

Absence of the μ -problem in grand unification

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(Received 23 September 2021; accepted 23 December 2021; published 11 January 2022)

Using properties of Goldstino, we show that in generic grand unified theories with gravity-mediated supersymmetry breaking the μ -problem is nonexistent. What happens is that supersymmetry breaking universally induces the shifts of the heavy fields that generate μ and B_μ terms. In the leading order, these are given by the mass of gravitino and are insensitive to the scale of grand unification. The mechanism works regardless whether doublet-triplet splitting is achieved via fine-tuning or not. Moreover, we illustrate this general phenomenon on explicit examples of theories that achieve doublet-triplet splitting dynamically. These include the theories with Higgs doublet as a pseudo-Goldstone boson, as well as the approach based on spontaneous decoupling of the light color triplet from quarks and leptons.

DOI: [10.1103/PhysRevD.105.016009](https://doi.org/10.1103/PhysRevD.105.016009)

I. INTRODUCTION

The hierarchy problem in supersymmetry (SUSY) comes in form of two puzzles. The first one is the origin of supersymmetry breaking. The second one goes under the name of the μ -problem. The question is what sets—at the same scale—the following three unrelated mass parameters: the supersymmetric mass of the Higgs doublet superfields in the superpotential

$$W = \mu H\bar{H}, \quad (1)$$

and the two types of the soft SUSY-breaking mass terms in the scalar potential

$$m_{\text{soft}}^2 (|H|^2 + |\bar{H}|^2) + B_\mu H\bar{H} + \text{c.c.} \quad (2)$$

Here and thereafter, we shall denote the superfields and their scalar components by the same symbols. At each occasion, the meaning of the notation will be clear from the context.

In the present paper we shall assume that supersymmetry breaking is due to standard gravity-mediated scenario [1–3]. In this framework, SUSY is spontaneously broken by some hidden sector superfield(s) X with no couplings to the Standard Model superfields in the superpotential. The supersymmetry breaking is communicated to the Standard Model fields via the supergravity couplings. These are

suppressed by the powers of the Planck mass, M_P . In the observable sector, this mediation results in effective soft SUSY-breaking terms set by the scale of the gravitino mass $m_{3/2}$.

It has been pointed out by Giudice and Masiero [4] that this framework offers a natural solution to the μ -problem. Namely, both μ and B_μ terms can be generated provided one postulates certain nonminimal couplings between the Higgs doublet and the hidden sector superfields in the Kähler function.

The goal of the present paper is to point out a distinct generic reason for the absence of the μ -problem, which does not require an assumption of a nonminimal Kähler function. Namely, we wish to show that the μ -problem generically gets nullified once the Standard Model becomes embedded in a grand unified theory (GUT) with a scale M much higher than $m_{3/2}$. With no further efforts, such theories generically deliver

$$\mu \sim m_{3/2} \quad \text{and} \quad B_\mu \sim m_{3/2}^2. \quad (3)$$

The dynamical mechanism behind this effect is the shift of the heavy fields—with masses and vacuum expectation values (VEVs) given by the scale M —induced by SUSY breaking. This shift then delivers the μ term (3), which in grand unification is protected solely by SUSY.

The underlying reason for the scaling (3) is that in the limit $M_P \rightarrow \infty$ the Goldstino must reside entirely within the hidden sector superfield X . This is true, even if $m_{3/2}$ is kept finite, so that the fermion and boson masses in the observable sector stay split. The bottom line is that the grand unified Higgs sector generates (3) irrespectively of how large the scale M is. In particular, (3) remains valid in the limit $M \rightarrow \infty$.

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Note, our results are generically applicable to any extension of the supersymmetric Standard Model with the scale $M \gg m_{3/2}$, provided below M the μ term is not forbidden by any conserved quantum number. An interesting thing about grand unification is that this condition is universally enforced by the phenomenon of the doublet-triplet splitting.

The essence of the problem is that in any GUT the Standard Model Higgs doublets H, \bar{H} acquire the color-triplet partners T, \bar{T} that reside within the same irreducible representation of the grand unified group. Because of this, the color triplets can mediate an unacceptably fast proton decay, unless some measures are taken. This difficulty goes under the name of the doublet-triplet splitting problem. Now the point is that, usually, the same mechanism that renders the color triplet harmless, below the GUT scale leaves no protective symmetries—other than supersymmetry (and possibly R symmetry)—for the μ term. As a result, the μ term of the form (3) is generated after SUSY breaking due to the shifts of the VEVs of the heavy fields.

For the above reason, we shall mostly be motivated by grand unification but the readers can apply the present mechanism to their favored high scale extensions of the SUSY Standard Model.

The traditional approach to doublet-triplet splitting problem is to split masses of doublets and triplets at the GUT scale. That is, upon Higgsing the grand unified group, the color triplets gain the masses of order the GUT scale M , whereas the doublets remain massless. The underlying reason for such mass splitting is model dependent. Some examples shall be discussed below.

In an alternative, less traditional, approach [5], no mass splitting takes place. That is, the entire multiplet that houses doublets and triplets remains massless at the supersymmetric level and gains a small mass after. So, up to higher order corrections, the color-triplet partner remains as light as the Higgs doublet. However, their couplings with quarks and leptons become so strongly split that the color triplet is rendered effectively decoupled. Therefore, in this scenario, the proton decay is suppressed because the spontaneous breaking of GUT symmetry dynamically uncouples the Higgs doublet's color-triplet partner from quarks and leptons (see later).

One way or another, in GUTs, the existence of massless Higgs doublets in unbroken SUSY theory becomes intertwined with the doublet-triplet splitting problem. The results of the present paper are largely insensitive to a concrete mechanism that solves this problem. As long as the theory delivers a pair of massless Higgs doublets in supersymmetric limit, the generation of μ and B_μ of the form (3) is generic.

In fact, this way of generating μ has been incorporated in the past within particular scenarios, most notably, within the pseudo-Goldstone approach to the doublet-triplet splitting problem [6,7] (see also [8–15]). In these scenarios, in

SUSY limit, the Higgs doublets are massless by Goldstone theorem. They acquire the desired μ and B_μ terms after SUSY breaking. We shall explain that this case represents a particular manifestation of a generic shift scenario. For this, we shall reduce the pseudo-Goldstone Higgs idea to its bare essentials and interpret the generation of the μ term in (3) as a consequence of the Goldstino argument. In fact, this case is predictive due to the interplay of Goldstino and Goldstone theorems. While the former gives (3), the latter makes the relation more precise, demanding

$$B_\mu = 2\mu^2 = 2m_{3/2}^2. \quad (4)$$

This is because, by Goldstone theorem, at the tree level, the mass matrix of the Higgs doublets must have one zero eigenstate.

As a second example, we apply the shift mechanism to the model of [5] in which the light color triplet is uncoupled from quark and lepton superfields. As said, here in SUSY limit the doublets and triplets are both massless. We shall show that a generation of universal μ, B_μ terms of order (3) for both components takes place after SUSY breaking.

Before we move on, some comments are in order.

First, the gravity mediation of SUSY breaking is important for our arguments. The implementation of an analogous shift mechanism in gauge mediation requires a specific construction that has been done in [16].

Second, it is important that all the singlet superfields, coupled to the Higgs doublets in the superpotential, have large masses and that there are no sliding singlets [17] among them. As it is well known [18], in gravity mediation, such singlets destabilize the weak scale.¹

Finally, it is well known that the LHC data is pointing towards an increased hierarchy between the weak scale and soft masses. Many proponents of low energy SUSY (including us) find this alarming and rightfully so. However, compared to cancellation of 28 orders of magnitude required for doublet-triplet splitting in nonsupersymmetric GUTs, in our view, LHC data is not a sufficiently strong demotivator against the low energy supersymmetry. The “little hierarchy” is a valid issue that requires a separate study if one wishes to avoid a “mild” fine-tuning of order percent or so. In the present paper we shall only be concerned with the question of leading order correlation between μ and $m_{3/2}$.

II. GOLDSTINO ARGUMENT

We shall now present a general argument. An impatient reader may find it more useful to first go over a simple explicit example discussed in the next chapter and then come back to a general proof.

¹This situation can change in gauge-mediated scenario with low scale of SUSY breaking, see [19].

In order to set the stage, consider a prototype supersymmetric grand unified theory that at low energies delivers a pair of massless Higgs doublet superfields, H , \bar{H} . These Higgs doublets are coupled in a superpotential to a singlet superfield S with large VEV/mass. This superfield impersonates the component(s) of the heavy Higgs superfields that break the grand unified symmetry group down to the Standard Model, $SU(3) \times SU(2) \times U(1)$.

As it is customary, we assume that the primary source of spontaneous breaking of supersymmetry is an F_X term of a (canonically normalized) hidden sector superfield X . This sets the absolute scale of supersymmetry breaking as $M_{\text{SUSY}}^2 = \langle F_X \rangle$. In the absence of other fields, the fermionic component Ψ_X of the superfield X is a Goldstino, which becomes eaten up by gravitino. The resulting mass of gravitino is

$$m_{3/2}^2 = \frac{|\langle W \rangle|^2}{M_P^4} = \frac{|\langle F_X \rangle|^2}{3M_P^2}, \quad (5)$$

where $\langle W \rangle$ is the expectation value of the superpotential. This order parameter breaks R symmetry. The second equality in (5) is the condition for zero vacuum energy.

Now, it is assumed that the two sectors do not talk to each other in the superpotential. The superpotential therefore has the following generic form:

$$W = W(S) + (\tilde{M} + gS)H\bar{H} + W(X), \quad (6)$$

where g is a coupling constant and \tilde{M} is a mass parameter of order M that also sets the scale in $W(S)$. The standard couplings of the Higgs doublets to quark and lepton superfields are not shown explicitly.

For the moment, we shall not specify the form of the superpotential $W(S)$ of the heavy superfield S . The only assumption we make is that in globally supersymmetric limit it has a SUSY-preserving vacuum in which the field S receives a supersymmetric mass from the VEV(s) of the heavy field(s) given by some high scale M , for example, a grand unification scale, and that below this scale S carries no conserved quantum number(s), with the possible exception of R charge. However, the R invariance cannot serve as an exact protective symmetry as it is broken together with SUSY by the nonzero gravitino mass. At the same time, throughout the discussion, we shall keep the VEVs of the Higgs doublets H, \bar{H} zero. Obviously, in case of a single field S , with the above assumption, its mass must come from the self-coupling(s) in $W(S)$. We begin with this case.

Thus, in the limit of global supersymmetry ($M_P \rightarrow \infty$, all other scales finite), we would have

$$\begin{aligned} \left\langle \frac{\partial W}{\partial S} \right\rangle &= 0, \quad \text{for } \langle S \rangle \equiv S_0 \sim M, \\ \text{such that: } \left\langle \frac{\partial^2 W}{\partial S^2} \right\rangle &\sim M, \end{aligned} \quad (7)$$

and

$$0 \neq \left\langle \frac{\partial W}{\partial X} \right\rangle = M_{\text{SUSY}}^2. \quad (8)$$

The resulting μ term is represented by the S_0 -dependent supersymmetric mass of the Higgs doublets,

$$\mu = \tilde{M} + gS_0. \quad (9)$$

We shall assume that this μ term is zero. Obviously, having a small or zero μ in SUSY limit is our starting point. If μ is of order the GUT scale, the tiny shift shall make no difference. But such a situation is also uninteresting from doublet-triplet splitting perspective which is one of our main motivations.

The underlying reason for this cancellation is unimportant. For example, it may take place due to a dynamical reason, a group theoretic structure, or (in a least attractive case) by fine-tuning. Some explicit examples of cancellation mechanisms shall be discussed below.

We wish to show that, regardless the nature of the cancellation mechanism, for finite $m_{3/2}$, the VEV of the heavy singlet S is shifted in such a way that the μ term of order $m_{3/2}$ is generated. We shall prove this using the following Goldstino argument.

Consider the supergravity potential for S and X scalar fields (for simplicity, we assume the minimal Kähler):

$$V = e^{(|S|^2 + |X|^2)/M_P^2} (|F_S|^2 + |F_X|^2 - 3m_{3/2}^2 M_P^2), \quad (10)$$

where

$$F_S \equiv \frac{\partial W}{\partial S} + m_{3/2} S^*, \quad (11)$$

$$F_X \equiv \frac{\partial W}{\partial X} + m_{3/2} X^*, \quad (12)$$

and $m_{3/2} \equiv W/M_P^2$ should be understood as a function of the scalar components.

Let us now consider various scaling regimes. First, we take $M_P \rightarrow \infty$ while keeping the scale F_X finite. From (5), this means that $m_{3/2} \rightarrow 0$, while the product $m_{3/2} M_P$ is kept finite. We wish to find out the scaling of F_S in this regime. At first glance, this depends on the behavior of the scale M . For example, if we take $M/M_P \rightarrow 0$ (equivalently $m_{3/2} M \rightarrow 0$), the vacuum of the S superfield reduces

to a globally supersymmetric case (7), with $\langle S \rangle = S_0$ and $F_S = 0$.

What happens if we keep M/M_P finite (but, of course, small)? Despite the fact that this implies $m_{3/2}M = \text{finite}$, the supergravity F_S term must vanish. This can be understood from the following argument. Since $m_{3/2} = 0$ and $M_P = \infty$, gravitino is both massless and decoupled. Therefore, the Goldstone fermion of spontaneously broken supersymmetry must remain as a physical massless fermion. By the super-Goldstone theorem, Goldstino is given by the following combination of Ψ_S and Ψ_X :

$$\Psi_{\text{Gold}} = \frac{\langle F_S \rangle \Psi_S + \langle F_X \rangle \Psi_X}{\sqrt{\langle F_S \rangle^2 + \langle F_X \rangle^2}}. \quad (13)$$

This fermion must form a zero eigenstate of 2×2 fermion mass matrix. The second eigenvalue is of order M and, therefore, is infinite (recall that M/M_P was kept finite). Since the two sectors talk via supergravity, the mixing angle that diagonalizes the mass matrix, must vanish for $m_{3/2} = 0$. From (13) it is clear that this mixing angle is $\sim F_S/F_X$. We therefore conclude that for $m_{3/2} = 0$, we have $F_S = 0$, even if M is taken to be infinite. This indicates that, for the finite values of these parameters, F_S is controlled by the gravitino mass and not by the GUT scale M . In particular, for $M \gg m_{3/2} \neq 0$, the F_S term must behave as $F_S \sim m_{3/2}^2$.

Let us obtain the same result more explicitly. Since F_S depends only on the scales M and $m_{3/2}$, we can compute it in power series of an infinitesimal parameter $\frac{m_{3/2}}{M}$:

$$F_S = M^2 \left(c_0 + c_1 \frac{m_{3/2}}{M} + c_2 \frac{m_{3/2}^2}{M^2} + \dots \right). \quad (14)$$

Obviously, $c_0 = 0$, since, by assumption, $W(S)$ does not break SUSY in the global limit. The above Goldstino argument suggests that $c_1 = 0$ as well. Let us check this explicitly.

From (10) it is clear that for $m_{3/2} \rightarrow 0$ and $m_{3/2}M = \text{finite}$, the vacuum of the S superfield is determined by the condition

$$F_S = \frac{\partial W}{\partial S} + m_{3/2} S^* = 0. \quad (15)$$

Notice that this condition does not reduce to a globally supersymmetric condition (7). This is because $S_0 \sim M$ and, therefore, the term $m_{3/2} S^*$ cannot be ignored. Let us find the solution to the condition (15) in the form of a small shift around the globally supersymmetric value: $S = S_0 + \delta S$. Plugging this in (15), we obtain that the cancellation of the leading terms demands

$$\left(\frac{\partial^2 W}{\partial S^2} \right)_{S=S_0} \delta S + m_{3/2} S_0^* = 0. \quad (16)$$

Since $\left(\frac{\partial^2 W}{\partial S^2} \right)_{S=S_0} \sim S_0 \sim M$, we have

$$\delta S \sim m_{3/2}. \quad (17)$$

Thus, we have shown that the shift of a field that gets the supersymmetric mass from its large VEV, is of order $m_{3/2}$. We observe that this shift, in the leading order, is independent of the scales M and M_P .

This result implies that if the μ term (9) is zero in the supersymmetric limit after SUSY breaking, then $\mu \sim m_{3/2}$ is generated.

The generalization of the above reasoning for arbitrary number of heavy superfields $S_j, j = 1, 2, \dots$, is straightforward. It is convenient to work in the eigenstate basis of the global-SUSY mass matrix $M^{ij} \equiv \frac{\partial^2 W}{\partial S_i \partial S_j} |_{S_j=S_{0j}}$, where, as previously, S_{0j} denote the globally supersymmetric VEVs of the superfields that satisfy $\frac{\partial W}{\partial S_j} = 0$. Such a basis always exists since the matrix M^{ij} is holomorphic and symmetric. Equation (16) then becomes

$$M_{jj} \delta S_j + m_{3/2} S_{0j}^* = 0. \quad (18)$$

Notice, this is a leading order relation that ignores unimportant contributions of order $m_{3/2}^2$. Now, since by our starting assumption, $M_{jj} \sim M$ for all j s and $S_{0j} \sim M$ at least for some j s, the corresponding shifts are $\delta S_j \sim m_{3/2}$.

III. SIMPLE EXAMPLE

We can illustrate the action of the above general mechanism for an explicit form of the superpotential:

$$W(S) = \frac{M}{2} S^2 + \frac{\lambda}{3} S^3, \quad (19)$$

where M is a high scale and λ is a coupling constant of order one. In the global SUSY limit, the VEV of S is given by

$$\frac{\partial W(S)}{\partial S} = MS + \lambda S^2 = 0 \rightarrow S_0 = -\frac{M}{\lambda}. \quad (20)$$

Of course, there is also a minimum with $S_0 = 0$, which we shall disregard. In general, the existence of multiple degenerate minima, among which only one is phenomenologically acceptable, is generic in SUSY GUTs. We are not concerned by explaining why our Universe ended up in the right one. We wish to show that, once a minimum with the right pattern of GUT symmetry breaking and the correct doublet-triplet splitting is chosen, the shift of the μ term after SUSY breaking follows. Since in the above example,

S impersonates a GUT Higgs field, it is natural that we are not interested in the minimum with zero S .

We wish to determine the shift δS triggered by the gravity-mediated supersymmetry breaking. From the Goldstino argument presented above, this shift can be found from the condition (15), which in the present case translates as

$$MS + \lambda S^2 \simeq -m_{3/2} S^*. \quad (21)$$

This gives the shift $\delta S = -\frac{m_{3/2}}{\lambda}$. The explicit minimization of the entire potential including the standard gravity-mediated soft terms,

$$V = |\lambda S + MS|^2 + m_{3/2}^2 |S|^2 + \left(m_{3/2} A \frac{\lambda}{3} S^3 + m_{3/2} (A - 1) \frac{M}{2} S^2 + \text{c.c.} \right), \quad (22)$$

fully confirms this result.

Now, we recall that in global supersymmetry limit the μ term was assumed to be zero. That is, we have

$$\mu = \tilde{M} + gS_0 = \tilde{M} - \frac{g}{\lambda} M = 0. \quad (23)$$

Then, the shift of S , induced by the soft terms, generates a nonzero μ term given by

$$\mu = g\delta S = -\frac{g}{\lambda} m_{3/2}. \quad (24)$$

To conclude this section, we have argued that the generation of the μ term of order $m_{3/2}$, due to the shift of the VEV(s) of heavy field(s), is rather generic. This phenomenon is independent of a precise mechanism that sets $\mu = 0$ in the supersymmetric limit.

IV. DECOUPLING

The Goldstino argument indicates that it is in general wrong to work with the low energy superpotentials obtained by the substitution of the supersymmetric VEVs of the heavy fields. Indeed, in the above example, the observable sector described by the superpotential,

$$W = \frac{M}{2} S^2 + \frac{\lambda}{3} S^3 + (\tilde{M} + gS) H \bar{H}, \quad (25)$$

had a supersymmetric vacuum

$$S = S_0 = -\frac{M}{\lambda}, \quad H = \bar{H} = 0. \quad (26)$$

In this vacuum, the superfield S had a mass M , whereas the mass of the doublets (23) was fine-tuned to zero.

Now, since the mass of the superfield S is much higher than the supersymmetry breaking scale, it may be tempting to integrate S out while ignoring supersymmetry breaking.

This naive approach would give an effective low energy superpotential for the doublets H, \bar{H} with zero μ term plus high-dimensional operators suppressed by the scale M . Then, neglecting the high-dimensional operators, one would arrive to a low energy theory with $\mu = 0$. Such a description would completely miss the generation of the μ term due to the shift of the heavy field S .

This may seem a bit confusing, since we expect that the effects of the heavy fields must be suppressed by powers of their mass M . However, obviously, there is no conflict with the principle of decoupling. What is happening in reality is that, although the mass term of the heavy scalar S scales as $\sim M^2$, so does the tadpole generated by supersymmetry breaking, which goes as $\sim m_{3/2} M^2 S$. As a result, the shift $\delta S \sim m_{3/2}$ is finite even in the limit $M \rightarrow \infty$. This effect must be taken into account when integrating out the heavy fields properly.

V. GUTS

A. Example with fine-tuning: $SU(5)$

The essence of how the dynamical generation of the μ term is intertwined with doublet-triplet splitting can be illustrated on a prototype GUT example of minimal $SU(5)$. As it is well known, in this theory the Higgs doublets H, \bar{H} and their triplet partners T, \bar{T} are embedded in $5_H, \bar{5}_H$ representations, respectively. The breaking of $SU(5)$ symmetry down to the Standard Model group is achieved by the 24_H Higgs representation. The superpotential of the Higgs superfields is

$$W = \frac{M}{60} \text{tr} 24_H^2 - \frac{\lambda}{90} \text{tr} 24_H^3 + \tilde{M} \bar{5}_H 5_H - \frac{g}{3} \bar{5}_H 24_H 5_H. \quad (27)$$

Note, it is the necessity of the doublet-triplet splitting that excludes the possibility of setting \tilde{M} and g small. This eliminates any symmetry protection for the resulting μ term that—after GUT symmetry breaking—is left solely at the mercy of SUSY.

Substituting into (27) the only nonzero component $24_H = S \text{diag}(2, 2, 2, -3, -3)$, the system effectively reduces to the example (19) [and (25)] with an extra pair of T, \bar{T} superfields,²

$$W(S) = \frac{M}{2} S^2 + \frac{\lambda}{3} S^3 + (\tilde{M} + gS) \bar{H} H + \left(\tilde{M} - \frac{2}{3} gS \right) \bar{T} T. \quad (28)$$

²As it is well known, in the above theory there are several other minima, all exactly degenerate in the SUSY limit. For example, there is a trivial minimum, $24_H = 0$, the one with $SU(4) \times U(1)$ symmetry, $24_H \propto \text{diag}(1, 1, 1, 1, -4)$, and even some minima in which the Higgs doublets $\bar{5}_H, 5_H$ have large VEVs. As already mentioned, we are only interested in the one in which, in the SUSY limit, the symmetry is broken to $SU(3) \times SU(2) \times U(1)$. We shall study the shift of the VEVs around this minimum.

The supersymmetric VEV of S is given by (20) and the doublet-triplet splitting is achieved by fine-tuning (23). This fine-tuning gives the zero μ term for the doublets and simultaneously generates a large supersymmetric mass term for their color-triplet partners, $\mu_T = \frac{5g}{3\lambda} M$. This takes care of the suppression of proton decay.

Now, as already explained, after SUSY breaking the μ term of order $m_{3/2}$ is generated due to the shift of the VEV of the S superfield and is given by (24).³

One may argue that, in theory with fine-tuning, we are not gaining much by inducing the required μ term after SUSY breaking. After all, it is not clear why the fine-tuning $\mu = 0$ is any more natural than a fine-tuning to order TeV. One could try to dispute this by saying that for the superpotential (27)—which knows nothing about the weak scale—the scales M and zero are the two natural points.

However, we shall not do this. By default, such disputes usually take one to nowhere due to the lack of the guiding principle in theories with fine-tuning. This is why we are more attracted to scenarios in which $\mu = 0$ in SUSY limit is justified by the underlying structure of the GUT theory.

However, there is an important point that works regardless of fine-tuning: the value of μ in the low energy theory is shifted by

$$\delta\mu \sim m_{3/2}, \quad (29)$$

with respect to its SUSY value. This exposes an intrinsic sensitivity of the supersymmetric Standard Model towards the GUT completion.

This concludes the example with fine-tuning. In what follows, we shall illustrate the same effect on examples of theories that, in unbroken supersymmetry, achieve the vanishing μ dynamically.

B. Example: Higgs as a pseudo-Goldstone

As the first example, we consider class of theories in which the Higgs doublets H, \bar{H} are pseudo-Goldstone bosons [6,7]. In these models, before supersymmetry breaking, the μ term is dynamically adjusted to zero by the Goldstone theorem.

The idea is that the Higgs part of the GUT superpotential has large accidental global symmetry. This global symmetry is spontaneously broken at the GUT scale along with the local one. This breaking results into a pair of (pseudo-)Goldstone superfields with the quantum numbers of H, \bar{H} . Due to the global symmetry of the Higgs part of the superpotential, in supersymmetric limit, these superfields are exactly massless. Correspondingly, before SUSY breaking, $\mu = 0$. After the soft SUSY-breaking terms are included, the $\mu \sim m_{3/2}$ is generated. In the minimal case (with canonical Kähler metric), due to Goldstone theorem,

at the tree level, one combination of doublets H, \bar{H} remains massless even after supersymmetry breaking. This degree of freedom acquires a nonzero mass and a VEV via radiative corrections.

The above idea was realized in two main directions, [6] and [7], where [7] represents a justification of [6] from a more fundamental theory. We shall briefly discuss the key aspects of the two approaches.

The first proposal was a model by Inoue, Kakuto and Takano, and by Anselm and Johansen [6]. Both examples were based on a minimal $SU(5)$ GUT. As already discussed, the Higgs sector of this theory consists of an adjoint 24_H -plet and a pair of $5_H, \bar{5}_H$ -plet chiral superfields. This theory exhibits a textbook example of the doublet-triplet splitting problem. The required mass splitting between the color triplet and the weak doublet components of $5_H, \bar{5}_H$ is achieved at the expense of a severe fine-tuning discussed in the previous chapter.

The idea by the authors of [6] was that this fine-tuning admits an interpretation in terms of the Goldstone theorem, provided an additional gauge-singlet chiral superfield, 1_H , is added to the Higgs sector. In such a case, after careful adjustment of the parameters, the Higgs part of the superpotential becomes invariant under a global $SU(6)$ symmetry group. Under it, various $SU(5)$ Higgs multiplets combine into a single adjoint representation: $35_H = 24_H + 5_H + \bar{5}_H + 1_H$. In such a case, the Higgsing of the gauge $SU(5)$ symmetry is accompanied by a spontaneous breaking of the global $SU(6)$ symmetry. The latter breaking results into leftover pseudo-Goldstone multiplets H, \bar{H} with the quantum numbers of the electroweak Higgs doublets. These are the doublet components of $5_H, \bar{5}_H$, respectively.

The potential criticism against this scenario is that a severe fine-tuning among two large numbers is traded for more severe fine-tunings among several large parameters. In order to dissolve this criticism, one needs to justify the demanded global $SU(6)$ pseudosymmetry as an accidental symmetry emerging from a more fundamental theory. This was achieved in [7] by lifting (i.e., UV completing) the theory into a GUT with a gauged $SU(6)$ symmetry. The accidental global symmetry then emerges as a low energy remnant of this gauge symmetry. This happens in the following way.

The minimal set of chiral superfields necessary for Higgsing the $SU(6)$ gauge symmetry down to the Standard Model group, $SU(3)_c \times SU(2)_L \times U(1)_Y$, consists of an adjoint 35_H -plet and a pair of $\bar{6}_H, 6_H$ -plets. If the cross coupling $\bar{6}_H 35_H 6_H$ is absent, the renormalizable superpotential of the Higgs fields splits into two non-interacting parts,

$$W_H = W(35) + W(6_H, \bar{6}_H). \quad (30)$$

Obviously, this superpotential has a global symmetry $SU(6)_{35_H} \times SU(6)_{6_H}$ under independent $SU(6)$ transformations of the two sectors. The subscripts indicate the superfields on which the corresponding symmetries act.

³Cf. [2], Eq. (15).

Without entering into a discussion about naturalness, we note that the global symmetry $SU(6)_{35_H} \times SU(6)_{6_H}$ can be viewed as accidental. This is because it results from the absence of a single cross-coupling, as opposed to fine-tuned cancellations among several big numbers. In addition, there have been proposals of justifying the absence of this cross-coupling from more fundamental theory, such as, for example, a stringy anomalous $U(1)$ symmetry [14] or locality in the extra space [15].

The Higgsing of the gauge symmetry down to Standard Model group is triggered by the following VEVs:

$$\langle 35_H \rangle = \text{diag}(1, 1, 1, 1, -2, -2)v_{35}, \quad \langle 6_H \rangle = \langle \bar{6}_H \rangle = \begin{pmatrix} v_6 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad (31)$$

where the parameters v_{35} and v_6 are of order on the GUT scale.

Now, simultaneously with the Higgsing of the gauge group, the global symmetries are spontaneously broken in the following way:

$$SU(6)_{35_H} \rightarrow SU(4) \times SU(2) \times U(1), \quad (32a)$$

$$SU(6)_{6_H} \rightarrow SU(5). \quad (32b)$$

The straightforward count of the Goldstone bosons and the diagonalization of the mass matrix shows that one pair of chiral superfields, with the quantum numbers of electro-weak doublets, is left out “uneaten” by the gauge fields and remains exactly massless. These superfields are the two linear combinations of the doublets H_{35}, \bar{H}_{35} and H_6, \bar{H}_6 from the 35_H and $6_H, \bar{6}_H$ fields, respectively:

$$H = \frac{H_{35}v_6 - 3H_6v_{35}}{\sqrt{v_6^2 + 9v_{35}^2}}, \quad \bar{H} = \frac{\bar{H}_{35}v_6 - 3\bar{H}_6v_{35}}{\sqrt{v_6^2 + 9v_{35}^2}}. \quad (33)$$

At the same time, all colored components of the Higgs superfields acquire masses of order the GUT scale. Thus, the doublet-triplet splitting is achieved as a result of the Goldstone phenomenon. The two Higgs doublets (33) that represent the Goldstone modes of the accidental global symmetry are strictly massless in the limit of exact SUSY. Correspondingly, the μ term vanishes in this limit.

The $\mu = m_{3/2}$ is generated after supersymmetry breaking. In the previous analysis this was demonstrated by an explicit minimization of the Higgs potential in the presence of the soft SUSY-breaking terms. Our goal here is to view

the generation of the μ term in the pseudo-Goldstone model as a manifestation of the universal shift mechanism discussed in the present paper. We shall therefore reduce the Goldstone mechanism to its bare essentials.

The unified property of such models is that the Higgs doublets H, \bar{H} are connected by a continuous global (pseudo) symmetry to some GUT superfield(s) (N, \bar{N}) with large VEV(s). The latter fields Higgs the gauge GUT symmetry and simultaneously break spontaneously the continuous global symmetry that connects them with H, \bar{H} . For all practical purposes, we can effectively characterize the above global degeneracy as a $U(3)$ global symmetry in which the gauge $SU(2)_L \times U(1)_Y$ electro-weak symmetry enters as $U(2)$ subgroup. We must note that this simplified formulation of the pseudo-Goldstone mechanism was previously proposed in [16].

Naturally, under this $U(3)$, the $U(2)$ doublets H, \bar{H} and singlets N, \bar{N} form the triplet and antitriplet representations, respectively:

$$3_H = \begin{pmatrix} H \\ N \end{pmatrix}, \quad \bar{3}_H = \begin{pmatrix} \bar{H} \\ \bar{N} \end{pmatrix}.$$

The precise form of a $U(3)$ -invariant superpotential that leads to the desired symmetry breaking, $U(3) \rightarrow U(2)$, is not important. For simplicity, we can choose it as

$$W = \lambda S(3_H \bar{3}_H - M^2), \quad (34)$$

where S is a singlet superfield, λ is coupling constant and M is a scale of symmetry breaking. For simplicity, we take all parameters as real.

It is easy to check that the above superpotential gives a globally supersymmetric vacuum in which the continuous global symmetry is broken down to Standard Model $U(2)$ by the following VEVs:

$$\langle N \rangle = \langle \bar{N} \rangle = M.$$

At the same time, the VEVs of all other superfields are zero, $S = H = \bar{H} = 0$. Obviously, the μ term is also zero, $\mu = \lambda S = 0$. This happens because the doublets H, \bar{H} represent the Goldstone bosons of spontaneously broken global symmetry $U(3) \rightarrow U(2)$.

After the soft SUSY-breaking terms are included, the VEVs get shifted. These shifts induce nontrivial mass terms in the Higgs sector and both the μ and B_μ terms are generated.

In order to find them, we minimize the potential with soft supersymmetry breaking terms included:

$$\begin{aligned}
V &= |\lambda(H\bar{H} + N\bar{N} - M^2)|^2 \\
&+ |\lambda S|^2(|H|^2 + |\bar{H}|^2 + |N|^2 + |\bar{N}|^2) \\
&+ m_{3/2}^2(|S|^2 + |H|^2 + |\bar{H}|^2 + |N|^2 + |\bar{N}|^2) \\
&+ m_{3/2} A \lambda S(H\bar{H} + N\bar{N}) + m_{3/2}(2-A)\lambda M^2 S + \text{c.c.}
\end{aligned} \tag{35}$$

After a straightforward calculation, we obtain that, in the leading order in expansion in small parameter $\frac{m_{3/2}}{M}$, the VEVs are given by

$$\begin{aligned}
\langle N \rangle &= \langle \bar{N} \rangle = M + \frac{m_{3/2}^2}{2\lambda^2 M} (A - 2), \\
\langle S \rangle &= -\frac{1}{\lambda} m_{3/2}.
\end{aligned} \tag{36}$$

Next we insert the above VEVs in the Lagrangian relevant for the masses of H, \bar{H} :

$$\begin{aligned}
\mathcal{L} \supset & (|\lambda S|^2 + m_{3/2}^2)(|H|^2 + |\bar{H}|^2) \\
& + ((|\lambda|^2(N\bar{N} - M^2)^* + Am_{3/2}\lambda S)H\bar{H} + \text{c.c.}) \\
\equiv & (\mu^2 + m_{3/2}^2)(|H|^2 + |\bar{H}|^2) + (B_\mu H\bar{H} + \text{c.c.}),
\end{aligned} \tag{37}$$

where

$$B_\mu = -2m_{3/2}^2, \quad \mu = -m_{3/2}, \tag{38}$$

in accordance to (4).

Hence, the μ and B_μ terms are produced at the same scale and the resulting mass matrix has the following form:

$$\hat{M}_H^2 = \begin{array}{c} H \\ \bar{H}^* \end{array} \begin{array}{cc} H^* & \bar{H} \\ \left(\begin{array}{cc} 2m_{3/2}^2 & -2m_{3/2}^2 \\ -2m_{3/2}^2 & 2m_{3/2}^2 \end{array} \right) \end{array}. \tag{39}$$

Notice, due to the existence of a Goldstone mode at the tree level, the mass matrix (39) has a zero eigenvalue. The latter equality is specific to the pseudo-Goldstone approach. However, the universal feature shared by other approaches is that the gravity-mediated SUSY breaking generates B_μ and μ at the scale set by $m_{3/2}$, as given by (3).

C. Example: decoupled triplet

The last example in which we shall implement the generation of the μ term by the shift mechanism is the approach to doublet-triplet splitting problem developed in [5]. In this picture the weak doublets (H, \bar{H}) as well as their color-triplet partners (T, \bar{T}), are isolated from the VEVs that break GUT symmetry down to the Standard Model. As a result, all these superfields remain exactly massless in supersymmetric theory. That is, no mass splitting among

the doublets and triplets takes place. Instead, the entire GUT multiplet remains massless.

This may come as a surprise, since it is expected that light color triplets T, \bar{T} mediate proton decay at an unacceptable rate. However, in the scenario of [5] this potential problem is avoided by decoupling the color triplets T, \bar{T} from the quark and lepton superfields. Only the doublets H, \bar{H} maintain the usual coupling to quarks and leptons. To put it shortly, in this scenario the doublet-triplet splitting gets transported from the mass terms into the Yukawa couplings. As a result, the proton decay is equally strongly suppressed both at $d = 6$ and $d = 5$ operator levels.

Let us, following [5], consider a realization of this idea in a supersymmetric $SO(10)$ theory in which the Higgs doublet resides in 10_H representation. The quarks and leptons are placed in 16_F spinor representation. The idea is that 10_H couples to matter fermions via an intermediate heavy 45_H Higgs that has a VEV on $SO(6) \times SU(2) \times U(1)$ -invariant component:

$$\langle 45_H \rangle = M_{45} \text{diag}(0, 0, 0, \epsilon, \epsilon), \quad \epsilon \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \tag{40}$$

where M_{45} is of order on the GUT scale. The coupling with matter fermions is generated by the exchange of a pair of heavy $144, \overline{144}$ -dimensional multiplets with the following couplings in the superpotential:

$$W_F = g16\gamma_i \overline{144}_j 45_{Hij} + M_{144} 144_j \overline{144}_j + g' 10_{Hj} 144_j 16, \tag{41}$$

where $j = 1, 2, \dots, 10$ is the $SO(10)$ tensor index, γ_j are $SO(10)$ gamma matrixes and spinor indexes are not shown explicitly. M_{144} is a mass term of order the grand unification scale and g, g' are dimensionless coupling constants. The integration out of the 144-dimensional multiplets results in the following effective coupling in the superpotential,⁴

$$W_F \rightarrow \frac{gg'}{M_{144}} 10_{Hi} 45_{Hij} 16_F \gamma_j 16_F. \tag{42}$$

Taking into account the form (40) of the VEV of 45_H , it is clear that the electroweak doublet components H, \bar{H} of 10_H acquire the usual Yukawa couplings with the Standard Model fermions, given by $\frac{gg'M_{45}}{M_{144}}$. At the same time, their color-triplet partners, T, \bar{T} , decouple.

⁴For illustrative purposes, here we are only concerned with the minimal structure of the theory. The generation of realistic fermion masses, as usual, requires the enrichment of the horizontal structure which can be incorporated without changing any of our conclusions and shall not be attempted here.

In this way, the burden of generating the huge masses for the color triplets, while keeping their weak-doublet partners massless, is avoided. The entire 10_H multiplet can be kept massless in supersymmetric limit. All one needs for achieving this is to assume that solely the heavy fields with zero VEVs interact directly with 10_H -plet in the superpotential.

Let such a superfield be S . At the same time, S is free to (and in general will) interact with the Higgs multiplets that participate in the breaking of $SO(10)$ symmetry. Then, according to our arguments, SUSY-breaking shall result in the shift of the VEV $S \sim m_{3/2}$ and in the subsequent generation of the μ term.

An example of the superpotential that validates this mechanism is

$$W = S(\lambda 10_H^2 + \lambda' N^2 - M^2), \quad (43)$$

where N impersonates the heavy superfield(s) that Higgs the $SO(10)$ symmetry. M is a mass scale and λ and λ' are coupling constants. For definiteness, we take all parameters to be real and positive.

Of course, for achieving the right symmetry breaking pattern a lot more terms and a garden variety of representations are required. This is the standard ‘‘engineering’’ problem in $SO(10)$ GUT and is not specific to the present discussion. Our goal here is not in a construction of a fully functioning $SO(10)$ theory but rather in pointing out an universal shift mechanism for the μ term. We shall therefore focus on (43). For a detailed analysis of the Higgs sector leading to a desired symmetry breaking patterns, the reader is referred to [20].

Now, in supersymmetric limit we have $N = M$ and $S = 10_H = 0$. Therefore, the μ term that is set by the VEV of S is zero and the entire 10_H -plet is massless. It is straightforward to check that the soft supersymmetry breaking generates the shift $S = -\frac{m_{3/2}}{2\lambda}$. Correspondingly, the μ term generated as a result of this shift is $\mu = -\frac{\lambda}{2\lambda'} m_{3/2}$.

Notice, the same μ term is generated for the color-triplet partners T, \bar{T} since they share the 10_H multiplet with the Higgs doublets. Of course, the exact doublet-triplet mass degeneracy shall be lifted by radiative and other higher order corrections, but the color triplets shall remain light. Because these particles are essentially decoupled from the light fermions, they are extremely long lived. The existence of the long-lived colored multiplets, with their masses correlated with the Higgs doublets, is a prediction of the decoupled triplet scenario [5].

This latest topic gives us an opportunity to observe another crucial impact on the low energy physics from the shift of the heavy VEVs. Namely, without such a shift, the colored triplets T, \bar{T} would remain decoupled from the Standard Model fermions. As a result, they would remain stable. However, the SUSY-breaking shifts the VEV of the

heavy 45_H -plet and generates the entries $\sim m_{3/2}$ in three empty 2×2 diagonal blocks of (40).⁵ This component of the 45_H VEV breaks the $SO(6)$ symmetry down to $SU(3) \times U(1)$. Simultaneously, via (42), this generates the effective couplings of the color triplets T, \bar{T} to the quark and lepton superfields. From (42) it is clear that the resulting decay constant of each color triplet is by a factor $\sim \frac{m_{3/2}}{M_{45}}$ smaller as compared to its doublet partner Higgs. For $m_{3/2} \sim \text{TeV}$ and $M_{45} \sim 10^{16}$ GeV, the resulting decay time of a color triplet into Standard Model particles is $\tau \sim \text{sec}$ or so. Such a long-lived colored state has potentially interesting collider signatures [5,8,20–22].

VI. COMPARISON WITH LOW-ENERGY SOLUTIONS OF THE μ -PROBLEM

The goal of the present paper is to create awareness of an intrinsic correlation. Namely, within the grand unified framework, the μ -problem becomes entangled with the doublet-triplet splitting problem in the way that the solution of the latter at the GUT scale, generically, results into the solution of the former at the scale $\mu \sim m_{3/2}$.

In this respect, it is useful to confront this situation with the one encountered in the proposals in which the μ -problem is solved by a protective symmetry broken at low energies. The representatives of this class of solutions are for example the Kim-Nilles solution [23] or more recently suggested models [24,25] based on Z_{24}^R symmetry. A more inclusive list of references can be found in the review [26].

This class of theories is based on the assumption of a protective symmetry which is spontaneously broken at low energies. Such can be a discrete symmetry or a continuous symmetry of Peccei-Quinn type. This approach can successfully address the μ -problem in non-GUT extensions of the standard model.

However, embedding of such symmetries within GUTs faces a fundamental obstacle in form of the doublet-triplet splitting. The caveat is that any protective external symmetry must be equally shared by the Higgs doublet and its color-triplet partner. Due to this, any mechanism that splits triplet and doublet masses, inevitably breaks the protective symmetry at the GUT scale. Correspondingly, such a symmetry alone cannot be used for suppressing the μ term. This is an intrinsic difficulty for a straightforward embedding of the approaches based on protective external symmetries into GUTs.

In contrast, our focus is entirely on the correlation between doublet-triplet splitting and the μ term in GUTs. Here, the protective mechanism cannot rely on

⁵This can be easily checked by a straightforward explicit computation for simple superpotentials for various choices of $SO(10)$ Higgs multiplets which in the global SUSY limits deliver the VEV (40).

an external symmetry but rather must be a part of the grand unified setup, as they are in the examples considered. In this sense, the mechanism that we are pointing out addresses a fundamentally different side of the problem as compared to the approaches based on protective low energy symmetries. To that effect the two approaches are complementary.

VII. CONCLUSIONS

The purpose of the present paper is to provide evidence that in a large class of grand unified theories the μ -problem is nonexistent. What happens is the following. The doublet-triplet splitting mechanism that in supersymmetric theory delivers a pair of massless Higgs doublets after soft supersymmetry breaking generates the required μ and B_μ terms. This effect is independent of how the splitting was achieved in the first place. For example, the splitting can be arranged by a direct fine-tuning or via a more natural dynamical mechanism. We gave some general arguments, in particular based on Goldstino theorem, showing that the shifts in VEVs of the heavy fields universally result in generating the μ term of order $m_{3/2}$.

After giving general arguments, we have shown how the mechanism works both in a fine-tuned scenario as well as in two illustrative examples with dynamical solutions to the doublet-triplet splitting problem.

In the first class of theories [6,7] the Standard Model Higgs doublet is a pseudo-Goldstone boson. In this scenario, the generation of the required μ and B_μ terms from the shift of the heavy VEVs, can be understood from the combination of Goldstone and Goldstino theorems. The first theorem demands that the mass matrix of the Higgs doublets has one exact zero eigenstate at the tree level. At the same time the Goldstino argument presented here requires that the entries are of order $m_{3/2}^2$. An explicit computation for the minimal Kähler confirms this and gives the relation (4).

Our second example is based on the scenario of [5] in which no mass splitting takes place between the Higgs doublet and its color-triplet partner. Instead, in the supersymmetric limit both components remain exactly massless. The proton decay is nevertheless safely suppressed because the color triplet is decoupled from quark and lepton superfields. In this scenario too, after supersymmetry breaking, the universal μ term is generated both for the Higgs doublets and their color-triplet partners. The theory therefore predicts the existence of long-lived colored states with their masses correlated with the masses of the Higgs doublets.

This scenario illustrates another important low energy effect originating from the shift of the VEVs of the heavy Higgses. Namely, this shift is the sole source for generating the nonzero decay constant of the color-triplet partners of the Higgs doublets. As a result, these color triplets acquire the finite, albeit macroscopically long, lifetimes. This makes them into the subjects of phenomenological and cosmological interests.

Due to its generic nature, the presented shift mechanism is expected to generate the μ term in other group-theoretic solutions of the doublet-triplet splitting problem. In particular, we have explicitly checked this by constructing a simple realization of the Dimopoulos-Wilczek mechanism [27]. The details will be given elsewhere. The similar phenomenon is expected to work also for the missing partner mechanism [28–30].

Also note that, although we are mostly focused here on dynamic scenarios, the models that achieve doublet-triplet splitting via an explicit fine-tuning [such as the minimal $SU(5)$, considered above] should not be left out. An interesting example is provided by a predictive $SO(10)$ theory of [31]. The universal shift mechanism for generating the μ term discussed in present paper shall be equally operative in such scenarios.

The described phenomenon once again teaches us an important lesson that the heavy fields must be integrated out only after the effects of SUSY-breaking on their VEVs are properly taken into account. In the opposite case, an important impact on the low energy physics from the high energy sector could be overlooked. What we observe is that the μ term of the supersymmetric standard model is directly sensitive to the SUSY-breaking-induced shifts of the heavy VEVs. It is therefore meaningless to talk about the μ problem, at the least, without knowing the details of UV theory.

In grand unification, the Higgs doublets are unified with color-triplet partners in the same multiplet and share quantum numbers with them. Then, the mandatory doublet-triplet splitting usually strips the μ term of all unbroken quantum numbers that could potentially forbid its generation. As a result, the SUSY breaking generically induces the μ term by the dynamical shift mechanism discussed in this paper.

In the present paper we have focused on $N = 1$ supergravity. We see no *a priori* reason for the shift mechanism discussed here not to be operative in its UV completions, such as string theory. However, such analysis is beyond the scope of the present work.

Part of the content of this paper has been presented in the lectures on supersymmetry delivered at NYU and were shared with David Pirtskhalava. We thank him for discussions and involvement at the preliminary stage. We are grateful to Goran Senjanovic for valuable discussions and comments.

ACKNOWLEDGMENTS

This work was supported in part by the Humboldt Foundation under Humboldt Professorship Award, by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany's Excellence Strategy, Grant No. EXC-2111–390814868, and Germany's Excellence Strategy under Excellence Cluster Origins.

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