Generation and catalysis of coherence with scalar fields

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Employing time-dependent perturbation theory, we discuss the conditions under which a spatially extended two level Unruh-DeWitt detector, coupled to the proper time derivative of a scalar field, can drive itself into a coherent superposition of its energy basis. It is shown that this process, which is assisted due to the energy cost required for the implementation of the interaction, is only possible for a field with a nonzero coherent amplitude distribution. When the detector interacts instantaneously with the field through a delta coupling, the latter acts as a catalyst resulting in the extraction of the same amount of coherence each time at the cost of a positive amount of work. For a Gaussian smeared detector and a field in a coherent state the amount generated to lowest order in the coupling constant depends on the phase of the amplitude distribution, the initial energy of the field, the mean radius of the detector and the mean interaction duration between the two. We observe that for a detector moving at a constant velocity and with a mean radius of the same order as its transition wavelength, coherence swelling effects are present the intensity of which depends on the dimension of the underlying Minkowski spacetime.

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I. INTRODUCTION

Superposition is one of the most striking phenomena which distinguishes quantum from classical physics. The degree to which a system is superposed between different orthogonal states is known as *coherence* [1–3]. Much like entanglement [4], coherence is considered to be a valuable resource in quantum information processes. In quantum computing [5,6], where information is encoded in the states of two-level systems, algorithms designed to operate in superposition, are exponentially faster than their classical counterparts [7–9]. Coherence is so central to the development of a universal quantum computer that it is used as a metric for the quality of a quantum processor. The time that it takes for a qubit to effectively decohere due to noise is known as the *dephasing time* with current processors achieving times of a few hundred microseconds. Coherent phenomena are important in other fields of research also, such as quantum metrology [10] and thermodynamics [11–15] for example. Surprisingly, the possible presence of coherence in biological processes has been suggested as an explanation for the efficiency of energy transport during photosynthesis [16].

A simple method of obtaining coherence is by extracting it from another system. Driven by similar research on entanglement harvesting protocols (see, e.g., [17–26]) and the deep connection that exists between entanglement and coherence [27–29], it was recently demonstrated that a two-level pointlike Unruh-DeWitt (UDW) detector [30–32], initially in its ground energy state, interacting with a coherent massless scalar field in $1+1$ flat spacetime, can drive itself into a superposition of its energy basis [33]. As it turns out, the amount of coherence obtained depends on the initial energy of the field, the mean interaction duration and the detector's state of motion. For a detector moving at relativistic speeds when the gap between its energy levels is greater than the initial energy of the field, it is possible to extract a larger amount of coherence compared to a static detector, a phenomenon dubbed by the authors as "coherence swelling." These relativistic effects may therefore be exploited as a possible mechanism against environmentally induced decoherence and could find applications in future quantum information technologies based in space [34].

In this article, we provide a thorough study of the conditions under which coherence generation in the detector is possible for any initial state of the field in $n+1$ dimensional Minkowski spacetime. In order to achieve this and to avoid the problem of IR divergences that plague the $1+1$ dimensional case of a linear coupling between detector and field [35], we instead consider an interaction in which the detector is coupled to the proper time derivative of the field. Under the assumption that there is no exchange of angular momentum, both models contain all the essential features of matter interacting with radiation [36,37], so they provide a useful benchmark for studying possible applications of relativistic effects in quantum information processing. Acknowledging the fact that a

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pointlike detector is an abstraction and not a physical system—an atom or an elementary particle, for example, has a finite size—and to make our results as relevant as possible we take into consideration the spatial extension of the detector.

We will show that when the interaction between detector and field is instantaneous coherence generation is *catalytic* [38]. At the cost of some energy, which assists in the overall process, it is possible to repeatedly obtain the same amount of coherence each time. For an inertial detector moving at a constant velocity it is proven that, under suitable conditions, this is also the maximum amount that can be obtained. As an example we consider the case of generating coherence with the help of a coherent scalar field. We find that the process depends on the phase of the field's coherent amplitude distribution, its initial energy, the mean radius of the detector and the mean interaction duration between the two. We conclude that even in the case of a spatially extended detector swelling effects are still present but these are weaker in a $3+1$ compared to a $1+1$ dimensional spacetime.

II. QUANTUM COHERENCE

From a physical point of view coherence reflects the degree of superposition that a quantum system exhibits when it simultaneously occupies different orthogonal eigenstates of an observable of interest [3]. Coherent systems are considered to be valuable resources in quantum information processes, because with their help it is possible, at the cost of consuming some of the coherence that they contain, to simulate transformations that violate conservation laws associated with the corresponding observable.

Mathematically, let $\{|i\rangle\}$ denote a set of basis states spanning a finite discrete Hilbert space $\mathcal{H}$, which correspond to the eigenstates of an observable $\hat{O}$. Any state ρ which is diagonal in this basis

$$\rho = \sum_i p_i |i\rangle\langle i| \tag{1}$$

is called *incoherent* and commutes with $\hat{O}$. If ρ contains nondiagonal elements then it is called *coherent* [1]. In this case $[\rho, \hat{O}] \neq 0$ [39], and the state changes under the action of the one parameter group of symmetry transformations $U(s) = \exp(-is\hat{O})$ generated by the observable. This makes coherent systems useful as reference frames and reservoirs for the implementation of nonsymmetric transformations [40–43]. For example, for a fixed Hamiltonian $\hat{H}$, any system that possesses coherence with respect to the energy basis has a nonzero rate of change, $\dot{\rho}(t) \neq 0$, and can be used as a reference system in order to track the passage of time. Alternatively this system could be utilized as a coherent energy reservoir with the help of which it is possible to perform incoherent transformations on other systems [38].

The amount of coherence present in a system can be quantified with the help of a *coherence measure*. This is a real valued function $C(\cdot)$ on the set of density matrices $\mathcal{D}$ such that

$$C(\rho) \geq 0, \quad \forall \ \rho \in \mathcal{D} \tag{2}$$

with equality if and only if ρ is incoherent. A simple example of such a function is given by the ℓ_1-norm of coherence, which is equal to the sum of the modulus of the system's nondiagonal elements

$$C(\rho) = \sum_{i \neq j} |\rho_{ij}| \tag{3}$$

with values ranging between 0 for an incoherent state and $d-1$ for the maximally coherent d-dimensional pure state

$$|\psi\rangle = \frac{1}{\sqrt{d}} \sum_{i=0}^{d-1} |i\rangle. \tag{4}$$

In order to extract coherence from a coherent system σ to an incoherent system ρ it is necessary to bring the two in contact and make them interact through a completely positive and trace preserving quantum operation. When the operation obeys the conservation law associated with the observable and is strictly incoherent (in the sense that it maps the set of incoherent states to itself) the process is called *faithful* [44]. When this is no longer the case the operation generates extra coherence, which increases the amount stored in the combined system and can assist in the extraction process [45,46], in much the same way that a quantum operation which is nonlocal can create entanglement between two spacelike separated systems. Since the interaction no longer obeys the conservation law, this assisted protocol requires a cost in the physical quantity represented by $\hat{O}$ which needs to be taken into account.

We shall now demonstrate how to construct such a protocol for generating coherence onto an UDW detector with the help of a scalar field. In what follows we shall assume a flat $n+1$ dimensional spacetime with metric signature $(- + \cdots +)$. We will denote spacetime vectors by sans-serif characters, and the scalar product of vectors x and y as $\mathsf{x} \cdot \mathsf{y}$. Boldface letters represent spatial n-vectors. Throughout, we make use of natural units in which $\hbar = c = 1$ and employ the interaction picture for operators and states.

III. UNRUH-DEWITT DETECTOR MODEL

To study the amount of coherence generated with a massless scalar field we will employ the help of an UDW detector which is coupled to the proper time derivative of the field [47–49]. In the simplest case considered here, the detector is modeled as a qubit with two energy levels, ground $|g\rangle$ and excited $|e\rangle$ with energy gap equal to Ω, and Hamiltonian

$$\hat{H}_D = \frac{\Omega}{2}(|e\rangle\langle e| - |g\rangle\langle g|) \tag{5}$$

which is moving along a worldline $\mathsf{x}(\tau)$ parametrized by its proper time τ. The detector is subsequently made to interact with a massless scalar field in $n+1$ dimensions

$$\hat{\phi}(\mathsf{x}) = \int \frac{d^n\mathbf{k}}{\sqrt{(2\pi)^n 2|\mathbf{k}|}}[\hat{a}_{\mathbf{k}} e^{i\mathsf{k}\cdot\mathsf{x}} + \text{H.c.}], \tag{6}$$

with a normal-ordered Hamiltonian of the form

$$\hat{H}_\phi = \int |\mathbf{k}|\hat{a}^\dagger_{\mathbf{k}}\hat{a}_{\mathbf{k}} d^n\mathbf{k}, \tag{7}$$

where $\hat{a}_{\mathbf{k}}$, and $\hat{a}^\dagger_{\mathbf{k}}$ are the creation and annihilation operators of the field mode with momentum $\mathbf{k}$ that satisfy the canonical commutation relations

$$[\hat{a}_{\mathbf{k}}, \hat{a}_{\mathbf{k}'}] = [\hat{a}^\dagger_{\mathbf{k}}, \hat{a}^\dagger_{\mathbf{k}'}] = 0, \qquad [\hat{a}_{\mathbf{k}}, \hat{a}^\dagger_{\mathbf{k}'}] = \delta(\mathbf{k} - \mathbf{k}'). \tag{8}$$

The interaction between detector and field is constructed by coupling the monopole moment operator

$$\hat{\mu}(\tau) = e^{i\Omega\tau}|e\rangle\langle g| + e^{-i\Omega\tau}|g\rangle\langle e|, \tag{9}$$

to the momentum degrees of freedom of the field through the following interaction Hamiltonian

$$\hat{H}_{\text{int}}(\tau) = \lambda\chi(\tau)\hat{\mu}(\tau) \otimes \partial_\tau\hat{\phi}_f(\mathsf{x}(\tau)). \tag{10}$$

Here, λ is a coupling constant with dimensions $(\text{length})^{\frac{n+1}{2}}$, $\chi(\tau)$ is a real valued *switching function* with dimensions $(\text{time})^{-1}$ that describes the way the interaction is switched on and off; and $\hat{\phi}_f(\mathsf{x}(\tau))$ is a smeared field along the detector's center of mass worldline $\mathsf{x}(\tau) = (t(\tau), \mathbf{x}(\tau))$,

$$\hat{\phi}_f(\mathsf{x}(\tau)) = \int_{\mathcal{S}(\tau)} f(\boldsymbol{\xi})\hat{\phi}(\mathsf{x}(\tau, \boldsymbol{\xi}))d^n\boldsymbol{\xi}, \tag{11}$$

where

$$\mathsf{x}(\tau, \boldsymbol{\xi}) = \mathsf{x}(\tau) + \boldsymbol{\xi} \tag{12}$$

are the Fermi-Walker coordinates [50] on the simultaneity hyperplane $\mathcal{S}(\tau)$, which is defined by all those spacelike vectors $\boldsymbol{\xi}$ normal to the detector's four-velocity, $\mathcal{S}(\tau) = \{\boldsymbol{\xi}|\mathsf{u}\cdot\boldsymbol{\xi} = 0\}$ (see Fig. 1). The real valued function $f(\boldsymbol{\xi})$ with dimensions $(\text{length})^{-n}$ in Eq. (11) is known as the *smearing function* and is a physical reflection of the finite size and shape of the detector [36,37,51,52].

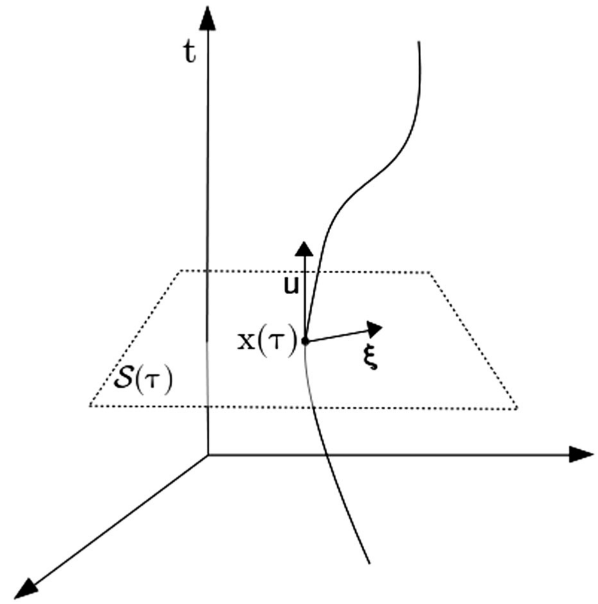


FIG. 1. Any point in the neighborhood of the detector's worldline can be described by its Fermi-Walker coordinates $(\tau, \boldsymbol{\xi})$, where the proper time τ indicates its position along the trajectory and $\boldsymbol{\xi}$ is the displacement vector from this point lying on the simultaneity hyperplane $\mathcal{S}(\tau)$ consisting of all those spacelike vectors normal to its four-velocity u.

Compared to the usual UDW interaction in which the detector is linearly coupled to the field, the derivative coupling is free of IR divergences which arise due to the massless nature of the field in the $1+1$ dimensional case [35]. The Hamiltonian in Eq. (10) resembles closely the dipole interaction between an atom with dipole moment $\mathbf{d}$ and an external electromagnetic field. In this case the electric field operator is defined, in the Coulomb gauge, by means of the vector potential $\hat{\mathbf{A}}(t, \mathbf{x})$ as $\hat{\mathbf{E}}(t, \mathbf{x}) = -\partial_t\hat{\mathbf{A}}(t, \mathbf{x})$ [53].

Combining Eq. (6) with Eqs. (11) and (12) the smeared field operator can be written as

$$\hat{\phi}_f(\mathsf{x}(\tau)) = \int \frac{d^n\mathbf{k}}{\sqrt{(2\pi)^n 2|\mathbf{k}|}}[F(\mathsf{k}, \tau)\hat{a}_{\mathbf{k}} e^{i\mathsf{k}\cdot\mathsf{x}(\tau)} + \text{H.c.}], \tag{13}$$

where

$$F(\mathsf{k}, \tau) = \int_{\mathcal{S}(\tau)} f(\boldsymbol{\xi})e^{i\mathsf{k}\cdot\boldsymbol{\xi}}d^n\boldsymbol{\xi} \tag{14}$$

is the Fourier transform of the smearing function. Decomposing the momentum vector k as

$$\mathsf{k} = -(\mathsf{k}\cdot\mathsf{u})\mathsf{u} + (\mathsf{k}\cdot\boldsymbol{\zeta})\boldsymbol{\zeta} \tag{15}$$

for some unit vector $\boldsymbol{\zeta} \in \mathcal{S}(\tau)$, it is easy to show that since for a massless scalar field k is light-like, $(\mathsf{k}\cdot\mathsf{u})^2 = (\mathsf{k}\cdot\boldsymbol{\zeta})^2$. This means that for a spherically symmetric smearing function the Fourier transform in Eq. (14) is real and depends only on $|\mathsf{k}\cdot\mathsf{u}|$,

$$F(\mathsf{k},\tau) = F(|\mathsf{k}\cdot\mathsf{u}|). \tag{16}$$

IV. GENERATION AND CATALYSIS OF COHERENCE WITH SCALAR FIELDS

Suppose now that before the interaction is switched on the combined system of detector and field starts out in a separable state of the form

$$|g\rangle\langle g| \otimes \sigma_\phi, \tag{17}$$

where the detector occupies its lowest energy level and the field is in a state σ_ϕ. The state of the system after a time at which the interaction is switched off can be obtained by evolving Eq. (17) with the unitary operator

$$\hat{U} = \mathcal{T}\exp\left(-i\int_{-\infty}^{+\infty}\hat{H}_{\text{int}}(\tau)d\tau\right), \tag{18}$$

where $\mathcal{T}$ denotes time ordering. When dealing with an extended detector interacting with a quantum field, issues concerning the covariance of the model arise due to the ambiguity of the way that the time ordering operator acts in different frames of reference [54,55]. For a detector initially in a diagonal state of its energy basis, these effects are of order $\mathcal{O}(\lambda^3)$ to the coupling constant and can be safely ignored in a perturbative treatment like the one considered here. Setting

$$\hat{\Phi} = \int_{-\infty}^{+\infty}\chi(\tau)e^{-i\Omega\tau}\partial_\tau\hat{\phi}_f(\mathsf{x}(\tau))d\tau, \tag{19}$$

the evolution operator can then be rewritten as

$$\hat{U} = \mathcal{T}\exp\left[-i\lambda(|e\rangle\langle g|\otimes\hat{\Phi}^\dagger + |g\rangle\langle e|\otimes\hat{\Phi})\right]. \tag{20}$$

Tracing out the field degrees of freedom of the evolved system and noting that

$$\frac{1}{2}(\mathcal{T}(\hat{\Phi}\hat{\Phi}^\dagger) + [\mathcal{T}(\hat{\Phi}\hat{\Phi}^\dagger)]^\dagger) = \hat{\Phi}\hat{\Phi}^\dagger \tag{21}$$

one can obtain the state of the detector after the interaction which in this case is equal to

$$\rho = \begin{pmatrix} 1-\lambda^2\langle\hat{\Phi}\hat{\Phi}^\dagger\rangle_{\sigma_\phi} & i\lambda\langle\hat{\Phi}\rangle_{\sigma_\phi} \\ -i\lambda\langle\hat{\Phi}^\dagger\rangle_{\sigma_\phi} & \lambda^2\langle\hat{\Phi}\hat{\Phi}^\dagger\rangle_{\sigma_\phi}\end{pmatrix} + \mathcal{O}(\lambda^3), \tag{22}$$

where $\langle\hat{A}\rangle_{\sigma_\phi} = \text{tr}(\hat{A}\sigma_\phi)$. In a similar fashion, by taking the partial trace over the detector's Hilbert space, we can obtain the state of the field after it has interacted with the detector,

$$\sigma'_\phi = \sigma_\phi + \lambda^2\hat{\Phi}^\dagger\sigma_\phi\hat{\Phi} - \frac{\lambda^2}{2}(\mathcal{T}(\hat{\Phi}\hat{\Phi}^\dagger)\sigma_\phi + \sigma_\phi[\mathcal{T}(\hat{\Phi}\hat{\Phi}^\dagger)]^\dagger) + \mathcal{O}(\lambda^4). \tag{23}$$

With the help of Eqs. (3) and (22) it can be seen that the amount of coherence generated in the detector to lowest order in the coupling constant is equal to

$$C = 2\lambda|\text{tr}(\hat{\Phi}\sigma_\phi)|. \tag{24}$$

Defining

$$\mathcal{F}_\pm(\mathbf{k}) = \int_{-\infty}^{+\infty}\chi(\tau)e^{\pm i\Omega\tau}\partial_\tau[F(\mathsf{k},\tau)e^{i\mathsf{k}\cdot\mathsf{x}(\tau)}]d\tau, \tag{25}$$

Eq. (24) can be written as

$$C = 2\lambda\left|\int\frac{d^n\mathbf{k}}{\sqrt{(2\pi)^n 2|\mathbf{k}|}}[\mathcal{F}_-(\mathbf{k})a(\mathbf{k}) + \mathcal{F}_+^*(\mathbf{k})a^*(\mathbf{k})]\right|, \tag{26}$$

where

$$a(\mathbf{k}) = \text{tr}(\hat{a}_\mathbf{k}\sigma_\phi) \tag{27}$$

is the *coherent amplitude distribution* of the field.

Suppose now that we wish to repeat the process and extract coherence onto a fresh detector copy. From Eqs. (21)–(24) it is easy to see that

$$\langle\hat{\Phi}\rangle_{\sigma'_\phi} = \langle\hat{\Phi}\rangle_{\sigma_\phi} + \frac{\lambda^2}{2}\langle[\hat{\Phi},[\mathcal{T}(\hat{\Phi}\hat{\Phi}^\dagger)]^\dagger]\rangle_{\sigma_\phi} + \mathcal{O}(\lambda^4). \tag{28}$$

This means that to lowest order the amount of coherence generated remains the same.

Let us focus our attention on normalized smearing and switching functions such that

$$\int_{-\infty}^{+\infty}\chi(\tau)d\tau = \int_{\mathcal{S}(\tau)}f(\boldsymbol{\xi})d^n\boldsymbol{\xi} = 1, \tag{29}$$

and define

$$R = \int_{\mathcal{S}(\tau)}|\boldsymbol{\xi}|f(\boldsymbol{\xi})d^n\boldsymbol{\xi} \tag{30}$$

as the *mean radius* of the detector and

$$T = \int_{-\infty}^{+\infty}|\tau|\chi(\tau)d\tau \tag{31}$$

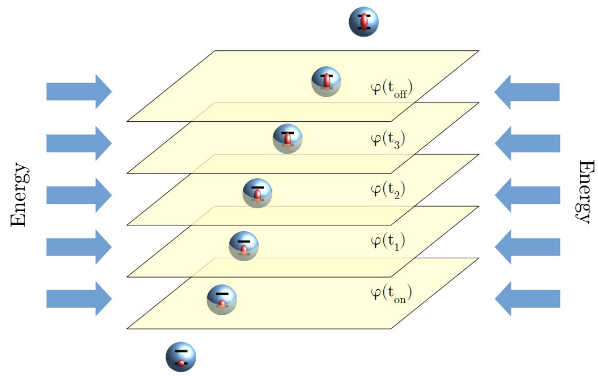


FIG. 2. Generating quantum coherence with scalar fields: A moving two-level system, initially in its ground state at some time $t < t_{\mathrm{on}}$, interacts with a massless scalar field through a derivative coupling. The interaction requires an external flow of energy which assists in the extraction process by increasing the combined system's coherence. After the interaction is switched off at a time t_{off} the detector will find itself in a superposition between its energy levels.

as the *mean interaction duration*. This will make it easier to compare different setups and will enable the study, in a unified way, of the effects that different sizes and finite interaction durations have on the extraction process.

In the limiting case of an instantaneous interaction for which $\chi(\tau) = \delta(\tau)$,

$$\hat{\Phi} = \hat{\Phi}^{\dagger} = \left.\frac{\mathrm{d}\hat{\phi}_f(\mathsf{x}(\tau))}{\mathrm{d}\tau}\right|_{\tau=0} \tag{32}$$

and the amount of coherence generated each time is exactly the same to any order (for more details see Appendix B). When the detector interacts with the field through a delta coupling, the process is catalytic [38,56,57]. Because the interaction Hamiltonian does not commute with the unperturbed part, $\hat{H}_D + \hat{H}_{\phi}$, of the total Hamiltonian, the process requires an outside supply of positive energy ΔE each time [58,59]. Energy nonconserving unitaries like the one in Eq. (18) can increase the coherence of the combined system assisting in the extraction process [45,46] (see Fig. 2). Roughly speaking the terms that are equal to $\lambda\mathcal{F}_{\pm}(\mathbf{k})/\sqrt{(2\pi)^n 2|\mathbf{k}|}$ in Eq. (26) correspond to the contribution to the amount of obtained coherence due to the interaction while $a(\mathbf{k})$ is the same contribution due to the field. It is clear that a necessary condition for generating a non trivial amount of coherence is for the field to be in a state with a nonzero coherent amplitude distribution.

V. INERTIAL DETECTORS

We will now consider an inertial detector which is moving along a worldline with a constant velocity $\boldsymbol{v}$, and whose center of mass coordinates is given by

$$\mathsf{x}(\tau) = \mathsf{u}\tau, \tag{33}$$

where $\mathsf{u} = \gamma(1, \boldsymbol{v})$ is its four-velocity, with $\gamma = 1/\sqrt{1-v^2}$ the Lorentz factor. For a spherically symmetric smearing function with a positive Fourier transform, it can be proven that

Theorem.—*For a suitable choice of the coherent amplitude distribution's phase the maximum amount of generated coherence to lowest order, is obtained by a detector interacting instantaneously with the field.*

Proof.—Taking the absolute value inside the integral in Eq. (26) we find that

$$C \leq 2\lambda \int \frac{d^n\mathbf{k}}{\sqrt{(2\pi)^n 2|\mathbf{k}|}} |a(\mathbf{k})| [|\mathcal{F}_{-}(\mathbf{k})| + |\mathcal{F}_{+}(\mathbf{k})|]. \tag{34}$$

For a detector moving with a constant velocity the Fourier transform of the smearing function no longer depends on its proper time, in this case

$$\mathcal{F}_{-}(\mathbf{k}) = i(\mathsf{k}\cdot\mathsf{u}) F(|\mathsf{k}\cdot\mathsf{u}|) X^{*}(\Omega - \mathsf{k}\cdot\mathsf{u}) \tag{35}$$

and

$$\mathcal{F}_{+}(\mathbf{k}) = i(\mathsf{k}\cdot\mathsf{u}) F(|\mathsf{k}\cdot\mathsf{u}|) X(\Omega + \mathsf{k}\cdot\mathsf{u}) \tag{36}$$

where

$$X(\Omega \pm \mathsf{k}\cdot\mathsf{u}) = \int_{-\infty}^{+\infty} \chi(\tau) e^{i(\Omega \pm \mathsf{k}\cdot\mathsf{u})\tau} d\tau. \tag{37}$$

Because of the normalization property in Eq. (29), $|X(\Omega \pm \mathsf{k}\cdot\mathsf{u})| \leq 1$, this means that

$$C \leq 4\lambda \int \frac{(-\mathsf{k}\cdot\mathsf{u})}{\sqrt{(2\pi)^n 2|\mathbf{k}|}} F(|\mathsf{k}\cdot\mathsf{u}|) |a(\mathbf{k})| d^n\mathbf{k}, \tag{38}$$

where equality holds for $\chi(\tau) = \delta(\tau)$ and a coherent amplitude distribution with phase, $\arg(a(\mathbf{k})) = \frac{\pi}{2}$.

Note that in the linear interaction where the factor $(-\mathsf{k}\cdot\mathsf{u})$ in the numerator is absent, the above Theorem holds for an arbitrary motion of the detector as long as $\mathsf{x}(0) = 0$. ∎

If the Fourier transform of the smearing function is not positive then Eq. (38) is only an upper bound on the amount of obtained coherence. It is noteworthy to point out that coupling the detector locally with the field through an instantaneous interaction is capable of generating coherence, while in the context of entanglement harvesting two spacelike separated detectors, each interacting instantaneously with the field, are unable to extract any entanglement [60].

If the amplitude distribution is also spherically symmetric then

$$C = 2\lambda\left|\int \frac{(-\mathsf{k}\cdot\mathsf{u})F(|\mathsf{k}\cdot\mathsf{u}|)}{\sqrt{(2\pi)^n 2|\mathbf{k}|}}[a(|\mathbf{k}|)X^*(\Omega-\mathsf{k}\cdot\mathsf{u}) - a^*(|\mathbf{k}|)X(\Omega+\mathsf{k}\cdot\mathsf{u})]d^n\mathbf{k}\right|, \quad (39)$$

which for a static detector reduces to

$$C = \frac{2\lambda s_n}{\sqrt{2(2\pi)^n}}\left|\int_0^\infty k^{n-\frac{1}{2}}F(k)[a(k)X^*(\Omega+k) - a^*(k)X(\Omega-k)]dk\right|, \quad (40)$$

where $s_n = 2\pi^{n/2}/\Gamma(n/2)$ denotes the surface area of the unit n-sphere. By boosting the four-momentum k to the detector's frame of reference it can be shown that Eq. (39) is equivalent to Eq. (40) with a symmetric coherent amplitude distribution $a_\upsilon(k)$ of the form

$$a_\upsilon(k) = \frac{1}{s_n}\int a\left(\frac{k}{\gamma(1-\boldsymbol{\upsilon}\cdot\hat{\mathbf{k}})}\right)\frac{d\hat{\mathbf{k}}}{[\gamma(1-\boldsymbol{\upsilon}\cdot\hat{\mathbf{k}})]^{n-\frac{1}{2}}}. \quad (41)$$

From the detector's point of view then, the field's coherent amplitude is perceived as a continuous mixture of Doppler shifted distributions with weight equal to $[s_n(\gamma-\gamma\boldsymbol{\upsilon}\cdot\hat{\mathbf{k}})^{n-\frac{1}{2}}]^{-1}$. For a similar result regarding the interaction of an inertial detector with a heat bath see [61].

VI. GENERATION AND CATALYSIS OF COHERENCE WITH A COHERENT FIELD

For a coherent state $|a\rangle$ of the field, the coherent amplitude distribution in Eq. (27) evaluated at $\mathbf{k}$ is equal to the eigenvalue of the annihilation operator with the same mode

$$\hat{a}_{\mathbf{k}}|a\rangle = a(\mathbf{k})|a\rangle, \quad (42)$$

in this case the amount of coherence generated to lowest order is given by the expectation value of the field operator $\hat{\Phi}$

$$C = 2\lambda|\langle a|\hat{\Phi}|a\rangle|. \quad (43)$$

Let us consider an inertial detector and an extraction process in which the switching and smearing functions are given by Gaussians

$$\chi(\tau) = \frac{\exp\left(-\frac{\tau^2}{\pi T^2}\right)}{\pi T} \quad (44)$$

$$f(\boldsymbol{\xi}) = \frac{\exp\left(-\frac{\xi^2}{\pi R_n^2}\right)}{(\pi R_n)^n}, \quad (45)$$

with widths $\sqrt{\frac{\pi}{2}}T$ and $\sqrt{\frac{\pi}{2}}R_n$ respectively while the state of the field is described by a coherent amplitude distribution with a unit average number of excited quanta of the form

$$a(\mathbf{k}) = \frac{\exp\left(-\frac{k^2}{2\pi E_n^2} + i\frac{\pi r}{2}\right)}{(\pi E_n)^{n/2}}, \qquad r = 0, 1 \quad (46)$$

where

$$E_n = \frac{s_{n+1}}{\pi s_n}E \quad \text{and} \quad R_n = \frac{s_{n+1}}{\pi s_n}R, \quad (47)$$

with $E = \langle a|\hat{H}_\phi|a\rangle$ the mean initial energy of the field. Note that even though technically speaking the support of Eq. (44) is no longer compact the analysis is expected to present a good approximation to a compact switching function of the form

$$\chi(\tau) = \begin{cases} \exp\left(-\frac{\tau^2}{\pi T^2}\right)/(\pi T), & |\tau| \le 4\sqrt{\pi}T \\ 0, & \text{otherwise} \end{cases} \quad (48)$$

We will now treat the static and moving cases separately.

A. Static detector

For $\upsilon = 0$ the Fourier transforms of the switching and smearing functions are equal to

$$X(\Omega \pm k) = \exp\left[-\frac{\pi(\Omega \pm k)^2 T^2}{4}\right] \quad (49)$$

and

$$F(\mathsf{k}) = \exp\left[-\frac{\pi k^2 R_n^2}{4}\right] \quad (50)$$

respectively. Inserting these expressions into Eq. (40) we obtain that the amount of generated coherence, which now depends on the initial energy of the field, the mean interaction duration and the mean radius of the detector is equal to

$$C(E,T,R) = \frac{4\lambda s_n}{\sqrt{2(2\pi^2 E_n)^n}}e^{-\frac{\pi\Omega^2 T^2}{4}} \times \int_0^\infty k^{n-\frac{1}{2}}e^{-\mathrm{a}k^2}\sinh^{1-r}(bk)\cosh^r(bk)dk, \quad (51)$$

with

$$\mathrm{a} = \frac{1}{2\pi E_n^2}\left[1 + \frac{\pi^2 E_n^2(R_n^2 + T^2)}{2}\right], \qquad b = \frac{\pi\Omega T^2}{2}. \quad (52)$$

The integral on the right-hand side of Eq. (51) can be rewritten in terms of *parabolic cylinder functions* [62], $D_p(z)$, in the following way

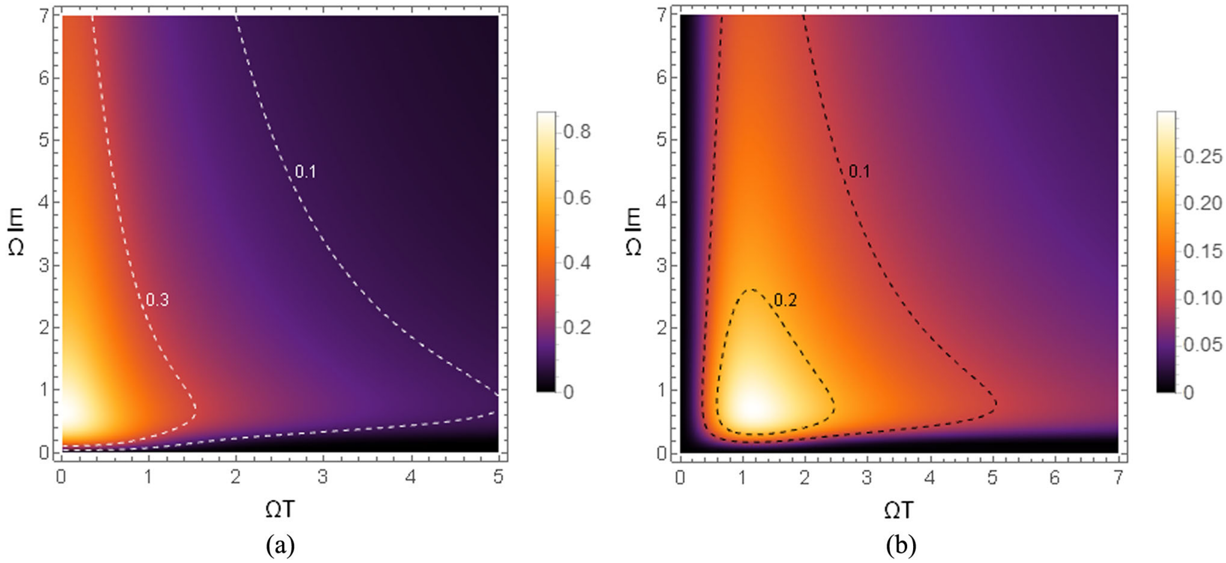


FIG. 3. Amount of coherence $C/\bar{\lambda}$ generated with the help of a coherent scalar field in $1+1$ dimensions and a Gaussian amplitude distribution with phase (a) $\phi = \frac{\pi}{2}$ and (b) $\phi = 0$, as a function of the mean initial energy of the field (in units Ω) and the mean interaction duration (in units $1/\Omega$), for a detector with mean radius $R = 1/\Omega$.

$$\int_0^\infty k^{n-\frac{1}{2}} e^{-ak^2} \sinh^{1-r}(bk)\cosh^r(bk)dk = \frac{\Gamma(n+\frac{1}{2})}{2(2a)^{\frac{n}{2}+\frac{1}{4}}} e^{\frac{b^2}{8a}} \left[D_{-n-\frac{1}{2}}\left(-\frac{b}{\sqrt{2a}}\right) - (-1)^r D_{-n-\frac{1}{2}}\left(\frac{b}{\sqrt{2a}}\right)\right], \qquad b > 0. \tag{53}$$

In Figs. 3 and 4 we present the amount of coherence obtained in this case, scaled by the dimensionless coupling constant $\bar{\lambda} = \lambda \Omega^{\frac{n+1}{2}}$, as a function of the initial mean energy E of the field (in units Ω) and the interaction duration T (in units $1/\Omega$) for a $1+1$ and a $3+1$ dimensional Mikowski spacetime respectively. In order to simplify the situation we will tacitly assume from now on that the mean radius of the detector is equal to its transition wavelength $R = 1/\Omega$. It is clear from both figures that the process depends strongly on the phase of the coherent amplitude distribution. For $r = 1$ and for a fixed initial field energy, the maximum amount that can be generated is achieved through the use of an instantaneous interaction ($T = 0$), in full agreement with the Theorem of Sec. V. On the contrary for a phaseless

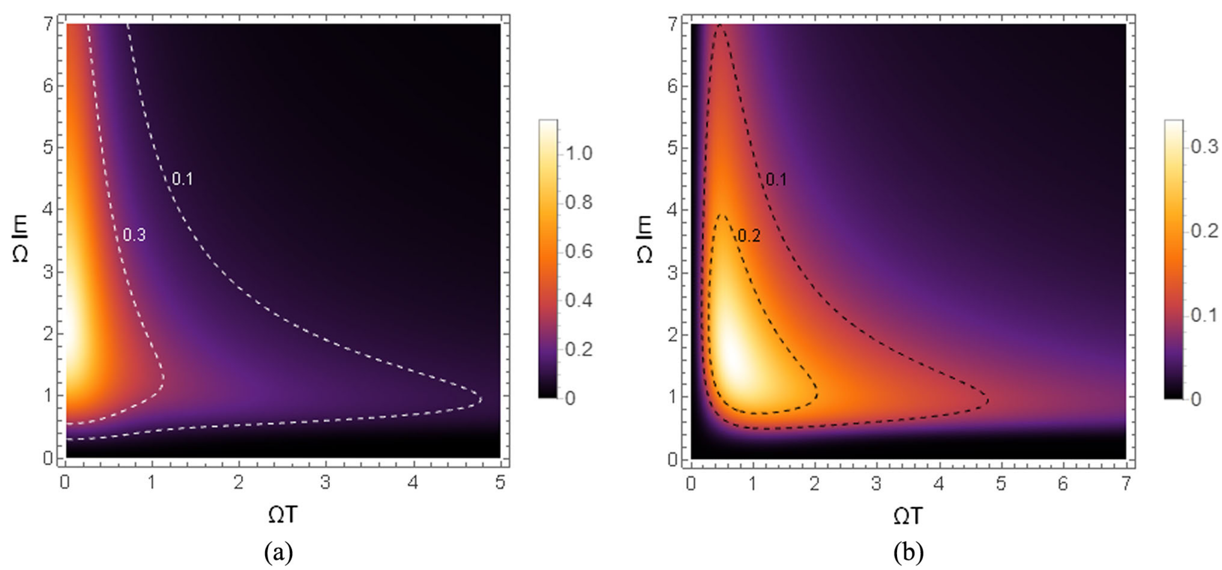


FIG. 4. Amount of coherence $C/\bar{\lambda}$ generated with the help of a coherent scalar field a coherent scalar field in $3+1$ dimensions and a Gaussian amplitude distribution with phase (a) $\phi = \frac{\pi}{2}$ and (b) $\phi = 0$, as a function of the mean initial energy of the field (in units Ω) and the mean interaction duration (in units $1/\Omega$), for a detector with mean radius $R = 1/\Omega$

coherent amplitude it is impossible to generate coherence to a detector which is interacting instantaneously with the field. In this case the maximum is obtained for interaction durations comparable to the mean radius. In both settings, if the initial energy of the field is zero the detector will remain incoherent after the interaction takes place. This is also true for very large energy values. Qualitatively, the process is more efficient for field energies comparable to the energy gap. For a resonant energy of the field, $E = \Omega$, it is possible to extend the duration of the process to greater interaction times compared to other energies and still extract a small amount of coherence.

B. Detector moving at a constant velocity

According to Eq. (41), a detector moving at a constant velocity still perceives the field as a coherent state but in a mixture of static coherent amplitude distributions of the form (46) with Doppler shifted energies equal to

$$E(v_{\hat{\mathbf{k}}}) = E\gamma(1 - \boldsymbol{v} \cdot \hat{\mathbf{k}}). \tag{54}$$

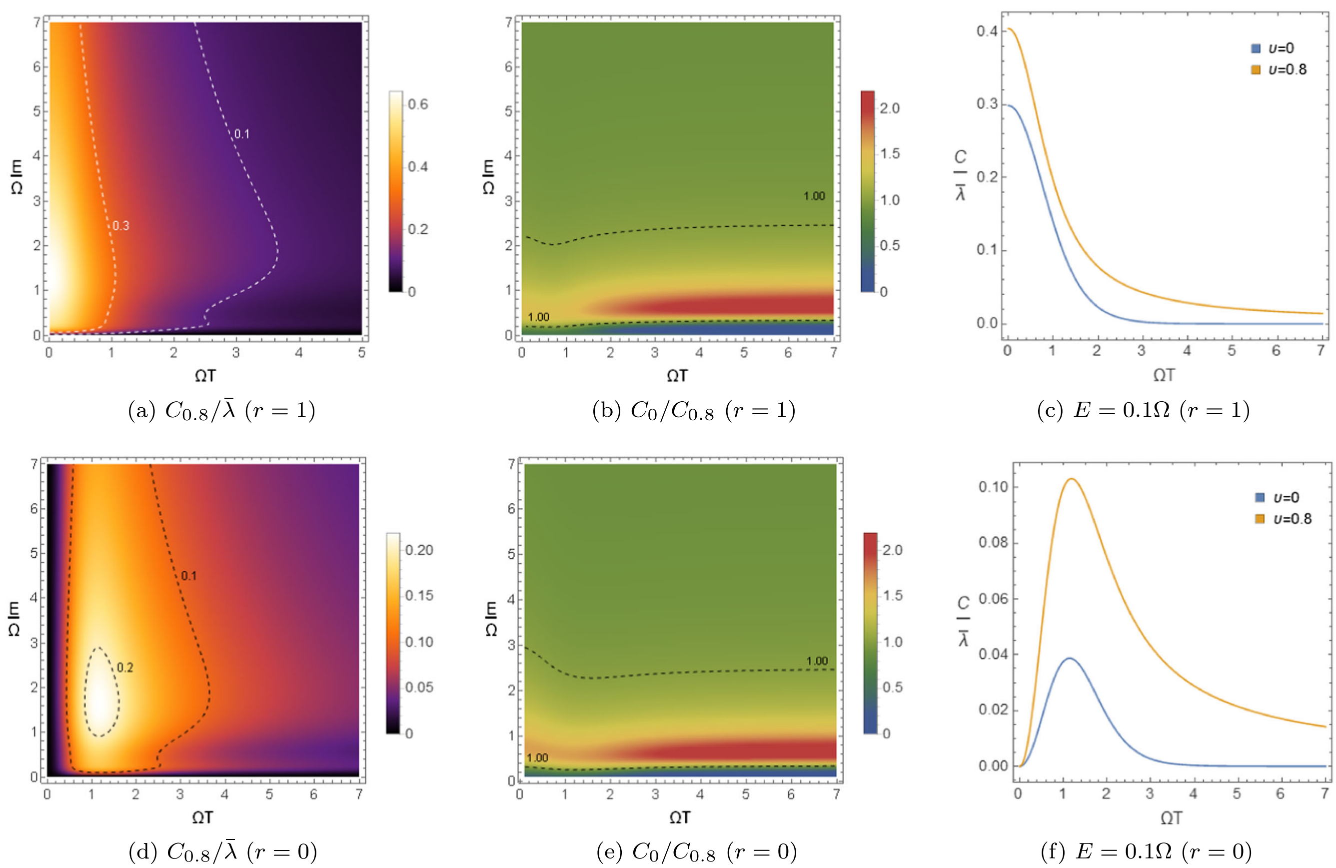
For the specific choice of phase for the coherent amplitude distribution the amount of generated coherence in this case can also be written as a continuous mixture of static coherences with mean field energy $E(v_{\hat{\mathbf{k}}})$ and weight equal to $\left[s_n(\gamma - \gamma\boldsymbol{v} \cdot \hat{\mathbf{k}})^{\frac{n-1}{2}}\right]^{-1}$

$$C_v(E, T, R) = \frac{1}{s_n} \int \frac{C(E(v_{\hat{\mathbf{k}}}), T, R)}{\left[\gamma(1 - \boldsymbol{v} \cdot \hat{\mathbf{k}})\right]^{\frac{n-1}{2}}} d\hat{\mathbf{k}}. \tag{55}$$

In Figs. 5 and 6 we numerically evaluate this amount for a detector moving at a constant relativistic speed of $v = 0.8$, in $1+1$ and $3+1$ dimensions respectively. We observe that close to resonance the amount of coherence obtained decreases when compared to the static case, a fact which holds true for any value of the detector's speed. As in [33], for lower and higher initial energies of the field there exist coherence "swelling" regions, where it is possible to extract more coherence to a moving than to a static detector. However, even though the parameter space in which swelling is present is larger in the $3+1$ dimensional case this effect is less prominent than a lower spacetime dimension.

C. Assisted catalysis

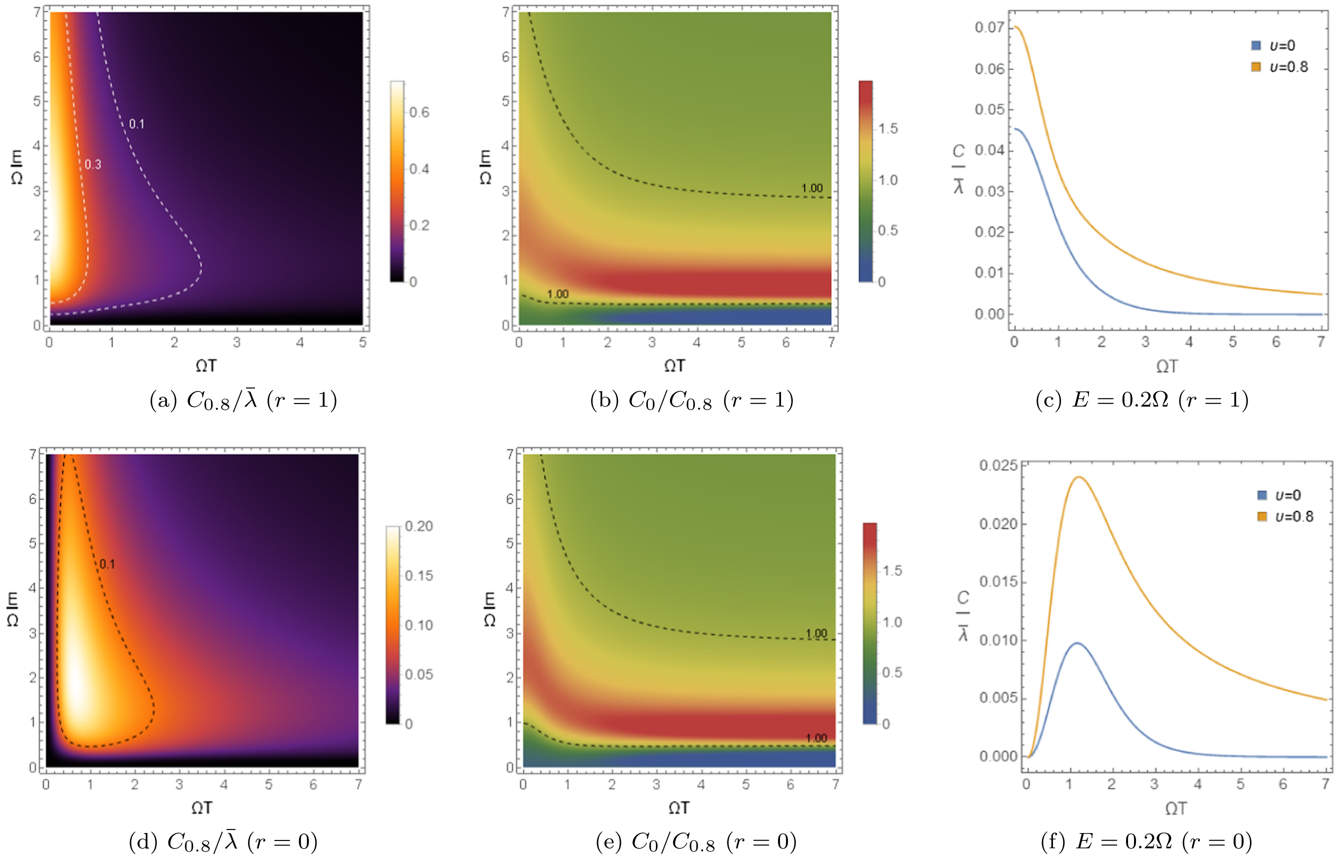
As it has already been mentioned, for an instantaneous interaction the process is catalytic. Despite the fact that



(a) $C_{0.8}/\bar{\lambda}$ $(r = 1)$ (b) $C_0/C_{0.8}$ $(r = 1)$ (c) $E = 0.1\Omega$ $(r = 1)$

(d) $C_{0.8}/\bar{\lambda}$ $(r = 0)$ (e) $C_0/C_{0.8}$ $(r = 0)$ (f) $E = 0.1\Omega$ $(r = 0)$

FIG. 5. Left: amount of generated coherence, $C_{0.8}/\bar{\lambda}$, in $1+1$ dimensions. Center: amount of coherence swelling $C_0/C_{0.8}$. Right: comparison between a static and a moving detector for an initial energy of the field $E = 0.1\Omega$.



(a) $C_{0.8}/\bar{\lambda}$ $(r=1)$ (b) $C_0/C_{0.8}$ $(r=1)$ (c) $E=0.2\Omega$ $(r=1)$

(d) $C_{0.8}/\bar{\lambda}$ $(r=0)$ (e) $C_0/C_{0.8}$ $(r=0)$ (f) $E=0.2\Omega$ $(r=0)$

FIG. 6. Left: amount of generated coherence, $C_{0.8}/\bar{\lambda}$, in $3+1$ dimensions. Center: amount of coherence swelling $C_0/C_{0.8}$. Right: comparison between a static and a moving detector for an initial energy of the field $E=0.2\Omega$.

after each extraction the state of the field has changed, it is possible to repeat the process in order to generate the same amount of coherence to a sequence of detectors. Ignoring the trivial case of a phaseless coherent amplitude distribution which does not yield any coherence, it is easy to show by setting $T=0$ in Eq. (51) that for a distribution with phase $\phi=\frac{\pi}{2}$ each static detector will generate

$$C(E)=\frac{4\lambda\Gamma(3/4)}{(2\pi)^{\frac{1}{4}}}\frac{E}{\left(1+\frac{\pi^2E^2}{2\Omega^2}\right)^{\frac{3}{4}}} \tag{56}$$

units of coherence in $1+1$ and

$$C(E)=\frac{2\lambda\Gamma(7/4)}{(2\pi)^{\frac{1}{4}}}\frac{E^2}{\left(1+\frac{\pi^2E^2}{32\Omega^2}\right)^{\frac{7}{4}}} \tag{57}$$

in $3+1$ dimensions respectively. Inserting the above expressions into Eq. (55), we obtain the amount of coherence generated in a detector moving at a constant velocity

$$C_\upsilon(E)=\frac{2\lambda\Gamma(3/4)}{(2\pi)^{\frac{1}{4}}}\left[\frac{E_+}{\left(1+\frac{\pi^2E_+^2}{2\Omega^2}\right)^{\frac{3}{4}}}+\frac{E_-}{\left(1+\frac{\pi^2E_-^2}{2\Omega^2}\right)^{\frac{3}{4}}}\right] \tag{58}$$

in $1+1$ and

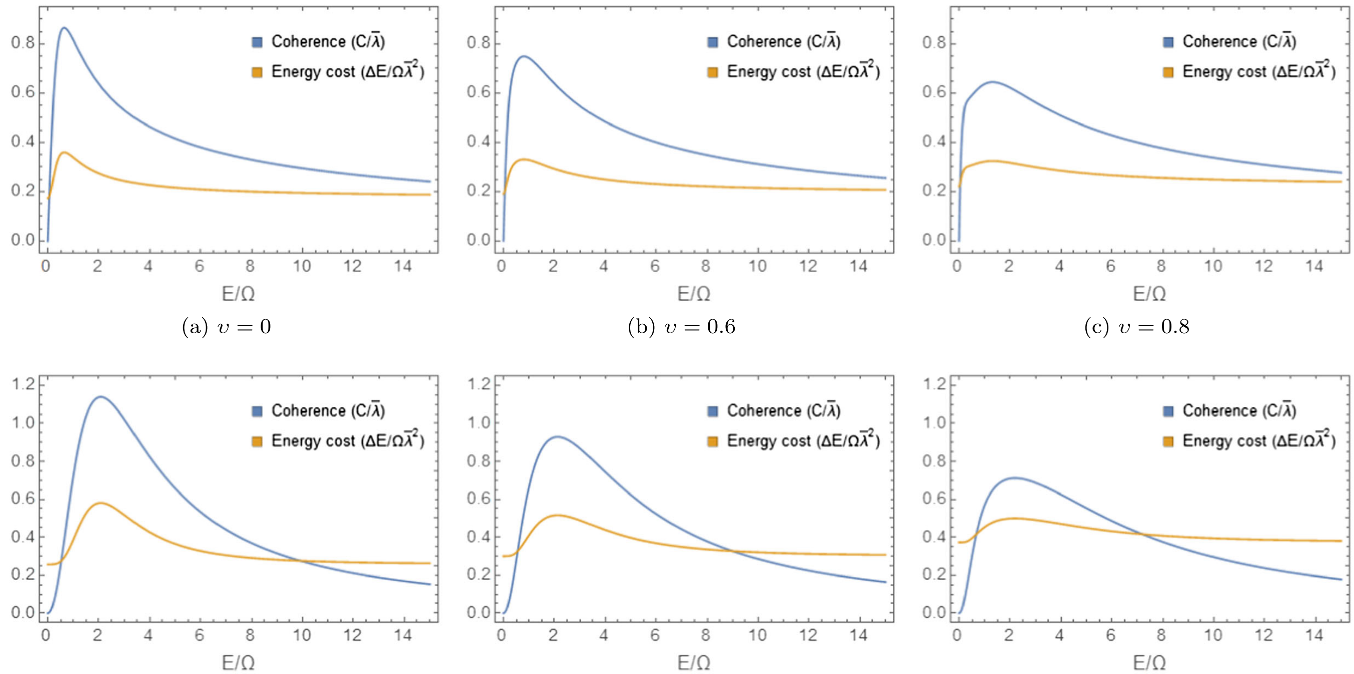
$$C_\upsilon(E)=\frac{16\bar{\lambda}\Gamma(3/4)}{(2\pi^9)^{\frac{1}{4}}\gamma\upsilon}\left[\left(1+\frac{\pi^2E_-^2}{32\Omega^2}\right)^{-\frac{3}{4}}-\left(1+\frac{\pi^2E_+^2}{32\Omega^2}\right)^{-\frac{3}{4}}\right] \tag{59}$$

in $3+1$ dimensions, where $E_\pm=E\gamma(1\pm\upsilon)$ denotes the field's relativistic Doppler shifted energies.

Due to the nature of the interaction, catalysis is an energy consuming process and requires a positive amount of work ΔE. This amount is equal to the difference between the final and initial energy of the combined system of detector and field

$$\Delta E=\mathrm{tr}(\hat{H}_D(\rho-|g\rangle\langle g|))+\mathrm{tr}(\hat{H}_\phi(\sigma'_\phi-|a\rangle\langle a|)). \tag{60}$$

which to lowest order, according to Eqs. (22) and (23), is equal to



(a) $\upsilon = 0$ (b) $\upsilon = 0.6$ (c) $\upsilon = 0.8$

(d) $\upsilon = 0$ (e) $\upsilon = 0.6$ (f) $\upsilon = 0.8$

FIG. 7. Amount of generated coherence $C_\upsilon/\bar{\lambda}$ and cost in energy $\Delta E/\Omega\bar{\lambda}^2$ as a function of the initial energy of the field (in units Ω) for various detector speeds. Upper: $1+1$ dimensions. Lower: $3+1$ dimensions.

$$\Delta E = \lambda^2 \langle a|\Omega \dot{\hat{\phi}}_f^2(\mathsf{x}(0)) + \frac{1}{2}[[\dot{\hat{\phi}}_f(\mathsf{x}(0)), \hat{H}_\phi], \dot{\hat{\phi}}_f(\mathsf{x}(0))]|a\rangle. \tag{61}$$

Keeping in mind that $C_\upsilon(E) = 2\lambda|\langle a|\dot{\hat{\phi}}_f(\mathsf{x}(0))|a\rangle|$, it can be shown that the above equation splits into two contributions

$$\Delta E = \Delta E_{\rm coh} + \Delta E_{\rm vac}, \tag{62}$$

where

$$\Delta E_{\rm coh} = \frac{C_\upsilon^2(E)\Omega}{4} \tag{63}$$

is the cost associated with generating coherence and

$$\Delta E_{\rm vac} = \frac{\lambda^2\gamma^2}{2(2\pi)^n}\int (1 - \boldsymbol{\upsilon}\cdot\hat{\mathbf{k}})^2(|\mathbf{k}^2| + \Omega|\mathbf{k}|)e^{-\frac{\pi(\mathsf{k}\cdot\mathsf{u})^2 R_n^2}{2}} d^n\mathbf{k}. \tag{64}$$

is the cost of interacting with the vacuum [22]. The cost of each extraction to lowest order is therefore equal to

$$\Delta E = \begin{cases} \frac{C_\upsilon^2(E)\Omega}{4} + \frac{\bar{\lambda}^2\Omega}{2\pi^2}\left(1 + \frac{\gamma}{\sqrt{2}}\right), & 1+1 \text{ dimensions} \\ \frac{C_\upsilon^2(E)\Omega}{4} + \frac{8\bar{\lambda}^2\Omega}{\pi^4}\left(1 + \frac{3\gamma}{\sqrt{2}}\right), & 3+1 \text{ dimensions.} \end{cases} \tag{65}$$

In Fig. 7 we present the amount of coherence generated through catalysis along with its energy cost (in units Ω) as a function of the initial energy of the field. For field energies close to resonance the amount obtained is maximized. Once again it can be seen that for an increasing value of the detector's speed this amount decreases. This is also true for the energy cost associated with coherence. On the other hand, the cost associated with interacting with the vacuum remains relatively constant.

VII. CONCLUSIONS

We have thoroughly investigated the conditions under which an Unruh DeWitt detector initially in its ground state and coupled to a massless scalar field through a derivative coupling, can be brought into a coherent superposition of its energy basis. It was proven that for an instantaneous interaction between detector and field, the process is catalytic, i.e., the same amount can be repeatedly extracted. For a suitable choice of the field's coherent amplitude distribution and an inertial detector, when the Fourier transform of the smearing function is positive this is also the maximum amount that can be obtained. By considering as an example a process in which the switching, smearing and coherent amplitude functions are Gaussian, it was demonstrated that for a coherent state of the field the amount of coherence that can be generated depends on the phase of the amplitude, the mean initial field energy, the mean interaction duration (which can be extended for a resonant energy of the field) and the mean radius of the detector.

For a detector moving at a constant velocity we verified the presence of swelling effects first reported in [33]. The increase on the amount of generated coherence due to the qubit's motion could be exploited in constructing protocols which would protect against decoherence effects induced by the environment, a major hurdle in current quantum technologies. Nonetheless, since energy nonconserving interactions such as the one considered here are coherence generating [45,46], it is uncertain whether this increase is an actual motion effect or simply due to the nature of the interaction. For this reason it would be interesting to study these protocols under energy conserving interactions such as the one given by the Glauber photodetection model [63,64] for example. From a fundamental point of view studying coherence generation under an energy conserved setting may provide a useful operational way of measuring the coherence stored in a quantum field since in this case the process would constitute a genuine method for extracting coherence similar to the one that exists for harvesting entanglement from the quantum vacuum.

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APPENDIX A: USEFUL RELATIONS

Let

$$\mathcal{F}_{\pm}(\mathbf{k}) = \int_{-\infty}^{+\infty} \chi(\tau) e^{\pm i\Omega\tau} \partial_\tau (F(\mathsf{k},\tau) e^{i\mathsf{k}\cdot\mathsf{x}(\tau)}) d\tau. \quad \text{(A1)}$$

Taking advantage of the commutation relations between the creation and annihilation operators in Eq. (8) and rewriting $\hat{\Phi}$ as

$$\hat{\Phi} = \int \frac{d^n\mathbf{k}}{\sqrt{(2\pi)^n 2|\mathbf{k}|}} (\mathcal{F}_-(\mathbf{k})\hat{a}_\mathbf{k} + \mathcal{F}_+^*(\mathbf{k})\hat{a}_\mathbf{k}^\dagger) \quad \text{(A2)}$$

we can easily compute the following commutators

$$[\hat{\Phi}, \hat{\Phi}^\dagger] = \int \frac{d^n\mathbf{k}}{(2\pi)^n 2|\mathbf{k}|} (|\mathcal{F}_-(\mathbf{k})^2| - |\mathcal{F}_+(\mathbf{k})^2|) \quad \text{(A3)}$$

$$[\hat{\Phi}, \hat{H}_\phi] = \int \frac{|\mathbf{k}| d^n\mathbf{k}}{\sqrt{(2\pi)^n 2|\mathbf{k}|}} (\mathcal{F}_-(\mathbf{k})\hat{a}_\mathbf{k} - \mathcal{F}_+^*(\mathbf{k})\hat{a}_\mathbf{k}^\dagger) \quad \text{(A4)}$$

$$[[\hat{\Phi}, \hat{H}_\phi], (\hat{\Phi}^\dagger)^m] = mc^2 (\hat{\Phi}^\dagger)^{m-1} \quad \text{(A5)}$$

where

$$c^2 = \frac{1}{2(2\pi)^n} \int (|\mathcal{F}_-(\mathbf{k})^2| + |\mathcal{F}_+(\mathbf{k})^2|) d^n\mathbf{k}. \quad \text{(A6)}$$

APPENDIX B: ASSISTED CATALYSIS FOR INSTANTANEOUS INTERACTIONS

For $\chi(\tau) = \delta(\tau)$ it is easy to see from Eq. (19) that $\hat{\Phi} = \hat{\Phi}^\dagger = \dot{\hat{\phi}}_f(\mathsf{x}(0))$. The unitary evolution operator in Eq. (18) can then be written as [23]

$$\hat{U} = I \otimes \cos(\lambda\hat{\Phi}) - i\sigma_x \otimes \sin(\lambda\hat{\Phi}) \quad \text{(B1)}$$

where $\sigma_x = |e\rangle\langle g| + |g\rangle\langle e|$. Evolving the separable state of the combined system of detector and field in Eq. (17) and tracing out the field degrees of freedom we find that the state of the detector after the interaction is equal to

$$\rho = \begin{pmatrix} \mathrm{tr}(\cos^2(\lambda\hat{\Phi})\sigma_\phi) & \frac{i}{2}\mathrm{tr}(\sin(2\lambda\hat{\Phi})\sigma_\phi) \\ -\frac{i}{2}\mathrm{tr}(\sin(2\lambda\hat{\Phi})\sigma_\phi) & \mathrm{tr}(\sin^2(\lambda\hat{\Phi})\sigma_\phi) \end{pmatrix}. \quad \text{(B2)}$$

Similarly the postinteraction state of the field is given by

$$\sigma'_\phi = \cos(\lambda\hat{\Phi})\sigma_\phi \cos(\lambda\hat{\Phi}) + \sin(\lambda\hat{\Phi})\sigma_\phi \sin(\lambda\hat{\Phi}). \quad \text{(B3)}$$

From Eqs. (B2) and (B3) and the definition of the ℓ_1-norm of coherence it can be seen that the amount of coherence extracted the second time remains the same

$$\begin{aligned} C' &= |\mathrm{tr}(\sin(2\lambda\hat{\Phi})\sigma'_\phi)| \\ &= |\mathrm{tr}(\sin(2\lambda\hat{\Phi})\sigma_\phi)| \end{aligned} \quad \text{(B4)}$$

where in the last equality we have taken advantage of the cyclic property of the trace and the fact that $\cos^2(\lambda\hat{\Phi}) + \sin^2(\lambda\hat{\Phi}) = I_\phi$.

We will now compute the energy difference ΔE between the initial and final states of the combined system of field plus detector and show that it is always positive. This means that for a detector coupled instantaneously to the time derivative of the field, catalysis is an energy consuming process so it cannot be repeated indefinitely.

From Eqs. (B2) and (B3) it is easy to see that the difference in energy before and after extraction is equal to

$$\Delta E = \mathrm{tr}(\hat{H}_D(\rho_D - |g\rangle\langle g|)) + \mathrm{tr}(\hat{H}_\phi(\sigma'_\phi - \sigma_\phi))$$
$$= \Omega \mathrm{tr}(\sin^2(\lambda\hat{\Phi})\sigma_\phi)$$
$$+ \frac{1}{2}\mathrm{tr}([[\cos(\lambda\hat{\Phi}), \hat{H}_\phi], \cos(\lambda\hat{\Phi})]\sigma_\phi)$$
$$+ \frac{1}{2}\mathrm{tr}([[\sin(\lambda\hat{\Phi}), \hat{H}_\phi], \sin(\lambda\hat{\Phi})]\sigma_\phi). \quad \text{(B5)}$$

The first term on the right-hand side as a product of two positive definite operators is evidently positive, indeed this must be the case since the qubit started out in its ground state and can only gain energy. On the other hand from Eq. (A5) it can be shown by induction that

$$[[\hat{\Phi}^\ell, \hat{H}_\phi], \hat{\Phi}^m] = \ell m c^2 \hat{\Phi}^{\ell+m-2}. \quad \text{(B6)}$$

This means that

$$[[\cos(\lambda\hat{\Phi}), \hat{H}_\phi], \cos(\lambda\hat{\Phi})] = c^2\lambda^2 \sin^2(\lambda\hat{\Phi}) \quad \text{(B7)}$$

and

$$[[\sin(\lambda\hat{\Phi}), \hat{H}_\phi], \sin(\lambda\hat{\Phi})] = c^2\lambda^2 \cos^2(\lambda\hat{\Phi}) \quad \text{(B8)}$$

so finally

$$\Delta E = \Omega \mathrm{tr}(\sin^2(\lambda\hat{\Phi})\sigma_\phi) + \frac{c^2\lambda^2}{2} \quad \text{(B9)}$$

which is always positive.

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