medium vector-axial mixing besides the mixing induced by the pion of Eq. (19) [28,29]. All these interaction terms emerge due to the anomalous trace of five Dirac matrices, Eq. (12). There is a similar momentum anisotropy to the case of zero magnetic field, being also dependent on the vector or axial meson polarization. The magnetic field contributions are suppressed by factors $1/M^*$ or $1/M^{*2}$ with respect to the zero magnetic field couplings, besides the factors $(eB_0)/M^{*2}$ were included too. Although the explicit magnetic field contribution is factorized and suppressed by a factor $(eB_0)/M^{*2}$, the effective quark masses also depend on the magnetic field in the gap equations. These anomalous form factors can be simply added to the anomalous form factors (9), (13) and (19) in the vacuum as it follows:

$$F(K,Q;B_0) = F(K,Q) + f\frac{(eB_0)}{M^{*2}}F^B(K,Q), \qquad (38)$$

where f is a factor that depends on the momentum components in Eq. (33) or (34). $F_{\mathrm{vja}}(K,Q)$ and $F^B_{\mathrm{vja}}(K,Q)$ also can receive magnetic field corrections from the gluon propagator and running coupling constant dependencies on B_0 being however neglected in the present work. Axial vector meson couplings to the quark vector currents, Eq. (13), receive analogous corrections due to the weak magnetic field to those shown above.

IV. NUMERICAL RESULTS

For the numerical estimations two different effective gluon propagators will be considered. The first one will be a transversal obtained from Schwinger-Dyson equations (SDE) that reproduce many hadron observables [9,62]. It can be written as

$$\begin{aligned} D_I(k) &= h_I g^2 R_T(k) \\ &= \frac{8\pi^2}{\omega^4} D e^{-k^2/\omega^2} + \frac{8\pi^2\gamma_m E(k^2)}{\ln[\tau + (1+k^2/\Lambda^2_{\mathrm{QCD}})^2]}, \end{aligned} \qquad (39)$$

where h_I is the factor that normalizes the coupling constant of the vector meson to the vector current, $\gamma_m = 12/(33-2N_f)$, $N_f = 4$, $\Lambda_{\mathrm{QCD}} = 0.234$ GeV, $\tau = e^2 - 1$, $E(k^2) = [1-\exp(-k^2/[4m_t^2])/k^2$, $m_t = 0.5$ GeV, $D = 0.55^3/\omega$ (GeV^2) and $\omega = 0.5$ GeV.

The second type is based in a longitudinal effective confining parametrization by Cornwall [64] that can be written as

$$R_L(k) = D_{II,z}(k) = h_{II,z}\frac{K_F}{(k^2+M_z^2)^2}, \qquad (40)$$

where $K_F = (2\pi M_z/3k_e)^2$ was considered in previous works [16,54]. In this equation, $k_e \simeq 0.15$, the normalization factor $h_{II,k}$ is considered to reproduce a reasonable value for the vector meson (rho or omega) coupling constant to the nucleon (or constituent quarks). This type of gluon effective propagator exhibits features of lattice QCD calculations [86]. Two choices were made for the effective gluon mass M_z^2: as a function of momentum ($z=6$ and $z=7$) for $M_6 = 0.5/(1+k^2)$ GeV and $M_7 = 0.5/(1+k^2/5)$ GeV.

In the figures below the following normalized spacelike form factors with respect to the usual vector meson coupling to the vector current will be drawn:

$$\begin{aligned} G_{\mathrm{vja}}(K,Q) &= \frac{F_{\mathrm{vja}}(K,Q)}{G_V(0,0)}, \\ G^B_{\mathrm{vja}}(K,Q) &= \frac{F^B_{\mathrm{vja}}(K,Q)}{G_V(0,0)}, \end{aligned} \qquad (41)$$

where the following normalizing parameters where chosen: $h_I = 1.4$ ($M^* = 0.33$ GeV), $h_I = 1.6$ ($M^* = 0.45$ GeV), and $h_{II,6} = h_{II,7} = 16$ ($M^* = 0.33$ GeV). These values for the effective masses and factors $h_{I,II}$ yielded $G_V(0,0) = 12$ in Ref. [16]. This way we can extract relative strengths of the couplings in a uniform way to make comparisons. Besides that, only one component of K and Q contributes, and, due to this, we considered (in a spacelike spherical coordinate basis) $Q = |Q|/4$ and $K \simeq |K|/4$.

In Fig. 1 the form factor $G_{\mathrm{vja}}(K,Q)$, calculated with the effective gluon propagator $D_I(k)$, is presented as a function of the vector meson momentum Q^2 for different values

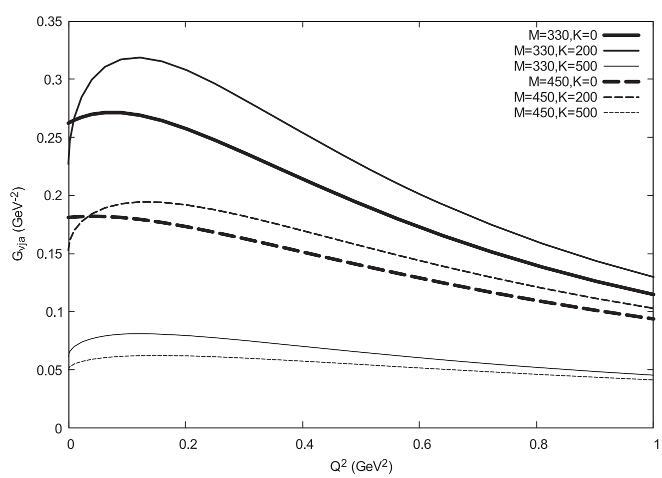

FIG. 1. Anomalous form factor $G_{\mathrm{vja}}(K,Q)$ for the effective gluon propagator $D_I(k)$ as a function Q^2 for different values of K. Two different quark effective masses are used $M^* = 0.33$ GeV and $M^* = 0.45$ GeV.

of the modulus of the constituent quark momentum $K = |K|/4$. Two different values of the quark effective mass, typical from the NJL model, are considered $M^* = 0.33$ GeV and $M^* = 0.45$ GeV. The momentum dependencies in K and Q are not very different, as it can be seen in Eq. (11). There is a slight increase up to $Q^2 \sim (0.2\ \text{GeV})^2$ or $K^2 \sim (0.2\ \text{GeV})^2$ and then a decrease with both Q^2 and K^2. For the zero or very low quark momenta $K \sim 0$ limit the behavior with Q^2 is almost monotonically decreasing. A larger quark effective mass leads to a suppression of the interaction and to a less strong dependence on momenta. This figure, and the following ones, show that there might be a weak coupling of the light vector mesons, ρ and/or ω, with the axial constituent quark current. The relative strength of the coupling $F_{\text{vja}}(K, Q)$ and the usual vector coupling close to $Q \sim K \sim 200$–500 MeV can be estimated to be

$$\left.\frac{F_{\text{vja}}(K, Q) \times |K||Q|}{G_V(K, Q)}\right|_{Q \sim K \sim 200\text{–}500\ \text{MeV}} \sim 0.1. \qquad (42)$$

This coupling function also provides an (anomalous) axial contribution for the vector meson form factor. Therefore only precision measurements of the vector meson interactions and form factors would identify such contributions. It is important to emphasize that $F_{\text{vja}}(K, Q)$ is strongly anisotropic, being that it selects a particular meson polarization. Furthermore, $F_{\text{vja}}(K, Q)$ for larger momenta is suppressed stronger than the vector coupling $G_V(K, Q)$. Its (anisotropic) effects could also be searched in the strength of the vector meson dominance VMD [87]. Besides that, these couplings correspond to anomalous axial (vector) corrections to the rho or omega (A_1 or f_1) form factors for which there are some estimations [16,88,89].

The same form factor $G_{\text{vja}}(K, Q)$ as a function of Q^2 is exhibited, for different values of $K = 0$, 200, 500 MeV, for the effective gluon propagators $D_{II,6}(k)$ and $D_{II,7}(k)$ in Fig. 2 for the quark effective mass $M^* = 0.33$ GeV. The behavior is very similar to the one found in Fig. 1 for the gluon propagator $D_I(k)$. The very small difference between the results from the gluon propagators only shows up for low-intermediary momenta, and it tends to zero for higher momenta $K > 400$ MeV and $Q^2 > 1\ \text{GeV}^2$.

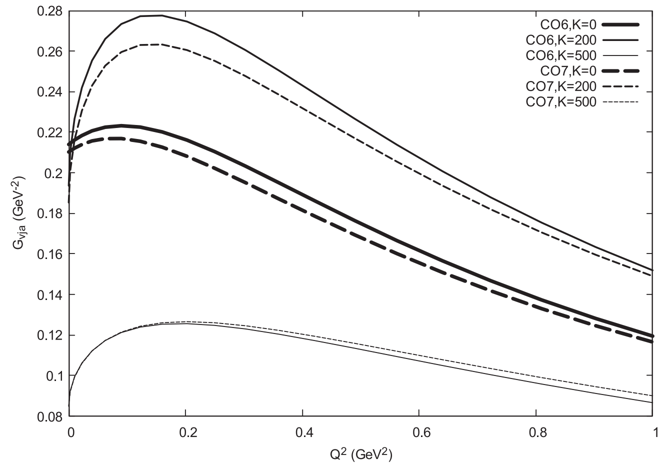

FIG. 2. Anomalous axial form factors $G_{\text{vja}}(K, Q)$ for the two effective gluon propagators $D_6(k)$ and $D_7(k)$, and with $M^* = 0.33$ GeV.

A. Magnetic field induced correction

In Fig. 3 the form factor $G^B_{\text{vja}}(K, Q)$ is drawn for the gluon propagator $D_I(k)$ and two different quark effective masses, $M^* = 0.33$ and 0.45 GeV, as a function of the vector meson momenta Q^2 for the same quark momentum as the previous figures. The effect of modifying the quark effective mass is much larger at low momenta Q and K. For higher momenta, the difference between the form factors with the different effective masses tends to become smaller with Q^2 and with K^2. Also, analogously to the zero magnetic field case exhibited above, there is a small increase with low momenta up to $K \sim 200$ MeV or $Q \sim 200$ MeV. In Fig. 4 the same form factor $G^B_{\text{vja}}(K, Q)$ is presented for the same values of $K = 0$, 200, 500 MeV, with $M^* = 0.33$ GeV and for the effective gluon propagators $D_{II,6}(k)$ and $D_{II,7}(k)$. The behavior with external momenta is basically the same as the form factor G_{vja} and the difference between results of the two different gluon propagators shows up in intermediary momenta. This anomalous constituent quark and vector meson coupling function could manifest in the low/intermediary vector meson energy regime in ultraperipheral collisions.

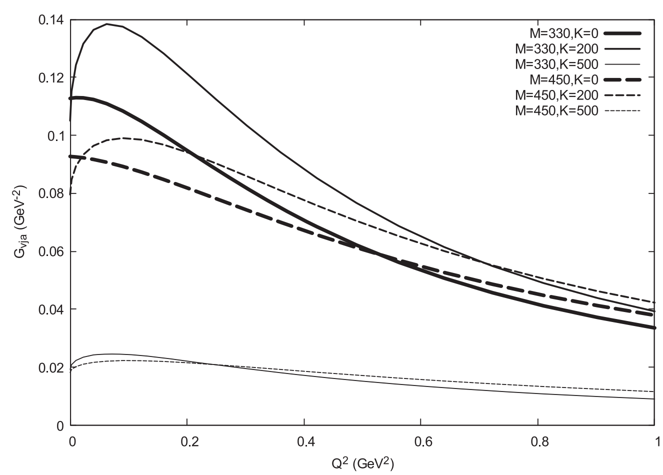

FIG. 3. Magnetic field induced correction to the anomalous axial form factors probed by the rho vector meson for the effective gluon propagator $D_I(k)$ and for two different quark effective masses $M^* = 0.33$ GeV and $M^* = 0.450$ GeV. The modulus of quark momentum was chosen to be the same as the previous figures: $K = 0$, 200, 500 MeV.

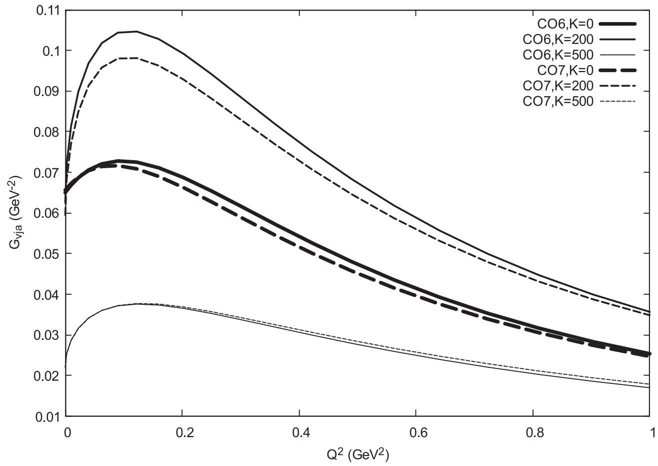

FIG. 4. The same as Fig. 3 for the two effective gluon propagators $D_6(k)$ and $D_7(k)$, with $M^* = 0.33$ GeV and $K = 0$, 200, 500 MeV.

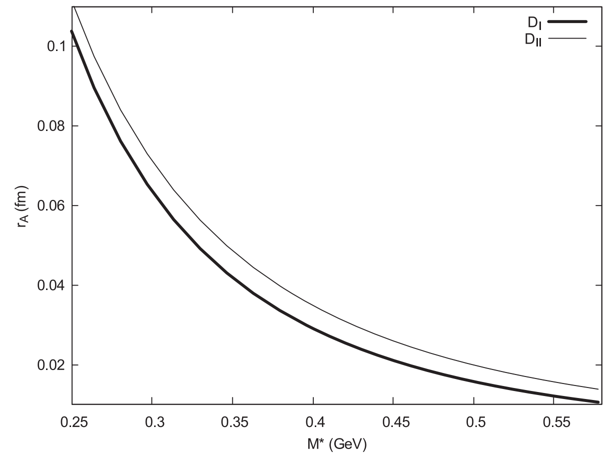

FIG. 5. The axial AQR, $r_A \equiv \sqrt{\Delta_A\langle r_\rho^2\rangle}$, obtained with D_I and $D_{II,6}$ (with $M_6 = 500$ MeV) as a function of the quark effective mass.

B. Averaged quadratic radius and on-shell estimates for coupling constants

Vector (and axial) mesons couple on shell although constituent quarks cannot be really said to do so. Below some estimates are provided for coupling constants with on-shell vector mesons. Because of the involved analytic structure, only gluon effective propagator (40) will be considered for which we adopted the value $M_6 = 500$ MeV. The momentum integral of the coupling constant $g_{\omega\rho A_1}$ is ultraviolet divergent. Because of that, a regularized truncated version of the quark propagator was considered [16,60]: $S_0(k) \to M^*/(k^2 - M^{*2})$. It may yield an effective momentum dependence close to the ones obtained from SDE. With that, the integral becomes finite and we denote the result by $G^{tr}_{\omega\rho A_1}(K,Q)$. All the singularities were taken into account by considering the average of the integration over the up and down complex semiplanes. The following ranges of values were obtained for a range of quark effective masses $M^* = 0.33$–0.35 GeV:

$$G_{\rm vja}(K_0 = M^*, Q_0 = M_\rho) = 0.6\text{–}0.4\ \mathrm{GeV}^{-2}, \qquad (43)$$

$$G^{\rm tr}_{\omega\rho A_1}(K_0 = M_\rho, Q_0 = M_\omega) = 0.2\text{–}0.3, \qquad (44)$$

$$G_{kv-a-\pi}(K_0 = M_\rho, Q_0 = M_{A_1}) = 0.9\text{–}1.2\ \mathrm{GeV}^{-1}, \qquad (45)$$

$$G^{B_1}_{v-a}(K_0 = M_\rho) = 0.1\text{–}0.3, \qquad (46)$$

Note that $G^{tr}_{v-a-\pi} = M^{*2} G_{kv-a-\pi}$. Also, from Eq. (25), one has for the vector mesons fusion channel $g_{\omega\rho A_1}(K_0,Q_0) \simeq g_{\omega\omega f_1}(K_0,Q_0) \simeq g_{\rho\rho A_1}(K_0,Q_0)$, since the mass difference $M_\rho - M_\omega \sim 12$ MeV is very small.

For the sake of comparison, few values for the coupling $G_{\omega\rho A_1}$ or $G_{VA\pi}$ considered in the literature are quoted next. The coupling constant $G_{\omega\rho A_1}$ has been considered for the investigations (of vector/axial mesons mixing) at finite density [27,28]. The following values were used: $g_{\omega\rho A_1}\langle\omega_0\rangle = C \simeq 0.1 \to 0.3$ GeV, at the saturation density ρ_0 where the quantity $\langle\omega_0\rangle$ is the omega mean field in the medium. From Ref. [20] the following values were considered for the dimensionless three-leg coupling given in (26): $G_{kv-a-\pi} \sim M^* G_{AVP} \sim 2M^* \sim 0.7$.

A simple estimation for the contribution of the form factor $F_{\rm vja}(K,Q)$ to the rho meson averaged quadratic radius can be also provided. It corresponds to a small axial component. For that, we can define a normalized dimensionless coupling function (form factor) as $\bar{G}_{\rm vja} = \bar{K}\,\bar{Q}\,G_{\rm vja}(K,Q)$, where $\bar{K}, \bar{Q} \sim 200$ MeV are averaged constituent quark and meson momenta. The usual definition of averaged quadratic radius (AQR) will be adopted:

$$\Delta_A\langle r_\rho^2\rangle = -6\frac{d\bar{G}_{\rm vja}}{dQ^2}\bigg|_{Q=0}. \qquad (47)$$

Results are shown in Fig. 5 for two gluon propagators D_I and $D_{II,6}$ with $M_6 = 500$ MeV. These values can be compared to estimations of the rho AQR: $\langle r_\rho^2\rangle \simeq 0.28$–$0.56$ fm^2 [13,16,88–90]. It very small, i.e., $\sqrt{\Delta_A\langle r_\rho^2\rangle}$ can be 1 order of magnitude smaller than the rho vector meson radius. The contribution of the weak magnetic field is very small, it is 1 order of magnitude smaller than the above contribution, i.e., $\Delta_A^B\langle r_\rho^2\rangle \sim \frac{\Delta_A\langle r_\rho^2\rangle}{10}$.

V. FINAL REMARKS

Unusual couplings of light vector mesons, rho and omega, to the constituent quark axial current were derived in this work. Their momentum dependencies were presented for different effective gluon propagators in the low momentum regime. The resulting coupling constants and form factors are suppressed at low momenta by $(K_\mu Q_\nu)/M^{*2}$ in comparison with the minimal vector meson couplings to the quark vector current [16]. K_μ and Q_ν are respectively the quark and vector meson momenta, being that the coupling functions are strongly anisotropic. The momentum dependencies of $F_{\rm vja}$ on Q and K are basically the same. The couplings are also dependent on the polarization of the vector meson. Although they go to zero for very large, infinite, meson momenta, the resulting equations are suppressed fast with respect to the leading form factors at higher energies. Besides that, at higher energies, quark effective masses should decrease and this might, to some extent, invalidate the large quark mass expansion. The aim of the present work is to provide a dynamical derivation for the low energy meson couplings. As such, the resulting equations can be written in terms of structureless (local) meson fields. When analyzing topological-based equations from Sec. II C there is a need to integrate over all the meson field profiles, or their momentum dependencies. However, since the structure of $F_{\rm vja}(K,Q)$ decreases quite strongly with increasing momenta, the overall high energy contributions from the meson profiles should be suppressed. This was not investigated further in the present work.

Numerically we have found $G_{\rm vja} = F_{\rm vja}/G_V \sim$ 0.1–0.3 GeV^{-2}, i.e., this coupling might produce small (anisotropic) effects in observables associated to the vector meson coupling to constituent quarks or to the nucleon. The corresponding light axial meson couplings to the constituent quark vector current were also derived and they have the same strength and overall momentum dependence. When comparing the behavior of the form factors $F_{\rm vja}(K,Q)$, calculated with different effective gluon propagators, very small (negligible) differences were found mostly for momenta $0.15\ \mathrm{GeV}^2 \leq Q^2 \leq 0.7\ \mathrm{GeV}^2$. Modifications in the value of the quark effective masses lead to shifts in the low momenta region of $F_{\rm vja}(K,Q)$ and to its weak magnetic field induced correction $F^B_{\rm vja}(K,Q)$. These form factors correspond to axial (vector) components of the vector (axial) mesons. Correspondingly a simple estimation of the axial averaged quadratic radius for the rho meson was provided. It is, at most, 1 order of magnitude smaller than the estimations for the rho AQR. There might appear other consequences; for example, a small (anisotropic) contribution for the so-called vector meson dominance VMD and related form factors [87].

Other three-leg meson vertices often considered, such as $\rho - A_1 - \pi$, $\rho\rho - A_1$ and $\omega - \rho - A_1$, were also derived being the estimated values in quite good agreement with other investigations. These three-meson coupling constants, or coupling functions, might be finite or logarithmic UV divergent. In this last case, they are renormalized by the same renormalization constants as the free vector meson kinetic terms [61]. Finally, mixings between light vector and axial mesons were also found. However, the corresponding anomalous mixing only can provide dynamical contributions in the presence of a third particle or in a finite density medium. This is due to the conservation of linear momentum. An incoming (or outgoing) vector meson and an outgoing (or incoming) axial meson must propagate in orthogonal directions. These couplings and mixings are strongly anisotropic and dependent on both meson momenta and polarizations.

Besides that, weak magnetic field contributions for the anomalous couplings were also presented with their momentum dependencies in the form factors, $F^B_{\rm vja}(K,Q)$. These corrections to the anomalous vector meson couplings to the axial currents are reasonably similar to the zero magnetic field case, $F_{\rm vja}$. These corrections are, however, numerically suppressed by the factor eB/M^{*2} that was assumed to be small. Two different mechanisms that generate weak magnetic field contributions for hadron couplings were considered: first, the leading correction to the quark propagator—Eq. (32)—and second, the background photon overall coupling to the quark-vector meson vertex. Both mechanisms provide similar contributions that add to each other in Eqs. (33) and (34). Vector-axial meson mixing induced by the magnetic field was also found.

By following the same method considered by Witten [35] for Wess-Zumino terms, conditions for the quantization of the anomalous couplings $F_{\rm vja}$ were found. The total axial current, however, is only partially conserved because of Lagrangian quark masses and of the ABJ anomaly. The vector current is not conserved due to the quark mass nondegeneracy. So, it is not clear how to cope with these small symmetry breakings, dictated by well-known low energy theorems, with the topologically conserved terms in the action given by Eqs. (27) or (30). They correspond to quantization conditions that contain the axial or vector currents, or a component of them. One way to, maybe, solve this apparent contradiction would be to consider that the light vector or axial mesons select components of, respectively, the axial or vector constituent quark currents that remain (topologically) preserved. This is not investigated further in the present work. For degenerate quark masses the quantization condition for the flavor singlet axial meson coupling to the vector current, with m_s, is the same as the flavor triplet one, with m_t. The nondegeneracy of quark masses does not prevent the quantization conditions to emerge, such as (27), (30) or (31), it slightly changes its shape. This case of nondegenerate quark mass will be addressed in another work.

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