de Sitter space as a BRST invariant coherent state of gravitons

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The S-matrix formulation indicates that a consistent embedding of the de Sitter state in quantum gravity is possible exclusively as an excited quantum state constructed on top of a valid S-matrix vacuum such as the Minkowski vacuum. In the present paper we offer such a construction of the de Sitter state in the form of a coherent state of gravitons. Unlike previous realizations of this idea, we focus on BRST invariance as the guiding principle for physicality. In order to establish the universal rules of gauge consistency, we study the BRST-invariant construction of coherent states created by classical and quantum sources in QED and in linearized gravity. Introduction of N copies of scalar matter coupled to gravity allows us to take a special double scaling limit, a so-called species limit, in which our construction of the de Sitter state becomes exact. In this limit, the irrelevant quantum gravitational effects vanish, whereas the collective phenomena, such as Gibbons-Hawking radiation, are calculable.

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I. INTRODUCTION

Classical general relativity (GR) gives no *a priori* preference to any space-time metric. All solutions of GR are equally legitimate, provided they satisfy the minimal consistency requirements, such as a valid causal structure. For example, there is no reason to give an advantage to the Minkowski space-time over a de Sitter one. This prompts the thought that, upon quantization, any valid classical background of GR can or must serve as a legitimate vacuum of the quantum theory.

However, the S-matrix formulation of quantum gravity, which, in particular, is organic to string theory, gives a very different perspective [1]. The consistency and double-scaling arguments show that having de Sitter space as a valid S-matrix vacuum inevitably trivializes the gravitational S matrix. The reason for this is that the rigidity of the geometry and the quantum coupling of gravitons (or closed strings) are controlled by one and the same parameter, the Planck mass, $M_{\rm pl}$. This simple but profound fact lies at the very foundation of quantum gravity.

Thus, the S matrix tells us that the only possibility for embedding de Sitter space in quantum theory is by treating it as an excited state constructed on top of a valid S-matrix vacuum, such as the Minkowski vacuum [1].

Originally, such a description has been applied to de Sitter and other cosmological space-times in [2–7]. In this approach, sometimes referred to as "corpuscular resolution," the nontrivial space-time geometry is treated as a state with a high occupation number of gravitons, which is approximately classical. The natural candidate for such a state is a coherent state.

Coherent states are thought to represent an adequate quantum description of classical systems. They were introduced in quantum field theory by Glauber [8]. They also play an important role in eliminating infrared divergencies by dressing asymptotic charged states with soft photons [9–14]. Coherent states have also been used to formulate the quantum picture of solitons [15].

In previous formulations [4–6] of de Sitter space as a coherent state, certain universal tendencies were observed. The known semiclassical features of de Sitter space are recovered as a result of time evolution of the coherent state. For example, a famous Gibbons-Hawking radiation with temperature set by the Hubble H emerges as a result of the actual decay of the coherent state of gravitons. It therefore uncovers new features that are not visible in ordinary treatments. In particular, as a result of backreaction, the de Sitter state gradually loses coherence and evolves into a self-entangled state. This results in a full departure from the classical description after the timescale of half-decay. In pure gravity, the corresponding timescale, a so-called quantum break-time, is $t_Q \sim M_{\rm pl}^2/H^3$, and it becomes shorter in the presence of more degrees of freedom. This phenomenon has important implications both fundamentally and observationally. For example, it limits the duration of any de Sitter Hubble patch by t_Q, thereby eliminating the possibility of an eternally inflating state. The significance of this statement for the efficient beginning of inflation was discussed in [16]. The S-matrix exclusion of the de Sitter landscape also has important implications for the concept of "naturalness" in particle physics [17]. In this light, it is crucial to have a satisfactory understanding of the quantum de Sitter state.

In the present paper we shall investigate the legitimacy of de Sitter space as a coherent state from the point of view of BRST symmetry (see e.g., [18]). Due to the fact that gravity is a gauge theory, it is imperative to have a rigorous understanding of the rules for building physical coherent states therein. In this work, we present such a construction within the canonical framework of BRST quantization. This work covers the analysis for scalar electrodynamics and linearized gravity, uncovering peculiarities of the procedure and capitalizing on physical implications.

We give a BRST-invariant formulation of the de Sitter coherent state in a quantum theory of massless spin-2, with a (positive) cosmological constant source and coupling to quantum scalar matter. We perform various consistency analyses of scaling properties of de Sitter space obtained in this way.

Next, we introduce the N species of scalar matter. It has been shown [19] that this theory allows for a special double-scaling limit with $N \to \infty$, the so-called "species limit," in which the quantum gravitational processes simplify significantly. All graviton nonlinearities, as well as their loop contributions, vanish as $1/N$. At the same time, the collective quantum gravitational effects, such as Gibbons-Hawking radiation, are finite and explicitly calculable. We observe that the BRST-invariant coherent state implemented in this setup correctly captures the quantum features of de Sitter space, such as the effects of Gibbons-Hawking particle creation.

Our results indicate that the process of treating de Sitter space as a coherent state of gravitons built on the Minkowski vacuum passes the essential quantum consistency tests.

II. COHERENT STATES WITH GLOBAL CHARGE

In the absence of gauge symmetry, the construction of coherent states is straightforward. They can be built around the vacuum of the theory by the action of the field displacement operator (see e.g., [20,21]), written in terms of canonical degrees of freedom without invoking their asymptotic representation. In particular, the coherent state for a complex scalar field takes the following form:

$$|C\rangle = e^{-i\int d^3x(\Phi_c\hat{\Pi}-\Pi_c\hat{\Phi}+\mathrm{H.c.})}|\Omega\rangle, \tag{1}$$

where $\Phi_c(x)$ and $\Pi_c(x)$ are the c-number functions of spatial coordinates, $\hat{\Phi}$ and $\hat{\Pi}$ stand for the field and canonical conjugate momentum operators, respectively, while $|\Omega\rangle$ denotes the vacuum (i.e., the lowest energy eigenstate of the Hamiltonian). This state is constructed in such a way that it satisfies the following initial conditions:

$$\langle C|\hat{\Phi}|C\rangle(t=0) = \Phi_c(x), \tag{2}$$

$$\langle C|\hat{\Pi}|C\rangle(t=0) = \Pi_c(x), \tag{3}$$

which follow solely from canonical commutation relations and the absence of the tadpoles in the vacuum. In the presence of global $U(1)$ symmetry that shifts the phase of $\hat{\Phi}$, the expectation value of the corresponding charge in the state in question is conserved and given by

$$\langle C|\hat{Q}|C\rangle = i\int d^3x(\Pi_c\Phi_c - \Pi_c^*\Phi_c^*) \equiv Q_c. \tag{4}$$

It must be pointed out that the coherent states are not the eigenstates of the global charge operator. Rather, they are a superposition of states with different charges. In particular, using canonical commutation relations and taking into account that the vacuum is a zero-charge eigenstate, one gets

$$\hat{Q}|C\rangle = Q_c|C\rangle + e^{-i\int d^3x(\Phi_c\hat{\Pi}-\Pi_c\hat{\Phi}+\mathrm{H.c.})} \times i\int d^3x'(\Phi_c\hat{\Pi}+\Pi_c\hat{\Phi}-\mathrm{H.c.})|\Omega\rangle. \tag{5}$$

The presence of the second term is the reason the state is not an eigenstate of Q. This will have peculiar ramifications for gauge theories.

III. QUANTUM ELECTRODYNAMICS

Upon gauging the $U(1)$ symmetry, the construction of coherent states needs to be revised, on account of the consistency requirement to be imposed on physical states. Our starting point is the BRST-invariant formulation of scalar electrodynamics,

$$\mathcal{L} = -\frac{1}{4}\hat{F}_{\mu\nu}^2 + |D_\mu\hat{\Phi}|^2 - m^2|\hat{\Phi}|^2 - \partial_\mu\hat{B}\hat{A}^\mu + \frac{1}{2}\xi\hat{B}^2 + \partial_\mu\hat{\bar{c}}\partial^\mu\hat{c}, \tag{6}$$

with $\hat{F}_{\mu\nu} \equiv \partial_\mu\hat{A}_\nu - \partial_\nu\hat{A}_\mu$ and $D_\mu\hat{\Phi} \equiv \partial_\mu\hat{\Phi} - ig\hat{A}_\mu\hat{\Phi}$ as usual and $\hat{c}, \hat{\bar{c}}$ being Fadeev-Popov ghosts that are anticommuting Lorentz scalars. Throughout this work, the repeated covariant space-time indices are contracted by the Minkowski metric $\eta_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$. The theory is invariant under the celebrated BRST transformations

$$\delta\hat{A}_\mu = \theta\partial_\mu\hat{c}, \qquad \delta\hat{\bar{c}} = \theta\hat{B}, \qquad \delta\hat{c} = \delta\hat{B} = 0,$$
$$\delta\Phi = ig\theta\hat{c}\,\hat{\Phi}, \qquad \delta\Phi^\dagger = -ig\theta\hat{c}\hat{\Phi}^\dagger, \tag{7}$$

where θ is a Grassmann number that serves as a parameter of transformation. The conserved charge associated with this symmetry, which is of utmost importance for the construction of the physical Hilbert space, takes the following form:

$$\hat{Q}_B = \int d^3x[\hat{c}(g\hat{\rho} - \partial_j\hat{E}_j) + \hat{B}\hat{\Pi}_{\bar{c}} + \partial_j(\hat{c}\,\hat{E}_j)], \quad (8)$$

with $\hat{\rho} \equiv i(\hat{\Phi}\,\hat{\Pi} - \hat{\Pi}^\dagger\hat{\Phi}^\dagger)$ representing the $U(1)$ charge density, $\hat{E}_j \equiv \hat{F}_{0j}$ being the electric field operator, and $\hat{\Pi}_{\bar{c}}$ denoting the conjugate momentum of the corresponding ghost field.

As it is well known, there are consistency requirements on physical states in gauge theories. For example, Gauss' law informs us that charges must be dressed with a corresponding gauge field configuration [22–24], as no naked charges can exist in nature. Within the adopted framework for quantization, the physical states carry vanishing BRST charge, together with vanishing ghost number. Although the photon states are effortless to construct, by means of the field displacement operator that satisfies Gauss' law, the dressing fields are somewhat more peculiar to incorporate.

The pure gauge field coherent state, devoid of ghosts, can be constructed as

$$|A\rangle = e^{-i\hat{f}_A}|\Omega\rangle, \quad (9)$$

with

$$\hat{f}_A \equiv \int d^3x(A_j^c\hat{E}_j - E_j^c\hat{A}_j + A_0^c\hat{B} - B^c\hat{A}_0), \quad (10)$$

and the quantities carrying the label "c" being the c-number functions that specify the initial field configuration. In particular, initial conditions for one-point expectation values simply follow from canonical commutation relations and are given by

$$\langle A|\hat{A}_\mu|A\rangle = A_\mu^c, \quad \langle A|\hat{E}_j|A\rangle = E_j^c, \quad \langle A|\hat{B}|A\rangle = B^c, \quad (11)$$

with other fields having vanishing initial expectation values. Taking into consideration that the proper vacuum must be annihilated by the BRST charge, the physicality condition $\hat{Q}_B|A\rangle = 0$ leads to the following constraint:

$$\int d^3x(-\hat{c}\partial_jE_j^c + \hat{\Pi}_{\bar{c}}B^c + \partial_j(\hat{c}E_j^c))|\Omega\rangle = 0. \quad (12)$$

For the derivation of this exact expression, we have taken advantage of $e^{-i\hat{f}_A}$ being the field displacement operator at the initial time (which in turn follows from the Baker-Cambell-Hausdorff formula and equal-time commutation relations) and the conservation of $\hat{Q}_B$. The last (boundary) term of (12) vanishes not only for localized electric field configurations but for more general ones as well. We see this by employing the momentum-space decomposition of the ghost operator $\hat{c}$, when relevant. The consistency condition (12) is straightforwardly satisfied for

$$\partial_jE_j^c = 0, \qquad B^c = 0. \quad (13)$$

In other words, the electric field must satisfy charge-free Gauss law. The introduction of nontrivial B^c seems possible but leads to unnecessary complications and will not be considered in this work.

Moving on to the dressing field, a naive attempt for converting an electrically charged coherent state (1) into a physical one would consist of dressing it with an appropriate coherent gauge field. In other words, one could consider the charged coherent states to be of the form

$$|A\rangle \otimes |C\rangle. \quad (14)$$

The inadequacy of this proposal can be straightforwardly demonstrated by showing that

$$\hat{Q}_B\{|A\rangle \otimes |C\rangle\} = \hat{Q}_B e^{-i\int d^3x(A_j^c\hat{E}_j - E_j^c\hat{A}_j + A_0^c\hat{B} - B^c\hat{A}_0)} \times e^{-i\int d^3x(\Phi_c\hat{\Pi} - \Pi_c\hat{\Phi} + \mathrm{H.c.})}|\Omega\rangle \neq 0, \quad (15)$$

as long as Φ_c and Π_c are nonvanishing. The inability to dress the scalar coherent state with a coherent electromagnetic configuration is connected to the fact that the former is not an eigenstate of $U(1)$ charge; see e.g., (5). Therefore, the matter states cannot be made BRST invariant by dressing them with coherent gauge field configurations. Instead, we begin the construction with the invariant operators.

The idea of defining gauge-invariant matter degrees of freedom goes back to Dirac [22] (see also [23]), who found that the operator undergoing merely a phase rotation under gauge transformations can be combined with the gauge field in the following invariant fashion:

$$\hat{\Phi}_g = \hat{\Phi}\cdot\exp\left(-ig\frac{1}{\nabla^2}\partial_j\hat{A}_j\right), \quad (16)$$

$$\hat{\Pi}_g = \hat{\Pi}\cdot\exp\left(+ig\frac{1}{\nabla^2}\partial_j\hat{A}_j\right), \quad (17)$$

with the subscript "g" indicating the gauge invariance. Similar nonlocal operators $\exp(-ig\frac{1}{\Box}\partial_\mu A^\mu)$ were used for maintaining gauge invariance in the case of anomalous symmetries [25].

It is important to notice that these operators satisfy canonical commutation relations,

$$[\hat{\Phi}_g(t,x), \hat{\Pi}_g(t,y)] = i\delta^{(3)}(x-y). \quad (18)$$

Moreover, it is straightforward to show explicitly that due to the outlined gauge-invariant construction,

$$[\hat{Q}_B, \hat{\Phi}_g(x)] = [\hat{Q}_B, \hat{\Pi}_g(x)] = 0. \tag{19}$$

Based on this observation, we can construct dressed coherent states for matter fields in analogy with (1), albeit with $\hat{\Phi}_g$ and $\hat{\Pi}_g$. In other words, the coherent state

$$|C_g\rangle = e^{-i\int d^3x(\Phi_c\hat{\Pi}_g - \Pi_c\hat{\Phi}_g + \mathrm{H.c.})}|\Omega\rangle \tag{20}$$

satisfies the physicality conditions for arbitrary $\Phi_c(x)$ and $\Pi_c(x)$. As for connection with the classical field configurations, it is straightforward to show that, at the initial moment,

$$\langle C_g|\hat{\Phi}_g|C_g\rangle(t=0) = \Phi_c, \quad \langle C_g|\hat{\Pi}_g|C_g\rangle(t=0) = \Pi_c, \tag{21}$$

$$\partial_j\langle C_g|\hat{E}_j|C_g\rangle(t=0) = ig(\Phi_c\Pi_c - \Pi_c^*\Phi_c^*), \tag{22}$$

$$\langle C_g|\hat{A}_\mu|C_g\rangle(t=0) = 0. \tag{23}$$

The state can be further supplemented with the coherent photon configuration using the gauge field displacement operator discussed above. Putting all the ingredients together, a physical coherent state that corresponds to a certain classical charge distribution and accounts for the dressing field, as well as for the additional electromagnetic field, is given by

$$|C_g, A\rangle = e^{-i\int d^3x(\Phi_c\hat{\Pi}_g - \Pi_c\hat{\Phi}_g + \mathrm{H.c.})} \times e^{-i\int d^3x(A_j^c\hat{E}_j - E_j^c\hat{A}_j + A_0^c\hat{B})}|\Omega\rangle. \tag{24}$$

It must be stressed that the order in which the exponentials appear in this definition of the coherent state is necessary for reproducing (21). The consistency of this state requires the transversality of E_j^c; thus (22) will be maintained, the only modification being the generation of the nonvanishing expectation value for $\hat{A}_\mu$.

The dynamics can be readily obtained from Heisenberg's equations, which follow from the Hamiltonian of the system

$$\hat{H} = \int d^3x\bigg[\frac{1}{2}\hat{E}_j^2 + \frac{1}{4}\hat{F}_{ij}^2 + |\hat{\Pi}|^2 + |D_j\hat{\Phi}|^2 + m^2|\hat{\Phi}|^2 + \hat{B}\partial_j\hat{A}_j - \frac{\xi}{2}\hat{B}^2 + \partial_j(\hat{A}_0\hat{E}_j - \hat{B}\hat{A}_j) + \hat{A}_0(-\partial_j\hat{E}_j + ig(\hat{\Phi}\hat{\Pi} - \hat{\Phi}^\dagger\hat{\Pi}^\dagger)) + \hat{\Pi}_c\hat{\Pi}_{\bar{c}} + \partial_j\hat{\bar{c}}\partial_j\hat{c}\bigg]. \tag{25}$$

Heisenberg's operator equations for the gauge sector lead to the following set of equations for one-point functions:

$$\partial_0\langle\hat{A}_0\rangle = \partial_j\langle\hat{A}_j\rangle - \xi\langle\hat{B}\rangle, \tag{26}$$

$$\partial_0\langle\hat{A}_j\rangle = \langle\hat{E}_j\rangle + \partial_j\langle\hat{A}_0\rangle, \tag{27}$$

$$\partial_0\langle\hat{E}_j\rangle - \partial_i\langle\hat{F}_{ij}\rangle - \partial_j\langle\hat{B}\rangle = ig\langle\hat{\Phi}^\dagger\hat{D}_j\hat{\Phi} - \mathrm{H.c.}\rangle, \tag{28}$$

$$\partial_0\langle\hat{B}\rangle = -\partial_j\langle\hat{E}_j\rangle + ig\langle\hat{\Phi}\hat{\Pi} - \hat{\Pi}^\dagger\hat{\Phi}^\dagger\rangle, \tag{29}$$

with $\langle\ldots\rangle$ denoting the expectation value of the enclosed Heisenberg picture operator in the coherent state (24). These equations represent quantum extensions of classical equations of motion that follow from (6). The nonlinear terms of (28) and (29), once evaluated over a coherent state, will contain both classical and quantum contributions. The latter can be quantified by

$$S_j \equiv i\langle\hat{\Phi}^\dagger\hat{D}_j\hat{\Phi} - (\hat{D}_j\hat{\Phi})^\dagger\hat{\Phi}\rangle - \bar{J}_j, \tag{30}$$

$$S_0 \equiv i\langle\hat{\Phi}\hat{\Pi} - \hat{\Pi}^\dagger\hat{\Phi}^\dagger\rangle - \bar{\rho}, \tag{31}$$

where $\bar{J}_\mu$ has been defined as the current constructed merely out of one-point functions, i.e.,

$$(-i)\bar{J}_j \equiv \langle\hat{\Phi}^\dagger\rangle(\partial_j\langle\hat{\Phi}\rangle - ig\langle A_j\rangle\langle\hat{\Phi}\rangle) - \mathrm{H.c.}, \tag{32}$$

$$(-i)\bar{\rho} \equiv \langle\hat{\Phi}\rangle\langle\hat{\Pi}\rangle - \langle\hat{\Pi}^\dagger\rangle\langle\hat{\Phi}^\dagger\rangle. \tag{33}$$

As a result, Eqs. (28) and (29) become

$$\partial_0\langle\hat{E}_j\rangle - \partial_i\langle\hat{F}_{ij}\rangle - \partial_j\langle\hat{B}\rangle = g\bar{J}_j + gS_j, \tag{34}$$

$$\partial_0\langle\hat{B}\rangle = -\partial_j\langle\hat{E}_j\rangle + g\bar{\rho} + gS_0. \tag{35}$$

In the absence of S_0 and S_j, these are classical equations of motions for the theory at hand. The computation of these quantum terms requires knowledge of two-point and three-point functions; however, we could instead follow [21] by evaluating them explicitly in the coherent state up to a desirable order in $\hbar$ and g. The explicit form of the equation for the scalar field will be useful when discussing the particle production and will be given when relevant. For now, let us keep in mind that the coherent state sets the stage by providing initial conditions for (26), (27), (34), (35) and for the scalar field.

IV. CLASSICAL CHARGES

In this section we discuss the possibility of introducing fundamentally classical sources as electromagnetic analogs of the cosmological constant (we borrow this analogy from [26]). We proceed by adding the following term to the Lagrangian:

$$\Delta\mathcal{L} = -\hat{A}_\mu J_{\mathrm{cl}}^\mu, \quad \text{with} \quad \partial_\mu J_{\mathrm{cl}}^\mu = 0, \tag{36}$$

where $J^\mu_{\rm cl}$ is a four-vector of predetermined c-number functions. Its presence does not alter the BRST transformation properties (7), under which the Lagrangian density is invariant up to a total derivative,

$$\delta(\mathcal{L}+\Delta\mathcal{L}) = -\partial_\mu(\theta\hat{c}J^\mu_{\rm cl}). \tag{37}$$

As a result, the Noether charge needs to be amended correspondingly, resulting in

$$\hat{Q}^J_B = \hat{Q}_B + \int d^3x\hat{c}J^0_{\rm cl}, \tag{38}$$

with $\hat{Q}_B$ denoting the BRST charge in the absence of the classical source, given by (15).

In the classical limit, the presence of $J_{\rm cl}$ would source classical electromagnetic field configuration. One might be tempted to associate this state to a vacuum of the theory. However, due to the fact that such a state is expected to precipitate particle production, the vacuum treatment is legitimate only in the limit of zero backreaction [1]. In gravity, where the existence of a valid S-matrix vacuum is vital, the analogous issue has profound consequences for a positive cosmological constant. Namely, it demands that de Sitter space be treated as a coherent state built around the Minkowski vacuum. For the theory at hand, this corresponds to constructing the coherent state around the vacuum of the Hamiltonian (25).[1]

Due to the fact that $\hat{Q}^J_B$ commutes with the total Hamiltonian

$$\hat{H}_J = \hat{H} + \int d^3x\hat{A}_\mu J^\mu_{\rm cl}, \tag{39}$$

we construct a coherent state of the electromagnetic field which satisfies

$$\hat{Q}^J_B|J\rangle = 0. \tag{40}$$

As per the arguments given above, we use the vacuum $|\Omega\rangle$ of $\hat{H}$ as the basis. It is straightforward to show that the state

$$|J\rangle = e^{-i\int d^3x(A^c_j\hat{E}_j - E^c_j\hat{A}_j + A^c_0\hat{B})}|\Omega\rangle \tag{41}$$

constructed in analogy with the pure photon state, satisfies the required constraint (40) if

$$\int d^3x[\hat{c}(-\partial_jE^c_j + J^0_{\rm cl}) + \partial_j(\hat{c}E^c_j)]|\Omega\rangle = 0. \tag{42}$$

This straightforwardly entails Gauss' law,

$$\partial_jE^c_j = J^0_{\rm cl}, \tag{43}$$

as long as the boundary term of (42) vanishes. The latter is trivially satisfied by localized classical sources, while its applicability to more general cases will be considered in the next section.

It must be stressed that $|\Omega\rangle$, being the vacuum of $\hat{H}$, is annihilated by $\hat{Q}_B$ and not by $\hat{Q}^J_B$. Consequently, the physicality condition it satisfies is not preserved by the Hamiltonian flow generated by $\hat{H}_J$.

Next, we ask if the coherent state $|J\rangle$ has a consistent dynamics. The evolution of one-point expectation values is governed by equations similar to the ones derived in the previous section, albeit with additional classical sources on the right-hand sides of (28) and (29). The process of interest is the Schwinger pair production of $\Phi\Phi^\dagger$ and the subsequent backreaction on the one-point function of the electric field. It is straightforward to see that to the leading order, the evolution of the quantum terms is determined by the time dependence of the tree-level mode functions for the scalar field, which in turn is governed by the electromagnetic background field. Moreover, the origin of the latter is immaterial for the former since the tree-level scalar mode functions are only sensitive to the classical background of the vector field. Therefore, as long as charges are lighter than the pair-creation threshold value, the Schwinger pair production will begin, and the backreaction on the electromagnetic field background will be governed by (26)–(29) supplemented with the external source. The process will continue until the produced particles decrease the electric field below the Schwinger threshold.

V. INFINITELY HOMOGENEOUS CLASSICAL SOURCE

Let us finish the discussion of quantum electrodynamics (QED) by focusing on a homogeneous constant source filling the entire space. This represents the closest electromagnetic analog of gravity with a cosmological constant [26].

The toy model consists of the scalar electrodynamics supplemented with a homogeneous external current $J^\mu_{\rm cl} = \delta^\mu_0\rho = {\rm const}$. The coherent electromagnetic state (41) sourced by this current must satisfy the physicality condition (42). As we saw, this condition straightforwardly leads to Gauss' law for the electric field configuration E^c_j parametrizing the coherent state produced by localized classical sources, for which the boundary term of (42) vanishes trivially. Let us now assess what happens for the homogeneous charge distribution.

The electric field produced by a constant charge is a linear function of coordinates. Such a field grows unbounded towards infinity. In the presence of any sort of dynamical charges in the theory, the field will be subjected to a discharge. It is therefore clear that a classical

[1]In different contexts, the coherent state of the electromagnetic field of classical charges has been discussed previously (see, e.g., [27,28]).

solution of a linearly growing field cannot be sustained in quantum theory. Nevertheless, it is useful to consider a regularized version of the story with an imposed spherical boundary which is gradually taken to infinity.

According to Gauss' law (43), E^c_j no longer vanishes on the boundary as it is given by

$$E^c_j = \frac{\rho}{3} x_j; \tag{44}$$

in fact, this quantity diverges at large distances. We can nevertheless demonstrate that the above-mentioned boundary term vanishes. For this, it suffices to notice that, without loss of generality, at any given moment in time the ghost field can be expanded into creation-annihilation operators

$$\hat{c}(x, t_0) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2k}} (\hat{a}_k e^{i\vec{k}\cdot\vec{x}} + \hat{b}^\dagger_k e^{-i\vec{k}\cdot\vec{x}}). \tag{45}$$

If we further take into account that the vacuum is annihilated by $\hat{a}_k$, we get the following equivalence:

$$\int d^3x \partial_j (\hat{c} E^c_j)|\Omega\rangle = 0 \Leftrightarrow \int d^3x \partial_j (e^{-i\vec{k}\cdot\vec{x}} x_j) = 0. \tag{46}$$

It is straightforward to see that the latter simplifies to

$$\frac{\partial}{\partial k_j} (k_j \delta^{(3)}(k)) = 0, \tag{47}$$

which holds as one of the properties of Dirac's delta function. Therefore, the coherent state of the electromagnetic field, produced by the external source in question, is consistent with the physicality conditions of BRST quantization. It must be stressed that this does not fully prove the legitimacy of the construction since the exact classicality of the source can lead to other inconsistencies such as unboundedness of the Hamiltonian from below. We set aside such issues and instead discuss the dynamical aspects of the constructed BRST-invariant coherent state.

Simplifying the construction, we take $A^c_\mu = 0$ in (41), which corresponds to $\langle \hat{A}_\mu \rangle = 0$ at the initial time, leaving us with the following coherent state:

$$|\rho\rangle = e^{i \int d^3x (E^c_j \hat{A}_j)} |\Omega\rangle. \tag{48}$$

Next we are interested in computing the leading quantum corrections to the dynamics of one-point expectation values for the gauge sector, resulting from the quantum pair creation of scalar particles. As was demonstrated explicitly in [21], the relevant dynamics follows from (26)–(29) (albeit with an additional classical source) by retaining only the expectation values of bilinear operators in S. For the setup in question, the equations take the following form:

$$\partial_0 \langle \hat{A}_0 \rangle = \partial_j \langle \hat{A}_j \rangle - \xi \langle \hat{B} \rangle, \tag{49}$$

$$\partial_0 \langle \hat{E}_j \rangle - \partial_i \langle \hat{F}_{ij} \rangle - \partial_j \langle \hat{B} \rangle = ig \langle \rho | \hat{\Phi}^\dagger D^{\rm cl}_j \hat{\Phi} - {\rm H.c.} | \rho \rangle, \tag{50}$$

$$\partial_0 \langle \hat{B} \rangle + \partial_j \langle \hat{E}_j \rangle - \rho = ig \langle \rho | \hat{\Phi} \hat{\Pi} - {\rm H.c.} | \rho \rangle, \tag{51}$$

where $D^{\rm cl}_\mu \equiv \partial_\mu - igA^{\rm cl}_\mu$. The terms on the right-hand side of (50) and (51) are already quantum since two-point functions are of order $\hbar$ at leading order. Therefore, we need to find the relevant correlators at tree level. The required dynamics of $\hat{\Phi}$ follows from solving the semi-classical equation

$$(D^{\rm cl}_\mu D^\mu_{\rm cl} + m^2) \hat{\Phi}(x, t) = 0, \tag{52}$$

with the classical electromagnetic configuration for our setup given by $A^{\rm cl}_j = \frac{1}{3}\rho t x_j$, $A^{\rm cl}_0 = \frac{1}{2}\rho t^2$, and $B^{\rm cl} = 0$. This equation needs to be solved with appropriate initial conditions, which for our coherent state correspond to

$$\langle \rho | \hat{\Phi}(x, 0) \hat{\Phi}(y, 0) | \rho \rangle = \langle \Omega | \hat{\Phi}(x, 0) \hat{\Phi}(y, 0) | \Omega \rangle. \tag{53}$$

In other words, even though the dynamics is given by the background-dependent equation of motion, the initial conditions for the mode functions are the ones in the Minkowski vacuum. See [20,21] for the discussion of this point and implications for perturbative dynamics.

The time dependence of the background field appearing in (52) will facilitate the particle production as expected.

In the process, the one-point function of the scalar field will remain zero, with nontrivial dynamics showing up in the two-point correlation function. The latter will backreact on the dynamics of the one-point functions of the gauge sector as per (50) and (51). Obviously, there will be divergences among the quantum terms, an obvious one emerging due to the appearance of the equal-point two-point function, which can be renormalized using standard prescriptions. The leftover finite quantum correction will amount to the physical backreaction.

There is a finite radius at which the field strength will cross the Schwinger pair-creation threshold. Beyond it, the efficient production of charges will take place until the background electric field is sufficiently reduced. This crossover radius can be estimated as

$$R \sim \frac{m^2}{g\rho}. \tag{54}$$

This radius marks the boundary of validity of the semi-classical approximation. Beyond it, a uniform charge distribution undergoes rapid quantum breaking.

VI. LINEAR GRAVITY

Next we would like to understand the ramifications of the BRST constraint for gravitational systems, specifically for the de Sitter space. We primarily work within linearized gravity, followed by the introduction of interactions with matter perturbatively.

The original construction of the de Sitter coherent state [4,6] was based on a correspondence between the classical de Sitter metric of linear Einstein theory and the source-free solution of massive Fierz-Pauli theory. This classical correspondence was established earlier in [26]. The idea [4,6] was to map the de Sitter space sourced by the cosmological constant on the solution of Fierz-Pauli theory, with subsequent quantum resolution of the latter. This allowed one to construct de Sitter space as a quantum state of high occupation number of nearly on-shell Fierz-Pauli gravitons on Minkowski space, accomplishing a first necessary step towards realization of the de Sitter state in the S-matrix formulation of quantum gravity. In this framework, processes such as Gibbons-Hawking particle creation are represented with the S-matrix process of decay of the graviton coherent state. At finite $M_{\rm pl}$, this leads to backreactions, such as loss of coherence. In this approach, the consistency of Hilbert space is guaranteed by gauge invariance of Fierz-Pauli gravitons. This invariance is due to additional polarizations of a massive graviton (as compared to the massless case), which play the role of Stückelberg fields that maintain the gauge invariance of the graviton state. Such invariance, of course, is not experienced by the states of massless Einstein gravitons. Therefore, construction of a consistent de Sitter coherent state directly in linear Einstein theory requires additional measures. Such measures will be developed in the present work in the form of BRST invariance of the state.

Our starting point is the BRST-invariant formulation of the theory of a free spin-2 field in Minkowski space,

$$\mathcal{L} = \frac{1}{2}(\partial_\alpha \hat{h}_{\mu\nu})^2 - \frac{1}{2}(\partial_\alpha \hat{h})^2 + \partial_\alpha \hat{h}\partial_\mu \hat{h}^{\mu\alpha} - \partial_\mu \hat{h}^{\mu\alpha}\partial_\nu \hat{h}^\nu_\alpha - \partial_\mu \hat{B}_\nu \left(\hat{h}^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu}\hat{h}\right) + \frac{1}{2}\xi \hat{B}_\mu^2 + \partial_\alpha \hat{\bar{C}}_\mu \partial^\alpha \hat{C}^\mu, \tag{55}$$

which is the linearized version of the framework developed in [29]. It follows from Einstein's theory of gravity in the limit of infinite Planck's mass. Here $\hat{h}_{\mu\nu}$, $\hat{C}_\mu$, and $\hat{B}_\mu$ are the massless spin-2 (graviton) field, the Fadeev-Popov ghost vectors, and an auxiliary vector, respectively, and ξ is the gauge fixing parameter. The theory at hand is invariant under the BRST transformation

$$\delta \hat{h}_{\mu\nu} = \theta(\partial_\mu \hat{C}_\nu + \partial_\nu \hat{C}_\mu), \tag{56}$$

$$\delta \hat{\bar{C}}_\mu = \theta \hat{B}_\mu, \tag{57}$$

where θ is a Grassmann variable, with the rest of the fields transforming trivially.

In analogy with the previous section, the construction of states is performed within the canonical Hamiltonian framework. Following the ADM formalism, we supplement the Lagrangian (55) with boundary terms appropriate for removing the time derivatives of $\hat{h}_{00}$ and $\hat{h}_{0j}$, making them manifestly nondynamical with vanishing conjugate momenta. As a result, the conjugate momentum of $\hat{h}_{ij}$ reduces to

$$\hat{\pi}_{ij} = \partial_0 h_{ij} - \delta_{ij}\partial_0 h_{kk} + 2\delta_{ij}\partial_k h_{k0} - \partial_i h_{j0} - \partial_j h_{i0}. \tag{58}$$

Its BRST transformation property follows from (56) and is given by

$$\delta \hat{\pi}_{ij} = 2\theta(\nabla^2 \delta_{ij} - \partial_i \partial_j)\hat{C}_0. \tag{59}$$

Following in the footsteps of our QED consideration, we construct the physical states in a BRST-invariant fashion. A pure-graviton coherent state, free of ghosts, can be built as

$$|h\rangle = e^{-i\int d^3x (h^c_{ij}\hat{\pi}_{ij} - \pi^c_{ij}\hat{h}_{ij} + B^c_\mu \hat{\Pi}^\mu - \Pi^\mu_c \hat{B}_\mu)}|\Omega\rangle, \tag{60}$$

$$\Pi^\mu \equiv -h^{0\mu} + \frac{1}{2}\eta^{0\mu} h$$

with c-number functions setting the initial expectation values of the corresponding operators and $|\Omega\rangle$ denoting the Minkowski vacuum. In analogy with QED, from the beginning, we take $B^c_\mu = 0$, as its presence introduces an unnecessary complication in the BRST condition which is easily satisfied otherwise. Imposing BRST invariance we immediately obtain

$$\hat{Q}_B|h\rangle = e^{-i\int d^3x (h^c_{ij}\hat{\pi}_{ij} - \pi^c_{ij}\hat{h}_{ij} - \Pi^\mu_c \hat{B}_\mu)} \times \int d^3x (-2h^c_{ij}(\nabla^2\delta_{ij} - \partial_i\partial_j)\hat{C}_0 + 2\pi^c_{ij}\partial_i \hat{C}_j)|\Omega\rangle. \tag{61}$$

The consistency of the state requires the above expression to vanish. Upon integrating the second line by parts, we obtain the relations reminiscent of the classical constraints of linear gravity,

$$(\nabla^2\delta_{ij} - \partial_i\partial_j)h^c_{ij} = 0 \quad \text{and} \quad \partial_i \pi^c_{ij} = 0, \tag{62}$$

which need to be satisfied by the physical configuration along with the following boundary conditions:

$$\int d^3x \partial_i(\pi^c_{ij}\hat{C}_j)|\Omega\rangle = 0, \tag{63}$$

$$\int d^3x \partial_i([\partial_j \hat{C}_0 - \hat{C}_0 \partial_j](\delta_{ij}h^c_{kk} - h^c_{ij}))|\Omega\rangle = 0. \tag{64}$$

Notice that $h^c_{0\mu}$ is unrestricted, due to the fact that the corresponding operators represent Lagrange multipliers, similar to A_0 in electrodynamics. A further parallel can be drawn with QED, by pointing out that (62) represents the gravitational counterpart to the charge-free Gauss' law, while (63) and (64) are equivalent to the boundary term of (12) and are automatically satisfied for configurations that vanish on the boundary.

Having demonstrated how to construct physical coherent states of gravitons over the Minkowski vacuum in a source-free theory at hand (55), we would like to begin introducing sources. As it has already been mentioned, the case of particular interest is the cosmological constant. It is a classical source of gravity that is incorporated in a linear theory by adding the following term to the Lagrangian:

$$\Delta\mathcal{L} = -\lambda\hat{h}. \tag{65}$$

Obviously, this addition does not alter the expressions for the canonical momenta. Nevertheless, since it is invariant under (56) only up to a total derivative, it gives the additional contribution to the Noether charge,

$$\hat{Q}^{\lambda}_{B} = \hat{Q}_B + \int d^3x 2\lambda\hat{C}_0. \tag{66}$$

This is the charge with respect to which we must define the physical Hilbert space since it commutes with the Hamiltonian of the system in the presence of the cosmological constant. That is, $[\hat{H}, \hat{Q}^{\lambda}_{B}] = 0$ while $[\hat{H}, \hat{Q}_B] \neq 0$. Here $\hat{Q}_B$ is the BRST charge in the absence of the cosmological constant, in complete analogy with our consideration of classical charges in QED. Notice that we have not provided the explicit expression for $\hat{Q}_B$ because it is not a necessity.

A classical solution of the linearized equation with a cosmological constant source is given by [26]

$$h_{ij} = -\frac{\lambda}{6}(t^2\delta_{ij} + x_i x_j), \qquad h_{00} = h_{0j} = 0, \tag{67}$$

which, for $\lambda > 0$, represents the short-scale approximation of the de Sitter space-time in closed slicing. In fact, switching to the dimensionless metric $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}/M_{\text{pl}}$, it is straightforward to see that (67) follows from the latter in the $M_{\text{pl}} \to \infty$ limit while λ is held fixed, as does (55) supplemented with (65) from the fully nonlinear Einstein-Hilbert action with the cosmological constant. It must be noted that in this limit the curvature of the space-time vanishes,

$$H^2 \simeq \frac{\lambda}{M_{\text{pl}}} \to 0. \tag{68}$$

This is equivalent to taking the Hubble radius to infinity. Thus, Eq. (67) is a good approximation of the de Sitter space-time only at deeply subhorizon scales.

As already commented, in the original proposal [4,6], the quantum resolution of the metric (67) as a multigraviton coherent state on Minkowski space was achieved via representing the Einstein graviton as a component of the Fierz-Pauli field. The gauge invariance of the latter served as assurance for the consistency of such a state. Here we take a different path.

The question we ask is whether the de Sitter background (67) can be represented as a BRST-invariant coherent state of gravitons built over the Minkowski vacuum. Let us begin by noticing that, due to the presence of the cosmological constant, the Minkowski vacuum $|\Omega\rangle$ is no longer annihilated by the full BRST charge given by (66) but only by $\hat{Q}_B$, which no longer commutes with the Hamiltonian. A similar observation was made for quantum electrodynamics in the presence of classical charges. There, it was nevertheless possible to construct a BRST-invariant coherent state over the vacuum of the theory free of external classical sources. To demonstrate the same for linear gravity, we move forward by constructing the coherent state corresponding to (67) at $t = 0$ as

$$|h\rangle = e^{-i\int d^3x(h^c_{ij}\hat{\pi}_{ij} + \frac{1}{2}h^c_{kk}\hat{B}_0)}|\Omega\rangle, \ \text{with}\ h^c_{ij} = -\frac{\lambda}{6}x_i x_j. \tag{69}$$

It is straightforward to see that in the presence of the cosmological constant, the first equation of (62) is modified to

$$(\nabla^2\delta_{ij} - \partial_i\partial_j)h^c_{ij} - \lambda = 0. \tag{70}$$

This is a direct consequence of the above-mentioned point about $|\Omega\rangle$ not being annihilated by $\hat{Q}^{\lambda}_{B}$. The constraint 70)) is readily satisfied by (67). The boundary conditions (63) and (64) persist without modification, the first of which is trivially satisfied for the configuration at hand. The demonstration for the second one, on the other hand, requires expansion of $\hat{C}_0(x, t)$ in ladder operators, as was done for electrodynamics in the previous section. It is nevertheless straightforward to show that it holds.

VII. SCALAR MATTER COUPLED TO GRAVITY

As the next step, we introduce coupling of the graviton to a scalar field. Truncating the theory of a scalar field coupled to Einstein gravity at the first nontrivial order in M^{-1}_{pl} expansion, we arrive at

$$\mathcal{L} = \mathcal{L}_\phi + \mathcal{L}_h - \frac{1}{2M_{\text{pl}}}\hat{h}_{\mu\nu}\hat{T}^{\mu\nu} + \mathcal{O}(M^{-2}_{\text{pl}}), \tag{71}$$

where $\mathcal{L}_\phi$ is a Lagrangian of the scalar field in the absence of gravity, $\mathcal{L}_h$ is given by (55) describing the free propagation of the graviton (albeit with additional nonlinear gauge fixing and ghost parts [29]), and $\hat{T}_{\mu\nu}$ stands for the energy-momentum tensor of the first two terms of (71).

The graviton contribution to $T_{\mu\nu}$ warrants the modification of the graviton's BRST transformation property (56) to include the "non-Abelian" correction. However, the latter will not contribute to the effects of order $M_{\rm pl}^{-1}$ that we are after, as will become clear shortly. The transformation property of the scalar field is given by

$$\delta\hat{\phi} = \frac{\theta}{M_{\rm pl}}\hat{C}^{\mu}\partial_{\mu}\hat{\phi}, \tag{72}$$

which takes the following form when rewritten in terms of canonical variables,

$$\delta\hat{\phi} = \frac{\theta}{M_{\rm pl}}(\hat{\Pi}_{\phi}\hat{C}_0 - \partial_j\hat{\phi}\hat{C}_j) + \mathcal{O}(M_{\rm pl}^{-2}), \tag{73}$$

assuming a canonical kinetic term for the scalar. (The generalization is straightforward and will not be pursued here.)

In analogy with electrodynamics, the BRST-invariant dressing of the scalar coherent state amounts to replacing the scalar field operators by their invariant counterparts. For example, what serves as the invariant version of $\hat{\phi}(x^{\mu})$ is simply $\hat{\phi}(x^{\mu}+\epsilon^{\mu})$, with ϵ being a function of the graviton field that transforms as

$$\delta\epsilon^{\mu} = -\frac{\theta}{M_{\rm pl}}C^{\mu}. \tag{74}$$

The explicit form of such ϵ is straightforward to find to the first nontrivial order in $M_{\rm pl}^{-1}$, and it is given by

$$\epsilon_0 = -\frac{1}{4M_{\rm pl}}\frac{1}{\nabla^2}\hat{\pi}_{kk} + \mathcal{O}(M_{\rm pl}^{-2}), \tag{75}$$

$$\epsilon_j = \frac{1}{M_{\rm pl}}\frac{1}{\nabla^2}\partial_i\left(\hat{h}_{ij} - \frac{1}{2}\delta_{ij}\hat{h}_{kk}\right) + \mathcal{O}(M_{\rm pl}^{-2}). \tag{76}$$

The transformation properties can be readily verified using (56) and (59). Therefore, a coherent state of the scalar field that satisfies the physicality condition of being annihilated by the BRST charge can be constructed as

$$|C\rangle = e^{i\int d^3x \Pi_{\phi}^{c}(x)\hat{\phi}(x^{\mu}+\epsilon^{\mu})}|\Omega\rangle, \tag{77}$$

with $\Pi_{\phi}^{c}(x)$ being an arbitrary function of the spatial coordinates and x^0 appearing in the argument of $\hat{\phi}$ setting the initial time (which we take to be at $x^0 = 0$). Notice that this state is not the most general one. In fact, the initial expectation value of $\hat{\phi}$ vanishes. On the other hand, the configuration at hand possesses nonzero kinetic energy. To be more specific, up to corrections of order $M_{\rm pl}^{-2}$, the nontrivial initial conditions for one-point expectation values are as follows:

$$\langle C|\hat{\Pi}_{\phi}|C\rangle(t=0) = \Pi_{\phi}^{c} + \mathcal{O}(M_{\rm pl}^{-2}), \tag{78}$$

$$\nabla^2\langle C|\hat{h}_{ij}|C\rangle(t=0) = \frac{\delta_{ij}}{8M_{\rm pl}}\Pi_{\phi}^{c\,2} + \mathcal{O}(M_{\rm pl}^{-2}). \tag{79}$$

Notice that (79) is precisely the equation one would expect in the Newtonian limit, for arbitrary $\Pi_{\phi}^{c}(x)$, while having $\phi_c(x) = 0$. The generalization of this argument for the BRST-invariant dressing of $\hat{\Pi}_{\phi}$ is straightforward, but the expression is cumbersome and will not be recited here.

VIII. NONLINEARITIES AND HUBBLE SCALE

So far we have discussed the de Sitter solution (67) and its coherent state realization in the massless spin-2 theory with a constant source. This theory, both at the classical and quantum levels, represents a fully self-consistent limit of Einstein gravity,

$$M_{\rm pl} \to \infty, \qquad \lambda = \text{finite}. \tag{80}$$

In this limit all nonlinearities vanish. Correspondingly, the solution (67), as well as its coherent state representation (69), is exact.

This is fully consistent with the fact that the Hubble scale (68) is vanishing, even though the cosmological constant source λ is nonzero. Correspondingly, the effects of de Sitter space are experienced neither by the graviton nor by the external particles coupled to it. For example, consider the scalar with energy momentum tensor $T^{\mu\nu}$ coupled to $\hat{h}_{\mu\nu}$ in (71). This scalar, while interacting with the coherent state of gravitons (69), effectively sees the following classical metric:

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{h_{\mu\nu}}{M_{\rm pl}} + \mathcal{O}(M_{\rm pl}^{-2}), \tag{81}$$

where the classical field $h_{\mu\nu}$ is given by (67). Obviously, because of the limit (80), this is just a flat Minkowski metric.

Let us now assume that $M_{\rm pl}$ is taken to be finite. This choice, of course, makes H nonzero. In the coordinates in question, a significant departure from the Minkowski geometry is felt immediately near the Hubble radius and beyond. At much shorter radii, on the other hand, the probe scalar starts experiencing the nontrivial metric after a finite time. This happens when $h_{\mu\nu}$ becomes of order $M_{\rm pl}$. As it is clear from the solution (67), the required timescale for such a growth near the origin is given by the Hubble time, $t \sim H^{-1}$.

Obviously, the same applies to all nonlinear self-interactions of the graviton. As it is well known, at tree level these interactions can be obtained by consistently coupling the graviton to its own energy momentum tensor, order by order in the $1/M_{\rm pl}$ expansion. The result fully coincides with the expansion of the Einstein-Hilbert action on the Minkowski background. At nth order, the nonlinear couplings exhibit the following power scaling with $M_{\rm pl}$,

$$\frac{h^n}{M_{\rm pl}^n}\partial h\partial h. \tag{82}$$

The notation is of course highly schematic but suffices for making an important point. As we can see, just as in the case of a probe scalar, the nonlinearities become important after the Hubble time. Of course, neither the classical solution (67) nor the corresponding quantum coherent state (69) is fit for adequately describing physics beyond this time. However, such long timescales are beyond the scope of the present work.

Our main point is that the BRST-invariant coherent state of gravitons (69) in linear theory of massless spin-2 correctly captures the correlation between nonlinearities and the Hubble time. This is a consistency check of the presented description.

Another consistency check is how the quantum depletion of the graviton coherent state captures the Gibbons-Hawking particle creation in de Sitter space. This is what we consider next.

IX. GIBBONS-HAWKING RADIATION IN LINEAR GRAVITY

The de Sitter space-time exhibits Gibbons-Hawking particle creation [30]. In a semiclassical treatment of de Sitter space, this effect is described as a vacuum process. The coherent state picture offers a different way of looking at the origin of this radiation. In this description, de Sitter space is not a vacuum but rather an excited coherent state, constructed on top of the S-matrix vacuum of Minkowski space. Therefore, particle creation represents a process of quantum decay of the graviton coherent state into different quanta. The rate of the decay [4,6],

$$\Gamma \sim H, \tag{83}$$

and the power of the emitted radiation,

$$P \sim H^2, \tag{84}$$

are in agreement with Gibbons-Hawking radiation of temperature H. As argued in [4,6], due to backreaction, this decay inevitably leads to a loss of coherence and generation of entanglement. This results in a complete departure from the classical picture after a half-decay. This timescale is called quantum break-time.

Can glimpses of the above process be captured by the coherent state (69) of linearized theory? The effect can be read in two different ways. The first way is by analyzing the equation for the probe scalar in the background metric (81), with the solution (67). Of course, this must be understood as the leading-order result in the expansion in $1/M_{\rm pl}$. During the computation, the scale $M_{\rm pl}$ must be kept finite. The limit (80) can be taken afterwards.

This approximation gives the particle creation rate (83) and the emission power (84). This indicates that the state produces, on average, one particle of energy $\sim H$ per time $t \sim H^{-1}$. Since we are not going beyond this time, the computation is informative, at least order-of-magnitude wise.

The above also indicates that within the exact validity of linear theory, the particle production vanishes since H vanishes in the limit (80). Remarkably, there is a way to capture a nonzero particle-creation rate even in this limit.

It has been shown in [19] that by taking a so-called "species limit," one can ensure that the linear gravity represents an exact description of full quantum theory while simultaneously maintaining the collective phenomena, such as particle creation in de Sitter space. This setup is ready-made for our purposes. Therefore, below, we shall follow the construction of [19] and implement our de Sitter coherent state (69) within this framework. This allows us to extract the Gibbons-Hawking particle-creation process in a BRST-invariant coherent state description of de Sitter space.

The idea of the species limit is the following. Instead of a single (massless) scalar, let us introduce N copies of them, ϕ_j, where $j = 1, 2, \ldots, N$ is the species index. For definiteness, we assume no self-interactions for scalars.

The introduction of species has the effect of lowering the gravitational cutoff $\Lambda_{\rm gr}$ [31,32] to

$$\Lambda_{\rm gr} = \frac{M_{\rm pl}}{\sqrt{N}}. \tag{85}$$

Now, the species limit is defined as

$$M_{\rm pl} \to \infty, \qquad N \to \infty, \qquad \Lambda_{\rm gr} = \text{finite}. \tag{86}$$

This limit is somewhat analogous to 't Hooft's planar limit in $SU(N)$ QCD [33], in the following sense. The quantum gravitational coupling, which at any finite energy scale q is defined as

$$\alpha_{\rm gr} \equiv \frac{q^2}{M_{\rm pl}^2}, \tag{87}$$

vanishes in the limit (86) as $1/N$. Correspondingly, all quantum gravitational processes in which $\alpha_{\rm gr}$ is not accompanied by N vanish. In a certain sense, this gives a greater simplification than the 't Hooft limit in QCD. This is due to the fact that the graviton, unlike gluons in QCD, carries no species index.

At loop level, this implies vanishing of all loop contributions except the renormalization of the graviton kinetic term, which is resummable. Of course, all nonlinear self-interactions of the graviton vanish. The resulting theory is linearized gravity coupled to N scalar species,

$$\mathcal{L} = \mathcal{L}_\phi + \mathcal{L}_h - \frac{1}{2M_{\rm pl}} \hat{h}_{\mu\nu} \sum_{j=1}^{N} \hat{T}_j^{\mu\nu}. \tag{88}$$

Notice the limit (86) ensures that the above form provides an exact description of any state in which the occupation number of quanta increases slower than N. Of course, the coupling of the graviton to each particular species is zero, but the collective effect is nonvanishing. Therefore, the coupling between the graviton and species must be kept in the Lagrangian, even in the species limit.

Next, we add a constant source λ and keep it nonzero and finite while simultaneously taking the species limit (86). The resulting state represents a linearized de Sitter coherent state (69) interacting with N particle species. The difference from the case of a single scalar is that the rate of particle creation is enhanced by the factor N,

$$\Gamma \sim HN. \tag{89}$$

Of course, in the present limit, Γ diverges due to the infinite Hubble volume. Despite this, the particle production rate per unit volume, which is given by

$$\frac{\Gamma}{V} \sim H^4 N = \left(\frac{\lambda}{\Lambda_{\rm gr}}\right)^2, \tag{90}$$

is finite.

From the quantum corpuscular point of view, the rate (89) has a clear physical meaning. Particle creation comes from rescattering of the constituent gravitons into the scalar species. Consider, for example, a scattering of two gravitons into a pair of scalars. The rate of the process is given by

$$\Gamma \sim H\alpha_{\rm gr}^2 N_{\rm gr}^2 N, \tag{91}$$

where $\alpha_{\rm gr} = H^2/M_{\rm pl}^2$ is the coupling between the coherent state gravitons and scalars and $N_{\rm gr} = M_{\rm pl}^2/H^2$ is the occupation number of gravitons in the coherent state per Hubble volume. The two exactly compensate each other, $\alpha_{\rm gr} N_{\rm gr} = 1$. What remains is the enhancement by an infinite factor N. The resulting particle-creation rate per unit volume (90) is finite.

We see that the species limit (86) allows for the computation of the Gibbons-Hawking radiation from the graviton coherent state (69). In this limit, the effect is finite, despite the fact that the Gibbons-Hawking temperature is zero and the Hubble time is infinite. This is a particular manifestation of a general phenomenon that species magnify the effects of quantum gravity.

As another check of consistency of our coherent state description, notice that the backreaction on the coherent state from particle creation vanishes despite the finite production rate per unit volume. This is due to the fact that in the limit (86), the mean number density of the constituents is infinite, and their frequencies ($\sim H$) are zero. This results in infinite quantum break-time since, in the species regime, the latter scales as [1,19]

$$t_Q \sim \frac{M_{\rm pl}^2}{H^3 N} \sim \frac{\Lambda_{\rm gr}^2}{H^3}, \tag{92}$$

which, in the present case, is infinite. The timescale for the half-decay of the coherent state is of the same order. This aspect is also correctly captured by linear theory.

X. SUMMARY

The S-matrix formulation of quantum gravity excludes de Sitter vacua [1]. The only remaining option for realizing de Sitter space in quantum gravity is by representing it as an excited state on top of a valid S-matrix vacuum, such as the Minkowski vacuum. Since the state must be close to classical, the natural candidate is a coherent state.

This approach, originally adopted in [4–6], offers a microscopic understanding of known de Sitter phenomena. At the same time, it reveals the new effects that are not visible in the ordinary semiclassical treatment. Processes such as Gibbons-Hawking radiation, which in the ordinary picture are viewed as vacuum processes, are described as the actual decay of the graviton coherent state in the S-matrix picture. This makes it clear that at finite $M_{\rm pl}$, de Sitter space is subjected to backreaction. This backreaction limits the duration of the classical de Sitter phase by its quantum break-time, t_Q. For finite values of $M_{\rm pl}$ and H, this time is finite, thereby eliminating the possibility of eternal de Sitter cosmology. This has important consequences both for cosmology and for particle physics.

The above gives fundamental importance to understanding the viability of consistent formulation of the de Sitter coherent state. In the present paper we have offered a BRST-invariant construction of such a state.

However, we approached the issue through the lens of a broader question, that of a BRST-invariant formulation of states produced by classical sources in quantum gauge theories. In this work, we focused on Abelian gauge theories such as QED and linear Einstein gravity.

First, starting with quantum electrodynamics, we discussed the coherent state description of the charged scalar field configuration. We showed that, in order to comply with physicality constraints, charges must be dressed in a special way. Namely, the nonasymptotic coherent state of the charged scalar field cannot be made BRST invariant by dressing it with the coherent state of the electromagnetic field, even though coherent states of photons on their own are fully consistent with the BRST condition. Instead, we built matter coherent states out of gauge-invariant operators *à la* Dirac [22]. Next, we studied the coherent state of photons produced in the presence of external classical charges.

An interesting toy example that shares certain qualitative properties with de Sitter space in gravity can be set up by introducing classical and quantum charges simultaneously. The role of the cosmological constant is played by the uniform source of constant charge density. In the absence of dynamical quantum charges, such a source of infinite extent produces a linearly growing (in space-time coordinates) electric field or, equivalently, a quadratically growing vector potential A_μ. This is similar to the graviton field produced by the cosmological constant in linearized Einstein gravity.

Introduction of quantum charges with finite couplings to corresponding gauge fields (photon in QED, graviton in Einstein) tames the unbounded growth in both cases. In QED this happens through Schwinger pair creation, which reduces the electric field below the threshold. In gravity, the growth is tamed by coupling of the graviton to itself and to other species, which becomes important on the Hubble scale. In addition, Gibbons-Hawking particle creation shares some similarity with the Schwinger discharge.

In gravity, we found that the de Sitter space can be constructed as a BRST-invariant coherent state of gravitons on top of Minkowski space within Gaussian theory of the massless spin-2 field. We also introduced gravitating scalar matter and showed how to dress its coherent state perturbatively in $M_{\rm pl}^{-1}$.

The introduction of N species of scalars allowed us to implement our coherent state construction in the species limit (86), which promotes the linear gravity coupled to the N species into an exact description. The only quantum gravitational effects surviving in this limit are the collective phenomena in which the powers of $1/M_{\rm pl}^2$ are compensated by corresponding powers of N. An important example of a nonzero collective effect surviving in the species limit is Gibbons-Hawking radiation. Correspondingly, the species limit allowed us to observe that the BRST-invariant coherent state, describing de Sitter space in linearized gravity, exhibits features of full nonlinear de Sitter space.

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Note added in the proof.—After submission of this work, it was brought to our attention that the gauge invariant observable $\phi(x+\varepsilon)$, discussed in Section VII, has been previously studied in a different context [34,35].

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