Constraining massless dilaton theory at Solar system scales with the planetary ephemeris INPOP

L. Bernus,[1,2] O. Minazzoli,[3] A. Fienga,[2,1] A. Hees,[4] M. Gastineau,[1] J. Laskar,[1] P. Deram,[2] and A. Di Ruscio[2,5]

[1]*IMCCE, Observatoire de Paris, PSL University, CNRS, Sorbonne Université, 77 avenue Denfert-Rochereau, 75014 Paris, France*
[2]*Géoazur, Observatoire de la Côte d'Azur, Université Côte d'Azur, IRD, 250 Rue Albert Einstein, 06560 Valbonne, France*
[3]*Artemis, Université Côte d'Azur, CNRS, Observatoire de la Côte d'Azur, BP4229, 06304, Nice Cedex 4, France*
[4]*SYRTE, Observatoire de Paris, Université PSL, CNRS, Sorbonne Université, LNE, 61 avenue de l'Observatoire, 75014 Paris, France*
[5]*Dipartimento di Ingegneria Meccanica e Aerospaziale, Sapienza Università di Roma, via Eudossiana 18, 00184 Rome, Italy*

(Received 3 October 2021; accepted 10 January 2022; published 28 February 2022)

We expose the phenomenology of the massless dilaton theory in the Solar system for a nonuniversal quadratic coupling between the scalar field which represents the dilaton and the matter. Modified post-Newtonian equations of motion of an N-body system and the light time travel are derived from the action of the theory. We use the physical properties of the main planets of the Solar system to reduce the number of parameters to be tested to three in the linear coupling case. In the linear case, we have a universal coupling constant α_0 and two coupling constants α_T and α_G related, respectively, to the telluric bodies and to the gaseous bodies. We then use the planetary ephemeris, INPOP19a, in order to constrain these constants. We succeeded to constrain the linear coupling scenario, and the constraints read $\alpha_0 = (1.01 \pm 23.7) \times 10^{-5}$, $\alpha_T = (0.00 \pm 24.5) \times 10^{-6}$, $\alpha_G = (-1.46 \pm 12.0) \times 10^{-5}$, at the 99.5% C.L.

DOI: 10.1103/PhysRevD.105.044057

I. INTRODUCTION

While the equivalence principle (EP) is at the heart of general relativity (GR), it has been argued that there actually exist no strong theoretical reasons to expect this principle to be valid in nature, notably suggesting that GR should be replaced by a more accurate theory of relativity [1]. This argument provides a strong motivation to search for an observational violation of the EP.

Among all the alternative theories of gravitation, scalar-tensor theories have been widely studied due to their simplicity as well as their manifest ability to give somewhat natural answers to apparently different issues in fundamental physics. In this class of theory, there is *a priori* no fundamental symmetry that can justify the EP to be valid. It is therefore natural to consider a nonminimal coupling between the scalar field and matter [1]. Furthermore, scalar fields with scalar-matter couplings are ubiquitous in several attempts to unify the whole fundamental interactions of physics [1], such as in superstring or Kaluza-Klein theories, for instance. Such a nonminimal coupling is also considered in some models of dark matter and dark energy [2,3]. In addition, it has been argued that scalar-tensor theories satisfying the EP at the classical level can exhibit a nonminimal coupling between the scalar field and matter due to quantum loop corrections [4]. This may actually lead to an impossible existence of a scalar field minimally coupled to matter fields in nature. Hence, all scalar-tensor theories are likely to violate the EP to some extent. In what follows, we generically name such a class of theories "*dilaton theory*", in reference to the massless dilaton field that is generic in superstring theories and which nonminimally couples to matter fields in the perturbative effective action [5,6].

From an experimental point of view, the violation of the EP has been mainly constrained by two types of experiments: (i) tests of the universality of free fall (UFF) and (ii) search for space-time variations in the constants of nature. UFF is tested on Earth by measuring the relative acceleration between two test masses at the level of 10^{-13} using torsion balances [7]. Recently, it has also been tested in space with the MICROSCOPE experiment at the level of 10^{-14} [8]. Tests at the astronomical scale are performed considering the free fall of the Earth and Moon towards the Sun. These experiments give limits at the level of 10^{-13} as well [9]. These results have been interpreted in the context of dilaton theories [10,11]. Besides UFF, there exist typically three different types of searches for variations of the constants of nature, all of them comparing the

 © 2022 American Physical Society

behavior of two collocated atomic clocks working on different atomic transitions [12]. First, comparing the long-term linear evolution of two atomic clocks gives a constraint on the cosmological evolution of the scalar field [6,12]. Such an experiment has been performed by several groups in the world [13] leading to constraints on a linear drift of several constants of nature at the relative level of 10^{-16} per year, such as for the fine structure constant, for instance. A second signature commonly searched for using an atomic clocks comparison is a harmonic temporal evolution of the constants of nature [14] motivated mainly by models of ultralight dark matter [2]. The last type of behavior consists of searching for a relative variation of the constants of nature as a function of the gravitational potential. Such a search has also been performed using atomic clocks [15] but also using astrophysical observations [16].

Given the ever increasing constraints on any EP violation from observations and experiments, this may seem to be a fatal blow for scalar-tensor theories, in general, and to superstring theories, in particular. Nevertheless, several types of decouplings exist that can suppress EP violations below experimental limits—may they be dynamical [5,6,17–20], intrinsic [21–23], or due to symmetries [24].

Hence, it seems important to continue to explore the phenomenology of scalar-tensor theories with nonminimal couplings to matter and to confront it to observations.

While UFF has been tested to an exquisite level with the MICROSCOPE experiment, the latter actually constrains a very narrow region of the parameter space when interpreted in the framework of a dilaton theory [10]. This is due to the very specific composition of the free falling masses in the experiment. On the other hand, planets in the Solar system may help to further expand the region of the parameter space being explored, due to their different compositions and scales. Nevertheless, while the dilaton theory has already been tested using atomic clocks, currently this theory has not been tested at the scale of the Solar system. In this paper, we show that a nonuniversally coupled massless dilaton induces a week equivalence principle (WEP) violation that can be constrained in the Solar system. We show that the physical nature of the Solar system allows one to simplify the dilaton modeling and reduce the number of parameters to be constrained. The dilaton theory introduces only a limited number of fundamental parameters to be tested: five in the case of a linear coupling between the scalar field and matter and ten in the case of a quadratic coupling. Nevertheless, we show that in the Solar system these fundamental parameters appear as combinations such that the number of coefficients to be tested can be reduced to three in the linear coupling scenario. These three parameters are derived constant from the fundamental coupling constants of the theory. This work is a first step to test dilaton theory in the Solar system. Finally, we present a data analysis of the recent planetary ephemeris INPOP19a that leads to a constraint on the dilaton parameters in the case of a nonuniversal linear coupling between the dilaton field and the matter fields.

In Sec. II, we summarize the phenomenology induced by a massless dilaton within the Solar system. We present the expression of the action, of the equations of motion for a N-body system and also the expression of the light time travel. The details of the mathematical derivations are provided in Appendix A, and some complements about the Lagrangian and Hamiltonian frameworks and about the first integrals of the equations of motion in the massless dilaton theory are given in Appendix B. In Sec. III, we expose the numerical methods used in our analysis. First, we show how the number of parameters of the dilaton theory can be efficiently reduced and present how we implemented the modified equations of motion to our Solar system planetary ephemeris INPOP19a to build a test of the theory of massless dilaton. We also expose the statistical criterion used to constrain the parameters of the theory. In Sec. IV, we present the residuals obtained as a function of the dilaton parameters and deduce a likelihood distribution of these parameters. In Sec. V, we deduce from the likelihood distributions our results for a linear coupling between the dilaton and the matter. These results are the posterior distributions of the parameters to be tested approximated by histograms. We also propose confidence intervals at 90% confidence limit (C.L.) and at 99.5% C.L. Finally, we summarize our results in the conclusion (Sec. VI).

II. PHENOMENOLOGY OF A MASSLESS DILATON IN THE SOLAR SYSTEM

A. Action of the theory and field equations

The difference between Einstein's theory of gravitation and massless dilaton theory consists in the existence of a light scalar field φ coupled to a gravity field and matter [22,25]. In the following, the term "dilaton" is identified with this scalar field. The massless dilaton theory contains four terms in its action. The first term consists in the Einstein-Hilbert action coupled to a differentiable function of the scalar field $f(\varphi)$, a kinetic term for the scalar field, the standard model Lagrangian for the matter fields, and a Lagrangian which parametrizes the interaction between the dilaton and matter. Let us consider a four-dimensional manifold $\mathcal{M}$ described by a map denoted generically (x^μ) (we identify the coordinates chart and the map of $\mathcal{M}$). This action reads [22]

$$S[\boldsymbol{g},\psi_i,\varphi]=\frac{1}{2\kappa c}\int\left(f(\varphi)R-\frac{\omega(\varphi)}{\varphi}\varphi^{,\mu}\varphi_{,\mu}\right)\sqrt{-g}\mathrm{d}^4x+\frac{1}{c}\int\left(\mathcal{L}_{\mathrm{SM}}[\boldsymbol{g},\psi_i]+\mathcal{L}_{\mathrm{int}}[\boldsymbol{g},\psi_i,\varphi]\right)\sqrt{-g}\mathrm{d}^4x, \tag{1}$$

where $\boldsymbol{g}$ is the metric tensor, g the determinant built with the matrix of the covariant components $g_{\mu\nu}$ of the metric tensor $\boldsymbol{g}$ in the map (x^μ), φ the scalar field named "dilaton", f and ω the two twice differentiable functions of one variable, $\mathcal{L}_{\rm SM}$ the Lagrangian density of matter described by the standard model, $\mathcal{L}_{\rm int}$ the Lagrangian density of the interaction between the dilaton and matter, and ψ_i the different matter fields. We could perform all the calculations with this action, but the field equations and the derivation of the post-Newtonian equations of motion can be simplified by performing a conformal transformation. Let us introduce a conformal transformation of the metric tensor, $\boldsymbol{g} \mapsto \boldsymbol{g}^*(\boldsymbol{g}, \varphi)$,

$$g_{\mu\nu} = \frac{f_0}{f(\varphi)} g^*_{\mu\nu}, \tag{2}$$

and a transformation of the scalar field $\varphi \mapsto \phi(\varphi)$ which satisfies

$$\left(\frac{\mathrm{d}\phi}{\mathrm{d}\varphi}\right)^2 = \frac{Z(\varphi)}{2} = \frac{\omega(\varphi)}{\varphi f(\varphi)} + \frac{3}{2}\left(\frac{f'(\varphi)}{f(\varphi)}\right)^2, \tag{3}$$

where f_0 denotes the value of $f(\varphi)$ when φ takes its background value φ_0 at infinite distance (a possible cosmological evolution of this background value is neglected in the present work). The map (x^μ) remains the same. Then, a straightforward calculation shows that in these new variables the action reads [22,26]

$$\begin{aligned}\tilde{S}[\boldsymbol{g}_*, \psi_i, \phi] &= S[\boldsymbol{g}(\boldsymbol{g}_*(\varphi(\phi))), \psi_i, \varphi(\phi)] \\ &= \frac{1}{2\kappa_* c}\int (R_* - 2g_*^{\mu\nu}\phi_{,\mu}\phi_{,\nu})\sqrt{-g_*}\mathrm{d}^4x \\ &\quad + \frac{1}{c}\int \mathcal{L}^*_m[\boldsymbol{g}_*, \psi_i, \phi]\sqrt{-g_*}\mathrm{d}^4x,\end{aligned} \tag{4}$$

where

$$\begin{aligned}\mathcal{L}^*_m[\boldsymbol{g}_*, \psi_i, \phi] = (&\mathcal{L}_{\rm SM}[\boldsymbol{g}(\boldsymbol{g}_*), \psi_i] \\ &+ \mathcal{L}_{\rm int}[\boldsymbol{g}(\boldsymbol{g}_*), \psi_i, \varphi(\phi)])\frac{\sqrt{-g}}{\sqrt{-g^*}},\end{aligned} \tag{5}$$

where R_* is the Ricci scalar computed with the new metric tensor $\boldsymbol{g}_*$, g_* the determinant computed with the covariant components $g^*_{\mu\nu}$ of the metric tensor $\boldsymbol{g}_*$ and

$$\kappa_* = \frac{\kappa}{f_0}. \tag{6}$$

The frame used after this transformation is usually called "Einstein frame". In this frame, a straightforward calculation shows that the field equations read [22,26]

$$R^*_{\mu\nu} - \frac{1}{2}R_* g^*_{\mu\nu} = \kappa_* T^*_{\mu\nu} + 2\phi_{,\mu}\phi_{,\nu} - g^*_{\mu\nu}\phi^{,\alpha}\phi_{,\alpha} \tag{7}$$

$$\Box_*\phi = -\frac{\kappa}{2}\frac{\partial(\mathcal{L}^*_m)}{\partial\phi}, \tag{8}$$

where

$$T^*_{\mu\nu} = -\frac{2}{\sqrt{-g_*}}\frac{\partial(\mathcal{L}_m\sqrt{-g_*})}{\partial g_*^{\mu\nu}}, \tag{9}$$

and $\Box_* = \nabla^*_\mu\nabla^\mu_*$, where $\boldsymbol{g}_*$ is used to compute the covariant derivatives.

B. Interaction between matter and the dilaton field

An effective Lagrangian describing the interactions between matter and the dilaton is assumed to be described by some differentiable functions of the dilaton multiplied by the main matter fields [22],

$$\begin{aligned}\mathcal{L}_{\rm int} = \frac{D_e(\varphi)}{4e^2}F_{\mu\nu}F^{\mu\nu} - \frac{D_g(\varphi)\beta_3(g_3)}{2g_3}G^a_{\mu\nu}G_a^{\mu\nu} \\ - \sum_{i=e,u,d}(D_{m_i}(\varphi) + \gamma_{m_i}D_g(\varphi))m_i\bar{\psi}_i\psi_i,\end{aligned} \tag{10}$$

where $F_{\mu\nu}$ is the Faraday tensor, $G^a_{\mu\nu}$ the gluons tensor, g_3 the strong force coupling constant, $\beta_3(g_3) = \mu\partial\ln g_3/\partial\mu$ its beta function relative to the quantum scale invariance violation, μ the energy scale of the relevant physical processes, m_i the fermions mass, ψ_i their spinor, and $\gamma_{m_i} = -\mu\partial\ln m/\partial\mu$ the beta function relative to the dimensional anomaly of the fermions masses coupled to the gluons. The $D_i(\varphi)$ functions describe the different couplings between the matter fields and the dilaton. D_e characterizes the φ dependency of the fine structure constant, D_g the φ dependency of the QCD mass scales Λ_3, and D_{m_i} the quarks masses. The Lagrangian density (10) is a straightforward nonlinear generalization of Damour and Donoghue theory [25]. The theory considered here becomes equivalent to the one of Damour and Donoghue [25] if we set $D_i(\varphi) = d_i\varphi$. Indeed, the dependency of the constant of nature to the scalar field reads [25]

$$\alpha(\varphi) = (1 + D_e(\varphi))\alpha, \tag{11a}$$

$$\Lambda_3(\varphi) = (1 + D_g(\varphi))\Lambda_3, \tag{11b}$$

$$m_e(\varphi) = (1 + D_{m_e}(\varphi))m_e, \tag{11c}$$

$$m_q(\varphi) = (1 + D_q(\varphi))m_q, \qquad q = u, d. \tag{11d}$$

Damour and Donoghue [25] have shown that the matter action of a point mass, at a macroscopic level, becomes

$$S_m = -c^2\int m_A(\varphi)\mathrm{d}\tau_A, \tag{12}$$

where A is the label of the considered body, $d\tau_A = \sqrt{-g(\vec{v_A}, \vec{v_A})}dt$, where $\vec{v_A} = c\overrightarrow{\mathbf{d}z_A}/dx^0$ the 4-velocity of body A [$z_A \in \mathcal{M}$ is the position of body A in the manifold and its coordinates in a generic map (x^μ) are z_A^μ]. The velocity vector in the tangent space $T_{z_A}\mathcal{M}$ is denoted $\overrightarrow{v_A}$, and its coordinates are $v_A^\alpha = dz_A^\alpha / \sqrt{-g_{\mu\nu} dz_A^\mu dz_A^\nu}$. The φ dependency of m_A depends on the internal structure of body A and is responsible for the violation of the WEP. All this violation can be encoded in a coupling parameter,

$$\alpha_A(\varphi) = \frac{d \ln m_A}{d\varphi}. \tag{13}$$

Damour and Donoghue have found a semianalytical expression of α_A with respect to its atomic composition [25]. The nonlinear generalization is straightforward. At first order, one needs to replace the d_i of Damour and Donoghue by $D_i'(\varphi)$. It is convenient to decompose α_A into a universal term α_u (which does not depend explicitly on the atomic composition of the various bodies) and a nonuniversal part $\bar{\alpha}_A$ (which depends explicitly on the atomic composition). The coupling function reads

$$\alpha_A(\varphi) = \alpha_u(\varphi) + \bar{\alpha}_A(\varphi), \tag{14}$$

where a straightforward nonlinear generalization of Eqs. (71) and (72) of [25] reads

$$\begin{aligned}\alpha_u(\varphi) = D_g'(\varphi) + [&9.3 \times 10^{-2}(D_{\hat{m}}'(\varphi) - D_g'(\varphi)) \\ &- 1.4 \times 10^{-4}(D_{m_e}'(\varphi) - D_g'(\varphi)]\end{aligned} \tag{15}$$

and

$$\begin{aligned}\bar{\alpha}_A(\varphi) = &(D_{\hat{m}}'(\varphi) - D_g'(\varphi))Q_{\hat{m}}^A + (D_{\delta m}'(\varphi) - D_g'(\varphi))Q_{\delta m}^A \\ &+ (D_{m_e}'(\varphi) - D_g'(\varphi))Q_{m_e}^A + D_e'(\varphi)Q_e^A.\end{aligned} \tag{16}$$

The coupling functions to the quarks have been redefined,

$$D_{\hat{m}}(\varphi) = \frac{m_u D_{m_u}(\varphi) + m_d D_{m_d}(\varphi)}{m_u + m_d} \tag{17}$$

$$D_{\delta m} = \frac{m_d D_{m_d}(\varphi) - m_u D_{m_u}(\varphi)}{m_d - m_u}. \tag{18}$$

In Eq. (16), the *dilatonic charges* appear as $Q_{\hat{m}}^A$, $Q_{\delta m}^A$, $Q_{m_e}^A$, and Q_e^A. They are responsible for the weak equivalence principle violation, because they depend explicitly on the atomic composition of body A. Charges $Q_{\hat{m}}^A$ and $Q_{\delta m}^A$ quantify the coupling between the quarks of body A and the dilaton field, Q_{m_e} the coupling with the mass of the electrons, and Q_e the coupling with the electromagnetic field. Let $\mathcal{A}$ be the average number of nucleons of body A and $\mathcal{Z}$ its average number of protons. Then, the dilatonic charges are the same as the one in Damour and Donoghue theory [25],

$$\begin{aligned}Q_{\hat{m}}^A = &\frac{-3.6 \times 10^{-2}}{\mathcal{A}^{1/3}} - 2.0 \times 10^{-2}\frac{(\mathcal{A} - 2\mathcal{Z})^2}{\mathcal{A}^2} - 1.4 \\ &\times 10^{-4}\frac{\mathcal{Z}(\mathcal{Z}-1)}{\mathcal{A}^{4/3}},\end{aligned} \tag{19}$$

$$Q_{\delta m}^A = 1.7 \times 10^{-3}\frac{\mathcal{A} - 2\mathcal{Z}}{\mathcal{A}}, \tag{20}$$

$$Q_{m_e}^A = 5.5 \times 10^{-4}\frac{\mathcal{Z}}{\mathcal{A}}, \tag{21}$$

$$Q_e = 8.2 \times 10^{-4}\frac{\mathcal{Z}}{\mathcal{A}} + 7.7 \times 10^{-4}\frac{\mathcal{Z}(\mathcal{Z}-1)}{\mathcal{A}^{4/3}}. \tag{22}$$

Note that compared to Damour and Donoghue's work [25], we have removed the constant part of the dilatonic charges, because they are taken into account in α_u, the universal coupling constant [see Eq. (15)].

We have computed some dilatonic charges by estimating the composition of the main bodies of the solar system. We report the values in Table I. Damour and Donoghue have already computed these charges [25,27].

Let us note that if we follow the work of Nitti and Piazza [28], the electromagnetic interaction should present a trace anomaly similarly to the other interactions, and we should

TABLE I. Dilatonic charges of some atoms computed with Eqs. (19)–(22). For SiO_2 dilatonic charges, we compute the average of oxygen and silicium, as did Damour and Donoghue [25,27].

Atom	$\mathcal{A}$	$\mathcal{Z}$	$Q_m \times 10^2$	Q_{dm}	$Q_{me} \times 10^4$	Q_e
Hydrogen	1	1	−5.60	-1.70×10^{-3}	5.50	8.20×10^{-4}
Helium	4	2	−2.27	0.00	2.75	6.53×10^{-4}
Oxygen	16	8	−1.45	0.00	2.75	1.48×10^{-3}
Silicium	28.10	14	−1.21	6.05×10^{-6}	2.74	0.205
Iron	56.00	26.00	−0.994	1.21×10^{-4}	2.55	2.72×10^{-3}
Magnesium	24.30	12.00	−1.27	2.10×10^{-5}	2.72	1.85×10^{-3}
SiO_2			−1.33	3.02×10^{-6}	2.75	1.76×10^{-3}

replace $D'_e(\varphi)$ by $D'_e(\varphi) - D'_g(\varphi)$ in Eq. (16). In this case, if the coupling is universal, which means if it appears as $\mathcal{L}_{\rm int} = D(\varphi)T_{\rm SM}$—where $T_{\rm SM}$ is the whole trace anomaly of the Standard Model, which includes a classical contribution from the fermion mass terms, plus a quantum contribution containing the beta functions of the theory and the fields' anomalous dimensions—then all the coupling functions are equal, and we get $\bar{\alpha}_A = 0$ such that the weak equivalence principle is still satisfied. Damour and Donoghue do not take the electromagnetic trace anomaly into account; therefore, in their theory, even with a universal coupling, the weak equivalence principle is broken. In our computations, it is always possible to replace $D'_e(\varphi)$ by $D'_e(\varphi) - D'_g(\varphi)$ if needed. In terms of Solar system phenomenology, if the coupling functions are different, then it changes nothing for testing alternative theories in an "agnostic" way, which are blind with respect to the choices of the definitions of coupling functions. Indeed, since we do not know anything about any coupling functions *a priori*, constraining $D'_e(\varphi)$ instead of $D'_e(\varphi) - D'_g(\varphi)$ is exactly the same when we perform experimental tests.

In our nonlinear dilaton theory, some second-order terms will appear at the post-Newtonian order. We introduce the quadratic coupling function

$$\beta_A(\varphi) = \frac{\mathrm{d}\alpha_A}{\mathrm{d}\varphi} = \frac{\mathrm{d}^2 \ln m_A}{\mathrm{d}\varphi^2}. \tag{23}$$

Similarly to the decomposition introduced in Eq. (14), we can decompose the quadratic coupling function in a universal part β_u and a nonuniversal part $\bar{\beta}_A$. It reads

$$\beta_A(\varphi) = \beta_u(\varphi) + \bar{\beta}_A(\varphi), \tag{24}$$

where

$$\beta_u(\varphi) = \alpha'_u(\varphi), \tag{25}$$

and

$$\bar{\beta}_A(\varphi) = (D''_{\hat{m}}(\varphi) - D''_g(\varphi))Q^A_{\hat{m}} + (D''_{\delta m}(\varphi) - D''_g(\varphi))Q^A_{\delta m} + (D''_{m_e}(\varphi) - D''_g(\varphi))Q^A_{m_e} + D''_e(\varphi)Q^A_e. \tag{26}$$

C. Modified Einstein-Infeld-Hoffmann-Droste-Lorentz equations of motion

In Appendix A, we present the derivation of the post-Newtonian equations of motion for N massive test particles from the fields equations (7) and (8), and the description of matter presented in Sec. II B. We show that after rescaling the constants as follows:

$$\alpha_0 = \alpha^*_u(\varphi_0) = \sqrt{\frac{2}{Z(\varphi_0)}}\left(\alpha_u(\varphi_0) - \frac{f'(\varphi_0)}{2f(\varphi_0)}\right), \tag{27a}$$

$$\beta_0 = \beta^*_u(\varphi_0) = \frac{2}{Z(\varphi_0)}\left(\beta_u(\varphi_0) + \frac{1}{2}\left(\frac{f'(\varphi_0)}{f(\varphi_0)}\right)^2 - \frac{1}{2}\frac{f''(\varphi_0)}{f(\varphi_0)}\right) - \frac{Z'(\varphi_0)}{Z(\varphi_0)^2}\left(\alpha_u(\varphi_0) - \frac{f'(\varphi_0)}{2f(\varphi_0)}\right), \tag{27b}$$

$$\tilde{\alpha}_A = \sqrt{\frac{2}{Z(\varphi_0)}}\bar{\alpha}_A(\varphi_0), \tag{27c}$$

$$\tilde{\beta}_A = \frac{2}{Z(\varphi_0)}\bar{\beta}_A(\varphi_0) - \frac{Z'(\varphi_0)}{Z^2(\varphi_0)}\bar{\alpha}_A(\varphi_0), \tag{27d}$$

$$d\beta_A = \frac{\tilde{\beta}_A}{2}\frac{\alpha_0^2}{(1+\alpha_0^2)^2}, \tag{27e}$$

$$\gamma = \frac{1-\alpha_0^2}{1+\alpha_0^2}, \qquad \beta = 1 + \frac{\beta_0}{2}\frac{\alpha_0^2}{(1+\alpha_0^2)^2}, \tag{27f}$$

$$\delta_A = \frac{\alpha_0\tilde{\alpha}_A}{1+\alpha_0^2}, \qquad \delta_{AB} = \frac{\tilde{\alpha}_A\tilde{\alpha}_B}{1+\alpha_0^2}, \tag{27g}$$

$$\mu_A = \frac{G}{f_0}(1+\alpha_0^2)(1+\delta_A)m_A(\varphi_0), \tag{27h}$$

where $Z(\varphi)$ is defined by Eq. (3) and where the α and β functions are defined in the previous section, the equations of motion read, at the first post-Newtonian order,

$$\begin{aligned}\boldsymbol{a}_T = &-\sum_{A\neq T}\frac{\mu_A}{r^3_{AT}}\boldsymbol{r}_{AT}(1+\delta_T+\delta_{AT}) - \sum_{A\neq T}\frac{\mu_A}{r^3_{AT}c^2}\boldsymbol{r}_{AT}\left\{\gamma v_T^2 + (\gamma+1)v_A^2 - 2(1+\gamma)\boldsymbol{v}_A.\boldsymbol{v}_T - \frac{3}{2}\left(\frac{\boldsymbol{r}_{AT}.\boldsymbol{v}_A}{r_{AT}}\right)^2 - \frac{1}{2}\boldsymbol{r}_{AT}.\boldsymbol{a}_A\right.\\ &\left.- 2(\gamma+\beta+\mathrm{d}\beta_T)\sum_{B\neq T}\frac{\mu_B}{r_{TB}} - (2\beta + 2\mathrm{d}\beta_A - 1)\sum_{B\neq A}\frac{\mu_B}{r_{AB}}\right\} + \sum_{A\neq T}\frac{\mu_A}{c^2 r^3_{AT}}[2(1+\gamma)\boldsymbol{r}_{AT}.\boldsymbol{v}_T - (1+2\gamma)\boldsymbol{r}_{AT}.\boldsymbol{v}_A](\boldsymbol{v}_T - \boldsymbol{v}_A)\\ &+ \frac{3+4\gamma}{2}\sum_{A\neq T}\frac{\mu_A}{c^2 r_{AT}}\boldsymbol{a}_A,\end{aligned} \tag{28}$$

where μ_A is the gravitational parameter of body A, $\boldsymbol{r}_{AT}$ the relative position of body T with respect to A, $r_{AT} = |\boldsymbol{r}_{AT}|$, and $\boldsymbol{v}_A$ the coordinate velocity of body A while $\boldsymbol{a}_A$ is its coordinate acceleration. These are the modified Einstein-Infeld-Hoffmann-Droste-Lorentz (EIHDL) equations of motion.

In Einstein theory of GR (that is to say without dilaton: $\gamma - 1 = \beta - 1 = \delta_T = \delta_{AT} = \mathrm{d}\beta_A = 0$), these equations of motion have been first written by Lorentz and Droste in 1917 ([29,30], for an English translation, see [31]), then by Einstein, Infeld, and Hoffmann in 1939 [32]. This name composed of five personalities (EIHDL) tells better science history than only the three first names [33]. In Appendix B, we give some more considerations about the dynamical system of N mass monopoles in the massless dilaton theory: global Lagrangian and Hamiltonian formulation and first integrals are derived.

D. Nordtvedt effect

So far we have only considered test particles and have neglected their self-gravitation. In Einstein's GR, it is possible to proceed like this by virtue of the strong equivalence principle. However, in any tensor-scalar theory, this principle is broken. The calculation of the strong equivalence principle violation was first done by Nordtvedt (1968) by considering extended bodies as a set of only gravitationally interacting points, and the auto-gravitation energy was integrated on this set in a formalism in which the strong equivalence principle is broken [34,35]. Later (1981), Will generalized this approach by modeling the bodies as perfect fluid [36].[1] More recently (2000), Klioner and Soffel have generalized this formalism by modeling the bodies as multipolar moments in the parametrized post-Newtonian formalism [37].

A heuristic argument allows us to implement the Nordtvedt effect without performing all these integrations [38]. In Appendix C, we show that the Nordtvedt effect can be integrated in a massless dilaton theory by substituting δ_A of EILDH equations of motion by δ'_A, where

$$\delta'_A = \delta_A - (4\beta - \gamma - 3)\frac{3\mu_A}{5R_A c^2}, \qquad (29)$$

where R_A is the average radius of body A. The quantity $\mu_A/R_A c^2$ corresponds to the self-gravitating energy of body A. We use this term in all our modeling of the Solar system in the following.

E. Modified time travel

In addition to impacting the trajectory of planets, the dilaton theory will also impact the light propagation. In tensor-scalar theory, it is known that the behavior of light in the geometric optic approximation does not depend on the frame chosen [39] (Einstein versus Jordan frames). We can then use the solutions of the field equations in the Einstein frame presented in Appendix A 2 and the fact light follows the null-geodetic curves. With these approximation, a classical calculation leads to the modified time travel between an emission event e and a reception event r

$$c(t_r - t_e) = \frac{R}{c} + \sum_A (\gamma + 1 - \delta_A)\frac{\mu_A}{c^2}\ln\frac{\boldsymbol{n}\cdot\boldsymbol{r}_{rA} + r_{rA}}{\boldsymbol{n}\cdot\boldsymbol{r}_{eA} + r_{eA}}, \qquad (30)$$

where the δ_A parameter is given by Eq. (A35), $\boldsymbol{n} = (\boldsymbol{r}_r - \boldsymbol{r}_e)/\|\boldsymbol{r}_r - \boldsymbol{r}_e\|$, $\boldsymbol{r}_{iA} = \boldsymbol{r}_i - \boldsymbol{z}_A$, and $r_{iA} = \|\boldsymbol{r}_{iA}\|$ with $i = e$ or r.

III. NUMERICAL METHODS WITH INPOP

We included the modifications to the equations of motion presented in Eq. (28) and to the light propagation presented in Eq. (30) in the INPOP planetary ephemerides to search for a possible signature induced by a massless dilaton.

A. Reduction of the number of parameters

Constraining the WEP at the Solar system scale using a totally general phenomenological approach is difficult, because without any physical hypothesis or without considering a specific underlying theory, there are too many parameters to be constrained—at least as many as the number of planets. For example, this would be the case if we tested the EP by including one parameter $\delta = (m_g/m_I) - 1$ for each body (m_G and m_I being, respectively, the gravitational and the inertial masses). On the other hand, considering a specific theory like the massless dilaton allows one to search for a specific violation of the WEP limiting the parameters space to be explored. The parameters that characterize the theory described by the action from Eq. (1) are the function $f(\varphi)$, $\omega(\varphi)$ and the coupling functions characterizing the interaction between the scalar field and matter $D_i(\phi)$. At the post-Newtonian level, only the background values for the function $Z(\varphi)$ defined in Eq. (3) and of the background values for the first and second derivatives of the coupling functions impact the measurements.

In the case of a linear coupling between the scalar field and matter, only the following coefficients enter the expression of the equations of motion and of the Shapiro time delay: $\gamma = \frac{1-\alpha_0^2}{1+\alpha_0^2}$, $\delta_A = \frac{\alpha_0\tilde{\alpha}_A}{1+\alpha_0^2} + (\gamma - 1)\frac{|\Omega_A|}{m_A c^2}$, and $\delta_{AB} = \tilde{\alpha}_A\tilde{\alpha}_B/(1 + \alpha_0^2)$. These effective parameters are related to the fundamental parameters of the theory through α_0 which is defined by Eq. (27b) and through $\tilde{\alpha}_A = d_{\hat{m}} Q^A_{\hat{m}} + d_{\delta m} Q^A_{\delta m} + d_{m_e} Q^A_{m_e} + d_e Q^A_e$ [see Eqs. (27c) and (16)] with

$$d_{\hat{m}} = \sqrt{\frac{2}{Z(\varphi_0)}}[D'_{\hat{m}}(\varphi_0) - D'_g(\varphi_0)], \qquad (31a)$$

$$d_{\delta m} = \sqrt{\frac{2}{Z(\varphi_0)}}[D'_{\delta m}(\varphi_0) - D'_g(\varphi_0)], \qquad (31b)$$

[1] The cited book was published in 2018, but the first edition was published in 1981.

$$d_{m_e} = \sqrt{\frac{2}{Z(\varphi_0)}}[D'_{m_e}(\varphi_0) - D'_g(\varphi_0)], \quad (31c)$$

$$d_e = \sqrt{\frac{2}{Z(\varphi_0)}}[D'_e(\varphi_0)]. \quad (31d)$$

In case of a nonlinear coupling, the following coefficients enter also the modeling of the observations: $\beta = 1 - \beta_0\alpha_0^2/2(1-\alpha_0^2)^2$ and $d\beta_A = \tilde{\beta}_A\alpha_0^2/2(1+\alpha_0^2)^2$. These effective parameters are related to the theory through α_0, β_0 [see Eq. (27b)] and $\tilde{\beta}_A = b_{\hat{m}}Q^A_{\hat{m}} + b_{\delta m}Q^A_{\delta m} + b_{m_e}Q^A_{m_e} + b_e Q^A_e$ [see Eqs. (27d) and (26)]with

$$b_{\hat{m}} = \frac{2}{Z(\varphi_0)}[D''_{\hat{m}}(\varphi_0) - D''_g(\varphi_0)] - \frac{Z'(\varphi_0)}{Z(\varphi_0)}[D'_{\hat{m}}(\varphi_0) - D'_g(\varphi_0)], \quad (32a)$$

$$b_{\delta m} = \frac{2}{Z(\varphi_0)}[D''_{\delta m}(\varphi_0) - D''_g(\varphi_0)] - \frac{Z'(\varphi_0)}{Z(\varphi_0)}[D'_{\delta m}(\varphi_0) - D'_g(\varphi_0)], \quad (32b)$$

$$b_{m_e} = \frac{2}{Z(\varphi_0)}[D''_{m_e}(\varphi_0) - D''_g(\varphi_0)] - \frac{Z'(\varphi_0)}{Z(\varphi_0)}[D'_{m_e}(\varphi_0) - D'_g(\varphi_0)], \quad (32c)$$

$$b_e = \frac{2}{Z(\varphi_0)}D''_e(\varphi_0) - \frac{Z'(\varphi_0)}{Z(\varphi_0)}D'_e(\varphi_0). \quad (32d)$$

Note that the Nordtvedt term is also modified when taking into account nonlinear coupling such that $\delta_A = \frac{\alpha_0\tilde{\alpha}_A}{1+\alpha_0^2} + (\gamma + 3 - 4\beta)\frac{|\Omega_A|}{m_A c^2}$.

In Table II, we give the values of the dilatonic charges estimated for the main bodies of the Solar system, using Table I. We note that, except for $Q_{\delta m}$ in telluric bodies, all dilatonic charges have a similar value for the various telluric planets and have a similar value for the gaseous planets. More precisely, the variations of the dilatonic charges are less than 10% for both types of planets. Moreover, we note that in telluric bodies, $Q_{\delta m}$ is one order of magnitude smaller than the other dilatonic charges. If we assume that all coupling coefficients are of the same order of magnitude, similarly to what has been assumed in [25], we can also safely neglect the dispersion of the value of $Q_{\delta m}$ for telluric planets. Therefore, in the following, we consider that all telluric planets share the same dilatonic charges and that all gaseous planets share the same dilatonic charges as well.

TABLE II. Dilatonic charges of the main gaseous then telluric bodies of the Solar system. For SiO_2, the weighting is computed based on the number of atoms. In the last column, we compute the relative dispersion of the dilatonic charges.

	Sun	Jupiter	Saturn	Uranus	Neptune	
Hydrogen	74%	90%	96%	83%	80%	
Helium	25%	10%	3%	15%	19%	
SiO_2	0%	0%	0%	0%	0%	
Iron	0%	0%	0%	0%	0%	
Magnesium	0%	0%	0%	0%	0%	$\frac{\sigma_Q}{\langle Q\rangle}$
$\langle Q_{\hat{m}}\rangle \times 10^2$	−4.76	−5.27	−5.50	−5.09	−4.96	5.6%
$\langle Q_{\delta m}\rangle \times 10^3$	−1.27	−1.53	−1.65	−1.44	−1.37	10%
$\langle Q_{m_e}\rangle \times 10^4$	4.81	5.23	5.42	5.08	4.97	4.6%
$\langle Q_e\rangle \times 10^4$	7.78	8.03	8.15	7.94	7.88	1.8%

	Mercury	Venus/Mars	Earth	Moon	
Hydrogen	0%	0%	0%	0%	
Helium	0%	0%	0%	0%	
SiO_2	40%	80%	45%	63%	
Iron	60%	20%	32%	13%	
Magnesium	0%	0%	14%	0%	$\frac{\sigma_Q}{\langle Q\rangle}$
$\langle Q_{\hat{m}}\rangle \times 10^2$	−1.13	−1.26	−1.20	−1.27	5.3%
$\langle Q_{\delta m}\rangle \times 10^5$	7.41	2.67	4.74	2.33	54.5%
$\langle Q_{m_e}\rangle \times 10^4$	2.63	2.71	2.67	2.71	1.4%
$\langle Q_e\rangle \times 10^3$	2.34	1.95	2.11	1.93	9.1%

TABLE III. Average values of the dilatonic charges for telluric and gaseous bodies, computed from Table II.

Dilatonic charge	$\langle Q_{\hat{m}}\rangle_T$	$\langle Q_{\delta m}\rangle_T$	$\langle Q_{m_e}\rangle_T$	$\langle Q_e\rangle_T$	$\langle Q_{\hat{m}}\rangle_G$	$\langle Q_{\delta m}\rangle_G$	$\langle Q_{m_e}\rangle_G$	$\langle Q_e\rangle_G$
Average value	-1.2×10^{-2}	4.3×10^{-5}	2.7×10^{-4}	2.1×10^{-3}	-5.1×10^{-2}	-1.5×10^{-3}	5.1×10^{-4}	8.0×10^{-4}

In Table III, we present the average values for $\langle Q_{\hat{m}}\rangle_i$, $\langle Q_{\delta m}\rangle_i$, $\langle Q_{m_e}\rangle_i$, and $\langle Q_e\rangle_i$, for the telluric ($i=T$) and gaseous bodies ($i=G$). These assumptions allow us to reduce significantly the computational time to explore the parameters space. This hypothesis can be criticized, because until we do not have any empirical positive detection of the coupling constant, nothing can be told about their ratio and the fact that they should have the same order of magnitude. While a more detailed modeling including more fitted parameters would be useful in the case of a positive detection, the assumptions described above are sufficient for this first exploration.

Under these assumptions, the number of effective parameters impacting planetary ephemerides is reduced to three for the linear coupling;:α_0 given by Eq. (27b) and $\tilde{\alpha}_T$ and $\tilde{\alpha}_G$ which are related to the fundamental parameters of the theory through

$$\tilde{\alpha}_T=\sqrt{\frac{2}{Z(\varphi_0)}}[-1.2\times10^{-2}(D'_{\hat{m}}(\varphi_0)-D'_g(\varphi_0))+4.3\times10^{-5}(D'_{\delta m}(\varphi_0)-D'_g(\varphi_0)) \\ +2.7\times10^{-4}(D'_{m_e}(\varphi_0)-D'_g(\varphi_0))+2.1\times10^{-3}D'_e(\varphi_0)], \tag{33a}$$

$$\tilde{\alpha}_G=\sqrt{\frac{2}{Z(\varphi_0)}}[-5.1\times10^{-2}(D'_{\hat{m}}(\varphi_0)-D'_g(\varphi_0))-1.5\times10^{-3}(D'_{\delta m}(\varphi_0)-D'_g(\varphi_0)) \\ +5.1\times10^{-4}(D'_{m_e}(\varphi_0)-D'_g(\varphi_0))+8.0\times10^{-4}D'_e(\varphi_0)]. \tag{33b}$$

From now, and until the end of the paper, in order to simplify the notations, we remove the tilde for the first-order coupling parameters; we set $\alpha_A=\tilde{\alpha}_A$ for $A\in\{T,G\}$. If one considers nonlinear couplings, three additional parameters impact planetary ephemerides: β_0, $d\beta_T$, which is $d\beta_A$ where A runs on the telluric bodies, and $d\beta_G$, which is $d\beta_A$ where A runs over the gaseous bodies.

B. Introduction in planetary ephemerides

INPOP (Intégration Numérique Planétaire de l'Observatoire de Paris) is a planetary ephemeris developed since 2003 [40]. It consists of integrating numerically the equations of motion of the main bodies and of more than 14,000 asteroids and adjusting the model parameters to the data with a least-squares algorithm. We model the solar system as a system of monopoles, except for the Sun, and the Earth-Moon system. For the Sun, we model its oblateness $J_{2\odot}$ defined as the dimensionless coefficient of the quadratic term of the multipole potential. For the Earth-Moon system, we model multipole interactions [41]. However, concerning GR and dilaton effects, only the monopoles terms are considered. Thus, for GR effects and dilaton effects, bodies are considered as homogeneous spheres. At the current level of observational accuracy, this is the level of modelization that is necessary to minimize the residuals in the planetary ephemeris INPOP. Additional planetary parameters have been found to have no impact on the residuals. INPOP19a includes recent data from Juno which provides accurate Jupiter barycenter positions, upgrades in the Cassini data sets which provides Saturn barycenter positions, and additional Mars orbiter observations. In addition, several asteroids and a trans-Neptunian objects belt have been added [42,43], and we have reanalyzed Cassini data, including the recent Grande Finale [43]. We have summarized the most sensitive data in Table IV. The complete documentation is available in [41].

INPOP is regularly used for testing fundamental physics [11,46–48]. For getting a realistic constraint, we have already shown, in particular, in [42,48], that the dilaton parameters have to be constrained together with the parameters of the reference model. The reference model contains the equations of motion and of light propagation at the first post-Newtonian approximation of GR. The method consists of adding the alternative terms in the acceleration *before* the adjustment. With the dilaton parameters being fixed, the parameters of the reference model are then adjusted. The adjustment explores the reference solution parameters space in the vicinity of the parameters of the reference model until the convergence is reached. A statistical criterion is then applied, and only the dilaton parameters whose best fit satisfies this criterion are kept to build the likelihood. We repeat this operation for a large number of dilaton parameters selected randomly with a uniform distribution, and after this,

TABLE IV. Summary of the data sets and their average observational uncertainties, σ_r, in meters. Messenger data where provided by [44]. Cassini JPL data are those provided by JPL [45]. Cassini Navigation and Gravity flybys data and Grand Finale are those reduced by [41,43].

Observations	#	Dates	σ_r (m)
Messenger	1065	2011–2014	4.1
Mars Express	27849	2005–2017	2.0
Mars Odyssey	18234	2002–2014	1.3
Cassini JPL	166	2004–2014	25
Cassini Navigation and Gravity flybys	614	2006–2016	6.1
Cassini Grand Finale	9	2017	2.7
Juno	9	2016–2018	18.5

we can get the final distribution. The method has been presented in [42] and relies on the computation of the adjustment likelihood (as defined in [42] or [48] for each of the ephemerides obtained with some randomly selected parameters and some fitted parameters). In the present work, we explore three parameters—in the linear coupling, we have α_0, α_T, and α_G. From the point of view of the numerical resources, a Monte Carlo exploration of these three parameters is more economic than a exhaustive exploration in a three-dimensional map (while in [42,48], we could do such an exhaustive one-dimensional and two-dimensional mapping, respectively). Indeed, for one set of dilaton parameters, we need to adjust all the parameters of the reference solution several times, which means integrating the equations of motion, simulating the observations, computing the residuals, and finding the modification of the parameters with the partial derivatives and doing it again until convergence. Once this is done, we can compute the likelihood of the solution as exposed in [42,48]. This procedure takes more than one hour for each set of dilaton parameters. To speed up the process, we actually do parallel computations and estimate the likelihoods of several parameter sets at the same time. With our resources (see Sec. VI), we could compute 28 800 sets of dilaton parameters. Once we have the likelihood $L(\alpha_0, \alpha_G, \alpha_T)$, as in [42,48], we can interpret it as the probability that the solution is better (if $L > 1/2$) or worse (if $L < 1/2$) than the reference solution. Here, we perform a rejection sampling (or accept-reject algorithm). This means that for each set of values of $(\alpha_0, \alpha_T, \alpha_G)$, we keep the set of values with a probability equal to $L(\alpha_0, \alpha_T, \alpha_G)$. If the ephemeris tested has lower or equal residuals than the reference ephemeris, it has more chances to be kept, and if the residuals are higher than the reference ephemeris, it has more chances to be rejected. We repeat the operation 1000 times in order to decrease the statistical fluctuations. At the end, the survival population can be interpreted as a sampling of the posterior distribution.

Initially, we have no idea of which initial intervals of values must be chosen for the dilaton parameters. So we had to test empirically several boundaries for the initial uniform distributions of the parameters. We modified them using the following criterions:

(i) If the final distribution is close to zero out of the peak, compared to the initial boundaries, or if there is almost no survivors with the rejection sampling, we choose to reduce the selection interval width. The goal of this operation is to focus on the peak to increase the accuracy of the histogram of the posterior distribution.

(ii) If the final distribution contains too many elements too close of the initial boundary, or equivalently, if we cannot see any peak in the histogram, then we choose to enlarge the selection interval width. The goal of this operation is to avoid missing some part of the final distribution.

IV. RESIDUALS AND LIKELIHOOD FOR A LINEAR COUPLING

We plot the residuals with respect to α_0, α_T, α_G in Fig. 1, after adjustment of the planetary parameters with the dilaton parameters randomly chosen from a uniform distribution but before the rejection sampling. It is also interesting to plot the residuals with respect to the derived parameters, see Fig. 2.

It appears that the parameters δ_A^d are the most constrained by the ephemeris. Indeed, in these residuals, we can see that these parameters are the only ones for which the residuals are always increasing when the parameters are far from zero. The smallness of $\gamma - 1$ compared to previously published results (around $|\gamma - 1| \lesssim 10^{-4}$ [46,47]) can be explained by the fact that we are considering a specific model where γ is not independent from the other fitted parameters due to the presence of α_0 in all the derived parameters, see Eqs. (27). By introducing a dependency between the parameters, one reduces mechanically the variability of the parameters. This leads to a reduction of the interval of possible values for γ in the dilaton framework as far as the INPOP likelihood is concerned.

We can see that the relative variation of the residuals with respect to the derived parameters has not the canonical quadratic behavior expected in order to perform a classical least square algorithm. This latest statement validates our approach by considering a partially

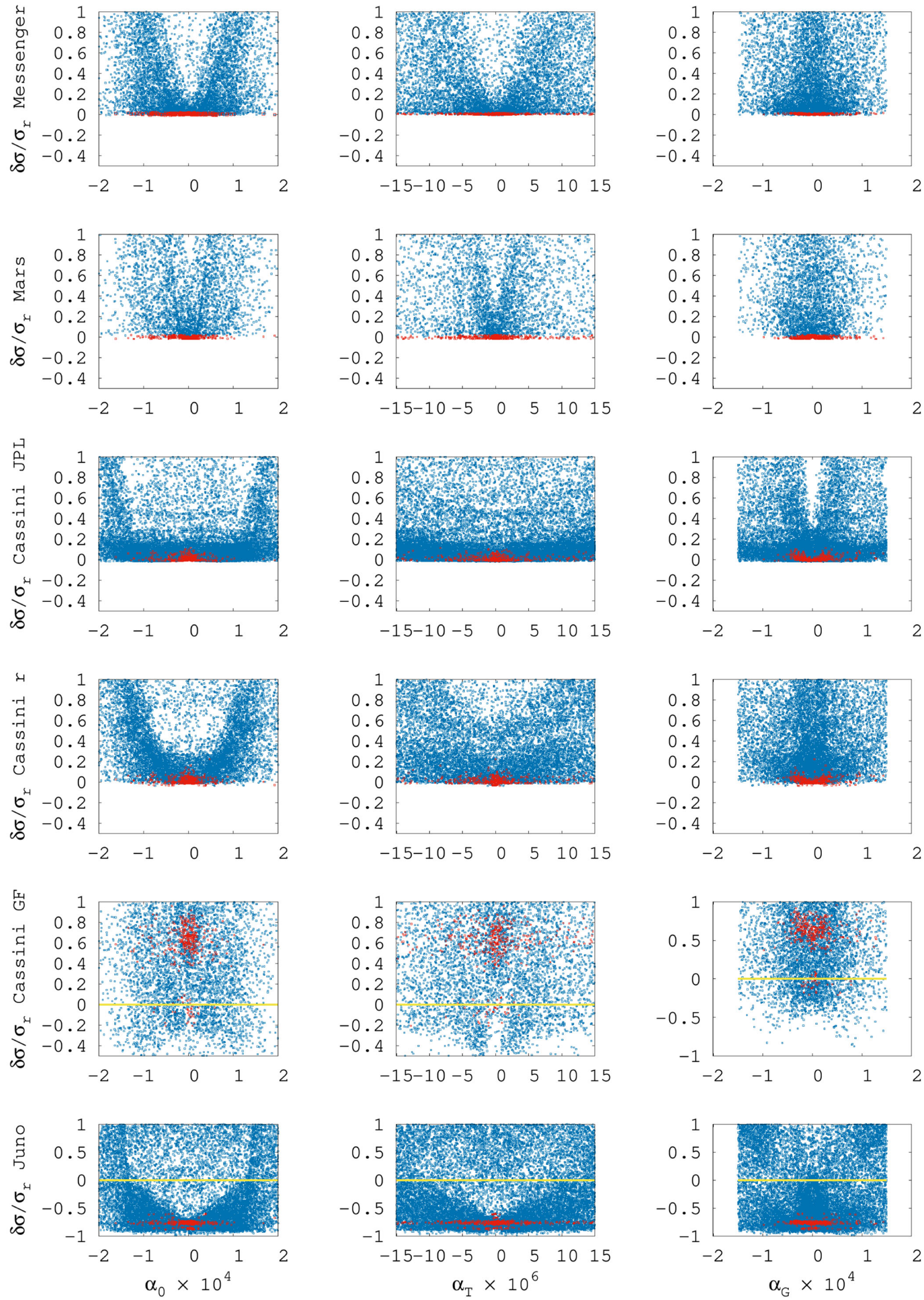

FIG. 1. Relative variations of the standard deviations ($(\sigma - \sigma_r)/\sigma_r$, where σ is the computed standard deviation, and σ_r is the standard deviation of the reference solution) of the residuals with respect to α_0, α_T, and α_G (respectively, first column, second column, and third column). We plot in red the residuals for which $L > 0.01$, that is to say the residuals of the ephemeris which have less than 99% chances to be rejected by the algorithm. Since some residuals are better than those of the reference solution, we plot a yellow line where they are equal.

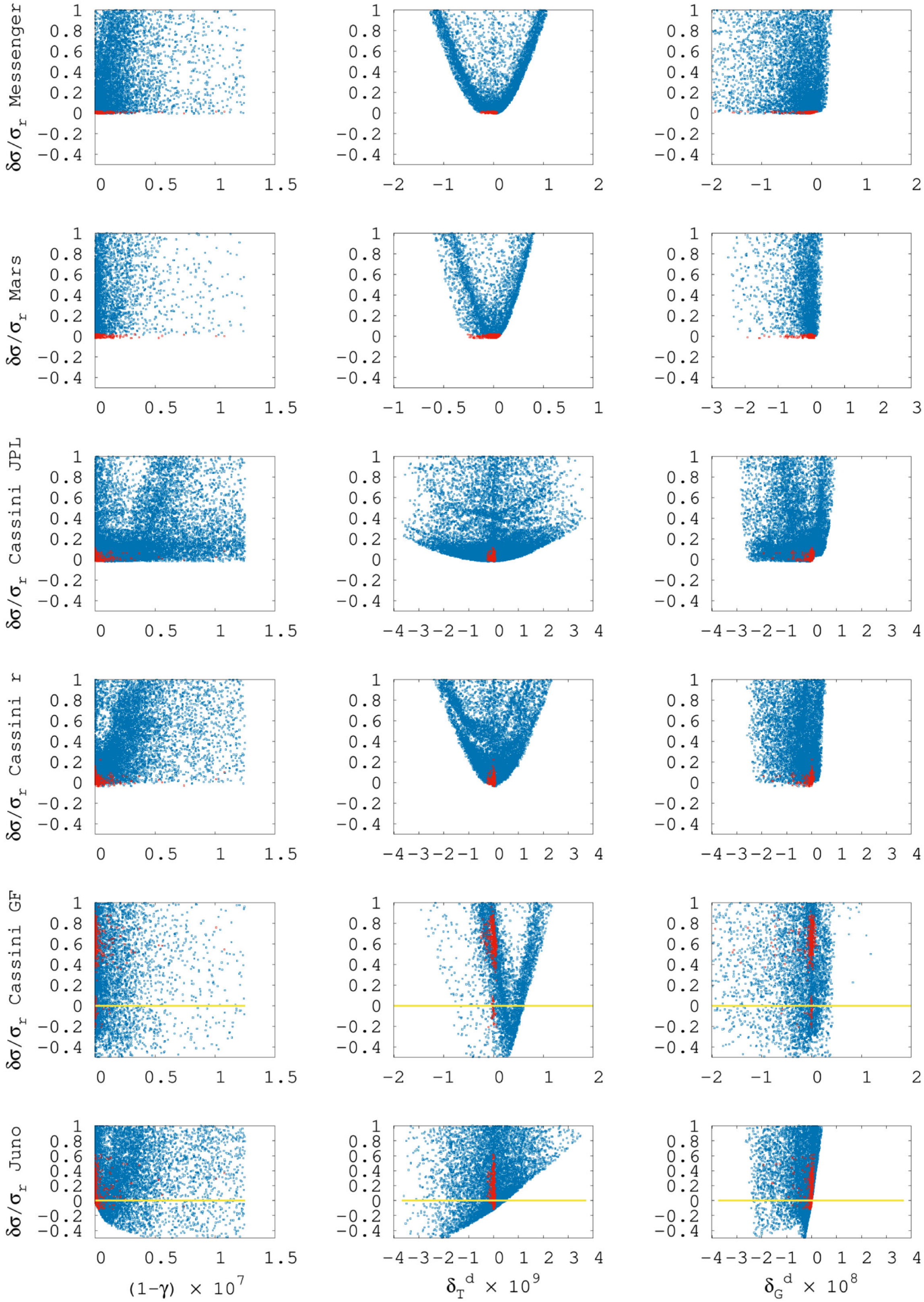

FIG. 2. Relative variations of the standard deviations ($(\sigma - \sigma_r)/\sigma_r$, where σ is the computed standard deviation, and σ_r is the standard deviation of the reference solution) of the residuals with respect to $\gamma - 1$, δ_T^d and δ_G^d (respectively, first column, second column, and third column). We plot in red the residuals for which $L > 0.01$, that is to say the residuals of the ephemeris which have less than 99% chances to be rejected by the algorithm. Since some residuals are better than those of the reference solution, we plot a yellow line where they are equal.

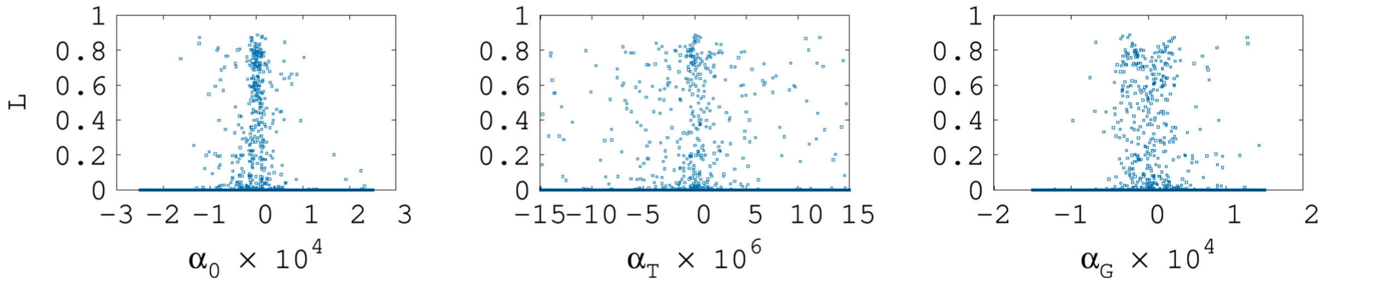

FIG. 3. Likelihood with respect to α_0, α_T, and α_G.

free-derivatives algorithm instead of a full direct least-squares inversion with heavy correlated parameters.

Finally, we plot the likelihood with respect to the tested parameters (Fig. 3) and with respect to the derived parameters (Fig. 4). We get Figs. 3 and 4.

We recall that for a given set of parameters tested $(\alpha_0, \alpha_G, \alpha_T)$ L represents the probability to be better than the reference solution, with respect to the observational χ^2 (for the reference solution itself, we have $L = 1/2$). We note a blue line confounded with the abscissa axis. This shows that a lot of sets of values for $(\alpha_0, \alpha_G, \alpha_T)$ have a very low probability to be better than the reference solution. We also see that all the nonzero values of L are located around 0 values for the abscissa axis (which represents the tested parameters). This means that all dilaton parameters acceptable zones are compatible with zero. At this step, we can already say that our results are compatible with Einstein's GR theory.

V. RESULTS FOR A LINEAR COUPLING

After the rejection sampling, an average of 163 solutions survive over 288 000 solutions tested. We repeat the rejection operation 1 000 times in order to average the statistical fluctuations and present the final results as histograms in Fig. 5.

The histograms of derived parameters (Fig. 6 shows that the constraint on $\gamma - 1$ is stronger than the usual constraints, between 10^{-4} and 10^{-5} [47,49]). As already explained, this is due to the fact that we are considering

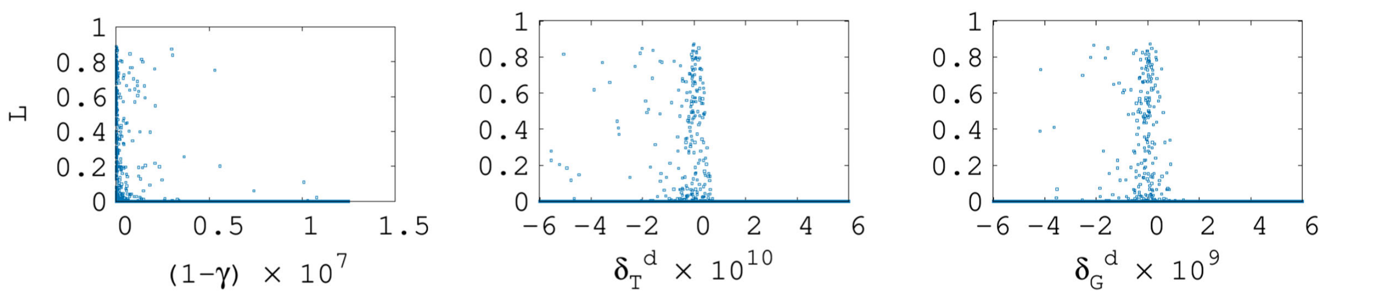

FIG. 4. Likelihood with respect to $1 - \gamma$, δ_T, and δ_G.

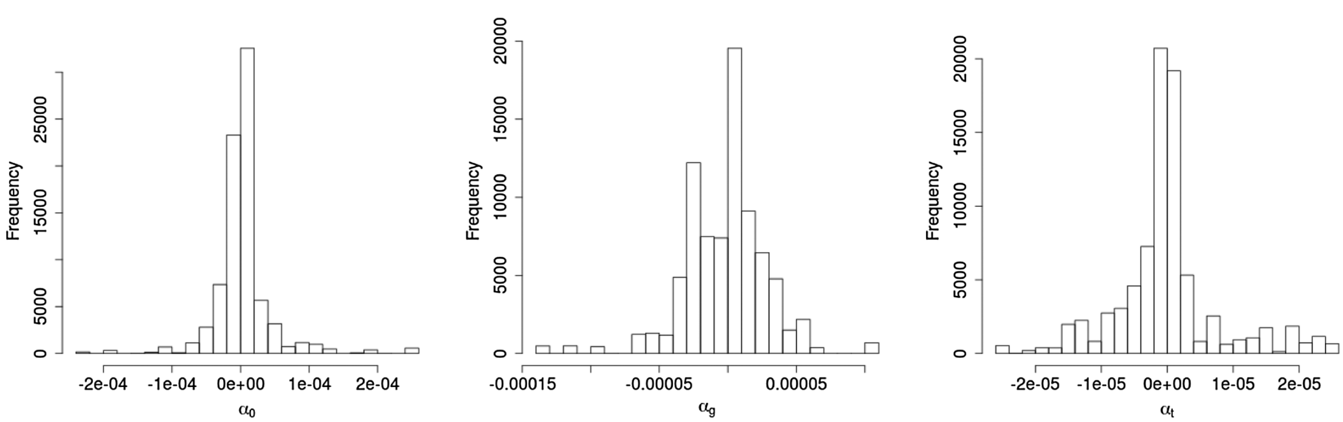

FIG. 5. Histograms of dilaton parameters in the case of a linear coupling, after averaged rejection sampling.

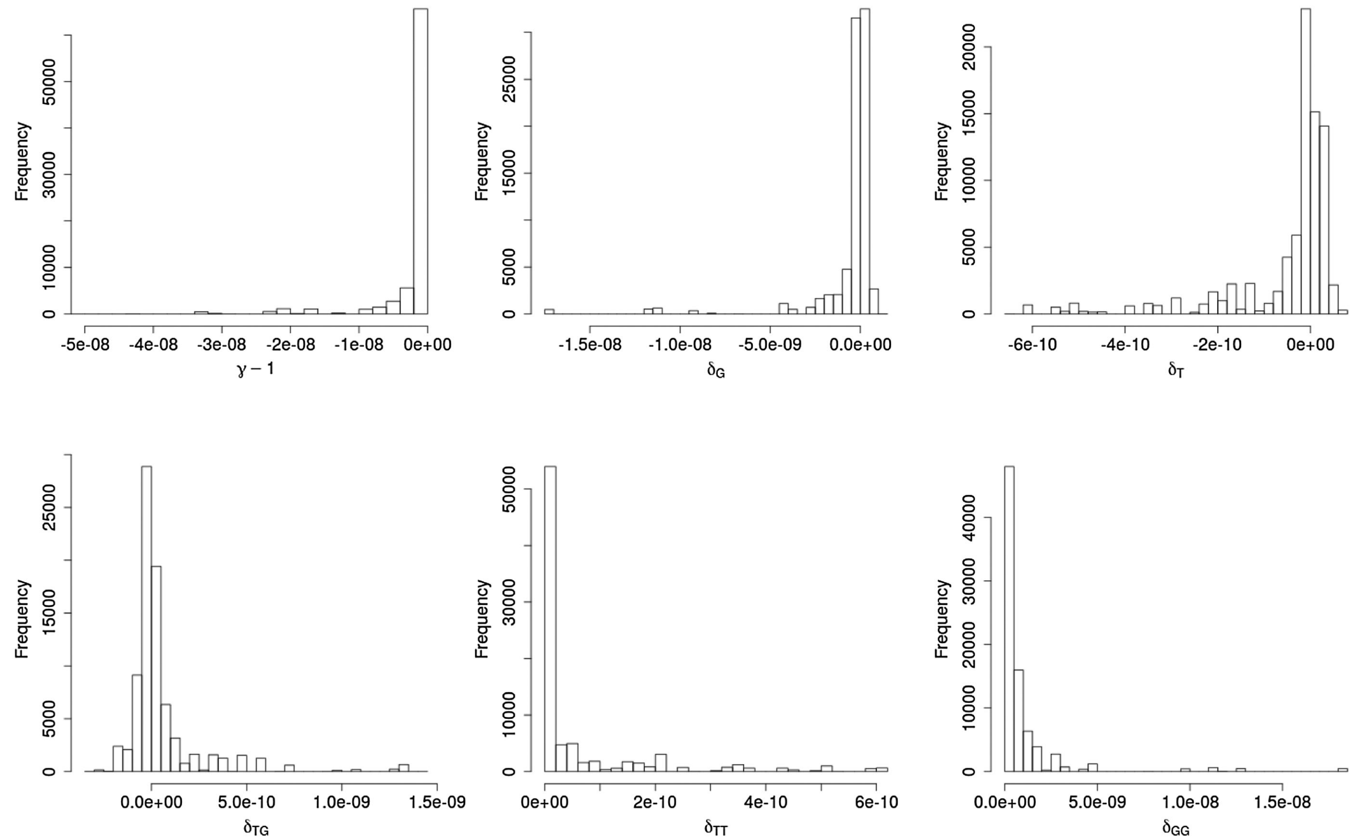

FIG. 6. Histograms of derived parameters: $\gamma - 1$, δ_G^d, δ_T^d, δ_{TG}, δ_{TT}, and δ_{GG}.

a specific model where relations are introduced in between tested parameters with the derived parameters, which all contain α_0.

In Table V, we present the different quantiles associated to the different confidence levels on the tested parameters of the massless dilaton theory considering only a linear coupling between matter and the scalar field. This is for now our constraint of the massless dilaton with nonuniversal linear coupling.

Let us note that the Lunar ephemeris alone constrains $\Delta_{ESM} = (\delta_E - \delta_{SE}) - (\delta_M - \delta_{SM})$ to be less than 10^{-13} [11], where S, E, and M stand for the Sun, Earth, and Moon respectively. However, with the set of approximations used in the present paper, one has $\Delta_{ESM} = 0$—due to the specific linear combination that defines Δ_{ESM} and to which the Lunar ephemeris is sensitive to. What is important to note is that the planetary ephemeris, on the other hand, allows one to constrain the values of δ_G, δ_T, and δ_{TG} individually—that is, not a linear combination of them—unlike the Lunar ephemeris alone, despite the greater accuracy that a Lunar ephemeris has over the whole planetary ephemeris—thanks to the much more precise data available regarding the sole position of the retroreflectors on the Moon with respect to the Earth.

TABLE V. Final results from our analysis: confidence intervals on the linear coupling parameters for the massless dilaton theory obtained using the rejection sampling.

Confidence:	90%	99.5%
$\alpha_0(\times 10^5)$	-0.94 ± 5.35	1.01 ± 23.7
$\alpha_T(\times 10^6)$	0.24 ± 1.62	0.00 ± 24.5
$\alpha_G(\times 10^5)$	0.01 ± 4.38	-1.46 ± 12.0

VI. CONCLUSION

The massless dilaton theory with nonuniversal coupling violates the WEP, the Einstein equivalence principle, and the strong equivalence principle. We have presented the phenomenology of this theory in the Solar system and have shown that because of the close chemical compositions of the telluric planets in one hand and of the gaseous planets in the other hand it was possible to reduce the number of parameters to be fitted to three in the case of a linear coupling between the dilaton and the matter and to six in the case of a quadratic coupling. In the case of a linear coupling, the parameters are α_0, a universal coupling constant, and α_T, α_G, two nonuniversal coupling constants related, respectively, to the telluric and gaseous bodies. In the quadratic coupling, one needs to add an additional universal constant, β_0, as well as two nonuniversal quadratic coupling constants related to the telluric bodies and gaseous bodies, $d\beta^T$ and $d\beta^G$, respectively.

We have tested these two scenarios with the planetary ephemeris INPOP19a. To do this, we have randomly selected many values of the parameters characterizing the dilaton theory and used the method of the likelihood based on the sensitive observations in order to get the distributions of the selected parameters. We have been able to constrain the linear coupling parameters but not the quadratic one. At 99.5% C.L., the results read $\alpha_0 = (1.01 \pm 23.7) \times 10^{-5}$, $\alpha_T = (0.00 \pm 24.5) \times 10^{-6}$, $\alpha_G = (-1.46 \pm 12.0) \times 10^{-5}$. The quadratic coupling is more difficult to constrain, due to a strong correlation between β and $J_{2\odot}$. We have some ideas on how to solve this difficulty, namely, to push the Taylor expansion of β^A_{BC} in the EILDH equations and/or to use external constraint on $J_{2\odot}$—such as the ones from the observation of the solar oblateness [50] or from helioseismic data [51].

ACKNOWLEDGMENTS

This work was granted access to the HPC resources of MesoPSL financed by the Region Ile de France and the project Equip@Meso (reference ANR-10-EQPX-29-01) of the programme Investissements d'Avenir supervised by the Agence Nationale pour la Recherche. It has also benefitted from the support of the French Space Agency (CNES).

APPENDIX A: POST-NEWTONIAN DERIVATION OF THE EQUATIONS OF MOTION

1. Stress-energy tensor

We consider a set of test particles which do not interact. The action of these points can be expressed as follows [22]:

$$S_m = -c^2 \sum_A \int m_A(\varphi) \mathrm{d}\tau_A = -c^2 \sum_A \int m_A^*(\Phi) \mathrm{d}\tau_A^*, \quad \text{(A1)}$$

where

$$m_A^*(\phi) = \sqrt{\frac{f_0}{f(\varphi)}} m_A(\varphi). \quad \text{(A2)}$$

We note that $m_A^*(\phi_0) = m_A(\varphi_0)$. The action can be expressed as a quadrivolumic integral,

$$S_m = -\int \sum_A \rho_A^* \sqrt{-g_*} \mathrm{d}^4 x, \quad \text{(A3)}$$

where

$$\rho_A^* = \frac{c^2 m_A^*(\phi(x^\mu))}{\sqrt{-g_*} u^0_{A*}} \delta^{(3)}(\boldsymbol{x} - \boldsymbol{z}_A(t)), \quad \text{(A4)}$$

where $\delta^{(3)}$ is the three-dimensional Dirac distribution, $z_A^\mu(t)$ the coordinates of A in the map (x^μ), and $u^\mu_{A*} = \mathrm{d}z^\mu_A / \sqrt{-g^*_{\alpha\beta} \mathrm{d}z^\alpha_A \mathrm{d}z^\beta_A}$ the coordinates of the 4-velocity of A. From here, we deduce

$$T_*^{\mu\nu} = \sum_A \rho_A{}^* u^\mu_{*A} u^\nu_{*A}, \quad \text{(A5)}$$

$$T_* = \sum_A \rho_A^*. \quad \text{(A6)}$$

The mass that appears in the density is a function of ϕ. In post-Newtonian formalism, the variation of ϕ is a Taylor expansion, such that we have $\phi - \phi_0 = O(c^{-2})$. Then it is also possible to express $m_*(\phi)$ as a Taylor expansion,

$$\frac{m_A^*(\phi)}{m_{A,0}^*} = 1 + (\phi - \phi_0)\alpha_A^*(\phi_0) + \frac{1}{2}(\phi - \phi_0)^2 \beta_A^*(\phi_0) + O(c^{-6}), \quad \text{(A7)}$$

where we have, from Eqs. (3), (14), (24), and (A2),

$$\alpha_A^* = \frac{\mathrm{d} \ln m_A^*}{\mathrm{d}\phi} = \alpha_u^*(\varphi) + \tilde{\alpha}_A(\varphi), \quad \text{(A8)}$$

$$\beta_A^*(\varphi) = \frac{\mathrm{d}\alpha_A^*}{\mathrm{d}\phi} = \beta_u^*(\varphi) + \tilde{\beta}_A(\varphi), \quad \text{(A9)}$$

where

$$\alpha_u^*(\varphi) = \sqrt{\frac{2}{Z(\varphi)}} \left(\alpha_u(\varphi) - \frac{f'(\varphi)}{2f(\varphi)} \right), \quad \text{(A10)}$$

$$\tilde{\alpha}_A = \sqrt{\frac{2}{Z(\varphi)}} \bar{\alpha}_A(\varphi), \quad \text{(A11)}$$

$$\beta_u^*(\phi) = \frac{2}{Z(\varphi)} \left(\beta_u(\phi) + \frac{1}{2} \left(\frac{f'(\varphi)}{f(\varphi)} \right)^2 - \frac{1}{2} \frac{f''(\varphi)}{f(\varphi)} \right) - \frac{Z'(\varphi)}{Z(\varphi)^2} \left(\alpha_u(\varphi) - \frac{f'(\varphi)}{2f(\varphi)} \right), \quad \text{(A12)}$$

$$\tilde{\beta}_A(\phi) = \frac{2}{Z(\varphi)} \bar{\beta}_A(\varphi) - \frac{Z'(\varphi)}{Z(\varphi)^2} \bar{\alpha}_A(\varphi). \quad \text{(A13)}$$

2. Post-Newtonian solution to the field equations in the Einstein frame

We consider N mass monopoles and their worldlines in $\mathcal{M}$. We define a map of $\mathcal{M}$, (x^μ), such that at infinity the metric tensor is the Cartesian Minkowski metric: $\|x^i\| \to \infty \Rightarrow g^*_{\mu\nu} \to \eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$. In this map, the worldlines of each body are described by four functions,

$$\mathcal{L}_A:\ t \mapsto z_A^\mu(t), \tag{A14}$$

that we can reduce to three if we consider t as the time coordinate. This choice is always possible for massive particles. We assume that in the gravitational fields the particles are weak ($GM/cr \ll 1$) and the velocities are also weak ($v/c \ll 1$). In these approximations, and in Einstein frame, the consequence is

$$T_*^{00} = O(c^{+2}), \qquad T_*^{0i} = O(c^{+1}), \qquad T_*^{ij} = O(c^0). \tag{A15}$$

Moreover, since we know that $\phi - \phi_0 = O(c^{-2})$, then we have $(\partial_i\phi, \partial_0\phi) = O(c^{-2}, c^{-3})$, such that from Eq. (7) one deduces that there exists a coordinate system that satisfies the strong isotropy condition as follows [52]:

$$g_{00}^* = -1 + 2\frac{w_*}{c^2} - 2\frac{w_*^2}{c^4} + O(c^{-6}), \tag{A16}$$

$$g_{0i}^* = -4\frac{w_*^i}{c^3} + O(c^{-5}), \tag{A17}$$

$$g_{ij}^* = \delta_{ij}\left(1 + 2\frac{w_*}{c^2}\right) + O(c^{-4}), \tag{A18}$$

where w_* and w_*^i are four fields that parametrize the metric tensor and that have to be determined. When linearizing Eq. (7) and using the time component of the harmonic gauge equation—that is, $g_*^{\alpha\beta}\Gamma^0_{*\alpha\beta} = 0$, where Γ_* is the Christoffel symbol constructed upon the Einstein-frame metric—we obtain partial differential equations that determine the potentials w_* and w_*^i,

$$\Box w_* = -4\pi G\frac{T_*^{00} + \delta_{ij}T_*^{ij}}{c^2} + O(c^{-4}), \tag{A19}$$

$$\Delta w_*^i = -4\pi G\frac{T_*^{0i}}{c} + O(c^{-4}). \tag{A20}$$

Linearizing Eq. (8) leads to a partial derivative equation that determines the scalar field ϕ,

$$\Box\phi = -4\pi G\frac{\partial T_*}{c^2\partial\phi} + O(c^{-6}). \tag{A21}$$

From here, a straightforward perturbative calculation—which can also be deduced from the method of Damour and Esposito-Farèse [26]—leads to the post-Newtonian solution of the field equations,

$$\phi = \phi_0 - \sum_A \alpha_{A0}^* \frac{G_* m_{A0}}{c^2 r_A}\left[1 + \frac{\boldsymbol{r}_A\cdot\boldsymbol{a}_A}{2c^2} + \frac{(\boldsymbol{r}_A\cdot\boldsymbol{v}_A)^2}{2c^2 r_A^2} + \frac{1}{c^2}\sum_{B\neq A}\frac{G_* m_{B0}}{r_{AB}}\left(1 + \alpha_{A0}^*\alpha_{B0}^* + \frac{\beta_{B0}^*\alpha_{B0}^*}{\alpha_{A0}^*}\right)\right], \tag{A22}$$

where

$$w_* = w_{0*} - \frac{1}{c^2}\Delta_* + O(c^{-4}), \tag{A23}$$

$$w_*^i = \sum_A \frac{G_* m_{A0}}{r_A} v_A^i + O(c^{-2}), \tag{A24}$$

where

$$w_0^* = \sum_A \frac{G_* m_{A0}}{r_A}, \tag{A25}$$

$$\Delta_* = \sum_A \frac{G_* m_{A0}}{r_A}\left[\frac{\boldsymbol{r}_A\cdot\boldsymbol{a}_A}{2} + \frac{(\boldsymbol{r}_A\cdot\boldsymbol{v}_A)^2}{2r_A^2} - 2v_A^2 + \sum_{B\neq A}(1 + \alpha_{A0}^*\alpha_{B0}^*)\frac{G_* m_{B0}}{r_{AB}}\right], \tag{A26}$$

where $\alpha_{A0}^* = \alpha_A^*(\phi_0)$, $\beta_{A0}^* = \beta_A^*(\phi_0)$, $\boldsymbol{r}_A = \boldsymbol{x} - \boldsymbol{z}_A$, $r_A = \|\boldsymbol{r}_A\|$, $\boldsymbol{a}_A = \mathrm{d}\boldsymbol{v}_A/\mathrm{d}t$.

3. Equations of motion

A way to deduce the equations of motion is to derive the Euler-Lagrange equation of a test particle in the gravitational field built by the other particle,

$$L_T = -c^2\int m_T^*(\phi)\sqrt{g_{\mu\nu}^* v_T^\mu v_T^\nu}. \tag{A27}$$

Performing a post-Newtonian expansion using the solution of the field equations and Eq. (A7) leads to the following Lagrangian:

$$\begin{aligned} L_T = &-m_{T0}c^2 + m_{T0}\frac{v_T^2}{2} + \sum_{A\neq T}\frac{G_{AT}m_{A0}m_{T0}}{r_{AT}} + \frac{m_{T0}v_{T0}^4}{8c^2} \\ &+ \frac{1}{c^2}\sum_{A\neq T}\frac{G_{AT}m_{A0}m_{T0}}{r_{AT}}\left[-2(1+\gamma_{AT})\boldsymbol{v}_A\cdot\boldsymbol{v}_T + \frac{2\gamma_{AT}+1}{2}v_T^2 + (\gamma_{AT}+1)v_A^2 - \frac{(\boldsymbol{v}_A\cdot\boldsymbol{r}_{AT})^2}{2r_{AT}^2} - \frac{\boldsymbol{r}_{AT}\cdot\boldsymbol{a}_A}{2}\right] \\ &- \frac{1}{c^2}\sum_{A\neq T}\frac{G_{AT}m_{A0}m_{T0}}{r_{AT}}\left[\sum_{B\neq A}\frac{G_{AB}m_{B0}}{r_{AB}}(2\beta_{BT}^A - 1) + \sum_{B\neq T}\frac{G_{BT}m_{B0}}{r_{BT}}\left(\beta_{AB}^T - \frac{1}{2}\right)\right], \end{aligned} \tag{A28}$$

where $\boldsymbol{r}_{AT} = \boldsymbol{x}_T - \boldsymbol{x}_A$, $r_{AT} = \|\boldsymbol{r}_{AT}\|$,

$$G_{AB} = \frac{G}{f_0}(1 + \alpha_{A0}^*\alpha_{B0}^*), \tag{A29}$$

$$\gamma_{AT} = \frac{1-\alpha^*_{A0}\alpha^*_{T0}}{1+\alpha^*_{A0}\alpha^*_{T0}}, \tag{A30}$$

$$\beta^T_{AB} = 1 + \frac{\beta^*_{T0}}{2}\frac{\alpha^*_{A0}}{1+\alpha^*_{A0}\alpha^*_{T0}}\frac{\alpha^*_{B0}}{1+\alpha^*_{B0}\alpha^*_{T0}}. \tag{A31}$$

Actually, this Lagrangian was already obtained by Damour and Esposito-Farèse [26] but with a universal conformal coupling and not a nonuniversal one. However, they considered the conformal coupling in a very general way and did all the calculations with $\alpha_A = \mathrm{d}\ln m_A/\mathrm{d}\phi$ without assuming anything about the nature of the coupling during the calculations, so we can use their work for checking our equations of motion.

Nevertheless, this Lagrangian can be simplified by taking into account the composition independent and dependent nature of some parameters. First, we can decompose the coupling constants in a universal part and a nonuniversal one,

$$\alpha^*_{A0} = \alpha_0 + \tilde{\alpha}_A, \tag{A32}$$

where $\alpha_0 = \alpha^*_u(\phi_0)$ [Eq. (A10)] is the universal coupling constant, and $\tilde{\alpha}_A$ has been defined in Eq. (A11). We can then redefine the gravitational constant as follows:

$$G_{AB} = \tilde{G}(1+\delta_A+\delta_B+\delta_{AB}), \tag{A33}$$

where

$$\tilde{G} = \frac{G}{f_0}(1+\alpha_0^2), \tag{A34}$$

$$\delta_A = \frac{\alpha_0\tilde{\alpha}_A}{1+\alpha_0^2}, \tag{A35}$$

$$\delta_{AB} = \frac{\tilde{\alpha}_A\tilde{\alpha}_B}{1+\alpha_0^2}. \tag{A36}$$

Most of the alternative constants can be absorbed by redefining the masses as follows:

$$\tilde{m}_A = m_{A0}(1+\delta_A). \tag{A37}$$

Then, the Lagrangian of a particle test reads, at the Newtonian approximation,

$$L_T = -\tilde{m}_T(1-\delta_T)c^2 + \tilde{m}_T\frac{v_T^2}{2}(1-\delta_T) + \sum_{A\neq T}\frac{\tilde{G}\tilde{m}_A\tilde{m}_T}{r_{AT}}(1+\delta_{AT}) + O(c^{-2}) + O(\delta_i^2). \tag{A38}$$

The equations of motion read, at the Newtonian order,

$$\boldsymbol{a}_T = \sum_{A\neq T}\tilde{G}\,\tilde{m}_A\frac{\boldsymbol{r}_{AT}}{r_{AT}^3}(1+\delta_T+\delta_{AT}). \tag{A39}$$

At this step, we note that the weak equivalence principle is broken at several levels. The variation of the ratio between the inertial mass and the gravitational mass is encoded in the parameter δ_T. But here a new violation appears through the parameter δ_{AT}. In some cases, it is possible that $\alpha_0 = 0$ such that $\delta_T = 0$ and only $\delta_{AT} \neq 0$, such that the weak equivalence principle violation cannot be reduced to a variation of the inertial/gravitational masses ratio [11,22]. Let us note that only the scalars $\mu_A = \tilde{G}\tilde{m}_A$ can be measured by a gravitation experiment. We can multiply L_T by $\tilde{G}$, and then, only the scalars μ_A appear. At the post-Newtonian level, we can neglect the terms of order $O(\delta_T c^{-2})$ and $O(\delta_{AT}c^{-2})$. Indeed, if they exist, they will be detected first at the Newtonian level. Thus, we keep only the universal coupling constant in the post-Newtonian approximation, since they have been absorbed at the Newtonian level by a unobservable redefinition of masses and gravitational constant. The resulting Lagrangian reads

$$\begin{aligned} L_T = {}& -\mu_T(1-\delta_T)c^2 + \frac{\mu_T v_T^2}{2}(1-\delta_T) \\ & + \sum_{A\neq T}\frac{\mu_A\mu_T}{r_{AT}}(1+\delta_{AT}) + \frac{\mu_T v_T^4}{8c^2} \\ & + \sum_{A\neq T}\frac{\mu_A\mu_T}{r_{AT}c^2}\Bigg[-2(\gamma+1)\boldsymbol{v}_T\cdot\boldsymbol{v}_A + \frac{2\gamma+1}{2}v_T^2 \\ & + (\gamma+1)v_A^2 - \frac{\boldsymbol{r}_{AT}\cdot\boldsymbol{a}_A}{2} - \frac{(\boldsymbol{r}_{AT}\cdot\boldsymbol{v}_A)^2}{2r_{AT}^2}\Bigg] \\ & - \sum_{A\neq T}\frac{\mu_A}{r_{AT}}\Bigg[\sum_{B\neq T}\frac{2\beta_T-1}{2c^2}\frac{\mu_B}{r_{BT}} + \sum_{B\neq A}\frac{2\beta_A-1}{c^2}\frac{\mu_B}{r_{AB}}\Bigg)\Bigg] \\ & + O(c^{-4}) + O(c^{-2}\delta_i) + O(\delta_i^2), \end{aligned} \tag{A40}$$

where

$$\gamma = \frac{1-\alpha_0^2}{1+\alpha_0^2}, \tag{A41}$$

and where β_A can be decomposed in a universal and nonuniversal part,

$$\beta_A = 1 + \frac{\beta_0+\tilde{\beta}_A}{2}\frac{\alpha_0^2}{(1+\alpha_0^2)^2} = \beta + d\beta_A, \tag{A42}$$

where $\beta = 1+\beta_0\alpha_0^2/2(1+\alpha_0^2)^2$ and $d\beta_A = \tilde{\beta}_A\alpha_0^2/2(1+\alpha_0^2)^2$. We can make the post-Newtonian expansion of the nonuniversal part of β_A since it does not appear in the Newtonian part. We find the well-known γ and β post-Newtonian parameters [36,37], to which we add nonuniversal coupling constants δ_A, δ_{AB}, and $d\beta_A$. The derivation of this Lagrangian leads to the modified Einstein-Infeld-Hoffmann-Droste-Lorentz (EIHDL) equations, which are Eq. (28).

APPENDIX B: GLOBAL LAGRANGIAN AND HAMILTONIAN FORMULATION AND FIRST INTEGRALS

We give a global Lagrangian and Hamiltonian formulation of the modified EIHDL equations of motion in massless dilaton theory. Then, we give an explicit form of the Euler-Lagrange equation in post-Newtonian formalism.

1. Global Lagrangian formulation

We can write the global post-Newtonian Lagrangian of a N-body system. The total Lagrange function L is not equal to the sum of the one-body Lagrangians. To get the same equations of motion, we need to check that $\partial L/\partial \boldsymbol{r}_A = \partial L_A/\partial \boldsymbol{r}|_{r=r_A}$. To do so, one just has to sum L_T over T, then symmetrize the explicit expressions including A and T interms of $\boldsymbol{z}_A$ and $\boldsymbol{v}_A$ [33]. Moreover, we can simplify the Lagrangian and make the term $\boldsymbol{a}_A$ disappear by remarking that

$$-\frac{\boldsymbol{r}_{AT}}{r_{AT}}\cdot\boldsymbol{a}_A = -\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\boldsymbol{r}_{AT}}{r_{AT}}\cdot\boldsymbol{v}_A\right) + \frac{(\boldsymbol{v}_T-\boldsymbol{v}_A)}{r_{AT}}\cdot\boldsymbol{v}_A - \frac{(\boldsymbol{r}_{AT}\cdot\boldsymbol{v}_A)}{r_{AT}}\frac{(\boldsymbol{r}_{AT}\cdot\boldsymbol{v}_T)}{r_{AT}} + \left(\frac{\boldsymbol{r}_{AT}\cdot\boldsymbol{v}_A}{r_{AT}}\right)^2. \tag{B1}$$

One can then replace the term containing the acceleration by the right-hand side ignoring the total derivative. We can also get the Lagrangian from the global Lagrangian of Damour and Esposito-Farèse [26] in the case of a conformal coupling by substituting our redefinitions of the constants. The result is the following where we have replaced label T by label A in order to get a more "canonical" expression of the Lagrangian, according to "canonical" post-Newtonian litterature, for example, [33,36,37]:

$$L = -\sum_A \mu_A(1-\delta_A)c^2 + L_N + \frac{1}{c^2}\sum_A L_A + \frac{1}{2c^2}\sum_A\sum_{B\neq A} L_{AB} + \frac{1}{2c^2}\sum_A\sum_{B\neq A}\sum_{C\neq A} L^A_{BC}, \tag{B2}$$

where

$$L_N = \sum_A \mu_A(1-\delta_A)\frac{v_A^2}{2} + \frac{1}{2}\sum_A \frac{\mu_A\mu_B}{r_{AB}}(1+\delta_{AB}) \tag{B3}$$

$$L_A = \frac{\mu_A v_A^4}{8}, \tag{B4}$$

$$L_{AB} = \frac{\mu_A\mu_B}{r_{AB}}\left[(2\gamma+1)v_A^2 - \frac{4\gamma+3}{2}\boldsymbol{v}_A\cdot\boldsymbol{v}_B - \frac{1}{2}(\boldsymbol{v}_A\cdot\boldsymbol{n}_{AB})(\boldsymbol{v}_B\cdot\boldsymbol{n}_{AB})\right], \tag{B5}$$

and

$$L^A_{BC} = -\frac{\mu_A\mu_B\mu_C}{r_{AB}r_{AC}}(2\beta + 2d\beta^A - 1), \tag{B6}$$

where $\boldsymbol{n}_{AB} = \boldsymbol{r}_{AB}/r_{AB}$. The first mass term $-\sum_A \mu_A(1-\delta_A)c^2$ is useless to derive the equations of motion but will be useful to express simply the first integrals, in particular, the barycenter. The Newtonian part L_N of the Lagrange function expresses the weak equivalence principle violation because, first, the inertial masses $\mu_A(1-\delta_A)$ are not equal to the gravitational masses, which become inseparable because their meaning is expressed only by the interaction of two bodies and are expressed by $\mu_A\mu_B(1+\delta_{AB})$. The last three-body term L^A_{BC} is also responsible for a weak equivalence principle violation because of $d\beta^A$, which expresses that the three-body interaction depends on the internal composition of the body which is in motion but also of the two bodies which generate the gravitational in which the body is in motion. Indeed, when we derive this term with respect to $\boldsymbol{r}_T$, terms proportional to $d\beta^T$ appear but also terms proportional to $d\beta^A$ where $A\neq T$. We note finally that L_{AB} is symmetric by permutation of A and B, but L^A_{BC} is only symmetric by permutation of B and C. It is because of these symmetries that we had to divide the sums by two.

Even by violating the equivalence principle in all these ways, this Lagrange function conserves the same symmetries as the modified GR with post-Newtonian parameters β and γ [53]. In principle, if the massless dilaton theory was well respected, each mass should be modified, but here we neglect the terms at the order $O(c^{-2}\delta_A)$.

The linear momentum of body A is

$$\boldsymbol{p}_A = \mu_A(1-\delta_A)\boldsymbol{v}_A + \frac{\mu_A v_A^2}{2c^2}\boldsymbol{v}_A + \frac{1}{c^2}\sum_{B\neq A}\frac{\mu_A\mu_B}{r_{AB}}\left[(2\gamma+1)\boldsymbol{v}_A - \frac{4\gamma+3}{2}\boldsymbol{v}_B - \frac{1}{2}(\boldsymbol{v}_B\cdot\boldsymbol{n}_{AB})\boldsymbol{n}_{AB}\right]. \tag{B7}$$

The Lagrangian (B2) is invariant by spatial rotation and translation and by temporal translation. The seven classical first integrals are well conserved. Linear momentum is

$$\begin{aligned}\boldsymbol{P} &= \sum_A \boldsymbol{p}_A \\ &= \sum_A \mu_A \boldsymbol{v}_A \left[1 - \delta_A + \frac{1}{2c^2}\left(v_A^2 - \sum_{B\neq A}\frac{\mu_B}{r_{AB}}\right)\right] \\ &\quad - \frac{1}{2c^2}\sum_A\sum_{B\neq A}\frac{\mu_A\mu_B}{r_{AB}}(\boldsymbol{n}_{AB}\cdot\boldsymbol{v}_A)\boldsymbol{n}_{AB}. \end{aligned} \tag{B8}$$

Angular momentum is

$$\begin{aligned}\boldsymbol{J} &= \sum_A \boldsymbol{z}_A \times \boldsymbol{p}_A \\ &= \sum_A \mu_A \boldsymbol{z}_A \times \boldsymbol{v}_A\left(1 - \delta_A + \frac{v_A^2}{2c^2}\right) \\ &\quad + \frac{1}{c^2}\sum_A\sum_{B\neq A}\frac{\mu_A\mu_B}{r_{AB}}\left[(2\gamma+1)\boldsymbol{z}_A\times\boldsymbol{v}_A \right. \\ &\quad \left. - \frac{4\gamma+3}{2}\boldsymbol{z}_A\times\boldsymbol{v}_B - \frac{1}{2}(\boldsymbol{v}_B\cdot\boldsymbol{n}_{AB})\boldsymbol{z}_A\times\boldsymbol{n}_{AB}\right]. \end{aligned} \tag{B9}$$

Energy is

$$\begin{aligned} h &= \sum_A \boldsymbol{p}_A\cdot\boldsymbol{v}_A - L \\ &= \sum_A (1-\delta_A)\mu_A\left(c^2 + \frac{v_A^2}{2}\right) + \sum_A \frac{3\mu_A v_A^4}{8c^2} \\ &\quad + \frac{1}{2c^2}\sum_A\sum_{B\neq A}\sum_{C\neq A}\frac{\mu_A\mu_B\mu_C}{x_{AB}x_{AC}}(2\beta + 2d\beta^A - 1) \\ &\quad - \frac{1}{2}\sum_A\sum_{B\neq A}\frac{\mu_A\mu_B}{x_{AB}}\left[1 + \delta_{AB} - \frac{2\gamma+1}{c^2}v_A^2 \right. \\ &\quad \left. + \frac{4\gamma+3}{2c^2}\boldsymbol{v}_A\cdot\boldsymbol{v}_B + \frac{1}{2c^2}(\boldsymbol{v}_A\cdot\boldsymbol{n}_{AB})(\boldsymbol{v}_B\cdot\boldsymbol{n}_{AB})\right]. \end{aligned} \tag{B10}$$

Three more first integrals can be obtained; they actually correspond to the Lorentz invariance. A direct derivation shows that the following vector is a first integral:

$$\boldsymbol{q} = \boldsymbol{G} - \boldsymbol{V}t, \tag{B11}$$

where

$$\boldsymbol{G} = \frac{c^2}{h}\sum_A \mu_A \boldsymbol{z}_A\left(1 - \delta_A + \frac{v_A^2}{2c^2} - \frac{1}{2c^2}\sum_{B\neq A}\frac{\mu_B}{r_{AB}}\right) \tag{B12}$$

are the coordinates of the relativistic barycenter of the system, and

$$\boldsymbol{V} = \frac{c^2\boldsymbol{P}}{h} \tag{B13}$$

is the velocity of the barycenter motion. $\boldsymbol{q}$ is called barycenter constant, since we have

$$\boldsymbol{G} = \boldsymbol{q} + \boldsymbol{V}t, \tag{B14}$$

and $\boldsymbol{q}$ is the constant component of $\boldsymbol{G}$.

2. Derivation of Euler-Lagrange equations of motion

In order to avoid indexes of confusion when we derive, we derive L with respect to $\boldsymbol{z}_X$ and $\boldsymbol{v}_X = \mathrm{d}\boldsymbol{z}_X/\mathrm{d}t$. This work uses the same method as [53] but is a generalization in the massless dilaton framework.

Euler-Lagrange equations of motion are

$$\begin{aligned}\boldsymbol{a}_X &= F_X^{-1}\left[\boldsymbol{H}_X - \frac{1}{c^2}(\boldsymbol{v}_X\cdot\boldsymbol{a}_X)\boldsymbol{v}_X + \frac{1}{c^2}\sum_{B\neq X}\frac{\mu_B}{r_{XB}}\left(\frac{4\gamma+3}{2}\boldsymbol{a}_B \right.\right. \\ &\quad \left.\left. + \frac{1}{2}(\boldsymbol{a}_B\cdot\boldsymbol{n}_{XB})\boldsymbol{n}_{XB}\right)\right], \end{aligned} \tag{B15}$$

where

$$F_X = 1 - \delta_X + \frac{v_X^2}{2c^2} + \frac{2\gamma+1}{c^2}\sum_{B\neq X}\frac{\mu_B}{r_{XB}} \tag{B16}$$

and

$$\begin{aligned}\boldsymbol{H}_X &= \sum_{B\neq X}\mu_B\frac{\boldsymbol{r}_{XB}}{r_{XB}^3}\left[1 + \delta_{BX} - \frac{3}{2}(\boldsymbol{v}_B\cdot\boldsymbol{n}_{XB})^2 \right. \\ &\quad \left. - (2\gamma+2)\boldsymbol{v}_X\cdot\boldsymbol{v}_B + (\gamma+1)v_B^2 + \frac{2\gamma+1}{2}v_X^2\right] \\ &\quad - \frac{1}{c^2}\sum_{B\neq X}\frac{\mu_B}{r_{XB}^2}[(2\gamma+1)(\boldsymbol{n}_{XB}\cdot(\boldsymbol{v}_X - \boldsymbol{v}_B))(\boldsymbol{v}_X - \boldsymbol{v}_B) \\ &\quad - (\boldsymbol{n}_{XB}\cdot\boldsymbol{v}_B)\boldsymbol{v}_B] \\ &\quad - \frac{1}{c^2}\sum_{B\neq X}\mu_B\frac{\boldsymbol{r}_{XB}}{r_{XB}^3}\left((2\beta + 2d\beta^X - 1)\sum_{C\neq X}\frac{\mu_C}{r_{XC}} \right. \\ &\quad \left. + (2\beta + 2d\beta^B - 1)\sum_{C\neq B}\frac{\mu_C}{r_{BC}}\right). \end{aligned} \tag{B17}$$

We note that like in [53] the exact resolution of Euler-Lagrange equations (B15) conserves exactly the following first integrals: linear momentum [Eq. (B8)], angular momentum [Eq. (B9)], and energy [Eq. (B10)]. However, they are not easy to integrate because they do not appear in the form of an ordinary differential equation. To solve it numerically, one has to invert a matrix on each step of time. However, in the first post-Newtonian approximation $O(c^{-2})$, one can get a second-order ordinary differential equation, but we lose the property of exactitude of the first integrals. A smooth signal remains at the order $O(c^{-4})$. By setting

$$F_X^{-1} = 1 + \delta_X - \frac{v_X^2}{2c^2} - \frac{2\gamma+1}{c^2}\sum_{B\neq X}\frac{\mu_B}{r_{XB}} + O(c^{-4}) \tag{B18}$$

and by neglecting all the second post-Newstonian terms at order $O(c^{-4})$ that appear when the products are developed, one finds good EIHDL modified equations (28).

3. Global Hamiltonian formulation

To get the global Hamiltonian, we perform a Legendre transformation. In this framework, we express energy (B10) with respect to the conjugated variables $(\boldsymbol{z}_A, \boldsymbol{p}_A)$ instead of $(\boldsymbol{z}_A, \boldsymbol{v}_A)$. To do so, we have to invert Eq. (B7) which is always possible by perturbation at order $O(c^{-2})$, then by substituting in Eq. (B10), still neglecting terms at order $O(c^{-4})$, $O(\delta_A^2)$, and $O(c^{-2}\delta_A)$. The result is

$$\begin{aligned}\mathcal{H} = &\sum_A \left(\mu_A(1-\delta_A)c^2 + \frac{p_A^2}{2\mu_A}(1+\delta_A) - \frac{p_A^4}{8\mu_A^3 c^2}\right)\\ &-\frac{1}{2}\sum_A\sum_{B\neq A}\frac{1}{x_{AB}}\left(\mu_A\mu_B(1+\delta_{AB})\right.\\ &+\frac{1}{c^2}\frac{\mu_B}{\mu_A}(2\gamma+1)p_A^2 - \frac{4\gamma+3}{2c^2}\boldsymbol{p}_A\cdot\boldsymbol{p}_B\\ &\left.-\frac{1}{2c^2}(\boldsymbol{p}_A\cdot\boldsymbol{n}_{AB})(\boldsymbol{p}_B\cdot\boldsymbol{n}_{AB})\right)\\ &+\frac{1}{2c^2}\sum_A\sum_{B\neq A}\sum_{C\neq A}\frac{\mu_A\mu_B\mu_C}{x_{AB}x_{AC}}(2\beta+2d\beta^A-1), \end{aligned} \tag{B19}$$

The mass term $\sum_A \mu_A(1-\delta_A)c^2$ is useless for derivating equations of motion but will be useful for the first integrals. The Newtonian part of this Hamiltonian is

$$\mathcal{H}_N = \sum_A \frac{p_A^2}{2\mu_A}(1+\delta_A) - \frac{1}{2}\sum_A\sum_{B\neq A}\frac{\mu_A\mu_B}{r_{AB}}(1+\delta_{AB}). \tag{B20}$$

In the conjugated variables, the first integrals have simpler expressions. Linear momentum is

$$\boldsymbol{P} = \sum_A \boldsymbol{p}_A. \tag{B21}$$

Angular momentum is

$$\boldsymbol{J} = \sum_A \boldsymbol{z}_A \times \boldsymbol{p}_A. \tag{B22}$$

The energy is $\mathcal{H}$ itself, and the barycenter constant is

$$\boldsymbol{q} = \boldsymbol{G} - \boldsymbol{V}t, \tag{B23}$$

where

$$\boldsymbol{G} = \frac{c^2}{\mathcal{H}}\sum_A \mu_A \boldsymbol{z}_A\left(1 - \delta_A + \frac{p_A^2}{2\mu_A c^2} - \frac{1}{2c^2}\sum_{B\neq A}\frac{\mu_B}{r_{AB}}\right) \tag{B24}$$

and

$$\boldsymbol{V} = \frac{c^2\boldsymbol{P}}{\mathcal{H}}. \tag{B25}$$

Here, as in the Lagrangian formalism, linear momentum, angular momentum, and energy are exactly conserved when Hamilton equations of motion,

$$\frac{\mathrm{d}\boldsymbol{z}_A}{\mathrm{d}t} = \frac{\partial\mathcal{H}}{\partial\boldsymbol{p}_A}, \qquad \frac{\mathrm{d}\boldsymbol{p}_A}{\mathrm{d}t} = -\frac{\partial\mathcal{H}}{\partial\boldsymbol{z}_A}, \tag{B26}$$

are integrated exactly.

APPENDIX C: NORDTVEDT EFFECT IN LIGHT DILATON FRAMEWORK

We derive the Nordtvedt effect in the dilaton framework in order to derive Eq. (29). As did Damour and Esposito-Farèse [26], the idea consists of introducing a sensitivity parameter,

$$s_A = -\frac{\partial \ln m_A}{\partial \ln G_L}, \tag{C1}$$

where G_L is the locally measured gravitational constant. In the weak-field limit, this sensitivity is $s_A = |\Omega_A|/m_A c^2$, where

$$\Omega_A = G\int_A\int_A \frac{\rho_A(\boldsymbol{r})\rho_A(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|}\mathrm{d}^3 r \mathrm{d}^3 r' \tag{C2}$$

is the autogravitation energy. In our parametrization, $\tilde{\alpha}_A$ should be modified as

$$\tilde{\alpha}'_A = \frac{\partial \ln \tilde{m}_A}{\partial\phi} + \frac{\partial \ln m_A}{\partial \ln \tilde{G}}\frac{\mathrm{d}\ln\tilde{G}}{\mathrm{d}\phi} = \tilde{\alpha}_A - \frac{|\Omega_A|}{\tilde{m}_A c^2}\frac{\mathrm{d}\ln\tilde{G}}{\mathrm{d}\phi}, \tag{C3}$$

where $\tilde{\alpha}_A$ is the coefficient already computed with respect to the dilatonic charges. We have also

$$\tilde{G} = \frac{G}{f(\phi_0)}(1+\alpha_u^*(\phi_0)^2), \tag{C4}$$

thus,

$$\frac{\mathrm{d}\ln\tilde{G}}{\mathrm{d}\phi} = -\sqrt{\frac{2}{Z(\varphi)}}\frac{f'(\varphi)}{f(\varphi)} + \frac{2\alpha_0\beta_0}{1+\alpha_0^2}. \tag{C5}$$

To find the classical parametrization, we can redefine the constant without measurable modification,

$$\tilde{G} \mapsto \mathrm{e}^{2D(\varphi)}\tilde{G}, \tag{C6}$$

$$\tilde{m}_A \mapsto \mathrm{e}^{D(\varphi)}\tilde{m}_A, \tag{C7}$$

where

$$D(\varphi) = \int_{\varphi_0}^{\varphi} \alpha_u(x)\mathrm{d}x. \tag{C8}$$

Then,

$$\left.\frac{\mathrm{d}\ln\tilde{G}}{\mathrm{d}\phi}\right|_{\phi_0} = \sqrt{\frac{2}{Z(\varphi_0)}}\left(2\alpha_u(\varphi_0) - \frac{f'(\varphi_0)}{f_0}\right) + \frac{2\alpha_0\beta_0}{1+\alpha_0^2}$$
$$= 2\alpha_0 + \frac{2\alpha_0\beta_0}{1+\alpha_0^2}. \tag{C9}$$

On the other hand, we have

$$\frac{\partial\ln\tilde{m}_A}{\partial\phi} = \tilde{\alpha}_A + \frac{\mathrm{d}\delta_A/\mathrm{d}\phi}{1+\delta_A}. \tag{C10}$$

The absence of α_0 comes from the redefinition $\tilde{m}_A \mapsto \mathrm{e}^{-D(\phi)}\tilde{m}_A$. We have then

$$\delta'_A = \frac{\alpha_0\tilde{\alpha}'_A}{1+\alpha_0^2} \tag{C11}$$

$$= \frac{\alpha_0\tilde{\alpha}_A}{1+\alpha_0^2} - \left[2\frac{\alpha_0^2}{1+\alpha_0^2} + \frac{2\alpha_0^2\beta_0}{(1+\alpha_0^2)^2}\right]\frac{|\Omega_A|}{\tilde{m}_A c^2} \tag{C12}$$

$$= \delta_A - (4\beta - \gamma - 3)\frac{|\Omega_A|}{\tilde{m}_A c^2}, \tag{C13}$$

where δ_A is the deviation from the weak equivalence principle. We have neglected the term $\alpha_0\mathrm{d}\delta_A/\mathrm{d}\phi/((1+\delta_A)(1+\alpha_0^2))$ because it would make appear terms of order three in α_i. For roughly spherical bodies, we can estimate Ω_A,

$$\frac{\Omega_A}{\tilde{m}_A c^2} = -\frac{\tilde{G}}{2\tilde{m}_A c^2}\int_A\int_A \frac{\rho(\boldsymbol{x})\rho(\boldsymbol{x}')}{\|\boldsymbol{x}-\boldsymbol{x}'\|}\mathrm{d}^3x\mathrm{d}^3x' + O(c^{-2}) \tag{C14}$$

$$\approx -\frac{3\mu_A}{5R_A c^2}, \tag{C15}$$

where R_A is the average radius of the body. We can limit our calculation to this approximation until we have a positive detection of the strong equivalence principle violation.

[1] T. Damour, Theoretical aspects of the equivalence principle, Classical Quant. Grav. **29**, 184001 (2012).

[2] Y. V. Stadnik and V. V. Flambaum, Can Dark Matter Induce Cosmological Evolution of the Fundamental Constants of Nature?, Phys. Rev. Lett. **115**, 201301 (2015).

[3] A. Arvanitaki, J. Huang, and K. Van Tilburg, Searching for dilaton dark matter with atomic clocks, Phys. Rev. D **91**, 015015 (2015).

[4] C. Armendariz-Picon and R. Penco, Quantum equivalence principle violations in scalar-tensor theories, Phys. Rev. D **85**, 044052 (2012).

[5] T. Damour and A. M. Polyakov, The string dilation and a least coupling principle, Nucl. Phys. **B423**, 532 (1994); B. Greene, S. Judes, J. Levin, S. Watson, and A. Weltman, Cosmological moduli dynamics, J. High Energy Phys. 07 (2007) 060.

[6] J.-M. Gérard and I. Mahara, Asymptotic solutions to a tensor-scalar theory of gravity, Phys. Lett. B **346**, 35 (1995); The attractor mechanism in the minimal tensor–scalar theory of gravity, Mod. Phys. Lett. A **12**, 631 (1997); T. Damour, F. Piazza, and G. Veneziano, Violations of the equivalence principle in a dilaton-runaway scenario, Phys. Rev. D **66**, 046007 (2002); L. Kraiselburd, S. J. Landau, M. Salgado, D. Sudarsky, and H. Vucetich, Equivalence principle in chameleon models, Phys. Rev. D **97**, 104044 (2018).

[7] S. Schlamminger, K.-Y. Choi, T. A. Wagner, J. H. Gundlach, and E. G. Adelberger, Test of the Equivalence Principle Using a Rotating Torsion Balance, Phys. Rev. Lett. **100**, 041101 (2008); T. A. Wagner, S. Schlamminger, J. H. Gundlach, and E. G. Adelberger, Torsion-balance tests of the weak equivalence principle, Classical Quant. Grav. **29**, 184002 (2012).

[8] P. Touboul *et al.*, Space test of the equivalence principle: First results of the MICROSCOPE mission, Classical Quant. Grav. **36**, 225006 (2019).

[9] J. G. Williams, S. G. Turyshev, and D. H. Boggs, Lunar Laser Ranging Tests of the Equivalence Principle with the Earth and Moon, Int. J. Mod. Phys. D **18**, 1129 (2009).

[10] J. Bergé, P. Brax, G. Métris, M. Pernot-Borràs, P. Touboul, and J.-P. Uzan, MICROSCOPE Mission: First Constraints on the Violation of the Weak Equivalence Principle by a Light Scalar Dilaton, Phys. Rev. Lett. **120**, 141101 (2018); A. Hees, O. Minazzoli, E. Savalle, Y. V. Stadnik, and P. Wolf, Violation of the equivalence principle from light scalar dark matter, Phys. Rev. D **98**, 064051 (2018).

[11] V. Viswanathan, A. Fienga, O. Minazzoli, L. Bernus, J. Laskar, and M. Gastineau, The new lunar ephemeris INPOP17a and its application to fundamental physics, Mon. Not. R. Astron. Soc. **476**, 1877 (2018).

[12] J.-P. Uzan, Varying constants, gravitation and cosmology, Living Rev. Relativity **14**, 2 (2011); T. Damour, F. Piazza, and G. Veneziano, Runaway Dilaton and Equivalence Principle Violations, Phys. Rev. Lett. **89**, 081601 (2002).

[13] J. Guéna, M. Abgrall, D. Rovera, P. Rosenbusch, M. E. Tobar, P. Laurent, A. Clairon, and S. Bize, Improved Tests of Local Position Invariance Using Rb87 and Cs133 Fountains, Phys. Rev. Lett. **109**, 080801 (2012); T. Rosenband, D. B. Hume, P. O. Schmidt, C. W. Chou, A. Brusch, L. Lorini, W. H. Oskay, R. E. Drullinger, T. M. Fortier, J. E.

Stalnaker, S. A. Diddams, W. C. Swann, N. R. Newbury, W. M. Itano, D. J. Wineland, and J. C. Bergquist, Frequency Ratio of Al^+ and Hg^+ Single-Ion Optical Clocks; Metrology at the 17th Decimal Place, Science **319**, 1808 (2008); N. Leefer, C. T. M. Weber, A. Cingöz, J. R. Torgerson, and D. Budker, New Limits on Variation of the Fine-Structure Constant Using Atomic Dysprosium, Phys. Rev. Lett. **111**, 060801 (2013); R. M. Godun, P. B. R. Nisbet-Jones, J. M. Jones, S. A. King, L. A. M. Johnson, H. S. Margolis, K. Szymaniec, S. N. Lea, K. Bongs, and P. Gill, Frequency Ratio of Two Optical Clock Transitions in $^{171}Yb^+$ and Constraints on the Time Variation of Fundamental Constants, Phys. Rev. Lett. **113**, 210801 (2014); N. Huntemann, B. Lipphardt, C. Tamm, V. Gerginov, S. Weyers, and E. Peik, Improved Limit on a Temporal Variation of m_p/m_e from Comparisons of Yb^+ and Cs Atomic Clocks, Phys. Rev. Lett. **113**, 210802 (2014).

[14] K. Van Tilburg, N. Leefer, L. Bougas, and D. Budker, Search for Ultralight Scalar Dark Matter with Atomic Spectroscopy, Phys. Rev. Lett. **115**, 011802 (2015); A. Hees, J. Guéna, M. Abgrall, S. Bize, and P. Wolf, Searching for an Oscillating Massive Scalar Field as a Dark Matter Candidate Using Atomic Hyperfine Frequency Comparisons, Phys. Rev. Lett. **117**, 061301 (2016); C. J. Kennedy, E. Oelker, J. M. Robinson, T. Bothwell, D. Kedar, W. R. Milner, G. E. Marti, A. Derevianko, and J. Ye, Precision Metrology Meets Cosmology: Improved Constraints on Ultralight Dark Matter from Atom-Cavity Frequency Comparisons, Phys. Rev. Lett. **125**, 201302 (2020); E. Savalle, A. Hees, F. Frank, E. Cantin, P.-E. Pottie, B. M. Roberts, L. Cros, B. T. McAllister, and P. Wolf, Searching for Dark Matter with an Optical Cavity and an Unequal-Delay Interferometer, Phys. Rev. Lett. **126**, 051301 (2021).

[15] S. Peil, S. Crane, J. L. Hanssen, T. B. Swanson, and C. R. Ekstrom, Tests of local position invariance using continuously running atomic clocks, Phys. Rev. A **87**, 010102 (2013); N. Ashby, T. E. Parker, and B. R. Patla, A null test of general relativity based on a long-term comparison of atomic transition frequencies, Nat. Phys. **14**, 822 (2018).

[16] J. Hu, J. K. Webb, T. R. Ayres, M. B. Bainbridge, J. D. Barrow, M. A. Barstow, J. C. Berengut, R. F. Carswell, V. Dumont, V. Dzuba *et al.*, Measuring the fine-structure constant on a white dwarf surface; a detailed analysis of Fe V absorption in G191-B2B, Mon. Not. R. Astron. Soc. **500**, 1466 (2020); A. Hees, T. Do, B. M. Roberts, A. M. Ghez, S. Nishiyama, R. O. Bentley, A. K. Gautam, S. Jia, T. Kara, J. R. Lu, H. Saida, S. Sakai, M. Takahashi, and Y. Takamori, Search for a Variation of the Fine Structure Constant around the Supermassive Black Hole in Our Galactic Center, Phys. Rev. Lett. **124**, 081101 (2020).

[17] J. Khoury and A. Weltman, Chameleon cosmology, Phys. Rev. D **69**, 044026 (2004); Chameleon Fields: Awaiting Surprises for Tests of Gravity in Space, Phys. Rev. Lett. **93**, 171104 (2004).

[18] K. Hinterbichler and J. Khoury, Screening Long-Range Forces through Local Symmetry Restoration, Phys. Rev. Lett. **104**, 231301 (2010); K. Hinterbichler, J. Khoury, A. Levy, and A. Matas, Symmetron cosmology, Phys. Rev. D **84**, 103521 (2011).

[19] A. I. Vainshtein, To the problem of nonvanishing gravitation mass, Phys. Lett. B **39**, 393 (1972); E. Babichev, C. Deffayet, and R. Ziour, The Vainshtein mechanism in the decoupling limit of massive gravity, J. High Energy Phys. 05 (2009) 098.

[20] O. Minazzoli and A. Hees, Intrinsic Solar System decoupling of a scalar-tensor theory with a universal coupling between the scalar field and the matter Lagrangian, Phys. Rev. D **88**, 041504 (2013).

[21] H. Ludwig, O. Minazzoli, and S. Capozziello, Merging matter and geometry in the same Lagrangian, Phys. Lett. B **751**, 576 (2015).

[22] O. Minazzoli and A. Hees, Dilatons with intrinsic decouplings, Phys. Rev. D **94**, 064038 (2016).

[23] O. Minazzoli, De sitter space-times in entangled relativity, Classical Quant. Grav. **38**, 137003 (2021).

[24] S. M. Carroll, Quintessence and the Rest of the World: Suppressing Long-Range Interactions, Phys. Rev. Lett. **81**, 3067 (1998).

[25] T. Damour and J. F. Donoghue, Equivalence principle violations and couplings of a light dilaton, Phys. Rev. D **82**, 084033 (2010).

[26] T. Damour and G. Esposito-Farèse, Tensor-multi-scalar theories of gravitation, Classical Quant. Grav. **9**, 2093 (1992).

[27] T. Damour and J. F. Donoghue, Spatial variation of fundamental couplings and lunar laser ranging, Classical Quant. Grav. **28**, 162001 (2011).

[28] F. Nitti and F. Piazza, Scalar-tensor theories, trace anomalies, and the QCD frame, Phys. Rev. D **86**, 122002 (2012).

[29] H. A. Lorentz and J. Droste, Versl. K. Akad. Wet. Amsterdam **26**, 392 (1917).

[30] H. A. Lorentz and J. Droste, Versl. K. Akad. Wet. Amsterdam **26**, 649 (1917).

[31] H. A. Lorentz and J. Droste, *The Motion of a System of Bodies under the Influence of their Mutual Attraction, According to Einstein's Theory* (Springer Netherlands, Dordrecht, 1937), pp. 330–355, 10.1007/978-94-015-3445-1_11.

[32] A. Einstein, L. Infeld, and B. Hoffmann, The gravitational equations and the problem of motion, Ann. Math. **39**, 65 (1938).

[33] T. Damour, M. Soffel, and C. Xu, General-relativistic celestial mechanics. I. Method and definition of reference systems, Phys. Rev. D **43**, 3273 (1991).

[34] K. Nordtvedt, Equivalence principle for massive bodies. I. Phenomenology, Phys. Rev. **169**, 1014 (1968).

[35] K. Nordtvedt, Equivalence principle for massive bodies. II. Theory, Phys. Rev. **169**, 1017 (1968).

[36] C. M. Will, *Theory and Experiment in Gravitational Physics*, 2nd ed. (Cambridge University Press, Cambridge, England, 2018).

[37] S. A. Klioner and M. H. Soffel, Relativistic celestial mechanics with PPN parameters, Phys. Rev. D **62**, 024019 (2000).

[38] J. Alsing, E. Berti, C. M. Will, and H. Zaglauer, Gravitational radiation from compact binary systems in the massive Brans-Dicke theory of gravity, Phys. Rev. D **85**, 064041 (2012).

[39] O. Minazzoli and A. Hees, Late-time cosmology of a scalar-tensor theory with a universal multiplicative coupling

between the scalar field and the matter Lagrangian, Phys. Rev. D **90**, 023017 (2014).

[40] A. Fienga, H. Manche, J. Laskar, and M. Gastineau, INPOP06: A new numerical planetary ephemeris, Astron. Astrophys. **477**, 315 (2008).

[41] A. Fienga, P. Deram, V. Viswanathan, A. Di Ruscio, L. Bernus, D. Durante, M. Gastineau, and J. Laskar, INPOP19a planetary ephemeris, Notes Scientifiques et Techniques de l'Institut de Mecanique Celeste, 109, 2019.

[42] A. Fienga, C. Avdellidou, and J. Hanuš, Asteroid masses obtained with INPOP planetary ephemerides, Mon. Not. R. Astron. Soc. **492**, 589 (2020).

[43] A. Di Ruscio, A. Fienga, D. Durante, L. Iess, J. Laskar, and M. Gastineau, Analysis of Cassini radio tracking data for the construction of INPOP19a: A new estimate of the Kuiper belt mass, Astron. Astrophys. **640**, A7 (2020).

[44] A. K. Verma and J.-L. Margot, Mercury's gravity, tides, and spin from MESSENGER radio science data, J. Geophys. Res. (Planets) **121**, 1627 (2016).

[45] A. Hees, W. M. Folkner, R. A. Jacobson, and R. S. Park, Constraints on modified Newtonian dynamics theories from radio tracking data of the Cassini spacecraft, Phys. Rev. D **89**, 102002 (2014).

[46] A. K. Verma, A. Fienga, J. Laskar, H. Manche, and M. Gastineau, Use of MESSENGER radioscience data to improve planetary ephemeris and to test general relativity, Astron. Astrophys. **561**, A115 (2014).

[47] A. Fienga, J. Laskar, P. Exertier, H. Manche, and M. Gastineau, Numerical estimation of the sensitivity of INPOP planetary ephemerides to general relativity parameters, Celest. Mech. Dyn. Astron. **123**, 325 (2015).

[48] L. Bernus, O. Minazzoli, A. Fienga, M. Gastineau, J. Laskar, and P. Deram, Constraining the Mass of the Graviton with the Planetary Ephemeris Inpop, Phys. Rev. Lett. **123**, 161103 (2019); L. Bernus, O. Minazzoli, A. Fienga, M. Gastineau, J. Laskar, P. Deram, and A. Di Ruscio, Constraint on the yukawa suppression of the newtonian potential from the planetary ephemeris INPOP19A, Phys. Rev. D **102**, 021501 (2020).

[49] B. Bertotti, L. Iess, and P. Tortora, A test of general relativity using radio links with the Cassini spacecraft, Nature (London) **425**, 374 (2003).

[50] M. Meftah, A. Irbah, A. Hauchecorne, T. Corbard, S. Turck-Chièze, J. F. Hochedez, P. Boumier, A. Chevalier, S. Dewitte, S. Mekaoui, and D. Salabert, On the determination and constancy of the solar oblateness, Sol. Phys. **290**, 673 (2015).

[51] R. Mecheri and M. Meftah, Updated values of solar gravitational moments J_{2n} using HMI helioseismic inference of internal rotation, Mon. Not. R. Astron. Soc. **506**, 2671 (2021).

[52] T. Damour, M. Soffel, and C. Xu, General-relativistic celestial mechanics. I. Method and definition of reference systems, Phys. Rev. D **43**, 3273 (1991).

[53] S. A. Klioner, Parametrized post-Newtonian equations of motion of N mass monopoles with the SEP violation, arXiv:1607.00183.