

Loop-induced determinations of V_{ub} and V_{cb}

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We use loop induced processes like meson oscillations and rare b hadron decays to determine the absolute values of the Cabibbo-Kobayashi-Maskawa matrix elements $|V_{ub}|$ and $|V_{cb}|$ and compare our results to the standard determinations based on inclusive and exclusive semileptonic tree-level decays of B mesons. For many years there have been tensions between the inclusive and exclusive determinations. Assuming the absence of new physics, we find that meson oscillation data shows a slight preference for the inclusive value of $|V_{cb}|$ and the exclusive value for $|V_{ub}|$. Rare b decay data prefers values for $|V_{cb}|$ far below the inclusive and exclusive determinations, offering a new perspective on some of the persistent rare b decay anomalies.

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I. INTRODUCTION

The elements V_{ub} and V_{cb} of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1,2] are crucial input parameters for the theory predictions of many observables in the flavor sector. A standard strategy to determine the absolute values of V_{ub} and V_{cb} is to use tree-level $b \rightarrow c\ell\nu$ and $b \rightarrow u\ell\nu$ decays, assuming no new physics at tree level. These values are then used to make Standard Model (SM) predictions for loop level flavor changing neutral current processes that are highly sensitive to new physics effects.

Since many years, discrepancies exist between the tree-level determinations using the inclusive $B \rightarrow X_c\ell\nu$ and $B \rightarrow X_u\ell\nu$ decays on the one side, and exclusive decays, like $B \rightarrow D^{(*)}\ell\nu$ and $B \rightarrow \pi\ell\nu$ on the other [3–5] (also $B \rightarrow \pi\pi\ell\nu$ decays can be used to determine $|V_{ub}|$ [6]). These discrepancies limit the sensitivity of various loop processes to new physics. At the same time, explaining the discrepancies by new physics is challenging [7–9]. Improving the theoretical description of inclusive and exclusive $b \rightarrow c\ell\nu$ and $b \rightarrow u\ell\nu$ decays is thus of high priority (see [10–23] for recent progress). Alternatively, one can focus on combinations of loop observables that are

independent of V_{cb} and/or V_{ub} as emphasized for example in [24–26].

In this paper, we follow a different approach. Assuming that there is no new physics in loop induced flavor changing neutral current processes, we use those processes to determine $|V_{ub}|$ and $|V_{cb}|$ and compare them to the tree-level determinations. The main results of our study are loop level determinations of $|V_{ub}|$ and $|V_{cb}|$ based on meson oscillations and determinations of $|V_{cb}|$ from rare b hadron decays.

On the one hand, we are motivated by recent progress in the SM calculations of the parameter ϵ_K that quantifies indirect CP violation in neutral Kaon oscillations [27,28]. As $|V_{cb}|$ is the largest uncertainty of the SM prediction, the precisely known experimental value of ϵ_K gives one of the most precise determinations of $|V_{cb}|$. On the other hand, the observation of anomalously low branching ratios of the rare B meson decays $B \rightarrow K\mu^+\mu^-$, $B \rightarrow K^*\mu^+\mu^-$, and $B_s \rightarrow \phi\mu^+\mu^-$ by the LHCb collaboration [29–31] provides motivation to reexamine the possible role of $|V_{cb}|$ in these B anomalies (see e.g. [32] for a previous study).

We stress that our results for $|V_{ub}|$ and $|V_{cb}|$ should not be used in studies of new physics in flavor changing processes. Instead, they offer a new perspective on the tension between inclusive and exclusive $|V_{ub}|$ and $|V_{cb}|$ determinations, the role of ϵ_K in CKM fits, and some of the persistent anomalies in rare B meson decays.

This paper is organized as follows: in Sec. II we review $|V_{ub}|$ and $|V_{cb}|$ determinations from tree-level b hadron decays, establishing a baseline for the comparison with our loop level determinations. In Sec. III we discuss a number of loop observables that are sensitive to $|V_{ub}|$ and $|V_{cb}|$.

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In particular, we consider neutral Kaon and B meson mixing as well as rare decays of b hadrons. Our results of the loop level determinations are discussed in Sec. IV. We conclude in Sec. V.

II. TREE-LEVEL DETERMINATIONS OF V_{cb} AND V_{ub}

Direct tree-level determinations of the CKM matrix elements $|V_{ub}|$ and $|V_{cb}|$ come from measurements of inclusive and exclusive semileptonic decays based on the $b \rightarrow u\ell\nu$ and $b \rightarrow c\ell\nu$ transitions. For many years, notable discrepancies have existed between inclusive and exclusive determinations.

The Particle Data Group (PDG) quotes the following inclusive values [5]

$$\begin{aligned} |V_{cb}|_{\text{incl}} &= (42.2 \pm 0.8) \times 10^{-3}, \\ |V_{ub}|_{\text{incl}} &= (4.25 \pm 0.30) \times 10^{-3}, \end{aligned} \quad (1)$$

where we added all individual uncertainties in quadrature. Note that the above value for $|V_{ub}|_{\text{incl}}$ does not yet take into account the latest determination from the Belle collaboration, $|V_{ub}|_{\text{incl}} = (4.1 \pm 0.28) \times 10^{-3}$, which is compatible within the uncertainties but has a slightly lower central value [33]. Also note that the PDG value for $|V_{cb}|_{\text{incl}}$ does not yet take into account improved theory calculations which give $|V_{cb}|_{\text{incl}} = (42.16 \pm 0.51) \times 10^{-3}$ [20]. For definiteness we will use the PDG values in (1) for the remainder of this work.

For exclusive determinations, the PDG gives averages based on $\bar{B} \rightarrow D\ell\nu$ and $\bar{B} \rightarrow D^*\ell\nu$ decays for $|V_{cb}|$ and $\bar{B} \rightarrow \pi\ell\nu$ decays for $|V_{ub}|$. The averages read [5]

$$\begin{aligned} |V_{cb}|_{\text{excl}} &= (39.5 \pm 0.9) \times 10^{-3}, \\ |V_{ub}|_{\text{excl}} &= (3.70 \pm 0.16) \times 10^{-3}, \end{aligned} \quad (2)$$

where, as above, we added all individual uncertainties in quadrature. We note that the PDG average of $|V_{cb}|_{\text{excl}}$ does not yet include the latest lattice results on the $B \rightarrow D^*$ form factors from [19]. The value for $|V_{cb}|$ determined in [19] is $|V_{cb}|_{B \rightarrow D^*} = (38.4 \pm 0.74) \times 10^{-3}$, very similar to the results from [34,35] that enter the PDG average, but with slightly smaller uncertainties. The discrepancies between the inclusive and exclusive values quoted by the PDG are 2.4σ for V_{cb} and 1.6σ for V_{ub} .

Exclusive decays are also used to independently determine the ratio $|V_{ub}|/|V_{cb}|$. A measurement from LHCb [36] of the branching ratios $\Lambda_b^0 \rightarrow p\mu^-\nu$ and $\Lambda_b^0 \rightarrow \Lambda_c^+\mu^-\nu$, combined with form factor ratios from [37], gives [38]

$$|V_{ub}|/|V_{cb}|_{\Lambda_b} = 0.079 \pm 0.006. \quad (3)$$

Combining recent results from LHCb on the decay modes $B_s \rightarrow K^-\mu^+\nu$ and $B_s \rightarrow D_s^-\mu^+\nu$ [39] with lattice form factors from [40–43] gives [44]

$$|V_{ub}|/|V_{cb}|_{B_s} = \begin{cases} 0.0819 \pm 0.0077, & \text{for } q^2 < 7 \text{ GeV}^2 \\ 0.0860 \pm 0.0053, & \text{for } q^2 > 7 \text{ GeV}^2 \end{cases}. \quad (4)$$

The values for the ratios show good agreement between the high q^2 and low q^2 region and with the values obtained from Λ_b decays quoted above. If alternative form factors based on light cone sum rules [45] are used to determine $|V_{ub}|/|V_{cb}|_{B_s}$ at low q^2 , then one finds a significantly lower value. We will work with the lattice results (4) in the following.

Finally, the exclusive leptonic decay mode $B^+ \rightarrow \tau^+\nu$ can also be used for a tree-level determination of $|V_{ub}|$. In the SM, the $B^+ \rightarrow \tau^+\nu$ branching ratio is given by

$$\text{BR}(B^+ \rightarrow \tau^+\nu)_{\text{SM}} = \tau_{B^+} \frac{G_F^2}{8\pi} f_{B^+}^2 m_{B^+} m_\tau^2 \left(1 - \frac{m_\tau^2}{m_{B^+}^2}\right)^2 |V_{ub}|^2. \quad (5)$$

The Fermi constant, the tau mass, and the B^+ mass and lifetime have negligible uncertainties [5]. The dominant uncertainty comes from the B^+ meson decay constant. For our numerical analysis, we use the value given by the flavor lattice averaging group (FLAG), $f_{B^+} \simeq f_B = (190.0 \pm 1.3) \text{ MeV}$ [44], ignoring small isospin breaking effects.

The experimental world average of the $B^+ \rightarrow \tau^+\nu$ branching ratio quoted by the heavy flavor averaging group (HFLAV) [38] is based on results by BABAR [46] and Belle [47] and reads

$$\text{BR}(B^+ \rightarrow \tau^+\nu)_{\text{exp}} = (1.06 \pm 0.19) \times 10^{-4}. \quad (6)$$

This allows us to extract the following value for $|V_{ub}|$

$$|V_{ub}|_{B \rightarrow \tau\nu} = (4.08 \pm 0.37 \pm 0.03) \times 10^{-3}, \quad (7)$$

where the first uncertainty is experimental and the second theoretical (i.e. from the B^+ meson decay constant). Our central value agrees very well with the values quoted in [44].

In Fig. 1, we compare the various determinations in the $|V_{cb}|$ - $|V_{ub}|$ plane. The black error bars correspond to the 1σ and 2σ ranges for the inclusive values given in (1). The 1σ and 2σ ranges of the exclusive values of V_{cb} and V_{ub} in (2) are represented by the blue and orange bands, respectively. The green band shows the combination of the ratios in (3) and (4), while the yellow band shows the result from the $B^+ \rightarrow \tau^+\nu$ decay in (7). Finally, the red region corresponds to the combination of all exclusive determinations. Approximating this region by a two-dimensional Gaussian distribution, we find

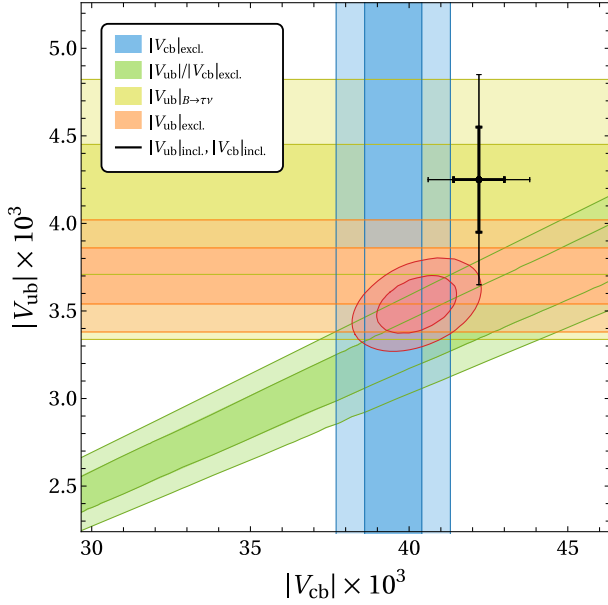


FIG. 1. Comparison of inclusive and exclusive determinations of the CKM matrix elements $|V_{cb}|$ and $|V_{ub}|$. The black error bars show the PDG average of the inclusive values at 1σ and 2σ , while the red region shows our combination of all exclusive determinations at 1σ and 2σ .

$$\begin{aligned} |V_{cb}|_{\text{excl}} &= (40.24 \pm 0.83) \times 10^{-3}, \\ |V_{ub}|_{\text{excl}} &= (3.54 \pm 0.11) \times 10^{-3}, \end{aligned} \quad (8)$$

with an error correlation of $\rho = +37\%$. We note that our results for $|V_{cb}|_{\text{excl}}$ and $|V_{ub}|_{\text{excl}}$ are very close to the results of the FLAG [44]. Our value for $|V_{cb}|_{\text{excl}}$ is slightly larger and has a $\sim 20\%$ larger uncertainty than the FLAG result ($|V_{cb}|_{\text{excl}}^{\text{FLAG}} = (39.48 \pm 0.68) \times 10^{-3}$ [44]). This is due to the fact that we use as input the more conservative PDG value from (2).

The results shown in Fig. 1 serve as a baseline for the comparison with alternative determinations of $|V_{cb}|$ and $|V_{ub}|$, which are discussed in the following sections.

III. LOOP OBSERVABLES SENSITIVE TO V_{cb} AND V_{ub}

While it is customary to work with the Wolfenstein parametrization of the CKM matrix [48], we decide to directly parametrize the CKM matrix in terms of the sine of the Cabibbo angle $\lambda = \sin \theta_C$, the absolute values $|V_{cb}|$ and $|V_{ub}|$, and the phase $\gamma = \arg(-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*)$. In this way, the $|V_{cb}|$ and $|V_{ub}|$ dependence of the various observables discussed below is made fully transparent. Working in the standard phase convention for the CKM matrix and expanding in λ , we have

$$\begin{aligned} V_{ud} &\simeq 1 - \frac{\lambda^2}{2}, & V_{us} &\simeq \lambda, & V_{ub} &\simeq |V_{ub}|e^{-i\gamma}, \\ V_{cd} &\simeq -\lambda, & V_{cs} &\simeq 1 - \frac{\lambda^2}{2}, & V_{cb} &= |V_{cb}|, \\ V_{td} &\simeq |V_{cb}|\lambda - |V_{ub}|e^{i\gamma} \left(1 - \frac{\lambda^2}{2}\right), \\ V_{ts} &\simeq -|V_{cb}| \left(1 - \frac{\lambda^2}{2}\right) - |V_{ub}|\lambda e^{i\gamma}, & V_{tb} &\simeq 1, \end{aligned} \quad (9)$$

where for each CKM element we take into account relative corrections up to $\mathcal{O}(\lambda^2)$.

A. CP violation in neutral kaon mixing

The parameter ϵ_K is a measure of CP violation in neutral kaon oscillations. Its SM prediction depends strongly on the value of $|V_{cb}|$. It has been shown in [27] that the short distance theory uncertainty of ϵ_K can be strongly reduced through a reorganization of the relevant effective Hamiltonian. The uncertainty of the SM prediction of ϵ_K is then dominated by the CKM matrix input, thus strengthening the role of ϵ_K in determining CKM parameters. Using the expressions from [27], we can write

$$\begin{aligned} |\epsilon_K|_{\text{SM}} &= \kappa_e C_e \hat{B}_K \left(-\frac{1}{2} \text{Im}((V_{ts}^* V_{td})^2) \eta_{tt} \mathcal{S}(x_t) \right. \\ &\quad \left. - \text{Im}(V_{us}^* V_{ud} V_{ts}^* V_{td}) \eta_{ut} \mathcal{S}(x_c, x_t) \right), \end{aligned} \quad (10)$$

where $x_q = m_q^2/m_W^2$ is the ratio of quark mass and the W boson mass squared. The loop functions include the leading terms in x_c and are given by

$$\mathcal{S}(x_t) = \frac{4x_t - 11x_t^2 + x_t^3}{4(1-x_t)^2} - \frac{3x_t^3 \log x_t}{2(1-x_t)^3}, \quad (11)$$

$$\mathcal{S}(x_c, x_t) = x_c \left(1 - \log(x_t/x_c) + \frac{3x_t}{4(1-x_t)} + \frac{3x_t^2 \log x_t}{4(1-x_t)^2} \right). \quad (12)$$

We follow [28] and include electroweak corrections of the top contribution by making the replacement $\eta_{tt} \rightarrow \eta_{tt}(1 - \Delta_{tt})$. To obtain numerical predictions we use the input $\Delta_{tt} = 0.01 \pm 0.004$ [28], $\eta_{tt} = 0.55 \pm 0.02$ [27], $\eta_{ut} = 0.402 \pm 0.005$ [27], $m_t = (163.48 \pm 0.86)$ GeV [27], $m_c = (1.27 \pm 0.02)$ GeV [5], $\kappa_e = 0.94 \pm 0.02$ [49], and $\hat{B}_K = 0.7625 \pm 0.0097$ [44]. The coefficient C_e is given by [27]

$$C_e = \frac{G_F^2 F_K^2 m_K^0 m_W^2}{6\sqrt{2}\pi^2 \Delta M_K} \simeq 3.634 \times 10^4, \quad (13)$$

with negligible uncertainty. Using these numerical values and expressing the CKM matrix elements in terms of $|V_{cb}|$, $|V_{ub}|$, λ , and γ as in (9), we find

$$|\epsilon_K|_{\text{SM}} = \left[\left(|V_{cb}|^2 \lambda \left(1 - \frac{\lambda^2}{2} \right) - |V_{cb}| |V_{ub}| (1 - 2\lambda^2) \cos \gamma - |V_{ub}|^2 \lambda \right) a_{\epsilon_K} + \lambda \left(1 - \frac{\lambda^2}{2} \right) b_{\epsilon_K} \right] |V_{cb}| |V_{ub}| \sin \gamma, \quad (14)$$

with the coefficients

$$a_{\epsilon_K} = (3.29 \pm 0.15) \times 10^4, \quad b_{\epsilon_K} = 20.55 \pm 0.81. \quad (15)$$

For the errors of the numerical coefficients a_{ϵ_K} and b_{ϵ_K} , we find a correlation of +34%, which we take into account in our numerical analysis in Sec. IV.

Experimentally, ϵ_K is known with very high precision [5]

$$|\epsilon_K|_{\text{exp}} = (2.228 \pm 0.011) \times 10^{-3}. \quad (16)$$

The measurement provides a stringent constraint on the CKM parameters in (14).

$$\sin(2\beta)_{\text{SM}} = \frac{2|V_{ub}|(|V_{cb}|\lambda - |V_{ub}|\cos\gamma)\sin\gamma}{(|V_{ub}|\cos\gamma - |V_{cb}|\lambda)^2 + |V_{ub}|^2\sin^2\gamma} \left(1 + \frac{|V_{cb}|\lambda^3}{2(|V_{ub}|\cos\gamma - |V_{cb}|\lambda)} \frac{(|V_{ub}|\cos\gamma - |V_{cb}|\lambda)^2 - |V_{ub}|^2\sin^2\gamma}{(|V_{ub}|\cos\gamma - |V_{cb}|\lambda)^2 + |V_{ub}|^2\sin^2\gamma} \right), \quad (18)$$

where we have neglected terms of $\mathcal{O}(\lambda^4)$.

Also the neutral B meson oscillation frequencies ΔM_d and ΔM_s can be used to determine CKM parameters. From the experimental side, the frequencies are known with remarkable precision

$$\begin{aligned} \Delta M_d^{\text{exp}} &= (0.5065 \pm 0.0019) \text{ ps}^{-1}, \\ \Delta M_s^{\text{exp}} &= (17.7656 \pm 0.0056) \text{ ps}^{-1}, \end{aligned} \quad (19)$$

where we quote the experimental world average for ΔM_d^{exp} from HFLAV [38] and the recent measurement for ΔM_s^{exp} by LHCb [50], which is the single most precise determination to date.

The SM predictions of the oscillation frequencies are given by the well-known expressions

$$\Delta M_d^{\text{SM}} = \frac{G_F^2 m_W^2}{6\pi^2} m_{B_d} |V_{td}^* V_{tb}|^2 S_0(m_t^2/m_W^2) \eta_B f_{B_d}^2 \hat{B}_{B_d}, \quad (20)$$

$$\Delta M_s^{\text{SM}} = \frac{G_F^2 m_W^2}{6\pi^2} m_{B_s} |V_{ts}^* V_{tb}|^2 S_0(m_t^2/m_W^2) \eta_B f_{B_s}^2 \hat{B}_{B_s}. \quad (21)$$

The dominant uncertainty stems from the values of the hadronic matrix elements, parametrized by the B meson decay constants f_{B_q} and the so-called bag parameters \hat{B}_{B_q} . We use the latest $N_f = 2 + 1 + 1$ lattice

B. Neutral B meson mixing

CP violation in $B^0 - \bar{B}^0$ mixing provides a very important ingredient for fits of the CKM matrix. In particular, the CKM angle $\beta = \text{Arg}(-V_{cd}V_{cb}^*/V_{td}V_{tb}^*)$ can be accessed through measurements of time-dependent CP asymmetries in $b \rightarrow c\bar{c}s$ transitions. The world average from HFLAV is [38]

$$\sin(2\beta)_{\text{exp}} = 0.699 \pm 0.017. \quad (17)$$

Expressing $\sin(2\beta)$ in terms of λ , $|V_{cb}|$, $|V_{ub}|$, and γ , we find the SM prediction

results from [51], $f_{B_s} \sqrt{\hat{B}_{B_s}} = (256.1 \pm 5.7) \text{ MeV}$, $f_{B_d} \sqrt{\hat{B}_{B_d}} = (210.6 \pm 5.5) \text{ MeV}$, and $\xi = f_{B_s} \sqrt{\hat{B}_{B_s}} / f_{B_d} \sqrt{\hat{B}_{B_d}} = 1.216 \pm 0.016$, neglecting unknown correlations between the uncertainties. The quoted lattice results for the hadronic matrix elements agree well with alternative determinations using sum rules [52]. (Note that the FLAG averages of the older $N_f = 2 + 1$ hadronic matrix elements [44] are instead significantly larger.) In our theory predictions of the mass differences, we also take into account the uncertainty from the top mass $m_t = (163.48 \pm 0.86) \text{ GeV}$ [27] that enters the loop function $S_0(x) = \mathcal{S}(x)$ given in (11). We do not take into account the uncertainty in the higher order QCD correction factor $\eta_B \simeq 0.552$ [53], as it is negligibly small. Using PDG values for G_F , m_W , and m_{B_q} [5], we find the expressions

$$\begin{aligned} \Delta M_d^{\text{SM}} &= (\lambda^2 |V_{cb}|^2 + |V_{ub}|^2 (1 - \lambda^2) \\ &\quad - \lambda |V_{cb}| |V_{ub}| (2 - \lambda^2) \cos \gamma) a_{\Delta M_d}, \end{aligned} \quad (22)$$

$$\Delta M_s^{\text{SM}} = (|V_{cb}|^2 (1 - \lambda^2) + 2\lambda |V_{cb}| |V_{ub}| \cos \gamma) a_{\Delta M_s}, \quad (23)$$

with the numerical coefficients

$$\begin{aligned} a_{\Delta M_d} &= (6.77 \pm 0.26) \times 10^3 \text{ ps}^{-1}, \\ a_{\Delta M_s} &= (10.18 \pm 0.37) \times 10^3 \text{ ps}^{-1}. \end{aligned} \quad (24)$$

We find a sizable error correlation of $\rho = +76\%$ between the two coefficients. The correlation arises from the precisely known ratio ξ .

Note that the dependence of the meson mixing observables on the top mass also leads in principle to a correlation between the coefficients $a_{\Delta M_q}$, which are relevant for B mixing, and the coefficients a_{e_K} , b_{e_K} , which enter the expression for e_K^{SM} discussed above. However, since the top mass uncertainty is not significant in both e_K^{SM} and the oscillation frequencies ΔM_q^{SM} , it is justified to neglect the error correlation across these observables.

C. Rare b hadron decays

The branching ratios of flavor changing neutral current b hadron decays depend on the CKM matrix element combinations $|V_{ts}^* V_{tb}|^2$ or $|V_{td}^* V_{tb}|^2$ and can be used to constrain $|V_{cb}|$ and, to a lesser extent, $|V_{ub}|$. In the following, we consider the radiative decay $B \rightarrow X_s \gamma$, the leptonic decays $B_s \rightarrow \mu^+ \mu^-$ and $B^0 \rightarrow \mu^+ \mu^-$, and the semileptonic decays $B^+ \rightarrow K^+ \mu^+ \mu^-$, $B^0 \rightarrow K^{*0} \mu^+ \mu^-$, $B_s \rightarrow \phi \mu^+ \mu^-$, and $\Lambda_b \rightarrow \Lambda \mu^+ \mu^-$. We do not include additional decay modes like $B^0 \rightarrow K^0 \mu^+ \mu^-$ or $B^+ \rightarrow K^{*+} \mu^+ \mu^-$, as they have larger experimental uncertainties and therefore have little impact on our results.

The inclusive radiative decay $B \rightarrow X_s \gamma$ is known to be a very sensitive probe of new physics. In the absence of new physics, it can be used to determine the CKM matrix element V_{cb} . The $B \rightarrow X_s \gamma$ rate is to a large extent determined by the product of the Wilson coefficient $|C_7^{\text{incl}}|$ and the CKM matrix combination $|V_{tb} V_{ts}^*|$. A recent global fit to inclusive $B \rightarrow X_s \gamma$ measurements finds [54]

$$|C_7^{\text{incl}} V_{tb} V_{ts}^*| = (14.77 \pm 0.78) \times 10^{-3}, \quad (25)$$

which can be compared to the SM prediction [54]

$$\begin{aligned} |C_7^{\text{incl}} V_{tb} V_{ts}^*|_{\text{SM}} &= \left(|V_{cb}| \left(1 - \frac{\lambda^2}{2} \right) + |V_{ub}| \lambda \cos \gamma \right) \\ &\times (0.3624 \pm 0.0151), \end{aligned} \quad (26)$$

where we added the uncertainties from $c\bar{c}$ loops and from scale variation in quadrature. Combining (25) with (26) allows one to constrain the value of $|V_{cb}|$. The term in (26) proportional to $|V_{ub}|$ plays a negligible role.

The leptonic decays $B_s \rightarrow \mu^+ \mu^-$ and $B^0 \rightarrow \mu^+ \mu^-$ are of particular interest, as they are theoretically very clean, with the most relevant hadronic uncertainty coming from the B_s and B^0 meson decay constants. The latest lattice results for decay constants [55] have reached subpercent precision, implying that the dominant source of uncertainty in the SM

predictions of the branching ratios $\text{BR}(B_s \rightarrow \mu^+ \mu^-)$ and $\text{BR}(B^0 \rightarrow \mu^+ \mu^-)$ is the CKM matrix input, in particular $|V_{cb}|$. Assuming the absence of new physics, precision measurements of $\text{BR}(B_s \rightarrow \mu^+ \mu^-)$ and $\text{BR}(B^0 \rightarrow \mu^+ \mu^-)$ can therefore give important constraints on $|V_{cb}|$. We use the experimental world average of the branching ratios that has been determined in [56], based on the most recent experimental results on $B_s \rightarrow \mu^+ \mu^-$ and $B^0 \rightarrow \mu^+ \mu^-$ from the LHCb, ATLAS, and CMS collaborations [57–60]

$$\text{BR}(B_s \rightarrow \mu^+ \mu^-)_{\text{exp}} = (2.93 \pm 0.35) \times 10^{-9}, \quad (27)$$

$$\text{BR}(B^0 \rightarrow \mu^+ \mu^-)_{\text{exp}} = (0.56 \pm 0.70) \times 10^{-10}, \quad (28)$$

with an error correlation of $\rho = -27\%$.

The SM predictions for the branching ratios are proportional to the CKM factors $|V_{tb} V_{ts}^*|^2$ and $|V_{tb} V_{td}^*|^2$, respectively. Therefore, we can write the branching ratios as

$$\begin{aligned} \text{BR}(B_s \rightarrow \mu^+ \mu^-)_{\text{SM}} &= (|V_{cb}|^2 (1 - \lambda^2) \\ &+ 2\lambda |V_{cb}| |V_{ub}| \cos \gamma) a_{B_s \rightarrow \mu\mu}, \end{aligned} \quad (29)$$

$$\begin{aligned} \text{BR}(B^0 \rightarrow \mu^+ \mu^-)_{\text{SM}} &= (\lambda^2 |V_{cb}|^2 + |V_{ub}|^2 (1 - \lambda^2) \\ &- \lambda |V_{cb}| |V_{ub}| (2 - \lambda^2) \cos \gamma) a_{B^0 \rightarrow \mu\mu}. \end{aligned} \quad (30)$$

We determine the numerical coefficients $a_{B_s \rightarrow \mu\mu}$ and $a_{B^0 \rightarrow \mu\mu}$ following the SM calculation of the branching ratios in [61]. We use the averages of the B meson decay constants from FLAG assuming isospin symmetry, $f_{B_s} = (230.3 \pm 1.3) \text{ MeV}$, $f_B = (190.0 \pm 1.3) \text{ MeV}$ [44], as well as the very precisely known ratio $f_{B_s}/f_B = 1.209 \pm 0.005$ [44]. We also take into account the uncertainties from the B meson lifetimes $\tau_{B_s^H} = (1.615 \pm 0.009) \text{ ps}$, $\tau_{B^0} = (1.520 \pm 0.004) \text{ ps}$ [5], a relative uncertainty of 1.1% from the top mass [61], an uncertainty of 1.2% from scale variation and higher order corrections [61], and a 0.5% uncertainty from light cone distribution amplitudes that enter the QED corrections of the branching ratios [61]. In that way we obtain

$$\begin{aligned} a_{B_s \rightarrow \mu\mu} &= (2.15 \pm 0.04) \times 10^{-6}, \\ a_{B^0 \rightarrow \mu\mu} &= (1.36 \pm 0.03) \times 10^{-6}, \end{aligned} \quad (31)$$

with a large positive error correlation of $\rho = +85\%$.

Similarly, semileptonic decays of b hadrons can be used to determine $|V_{cb}|$, albeit with larger theoretical uncertainty. The branching ratios of the semileptonic rare b hadron decays are measured in bins of q^2 , the dilepton invariant mass squared. As in the case of the $B_s \rightarrow \mu^+ \mu^-$ decay, we factor out the CKM dependence and write the SM predictions for the $b \rightarrow s \mu\mu$ branching ratios of a hadron H_1 to a hadron H_2 and two muons in the following way:

$$\begin{aligned} \text{BR}(H_1 \rightarrow H_2 \mu^+ \mu^-)_{\text{SM}}^{[q_{\min}^2, q_{\max}^2]} \\ = (|V_{cb}|^2(1 - \lambda^2) + 2\lambda|V_{cb}||V_{ub}|\cos\gamma) a_{H_1 \rightarrow H_2 \mu\mu}^{[q_{\min}^2, q_{\max}^2]}, \end{aligned} \quad (32)$$

where the superscript $[q_{\min}^2, q_{\max}^2]$ indicates the q^2 bin. In our analysis, we consider the well measured $B^+ \rightarrow K^+ \mu^+ \mu^-$, $B^0 \rightarrow K^{*0} \mu^+ \mu^-$, $B_s \rightarrow \phi \mu^+ \mu^-$, and $\Lambda_b \rightarrow \Lambda \mu^+ \mu^-$ decays. For each decay mode, we take into account one broad q^2 bin below the narrow charmonium resonances and one broad bin above. Considering finer q^2 bins that are often also available has no advantage for the determination of the CKM matrix elements.

The main theoretical uncertainties of the semileptonic branching ratio predictions arise from form factors and from additional nonfactorizable effects. We use FLAVIO version 2.3.0 [62] with default hadronic parameters to determine the central values and the uncertainties of the branching ratios, as well as the correlations of the uncertainties. Note that FLAVIO does not take into account the QED corrections that were calculated in [63]. The form factors implemented in FLAVIO are based on a combined fit of light cone sum rule and lattice QCD results [64] (see also [65]) and therefore lead to sizeable error correlations of the branching ratio predictions in the low- q^2 and high- q^2 bins.

$$\begin{aligned} a_{B^0 \rightarrow K^{*0}}^{[1.1,6]} &= (1.36 \pm 0.16) \times 10^{-4} \\ a_{B^0 \rightarrow K^{*0}}^{[15,19]} &= (1.39 \pm 0.15) \times 10^{-4} \\ a_{B_s \rightarrow \phi}^{[1.1,6]} &= (1.54 \pm 0.14) \times 10^{-4} \\ a_{B_s \rightarrow \phi}^{[15,19]} &= (1.30 \pm 0.12) \times 10^{-4} \end{aligned} ,$$

On the experimental side, we use the latest results from the LHCb collaboration for the branching ratios of $B^+ \rightarrow K^+ \mu^+ \mu^-$ [29], $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ [30], $B_s \rightarrow \phi \mu^+ \mu^-$ [31], and $\Lambda_b \rightarrow \Lambda \mu^+ \mu^-$ [66]

$$\text{BR}(B^+ \rightarrow K^+ \mu^+ \mu^-)_{\text{exp}}^{[1.1,6]} = (1.19 \pm 0.07) \times 10^{-7}, \quad (36)$$

$$\text{BR}(B^+ \rightarrow K^+ \mu^+ \mu^-)_{\text{exp}}^{[15,22]} = (0.85 \pm 0.05) \times 10^{-7}, \quad (37)$$

$$\text{BR}(B^0 \rightarrow K^{*0} \mu^+ \mu^-)_{\text{exp}}^{[1.1,6]} = (1.68 \pm 0.15) \times 10^{-7}, \quad (38)$$

$$\text{BR}(B^0 \rightarrow K^{*0} \mu^+ \mu^-)_{\text{exp}}^{[15,19]} = (1.74 \pm 0.14) \times 10^{-7}, \quad (39)$$

$$\text{BR}(B_s \rightarrow \phi \mu^+ \mu^-)_{\text{exp}}^{[1.1,6]} = (1.41 \pm 0.10) \times 10^{-7}, \quad (40)$$

$$\text{BR}(B_s \rightarrow \phi \mu^+ \mu^-)_{\text{exp}}^{[15,19]} = (1.85 \pm 0.13) \times 10^{-7}, \quad (41)$$

Moreover, there are non-negligible correlations between the uncertainties of the two pseudoscalar to vector transitions $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ and $B_s \rightarrow \phi \mu^+ \mu^-$, due to the approximate $SU(3)$ flavor symmetry. We neglect additional percent level correlations between the uncertainties of the considered decay modes that are due to common input parameters. For the numerical coefficients in (32), we find for the $B^+ \rightarrow K^+ \mu^+ \mu^-$ decay

$$\begin{aligned} a_{B^+ \rightarrow K^+}^{[1.1,6]} &= (1.00 \pm 0.16) \times 10^{-4}, \\ a_{B^+ \rightarrow K^+}^{[15,22]} &= (0.61 \pm 0.06) \times 10^{-4}, \end{aligned} \quad (33)$$

with an error correlation of $\rho = +68\%$. For the baryonic decay $\Lambda_b \rightarrow \Lambda \mu^+ \mu^-$ we find

$$\begin{aligned} a_{\Lambda_b \rightarrow \Lambda}^{[1.1,6]} &= (0.30 \pm 0.16) \times 10^{-4}, \\ a_{\Lambda_b \rightarrow \Lambda}^{[15,20]} &= (2.07 \pm 0.21) \times 10^{-4}, \end{aligned} \quad (34)$$

with a modest error correlation of $\rho = +5\%$. Finally, for the decays $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ and $B_s \rightarrow \phi \mu^+ \mu^-$ we find the following coefficients and correlation matrix

$$\rho = \begin{pmatrix} 1 & 0.50 & 0.24 & 0.01 \\ & 1 & 0.00 & 0.22 \\ & & 1 & 0.36 \\ & & & 1 \end{pmatrix}. \quad (35)$$

$$\text{BR}(\Lambda_b \rightarrow \Lambda \mu^+ \mu^-)_{\text{exp}}^{[1.1,6]} = (0.44 \pm 0.31) \times 10^{-7}, \quad (42)$$

$$\text{BR}(\Lambda_b \rightarrow \Lambda \mu^+ \mu^-)_{\text{exp}}^{[15,20]} = (6.00 \pm 1.34) \times 10^{-7}. \quad (43)$$

We note that in most cases the experimental precision is better than $\sim 10\%$ and has already surpassed the precision of the SM predictions.

IV. RESULTS OF THE FITS

Based on the above SM expressions for the various observables in terms of the CKM parameters λ , $|V_{cb}|$, $|V_{ub}|$, and γ , and the corresponding experimental measurements, we construct a χ^2 function that takes into account the experimental uncertainties as well as the theoretical uncertainties and their correlations in the form of covariance matrices.

As the sine of the Cabibbo angle is known with excellent precision, we simply set it to its central value $\lambda \simeq 0.2248$ [67] in our numerical analysis. For the CKM angle γ , we

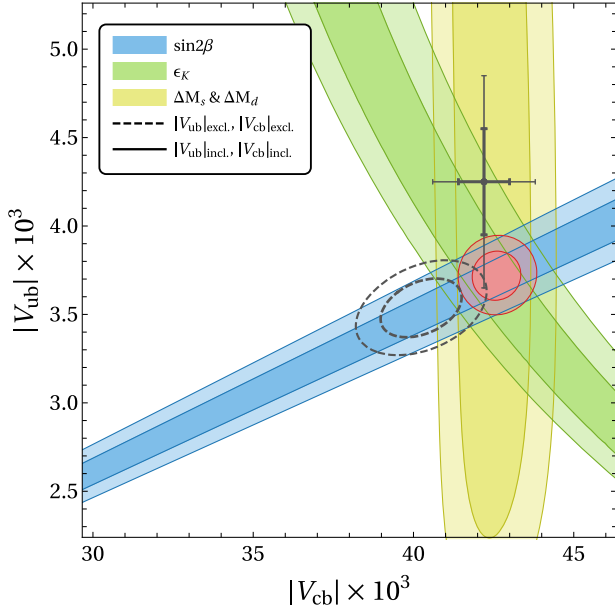


FIG. 2. Constraints in the $|V_{cb}|-|V_{ub}|$ plane from meson mixing observables. The blue, green, and yellow bands show the constraints from $\sin(2\beta)$, ϵ_K , and ΔM_d and ΔM_s , respectively. The combined best fit region is shown in red. The dashed contours and the gray error bars show for comparison the exclusive and inclusive values from Fig. 1.

use the HFLAV average $\gamma = (66.1^{+3.4}_{-3.6})^\circ$ [38], which is dominated by the precise LHCb results from [68] (see also the very recent update [69]). The importance of precise determinations of γ has recently been emphasized also in [26,70]. By profiling over γ we arrive at best fit contours in the $|V_{cb}|-|V_{ub}|$ plane.

A. Loop level determination from meson oscillations

We first focus on the meson oscillation observables, ϵ_K , ΔM_d , ΔM_s , and $\sin(2\beta)$ as discussed in Secs. III A and III B. In Fig. 2 we show the individual 1σ and 2σ constraints in the $|V_{cb}|-|V_{ub}|$ plane from ϵ_K (green), from $\sin(2\beta)$ (blue), and from the combination of ΔM_d and ΔM_s (yellow). The red region is the combination of those constraints and corresponds to the values

$$\begin{aligned} |V_{cb}|_{\text{meson mixing}} &= (42.6 \pm 0.5) \times 10^{-3}, \\ |V_{ub}|_{\text{meson mixing}} &= (3.72 \pm 0.09) \times 10^{-3}, \end{aligned} \quad (44)$$

with a negligibly small error correlation. These central values are very close to the results from global CKM fits [67,71]. Our values of $|V_{cb}|$ and $|V_{ub}|$ from meson mixing observables also agree very well with the ones recently found in [26].

We observe that the B meson oscillation frequencies ΔM_d and ΔM_s predict a value for $|V_{cb}|$ that is in excellent

agreement with the inclusive determination.¹ Also the combination of the CP violating observables ϵ_K and $\sin(2\beta)$ prefers a value for $|V_{cb}|$ close to the inclusive determination. The combination of all meson mixing observables shows a remarkable level of consistency. The combination is dominated by ϵ_K and $\sin(2\beta)$, while ΔM_d and ΔM_s play a slightly lesser role due to the slightly larger theory uncertainties. Continued improvements in the determination of the hadronic B mixing matrix elements are thus highly motivated.

Overall, we observe that our combination of meson mixing observables is compatible with both the inclusive and the exclusive determination of $|V_{cb}|$ and $|V_{ub}|$ to better than 2σ . There is a slight preference for the inclusive value of $|V_{cb}|$ and the exclusive value of $|V_{ub}|$.

B. Loop level determination from rare b decay rates

The rare B decays discussed in Sec. III C can be used to determine $|V_{cb}|$ but have only very weak sensitivity to the value of $|V_{ub}|$. We therefore do not show the results in the $|V_{cb}|-|V_{ub}|$ plane but directly as values for $|V_{cb}|$, profiling over $|V_{ub}|$ within the exclusive value from the PDG, cf. Eq. (2). We checked that using instead the inclusive value for $|V_{ub}|$ leads to negligible differences in our results.

The error bars in Fig. 3 show the best fit values for $|V_{cb}|$ based on the rare semileptonic decays $B^+ \rightarrow K^+\mu^+\mu^-$, $B^0 \rightarrow K^{*0}\mu^+\mu^-$, and $B_s \rightarrow \phi\mu^+\mu^-$ at low q^2 (red) and at high q^2 (orange), as well as from $B \rightarrow X_s\gamma$ (yellow) and $B_s \rightarrow \mu^+\mu^-$ (green). The colored bands in the figure show the results from the combinations of the semileptonic decays at low q^2 and high q^2 as well as the combination of all rare B decay results. The results from the baryonic decay $\Lambda_b \rightarrow \Lambda\mu^+\mu^-$ are not shown individually as they have large uncertainties but they are included in the combinations. The exclusive and inclusive values for $|V_{cb}|$ from the PDG are shown for comparison.

As is well known, the experimental results for the branching ratios of $B \rightarrow K\mu^+\mu^-$, $B \rightarrow K^*\mu^+\mu^-$ and $B_s \rightarrow \phi\mu^+\mu^-$ [29–31] are all significantly low compared to SM predictions, in particular in the low q^2 region. These results have been interpreted as possible signs of new physics effects in rare B decays. In the context of the SM, the results instead are an indication for a very small value of $|V_{cb}|$. For example, the $B_s \rightarrow \phi\mu^+\mu^-$ branching ratio at low q^2 points to a particularly low central value $|V_{cb}| \simeq 31 \times 10^{-3}$, much lower than the inclusive or exclusive tree-level determinations. The results of our combinations of the semileptonic decays at low q^2 and high q^2 are

¹As mentioned in Sec. III B, we use the $N_f = 2 + 1 + 1$ lattice results for the hadronic matrix elements relevant for B_s and B^0 mixing [44]. Comparable results are obtained if sum rule determinations of the hadronic matrix elements are used [72]. We checked that the preferred value for $|V_{cb}|$ is very close to the exclusive determination if we use the $N_f = 2 + 1$ lattice results.

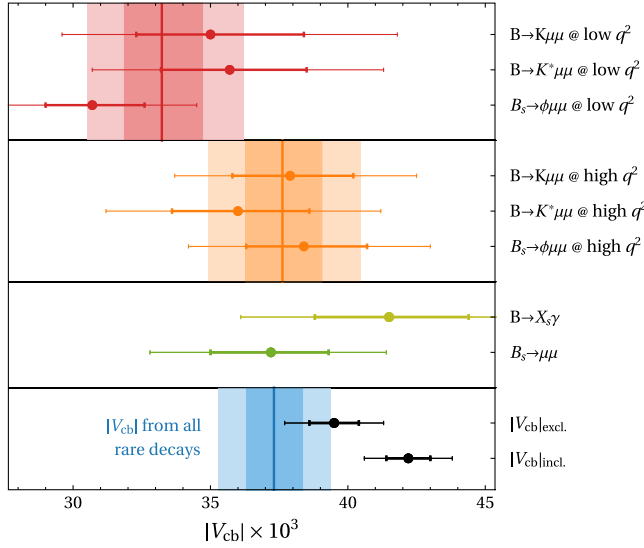


FIG. 3. Determinations of $|V_{cb}|$ from various rare decays of B mesons. The error bars correspond to the individual 1σ and 2σ uncertainties. The red and orange bands show the combination of the semileptonic decays at low q^2 and high q^2 , respectively. The blue band is the combination of all rare B decay data. For comparison, the PDG averages of the inclusive and exclusive determinations of $|V_{cb}|$ are shown as well.

$$\begin{aligned} |V_{cb}|_{\text{low } q^2} &= (33.2 \pm 1.5) \times 10^{-3}, \\ |V_{cb}|_{\text{high } q^2} &= (37.6 \pm 1.4) \times 10^{-3}. \end{aligned} \quad (45)$$

While the result from high q^2 is compatible with the tree-level exclusive determination, it is more than 2σ below the value from the inclusive determination. Our result from low q^2 is more than 3σ below the exclusive value and more than 5σ below the inclusive one. A small $|V_{cb}|$ thus appears to be an unlikely explanation of the low $b \rightarrow s\mu^+\mu^-$ branching ratio data.

For comparison, the value for $|V_{cb}|$ we find from the $B \rightarrow X_s\gamma$ decay sits between the inclusive and exclusive determinations and has a large uncertainty. It is compatible with both determinations to better than 1σ . The $B_s \rightarrow \mu^+\mu^-$ decay gives a $|V_{cb}|$ that is approximately 2σ below the inclusive determination. Interestingly, it is compatible with the exclusive determination to better than 1σ . This indicates that the 2σ tension that is observed between the SM prediction of $\text{BR}(B_s \rightarrow \mu^+\mu^-)$ [61] and the experimental measurements [57–60] can be largely resolved if the exclusive determination of $|V_{cb}|$ were used for the SM prediction. This has been also pointed out in [25].

Combining all rare B decay data, we arrive at the average

$$|V_{cb}|_{\text{rare decays}} = (37.3 \pm 1.0) \times 10^{-3}. \quad (46)$$

The central value is significantly below the inclusive determination of $|V_{cb}|$. It is also below the exclusive

determination, but compatible to better than 2σ . Interestingly, the uncertainty is only slightly larger than the uncertainty of the tree-level determination. However, the uncertainty is dominated by theory and challenging to improve.

C. Global fit of $|V_{cb}|$ and $|V_{ub}|$ from loops

Finally, we combine the various loop determinations of $|V_{cb}|$ and $|V_{ub}|$ in a global fit. We take into account the meson mixing observables discussed in Secs. III A and III B and the rare meson decays discussed in Sec. III C and compare to the tree-level determinations from Sec. II. The result is shown in Fig. 4.

We observe considerable tensions between the different determinations. In particular, there is a large tension between the rare decays (blue band) on the one side and the meson mixing observables (red ellipse) and the tree-level inclusive determination (black error bars) on the other side. Our average of the tree-level exclusive determinations (dashed ellipse) sits in between. To obtain the global loop combination shown in yellow, we follow the PDG prescription and inflate the uncertainty by $\sqrt{\chi_{\text{bf}}^2/N_{\text{dof}}} = \sqrt{40.5/13} = 1.76$. Employing a Gaussian approximation, we find

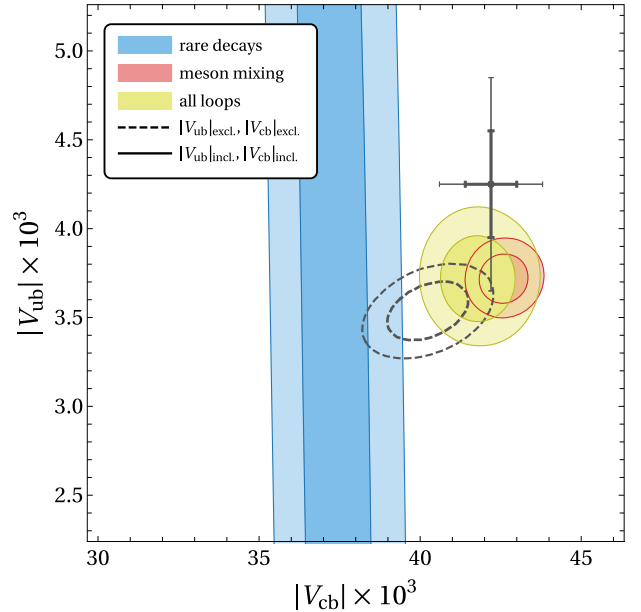


FIG. 4. Comparison of all $|V_{cb}|$ and $|V_{ub}|$ determinations discussed in this work. The black cross shows the inclusive values from the PDG, cf. Eq. (1). The dashed ellipse is our combination of all exclusive tree-level determinations as discussed in Sec. II. The red region corresponds to our fit of meson oscillation data from Sec. IV A. The blue band corresponds to the determination using rare B decays from Sec. IV B. Finally, the yellow region corresponds to a combined fit of all loop determinations with error contours inflated to account for the tensions in the fit.

$$\begin{aligned} |V_{cb}|_{\text{loop}} &= (41.75 \pm 0.76) \times 10^{-3}, \\ |V_{ub}|_{\text{loop}} &= (3.71 \pm 0.16) \times 10^{-3} \end{aligned} \quad (47)$$

with a negligible error correlation. The central values are in very good agreement with the results from global CKM fits [67,71] that do not take into account rare B decay data. The uncertainties of our loop level determinations are considerably larger, due to the omission of the tree-level information and the error inflation mentioned above.

V. CONCLUSIONS

Precise determinations of V_{cb} and V_{ub} are crucial inputs for testing the SM CKM picture of flavor and CP violation. For many years, discrepancies between determinations using exclusive and inclusive tree-level B decays have limited the precision of V_{cb} and V_{ub} determinations. A summary of the current status is provided in Fig. 1. In this paper, we used loop level processes to obtain the absolute values of V_{cb} and V_{ub} . Such a loop level strategy gives valid results in the absence of new physics effects in the considered observables. We focused on two classes of loop processes: neutral meson oscillations and rare b hadron decays.

Our main results are summarized in Figs. 2 and 3. We find that the combination of the observables ϵ_K (that parametrizes indirect CP violation in Kaon oscillations) and $\sin(2\beta)$ (that parameterizes indirect CP violation in B^0 oscillations) gives a precise determination of $|V_{cb}|$ and $|V_{ub}|$, with central values close to the inclusive value of

$|V_{cb}|$ and the exclusive value for $|V_{ub}|$. Also the B meson oscillation frequencies ΔM_d and ΔM_s show preference for the inclusive value of $|V_{cb}|$, but are also compatible with the exclusive value, given the current theory errors. To improve the constraining power of ΔM_d and ΔM_s , more precise determinations of the hadronic matrix elements in B meson mixing are required.

As is well known, the experimental measurements of several rare B decay branching ratios are significantly below the SM predictions. Therefore, $|V_{cb}|$ determinations based on rare b decay data give values far below the direct determinations from tree-level decays. The preference for small $|V_{cb}|$ is particularly pronounced if one focuses on rare semileptonic B decays at low dilepton invariant mass. In that case we find a value for $|V_{cb}|$ that is more than 3σ below the exclusive value and more than 5σ below the inclusive one. We thus consider it unlikely that $|V_{cb}|$ is the sole culprit behind the low $b \rightarrow s\mu^+\mu^-$ branching ratios.

Interestingly, the $\sim 2\sigma$ tension between the commonly quoted SM prediction for the $B_s \rightarrow \mu^+\mu^-$ branching ratio and the corresponding experimental world average largely disappears if the exclusive value of $|V_{cb}|$ were used in the SM prediction.

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