

# Probing nonunitary neutrino mixing via long-baseline neutrino oscillation experiments based at J-PARC

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This paper investigates the capability of long-baseline experiments, which are making use of neutrinos that are coming from Japan Proton Accelerator Research Complex, in establishing the unitarity of active-neutrino mixing by ruling out the nonunitary mixing scheme as a function of true values of  $CP$ -violating phase  $\delta_{CP}$ . It is found that T2HK can establish the unitarity of active neutrino mixing at above  $2\sigma$  C.L. irrespective of neutrino mass hierarchy and the true value of  $\delta_{CP}$ , if the nonunitary (NU) parameter  $\alpha_{21}$  is of the order of  $10^{-2}$ . Further, this paper also discusses the bound on the NU parameter in the 21 sector and the sensitivity limit of these experiments in determining the NU parameter. It is found that the bounds on  $(\alpha_{21}/2)$  are 0.028, 0.0026, 0.005 at  $2\sigma$  C.L., respectively, for T2K, T2HK, and T2HKK. Moreover, it is also found that the sensitivity limit of T2HK on NU parameter is far better than that of both T2HKK and T2K.

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## I. INTRODUCTION

Ever since the phenomenon of flavor transition of neutrino [1–7] has become one of the center attentions in particle physics, long-baseline (LBL) experiments have played a significant role to understand the hidden nature of the fundamental particle neutrino. With the confirmation of a nonzero reactor mixing angle by both accelerator and reactor neutrino experiments, the three flavor neutrino oscillation paradigm that is governed by two mass squared differences ( $\Delta m_{21}^2$  and  $\Delta m_{31}^2$ ) has become the most accepted theoretical model for the neutrino flavor transition. The unitary mixing of active neutrinos ( $\nu_e$ ,  $\nu_\mu$ , and  $\nu_\tau$ ) in this model is described by three mixing angles ( $\theta_{12}$ ,  $\theta_{13}$ ,  $\theta_{23}$ ) and one phase  $\delta_{CP}$ . Though the oscillation parameters in this paradigm are determined with an unprecedented accuracy, the information about  $CP$ -violating phase, hierarchy of neutrino masses, and octant of  $\theta_{23}$  (i.e., whether  $\theta_{23}$  is greater or lesser than  $45^\circ$ ) are not known. The determination of these unknowns by currently running LBL experiments (T2K and NO $\nu$ A) is quite challenging as there exists degeneracies among the oscillation parameters, and the data collected so far are not sufficient enough to resolve the degeneracies among the parameters. The phase II runs of current generation experiments along with future generation LBL experiments with greater energy resolution,

improved statistics, and magnificent matter effect like the Deep Underground Neutrino Experiment (DUNE), Tokai to Hyper-Kamioka (T2HK), Tokai to Hyper-Kamioka to Korea (T2HKK), and European Spallation Source Neutrino Super Beam (ESS $\nu$ SB) etc., are expected to shed light on remaining unknowns in the neutrino sector.

The discovery of neutrino oscillation not only opens up a way to probe the properties of neutrinos, but it also motivates us to explore physics beyond the Standard Model (SM) as it clearly indicates that the neutrino has a nonzero tiny mass, which is one of the shortcomings of the SM. Consequently, many viable mechanisms collectively known as seesaw mechanisms [8–17] are introduced in the literature to explain the lightness of neutrino mass. However, the models based on low-scale seesaw [18] are more captivating over the high-scale seesaw as these models require strong evidence to support them and within the experimental limit it is difficult to probe high-scale seesaw even using Large Hadron Collider experiments. Whereas in low-scale seesaw models for instance, inverse seesaw [19–21], linear seesaw etc., contain new neutrino states, which do not have any SM interactions, the so-called sterile neutrinos with a mass of the order of a GeV/TeV scale as the seesaw breaking scale in these models can be brought down to TeV/GeV scale. Therefore, signatures of such sterile neutrinos can be probed at LHC experiments. Moreover, existence of such sterile neutrinos and their mixing with active neutrinos leads to the nonunitary mixing of active neutrinos. As a result, one can also probe them at long-baseline neutrino experiments by looking at the deviation from the unitary mixing of active neutrinos.

Enormous studies regarding the nonunitary neutrino mixing have been already discussed in the literature in

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both phenomenological and theoretical point of views [22–32]. In [33], it has been shown that nonunitary effects originated from a minimal inverse seesaw model can be probed at neutrino factory experiment. The bounds on nonunitary mixing parameters are obtained in [34,35]. Moreover, the impact of nonunitarity mixing on the determination of various unknowns in neutrino sector such as neutrino mass hierarchy, octant of atmospheric mixing angle, and  $CP$ -violating phase by long-baseline experiments are discussed in [36–39]. A combined analysis of short and long-baseline neutrino oscillation data in nonunitary mixing scenario has been explored in [40], and it is found that there is no significant deviation from unitary mixing. The results of a combined analysis in neutrino oscillations without unitarity assumption in the three flavor mixing is presented in [41]. In [42], it is found that with the next-generation experimental data, the normalizations of all rows and columns of the lepton mixing matrix will be constrained to  $\leq 10\%$  precision, with the  $e$  row best measured at  $\leq 1\%$  and the  $\tau$  row worst measured at  $\approx 10\%$  precision. A recent study on nonunitary mixing using current generation experiments showed that the stronger tension that exists between the latest 2020 data of the T2K and NO $\nu$ A experiments gets reduced with nonunitary analysis [43]. Another study that obtained the constraints for nonunitarity coming from the observables: the neutrino-antineutrino gamma process and the invisible  $Z$  boson decay into neutrinos is presented in [44]. This paper address three basic questions regarding the nonunitary mixing for the first time, which are

- (i) Whether the long-baseline experiments based at J-PARC are capable of establishing the unitarity of active neutrino mixing matrix by ruling out nonunitary mixing or not?
- (ii) What are the bounds on the nonunitary mixing parameter that can be achieved by these experiments?
- (iii) What is the sensitivity limits of these experiments in determining the nonunitary mixing parameter?

This paper is organized as follows. The neutrino oscillation in presence of nonunitary mixing scheme is discussed in Sec. II. Section III discusses the simulation details of the experiments that are considered for this study. The capability of these experiment in establishing unitarity of active neutrino mixing, the bounds on the NU parameter and the sensitivity limits of these experiment in determining NU parameter are, respectively, discussed in Sec. IV. Finally, the summary and conclusions of this study is given in Sec. V.

## II. NEUTRINO OSCILLATION IN PRESENCE OF NONUNITARITY MIXING

The general form of a unitary neutrino mixing matrix in a model with  $n$  sterile neutrinos can be written as

$$\mathcal{U} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & \cdot & \cdot & U_{e(n+3)} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} & \cdot & \cdot & U_{\mu(n+3)} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} & \cdot & \cdot & U_{\tau(n+3)} \\ U_{s^1 1} & U_{s^1 2} & U_{s^1 3} & \cdot & \cdot & U_{s^1(n+3)} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ U_{s^n 1} & U_{s^n 2} & U_{s^n 3} & \cdot & \cdot & U_{s^n(n+3)} \end{pmatrix} \approx \begin{pmatrix} N_{3 \times 3} & \Theta_{3 \times n} \\ R_{n \times 3} & S_{n \times n} \end{pmatrix} \quad \text{with } \mathcal{U}\mathcal{U}^\dagger = I, \quad (1)$$

where  $N_{3 \times 3}$  is the active neutrino mixing matrix which is no more unitary,  $\Theta_{3 \times n}$  and  $R_{n \times 3}$  are active-sterile neutrino mixing matrices, and  $S_{n \times n}$  is sterile-sterile neutrino mixing matrix. It should be also noted that the submatrix  $\mathcal{W}(=[N \ \Theta])$  of  $\mathcal{U}$  satisfy the unitarity relation

$$\mathcal{W}\mathcal{W}^\dagger = NN^\dagger + \Theta\Theta^\dagger = I. \quad (2)$$

Generally, the nonunitary active neutrino mixing matrix  $N$  is decomposed in two ways:

- (i) In terms of  $\eta(=\frac{1}{2}\Theta^\dagger\Theta)$  parameters

$$N = (1 - \eta)U_{\text{PMNS}}. \quad (3)$$

- (ii) In terms of lower triangular matrix  $T$  with parameter  $\alpha$ ,

$$N = TU = (I - \alpha)U, \quad (4)$$

where  $U$  is the standard neutrino mixing matrix. The explicit form of triangular matrix  $T$  is given by

$$T = \begin{pmatrix} \alpha_{11} & 0 & 0 \\ \alpha_{21} & \alpha_{22} & 0 \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix}, \quad (5)$$

where the diagonal elements of  $T$  are of the form  $(1 - \alpha_{ii}) \rightarrow \alpha_{ii}$ .

The relation between these two parametrizations of nonunitary mixing is derived in [31] and is given by

$$\begin{pmatrix} \eta_{11} & 0 & 0 \\ 2\eta_{12}^* & \eta_{22} & 0 \\ 2\eta_{13}^* & 2\eta_{23}^* & \eta_{33} \end{pmatrix} = \begin{pmatrix} \alpha_{11} & 0 & 0 \\ \alpha_{21} & \alpha_{22} & 0 \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix}. \quad (6)$$

This paper follows the parametrization of  $N$  in terms of triangular matrix as it is the preferred one for oscillation studies. In presence of nonunitary mixing the flavor state of neutrino can be written as

$$|\nu_\alpha\rangle = \sum_i N_{\alpha i} |\nu_i\rangle. \quad (7)$$

As neutrino propagates the mass eigenstate evolves as

$$i \frac{d}{dt} |\nu_i\rangle = \mathcal{H}_0 |\nu_i\rangle, \quad (8)$$

where  $\mathcal{H}_0$  is a Hamiltonian in vacuum, i.e.,

$$\mathcal{H}_0 = \frac{1}{2E} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2 & 0 \\ 0 & 0 & \Delta m_{31}^2 \end{pmatrix}. \quad (9)$$

The nonunitary neutrino oscillation probability in vacuum is given by

$$P(\nu_\mu \rightarrow \nu_e) = \sum_{i,j}^3 N_{\mu i}^* N_{ei} N_{\mu j} N_{ej}^* - 4 \sum_{j>i}^3 \text{Re}[N_{\mu j} N_{ej} N_{\mu i} N_{ei}^*] \sin^2\left(\frac{\Delta m_{ji}^2 L}{4E}\right) + 2 \sum_{j>i}^3 \text{Im}[N_{\mu j}^* N_{ej} N_{\mu i} N_{ei}] \sin\left(\frac{\Delta m_{ji}^2 L}{2E}\right). \quad (10)$$

And the explicit form by neglecting cubic products of  $\alpha_{21}$ ,  $\sin\theta_{13}$ , and  $\Delta m_{21}^2$  gives [32]

$$P(\nu_\mu \rightarrow \nu_e) = (\alpha_{11}\alpha_{22})^2 P_{\mu e}^{SO} + \alpha_{11}^2 \alpha_{22} |\alpha_{21}| P_{\mu e}^I + \alpha_{11}^2 |\alpha_{21}|^2, \quad (11)$$

where  $P_{\mu e}^{SO}$  is the vacuum neutrino oscillation probability in standard three flavor oscillation framework, i.e.,

$$P_{\mu e}^{SO} = \sin^2 2\theta_{12} \cos^2 \theta_{23} \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right) + \sin^2 2\theta_{13} \sin^2 \theta_{23} \sin^2\left(\frac{\Delta m_{31}^2 L}{4E}\right) + \sin 2\theta_{12} \sin 2\theta_{23} \sin \theta_{13} \sin\left(\frac{\Delta m_{21}^2 L}{2E}\right) \times \sin\left(\frac{\Delta m_{31}^2 L}{4E}\right) \cos\left(\frac{\Delta m_{31}^2 L}{4E} - I_{123}\right), \quad (12)$$

and  $P_{\mu e}^I$  is the term which contain the new phase and explicit form is given by

$$P_{\mu e}^I = -2 \left[ \sin 2\theta_{13} \sin \theta_{23} \sin\left(\frac{\Delta m_{31}^2 L}{4E} + I_{NP} - I_{123}\right) \right] - \cos \theta_{13} \cos \theta_{23} \sin 2\theta_{12} \sin\left(\frac{\Delta m_{21}^2 L}{2E}\right) \sin(I_{NP}), \quad (13)$$

with  $I_{123} = -\delta_{CP} = \phi_{12} - \phi_{13} + \phi_{23}$  and  $I_{NP} = \phi_{12} - \text{Arg}(\alpha_{21})$ . The propagation of neutrino through matter is governed by the charged current ( $V_{CC} = \sqrt{2}G_F n_e$ ) and neutral current ( $V_{NC} = -G_F n_n / \sqrt{2}$ ) matter potentials. In presence of nonunitary neutrino mixing, the charged current (CC) and neutral current (NC) interaction Lagrangian becomes [22]

$$-\mathcal{L}_{\text{int}} = V_{CC} \sum_{i,j} N_{ei}^* N_{ej} \bar{\nu}_i \gamma^0 \nu_j + V_{NC} \sum_{\alpha,i,j} N_{\alpha i}^* N_{\alpha j} \bar{\nu}_i \gamma^0 \nu_j, \quad (14)$$

which yields the effective Hamiltonian as

$$\mathcal{H}_m^N = \frac{1}{2E} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2 & 0 \\ 0 & 0 & \Delta m_{31}^2 \end{pmatrix} + N^\dagger \begin{pmatrix} V_{CC} + V_{NC} & 0 & 0 \\ 0 & V_{NC} & 0 \\ 0 & 0 & V_{NC} \end{pmatrix} N. \quad (15)$$

Then the nonunitary oscillation probability after traveling a distance  $L$  yields

$$P_{\alpha\beta}(E, L) = |\langle \nu_\beta | \nu_\alpha(L) \rangle|^2 = |(N e^{-i\mathcal{H}_m^N L} N^\dagger)_{\beta\alpha}|^2. \quad (16)$$

An attempt to obtain the explicit analytical expression for neutrino oscillation probability in presence of nonunitary mixing is done in [45]. Further, an explicit perturbative calculation up to the first order in the  $\nu_\mu \rightarrow \nu_e$  oscillation channel has been done in [45,46]. However, in this paper numerical calculations is done by using the General Long Baseline Experiment Simulator (GLoBES) [47,48] package along with the plugin MonteCUBES [49]. The neutrino oscillation parameters that are considered in this analysis are given in the Table I. From the previous analysis on nonunitary parameters [50,51], it is identified that the parameters in the 21 sector plays major role in  $\nu_\mu$  to  $\nu_e$  oscillation channel. Therefore, now onwards, the discussion is focused on nonunitary parameters in the 21 sector, i.e.,  $\alpha_{21}$  and its corresponding new phase  $\phi_{21}$ . Though the

TABLE I. The values of neutrino oscillation parameters used in the analysis [52].

Parameters	$\sin^2 \theta_{12}$	$\sin^2 2\theta_{13}$	$\sin^2 \theta_{23}$	$\Delta m_{21}^2$	$\Delta m_{\text{atm}}^2$ NH (IH)	$\delta_{CP}$
Best fit	0.307	0.085	0.5	$7.4 \times 10^{-5} \text{ eV}^2$	$2.5(-2.4) \times 10^{-3} \text{ eV}^2$	$-90^\circ$

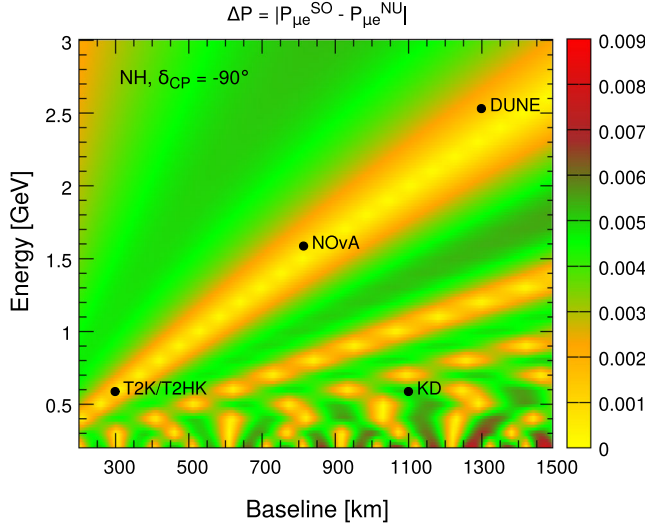


FIG. 1.  $\Delta P_{\mu e}$  in  $L$ - $E$  plane for the nonunitarity parameter  $\alpha_{21} = 0.01$ . The mass hierarchy of neutrino is assumed to be normal and other oscillation parameters are taken as given in Table I.

phase associated with the complex nonunitarity parameter can take values from  $-\pi$  to  $\pi$ , the new phase is assumed to be zero while doing the analysis unless otherwise mentioned.

To quantify the deviation from unitary mixing of neutrinos, one can define  $\Delta P_{\mu e} = |P_{\mu e}^{NU} - P_{\mu e}^{SO}|$ , where  $P_{\mu e}^{NU}$  and  $P_{\mu e}^{SO}$ , respectively, are the oscillation probability in the nonunitary and unitary mixing schemes. As the LBL experiments are mainly searching for  $\nu_{\mu} \rightarrow \nu_e$  and  $\bar{\nu}_{\mu} \rightarrow \bar{\nu}_e$  oscillation signals, the relative deviation in the  $\nu_{\mu} \rightarrow \nu_e$  oscillation probability due to the nonunitary mixing as a function of all possible neutrino energies and baselines is shown in Fig. 1. The color gradient corresponds to relative deviation in oscillation probability. From the figure, it can be seen that the nonunitary parameters in the 21 sector can be probed at LBL experiments like T2K, NO $\nu$ A, T2HK, T2HKK, and DUNE.

### III. SIMULATION DETAILS

This section briefly describes the experimental features of the LBL experiments at J-PARC: T2K, T2HK, and T2HKK, which are considered in this analysis.

T2K [53,54] is a currently running LBL experiment which has already started collecting data from 2010 onwards, whereas T2HK and T2HKK are the proposed experiments which are considered as the upgraded version of T2K experiment. T2K experiment completed its scheduled run and now it is upgraded to phase II and continuing to take neutrino data. The muon neutrino/antineutrino beam for all these experiments is produced in the J-PARC accelerator facility at Tokai. However, the Water-Cerenkov detectors of these experiments are located at different locations. The detectors of T2K and T2HK experiments are kept at 295 km away from the neutrino

beam source. T2HKK experiment has two detectors: the first detector so-called Japan Detector (JD), which plans to keep 295 km away from the source at Japan, and the second so-called Korean Detector (KD) plans to keep at Korea about 1100 km away from the source. Moreover, the fiducial mass of detectors of each experiments are different. The detector fiducial mass of T2K experiment is 22.5 kt. Initial plan of T2HK experiment is to consider 560 kt fiducial mass for the detector. However, the recent plan of this experiment is to consider 374 kt. Each of the two detectors of the T2HKK experiment (JD and KD) is having a fiducial mass of 187 kt. The detector of T2HK is also known as 2JD as the fiducial mass of T2HK detector is twice that of JD detector of T2HKK. The detector of all these experiments is kept at an off-axis angle  $2.5^\circ$  to the neutrino beam line which helps the neutrino flux to peak sharply at first (second) oscillation maximum of 0.6 GeV for detector, which is kept at Japan (Korea). Further, such off-axis beam nature also reduces the intrinsic  $\nu_e$  contamination in the beam and the background due to neutral current events and thus helps to improve the signal-to-background ratio by a great extent.

A proton beam power of 750 kW and with a proton energy of 30 GeV, which corresponds to a total exposure of  $7.8 \times 10^{21}$  protons on target (POT) with a 1:1 ratio of neutrino to antineutrino modes, is considered to simulate T2K experiment for this study [55]. The signal and background event spectra and rates are matched with those given in the recent publication of the T2K collaboration [56]. An uncorrelated 5% normalization error on signal and 10% normalization error on background for both the appearance and disappearance channels are considered as given in [56] to analyze the prospective data from the T2K experiment, and it is assumed that the set of systematics for both the neutrino and antineutrino channels are uncorrelated.

A total of 10 years of operation with 1.3 MW beam power with 1:3 ratio of neutrino to antineutrino modes which corresponds to  $27 \times 10^{21}$  POT by following [57,58] is considered to simulate both T2HK and T2HKK experiments. Moreover, both signal and background event spectra and rates are matched with those given in [58]. An uncorrelated 5% normalization error on signal and 10% normalization error on background for both the appearance and disappearance channels are used as the way considered for those of the T2K experiment. For the simulations, GLOBES along with MonteCUBES have been used. Further, the Poissonian  $\chi^2$  is evaluated using GLOBES package [59–61] and its explicit form is given by

$$\chi^2 = \min_{\vec{\xi}, \vec{P}} \sum_{i=1}^{N_E} \left[ 2N_i^{\text{th}}(\vec{P}, \xi) - 2N_i^{\text{dat}}(\vec{P}, \xi) - 2N_i^{\text{dat}}(\vec{P}, \xi) \ln \left( \frac{N_i^{\text{th}}(\vec{P}, \xi)}{N_i^{\text{dat}}(\vec{P}, \xi)} \right) \right] + \sum_{l=1}^2 \xi_l^2, \quad (17)$$

where  $N_i^{\text{th}}(\vec{P}, \xi)$  and  $N_i^{\text{dat}}(\vec{P}, \xi)$ , respectively, are the expected and observed events (both signal and background) for a considered  $i$ th energy bin. Further,  $N_i^{\text{th}}(\vec{P}, \xi) = N_i^0(1 + \sum_{l=1}^2 \pi_l^i \xi_l^2)$  with  $N_i^0$  as the number of events without systematics,  $N_E$  is the total number of energy bins,  $\xi_1$  and  $\xi_2$  are the systematic errors associated with signal and background events, respectively, with  $\vec{P} = \{\theta_{12}, \theta_{13}, \theta_{23}, \Delta m_{21}^2, \Delta m_{31}^2, \delta_{CP}\}$  representing all the fundamental oscillation parameters, while  $\vec{p} = \{\theta_{23}, \delta_{CP}, \Delta m_{31}^2\}$  is the subset of  $\vec{P}$  on which we perform marginalization. The marginalization range for  $\delta_{CP}$ ,  $\sin^2 \theta_{23}$ , and  $\Delta m_{31}^2$  are  $[-180^\circ:180^\circ]$ ,  $[0.4:0.6]$ , and  $[0.36:0.64]$ , respectively. Moreover, in this work,  $\Delta\chi^2$  is determined using the pull variable over the systematic uncertainties and a detailed discussion on this is given in [60,61].

#### IV. RESULTS AND DISCUSSIONS

This section mainly discuss the capability of LBL experiments based at J-PARC in establishing unitarity of neutrino mixing matrix. Further, this section also discusses the bounds on NU parameter and sensitivity limits of LBL experiment to determine NU parameter.

As this study is focusing on the nonunitarity parameters  $\alpha_{21}$  and its corresponding  $CP$ -violating phases, it is most important to know how these parameters affect the oscillation probability. Though it is always better to start with analysis by looking at the analytical expression of neutrino oscillation probability to have a deep understanding of physics, the oscillation probability is calculated numerically as a function of neutrino energy for this study, as is shown in Fig. 2. In the figure, the black dashed curve

corresponds to the oscillation probability in standard oscillation case with normal hierarchy and  $\delta_{CP} = -90^\circ$ , whereas the red, green, magenta, and blue solid curves correspond to oscillation probabilities in the presence of NU parameter  $\alpha_{21} = 0.01$  with  $\phi_{21} = -90^\circ, 0^\circ, 90^\circ$ , and  $180^\circ$ , respectively. The left (right) panel is for oscillation probability for neutrino (antineutrino). From the figure, it can be seen that for  $\phi_{21} = 90^\circ, -90^\circ$ , there is a significant deviation from the standard oscillation case, whereas there is no significant deviation for  $\phi_{21} = 0^\circ, 180^\circ$ . Therefore,  $\phi_{21} = 90^\circ, -90^\circ$  are the favorable values of new phase to rule out the nonunitary mixing and  $\phi_{21} = 0^\circ, 180^\circ$  are the unfavorable values as there is no much deviation from standard oscillation case. It should be also noted that the oscillation curves for  $\phi_{21} = 0^\circ, 180^\circ$  are touching the standard oscillation curve at some point where one cannot distinguish between nonunitary mixing and unitary mixing. However, such intersection point is different in neutrino and antineutrino oscillation channels. Therefore, an interplay of neutrino and antineutrino oscillation helps in distinguishing unitary mixing from nonunitary mixing.

Now the focus will be on the unfavorable values of new phase, i.e.,  $\phi = 0^\circ$ , as it is challenging for this value to distinguish nonunitary mixing from unitary mixing. In order to show the capability of a LBL experiment to establish unitary mixing as a function of true values of  $\delta_{CP}$ , one can define

$$\Delta\chi_{NU}^2 = \chi_{SO}^2 - \chi_{NU}^2, \quad (18)$$

where  $\chi_{SO}^2$  is evaluated using Eq. (17) by assuming both  $N^{\text{th}}$  and  $N^{\text{dat}}$  are with unitary mixing. Whereas,  $\chi_{NU}^2$  is calculated by assuming  $N^{\text{th}}$  with nonunitary mixing and

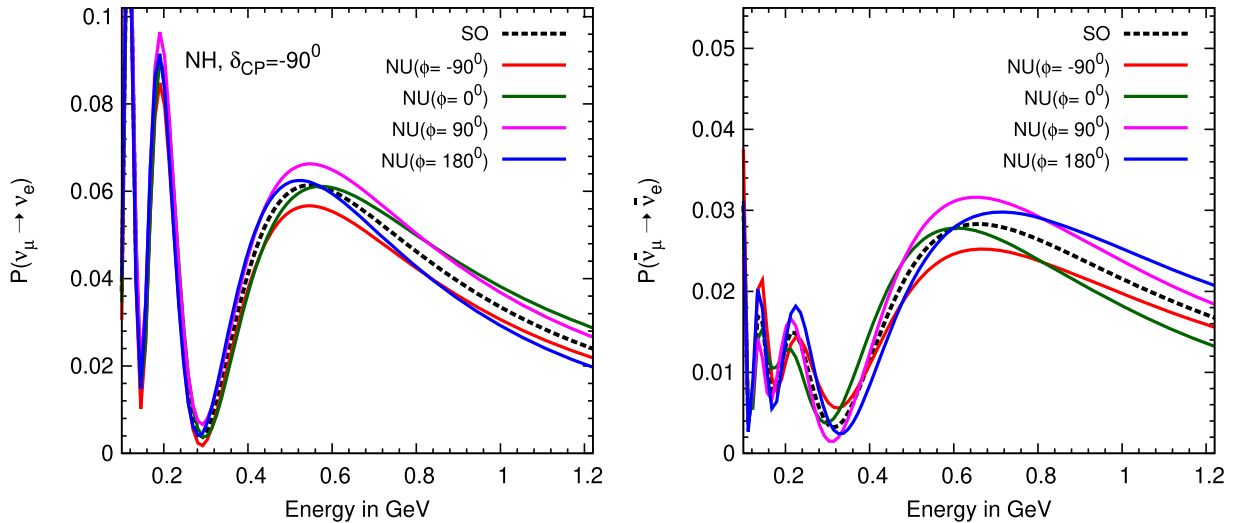


FIG. 2. The neutrino (antineutrino) oscillation probability as a function of neutrino energy for T2K/T2HK in the left (right) panel. The black dashed curve corresponds to the oscillation probability in standard oscillation paradigm with  $\delta_{CP} = -90^\circ$ , whereas the red, green, magenta, and blue solid curves correspond to oscillation probabilities in the presence of NU parameter  $\alpha_{21} = 0.01$  with  $\phi_{21} = -90^\circ, 0^\circ, 90^\circ$ , and  $180^\circ$ , respectively. Neutrino mass hierarchy is assumed to be normal. The left (right) panel is for oscillation probability for neutrino (antineutrino).

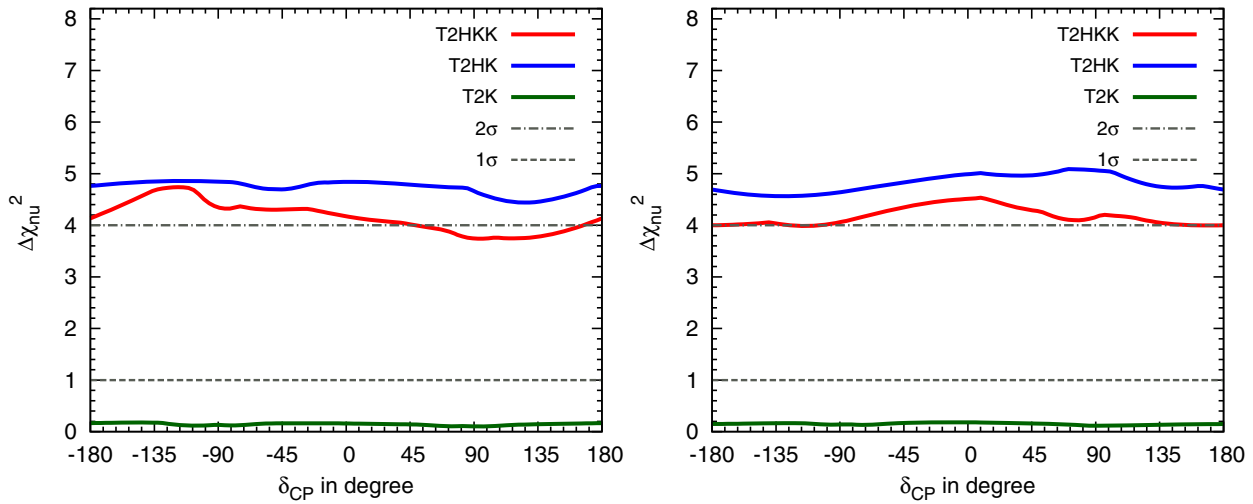


FIG. 3. The capability of LBL experiments in establishing the unitary mixing by ruling out the nonunitary mixing as a function of true values of  $\delta_{CP}$ . In the left (right) panel the mass hierarchy of neutrino is assumed to be normal (inverted).

$N^{\text{dat}}$  with unitary mixing. The minimum value for  $\Delta\chi_{NU}^2$  is obtained by doing marginalization over oscillation parameters including the new phase  $\phi_{21}$  in its allowed range  $[-180^\circ:180^\circ]$ . The minimized  $\Delta\chi_{NU}^2$  as a function of true

values of  $\delta_{CP}$  is shown in Fig. 3. In the left (right) panel of the figure, hierarchy is assumed to be normal (inverted) and the atmospheric mixing angle is set to maximal mixing. From the figure, it can be seen that the T2K experiment

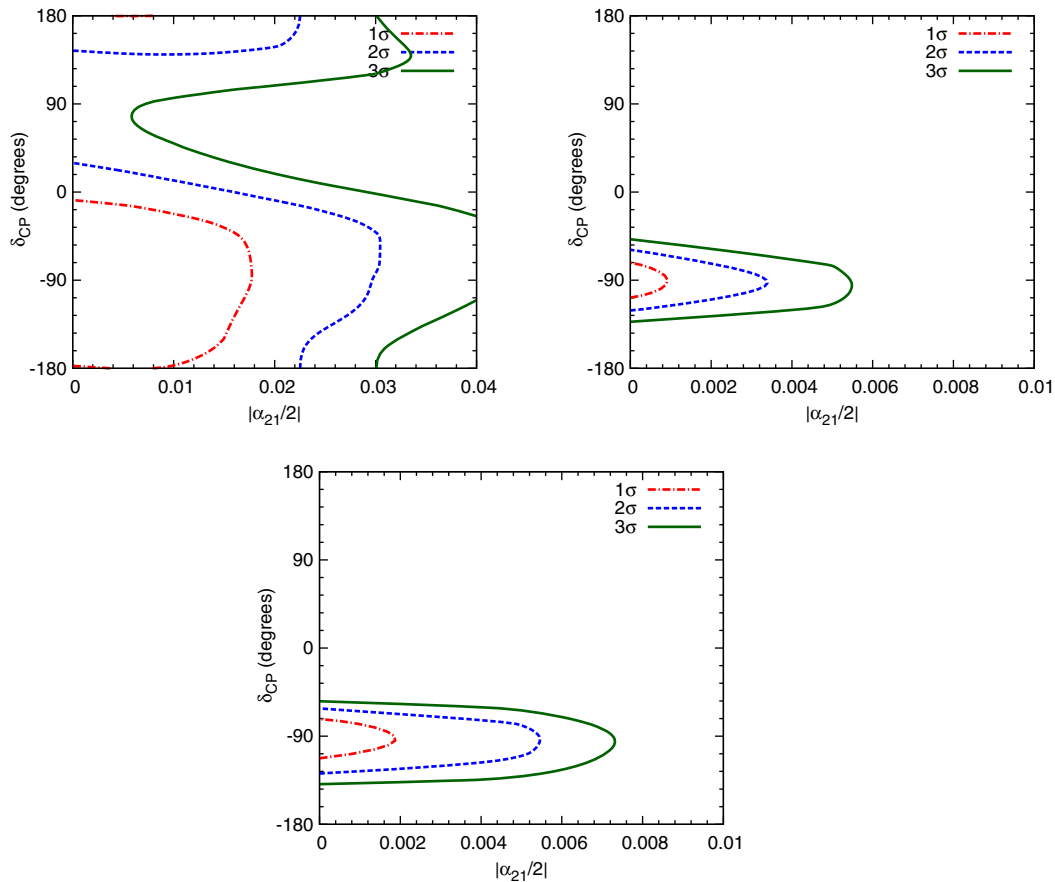


FIG. 4. The bounds on the NU parameter is obtained by comparing unitary mixing against nonunitary mixing. The upper left (right) panel is for T2K (T2HK) and lower panel is for T2HKK. The red, blue, and green curves are, respectively, for  $1\sigma$ ,  $2\sigma$ , and  $3\sigma$  C.L. contours. The neutrino mass hierarchy is assumed to be normal and other oscillation parameters are used as given in Table I.

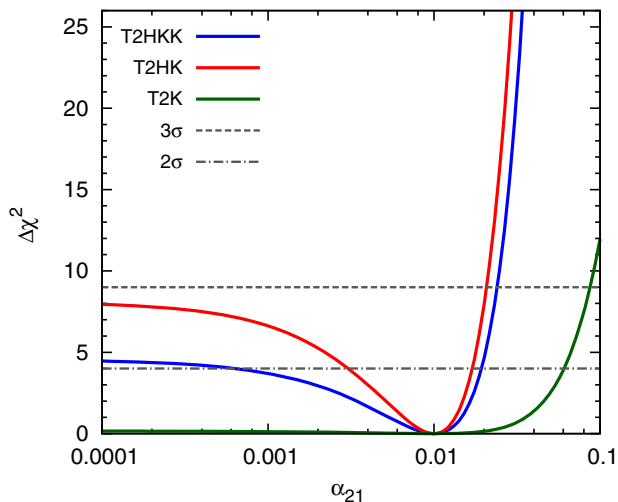


FIG. 5. The precision measurement of  $\alpha_{21}$  at LBL experiments. The mass hierarchy of neutrino is assumed to be normal and other oscillation parameters are set as given in Table I.

cannot rule out the nonunitary mixing with the data so far collected. Whereas T2HKK can rule out nonunitary mixing with a significance of more than  $2\sigma$  for most of the values of  $\delta_{CP}$ . Moreover, T2HK can rule out nonunitary mixing above  $2\sigma$  C.L. irrespective of mass hierarchy and the true value of  $\delta_{CP}$ . From the analysis, it is found that the interplay between neutrino and antineutrino runs helps in ruling out the nonunitary mixing.

The bounds on NU parameters using LBL experiment is obtained by comparing unitary mixing with oscillation parameters as shown in Table I against nonunitary mixing. The minimized  $\Delta\chi^2$  is evaluated by doing marginalization over oscillation parameters and it is shown in the  $\delta_{CP}^{\text{test}} - (\alpha_{21}/2)^{\text{test}}$  plane as given in Fig. 4. The red, blue, and green lines are, respectively,  $1\sigma$ ,  $2\sigma$ , and  $3\sigma$  C.L. contours. From the figure, it can be seen that the bounds from T2K experiment on NU parameter is not significantly constraint, whereas that for T2HK and T2HKK is severely constraint. Further, the bounds on  $\alpha_{21}/2$  are 0.028, 0.0026, 0.005 at  $2\sigma$  C.L., respectively, for T2K, T2HK, and T2HKK.

Next, analyzing the sensitivity limit of LBL experiments in determining NU parameters. In order to obtain this, the

experimental data is simulated by fixing true oscillation parameters as given in Table I and setting true value of  $\alpha_{21} = 0.01$ , then comparing it with theory by varying  $\alpha_{21}$  in the range [0.001:0.1]. Further, the  $\Delta\chi^2$  is minimized by doing marginalization over oscillation parameters and nonunitary phase and it is given in Fig. 5. From the figure, it can be seen that the precision in the measurement of  $\alpha_{21}$  by T2HK is better than that of both T2K and T2HKK.

## V. SUMMARY AND CONCLUSIONS

The measurement of neutrino oscillation parameters in a three flavor framework is usually done by assuming that the active neutrino mixing matrix is unitary. However, the extended theories to accommodate massive neutrinos indicate the existence of new neutrino states that can give rise to the nonunitary mixing of active neutrinos. This paper mainly scrutinized whether the long baseline experiments like T2K, T2HK, and T2HKK can establish the unitarity of active neutrino matrix by ruling out such nonunitary mixing in the 21 sector. It is found that T2HK can establish unitarity of active neutrino mixing above  $2\sigma$  C.L. irrespective of neutrino mass hierarchy and true value of  $\delta_{CP}$  if NU parameter  $\alpha_{21}$  of the order of  $10^{-2}$ . Further, this paper also investigated the bound on NU parameters that can be achieved from these LBL experiments and found that the bounds on  $\alpha_{21}/2$  are 0.028, 0.0026, 0.005 at  $2\sigma$  C.L., respectively, for T2K, T2HK, and T2HKK. Finally, it is also found that the sensitivity limit of T2HK on NU parameter is far better than that of both T2HKK and T2K.

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