

# Theoretical analysis of the leptonic decays $B \rightarrow \ell\ell\ell'\bar{\nu}_{\ell'}$

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We discuss the amplitude of the  $B \rightarrow l^+l'^-\nu'$  decays and the differential decay rate  $d^2\Gamma/dq^2dq'^2$ ,  $q$  the momentum of the  $l^+l^-$  pair emitted from the electromagnetic vertex and  $q'$  the momentum of the  $l'\nu'$  pair emitted from the weak vertex. For the relevant form factors, we construct dispersion representations in  $q^2$ , which consistently take into account the Ward identity constraints at  $q^2 = 0$  and the contributions of light vector resonances. This allows a consistent description of the form factors in the range  $0 < q^2 \leq 1 \text{ GeV}^2$  that saturates around 99% of the decay rate. The differential decay rate behaves at small  $q^2$  as  $d\Gamma(B \rightarrow l^+l'^-\nu')/dq^2 \propto 1/q^2$  in the limit  $m_l' = 0$  but contains also more singular contribution of order  $m_l'^2/q^4$ , which we take into account. For the case  $m_l' \leq m_l$ , the latter may be neglected, and one obtains a mild logarithmic dependence of  $\Gamma(B \rightarrow l^+l'^-\nu')$  on  $m_l$ . For the case  $m_l \ll m_l'$ , however, the  $m_l'^2/q^4$  terms dominate the decay rate, leading to  $\Gamma(B \rightarrow l^+l'^-\nu') \sim m_l'^2/m_l^2$ . We find the following features of the four-lepton  $B$  decays: i) The decay rates  $\Gamma(B \rightarrow \mu^+\mu^-(\mu\nu_\mu, e\nu_e))$  are fully dominated by the region of light vector resonances  $q^2 \simeq M_\rho^2, M_\omega^2$ . ii) The decay rate  $\Gamma(B \rightarrow e^+e^-e\nu_e)$  receives comparable contributions from the region near  $q^2 \sim 4m_e^2$  and from the resonance region. iii) One finds a strong enhancement of the decay rate  $\Gamma(B \rightarrow e^+e^-\mu\nu_\mu) \sim m_\mu^2/m_e^2$ , which is dominated by the region  $q^2 \sim 4m_e^2$  due to the terms  $O(m_\mu^2/q^4)$  in the differential distribution.

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## I. INTRODUCTION

In this paper, we revisit the amplitude  $B \rightarrow \gamma^*l'\nu'$ : we discuss constraints imposed by gauge invariance, construct dispersion representations for the corresponding form factors, and obtain predictions for the differential distributions in the  $B$ -meson decays into four leptons in the final state,  $B \rightarrow l^+l'^-\nu'$ . The latter reactions are being studied experimentally [1–4], thus requiring a proper theoretical understanding of the  $B$ -meson form factors in two currents. By now, there have been a few theoretical papers [5–8] in which  $B$  decays into two lepton pairs have been studied.

The  $B \rightarrow \gamma^*l'\nu'$  amplitude (see Fig. 1) may be parametrized via Lorenz-invariant form factors as

$$\begin{aligned} T_{av}(q, q'|p) &= i \int dx e^{iqx} \langle 0 | T \{ j_\alpha^{\text{e.m.}}(x), \bar{u}(0) \mathcal{O}_\nu b(0) \} | \bar{B}_u(p) \rangle \\ &= \sum_i L_{av}^{(i)}(q, q') F_i(q^2, q'^2) + \dots, p = q + q', \end{aligned} \quad (1.1)$$

with  $q'$  the momentum of the weak  $b \rightarrow u$  current and  $q$  the momentum of the electromagnetic current. In Eq. (1.1),  $\mathcal{O}_\nu = \gamma_\nu, \gamma_\nu \gamma_5$ , and  $j_\alpha^{\text{e.m.}}$  is the conserved electromagnetic current

$$j_\alpha^{\text{e.m.}}(0) = eQ_b \bar{b}(0) \gamma_\alpha b(0) + eQ_u \bar{u}(0) \gamma_\alpha u(0). \quad (1.2)$$

The quantities  $L_{av}^{(i)}(q, q')$  represent the transverse Lorentz structures,  $q^\alpha L_{av}^{(i)}(q, q') = 0$ , and the dots stand for the

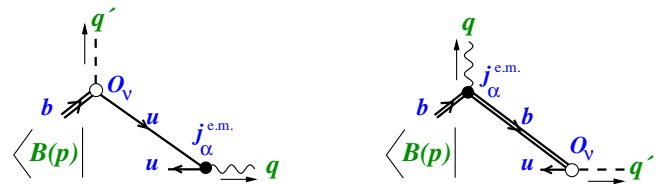


FIG. 1. Feynman diagrams describing the amplitude (1.1).

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longitudinal part, which is constrained by the conservation of the electromagnetic current,  $\partial_a J_a^{\text{em}} = 0$ , and the equal-time commutation relations.

The form factors  $F_i(q'^2, q^2)$  are complicated functions of two variables,  $q'^2$  and  $q^2$ ; the general properties of these objects in QCD have been studied recently in Ref. [9]. It is noteworthy that gauge invariance provides essential constraints on some of the form factors describing the transition of the  $B$  meson into the real photon, i.e., at  $q^2 = 0$  [10–13].

In the past, theoretical analyses focused on a family of similar reactions, namely, the  $B \rightarrow \gamma l^+ l^-$  and  $B \rightarrow \gamma l \nu$  decays (see, e.g., Refs. [14–24]); these processes are described by the same form factors as four-lepton  $B$  decays, but evaluated at a zero value of one of the momenta squared. The corresponding form factors depend on one variable  $q'^2$ ,  $q'$  the momentum of the weak current; for instance, for radiative leptonic decays  $B \rightarrow \gamma l' \nu'$ , one needs the form factors  $F_i(q'^2, q^2 = 0)$ .

The four-lepton decay of interest,  $B \rightarrow l^+ l^- l' \nu'$ , requires the form factors  $F_i(q'^2, q^2)$  for  $0 < q'^2, q^2 < M_B^2$ . The dependence of the form factors on the variable  $q'^2$  can be predicted reasonably well: there are no hadron resonances in the full decay region  $0 < q'^2 < M_B^2$ , and the  $q'^2$  dependence of the form factors is determined to a large extent by the influence of the beauty mesons with the appropriate quantum numbers; all these mesons are heavier than the  $B$  meson and therefore lie beyond the physical decay region of the variable  $q'^2$ . The calculation of the  $q'^2$  dependence of the form factors is a much more difficult task; light vector mesons  $V = \rho^0, \omega, \dots$  lie in the physical decay region and should be properly taken into account. At  $q'^2$  in the region of light vector meson resonances, the form factors cannot be obtained directly in perturbative Quantum Chromodynamics [9]. Here, considerations based on the explicit account of these light vector resonances—including their finite width effects—are mandatory; the resonance contributions of interest may be unambiguously expressed via the weak  $B \rightarrow V$  form factors. Then, at  $q^2 = 0$ , gauge invariance constrains the values of the form factors. These features allow us to calculate the form factors  $F_i(q'^2, q^2)$  in the region  $0 < q^2 \leq 1\text{--}2 \text{ GeV}^2$ , which dominates the four-meson decay rates, and to obtain consistent predictions for the latter.

Let us now turn to the differential distributions. After summing over the polarizations of the final leptons, the square of the amplitude of the  $B \rightarrow l^+ l^- l' \nu'$  decay may be written in the form

$$|A|^2 = |A|_0^2 + |A|_{m_l^2}^2 + \dots, \quad (1.3)$$

where  $|A|_0^2$  corresponds to the massless leptons,  $m_l = m_l' = 0$ ,  $|A|_{m_l^2}^2$  is the term proportional to  $m_l^2$  which provides the most singular behavior of the amplitude, and the dots stand for those terms which yield negligible

contributions to the differential and to the integrated decay rate compared to  $|A|_0^2$  and may be safely omitted. Among the terms given by the dots in Eq. (1.3), one finds also the terms  $O(m_l^2/q^4)$  [ $q^4 \equiv (q^2)^2$ ], but the contribution of the latter both to the differential and to the integrated decay rates may be neglected.

Because of the gauge-invariance constraints on the form factors, one finds

$$|A|_0^2 \propto 1/q^2. \quad (1.4)$$

This property was already emphasized in Ref. [8], in which it is pointed out that the naive behavior  $1/q^4$ , reported earlier in Ref. [6], is unphysical. Nevertheless, we find it useful to present here an explicit derivation of the constraints on the amplitude imposed by gauge invariance. The term  $|A|_0^2$  yields the contribution to the integrated decay rate  $\Gamma(B \rightarrow l^+ l^- l' \nu')$  that has a mild logarithmic dependence  $\propto \log(m_l^2)$ .

The term  $|A|_{m_l^2}^2$ , for which we derive an explicit expression, is proportional to  $m_l^2$  but has a more singular behavior at  $q^2 \rightarrow 0$  compared to  $|A|_0^2$ :

$$|A|_{m_l^2}^2 \propto m_l^2/q^4. \quad (1.5)$$

The  $|A|_{m_l^2}^2$  contribution to the differential decay rate is negligible compared to the contribution of  $|A|_0^2$  in the full kinematical region of  $B$  decay and may be safely omitted except for one case: if  $m_l \ll m_l'$ , the contribution of  $|A|_{m_l^2}^2$  dominates over  $|A|_0^2$  in the vicinity of the end point  $q^2 = 4m_l^2$ . Moreover, in this case,  $|A|_{m_l^2}^2$  gives the dominant contribution to the decay rate  $\Gamma(B \rightarrow l^+ l^- l' \nu') \propto m_l^2/m_l'^2$ .

We shall demonstrate that these essential qualitative features of the  $q^2$  distribution at small  $q^2$  yield important consequences for the theoretical estimates of the  $B \rightarrow l^+ l^- l' \nu'$  branching fractions:

- (i) The branching fraction  $\text{Br}(B \rightarrow \mu^+ \mu^- (\mu \nu_\mu, e \nu_e))$  is dominated by the region of  $q^2$  around the light vector resonances whereas the region of small  $q^2$  yields a much smaller contribution.
- (ii) The branching fraction  $\text{Br}(B \rightarrow e^+ e^- e \nu)$  receives comparable contributions from the resonance region and the end point region near  $q^2 = 4m_e^2$ .
- (iii) The branching fraction  $\text{Br}(B \rightarrow e^+ e^- \mu \nu_\mu)$  is fully dominated by the end point region  $q^2 = 4m_e^2$ .

It is noteworthy that in all these cases the region  $q^2 > 1 \text{ GeV}^2$  contributes less than 1% of the decay rate.

## II. CONSTRAINTS ON THE TRANSITION FORM FACTORS

We now discuss the requirements imposed by the electromagnetic gauge invariance on the  $B \rightarrow \gamma^*$  transition

amplitudes  $\langle \gamma^*(q) | \bar{u} \{ \gamma_\nu, \gamma_\nu \gamma_5 \} b | \bar{B}_u(p) \rangle$  induced by the vector and the axial-vector charged currents.<sup>1</sup> The corresponding form factors are functions of two variables,  $q^2$  and  $q'^2$ , where  $q'$  is the momentum of the weak  $b \rightarrow u$  current and  $q$  is the momentum of the electromagnetic current, and  $p = q + q'$ . Gauge invariance provides constraints on some of the form factors describing the transition of  $B_u$  to the real photon,  $q^2 = 0$ .

### A. Form factors of the vector weak current

In case of the vector charged quark current  $\bar{u} \gamma_\nu b$ , the gauge-invariant amplitude contains one Lorentz structure and one dimensionless form factor  $F_V(q'^2, q^2)$ :

$$\begin{aligned} T_{\alpha\nu} &= i \int dx e^{iqx} \langle 0 | T \{ j_\alpha^{e.m.}(x), \bar{u} \gamma_\nu b(0) \} | \bar{B}_u(p) \rangle \\ &= e \epsilon_{\nu\alpha q'q} \frac{F_V(q'^2, q^2)}{M_B}. \end{aligned} \quad (2.1)$$

The amplitude is transverse,  $q^\alpha T_{\alpha\nu} = 0$ , and contains no contact term. It is free of the kinematical singularities, so gauge invariance provides no constraints on  $F_V(q'^2, q^2 = 0)$ . The contribution of the vector charged quark current to the amplitude of the  $B \rightarrow \gamma^* l' \nu'$  decay reads

$$\begin{aligned} A_{\text{vector}}(B \rightarrow \gamma^* l' \nu') \\ = e \frac{G_F}{\sqrt{2}} V_{ub} \bar{l}' \gamma_\nu (1 - \gamma_5) \nu' \epsilon_\alpha^*(q) \epsilon_{\nu\alpha q'q} \frac{F_V(q'^2, q^2)}{M_B}. \end{aligned} \quad (2.2)$$

### B. Form factors of the axial-vector weak current

For the axial-vector current,  $\bar{u} \gamma_\nu \gamma_5 b$ , the corresponding amplitude is more complicated; it contains three independent gauge-invariant structures and three form factors,  $f_{1A}(q'^2, q^2)$ ,  $f_{2A}(q'^2, q^2)$ , and  $f_{3A}(q'^2, q^2)$ , and, in addition, it has the contact term which is fully determined by the conservation of the electromagnetic current,  $\partial^\alpha j_\alpha^{e.m.} = 0$ :

$$\begin{aligned} T_{\alpha\nu}^5 &= i \int dx e^{iqx} \langle 0 | T \{ j_\alpha^{e.m.}(x), \bar{u} \gamma_\nu \gamma_5 b(0) \} | \bar{B}_u(p) \rangle \\ &= ie \left( g_{\alpha\nu} - \frac{q_\alpha q_\nu}{q^2} \right) q' q f_{1A} + ie \left( q'_\alpha - \frac{q' q}{q^2} q_\alpha \right) \\ &\quad \times \{ p_\nu f_{2A} + q_\nu f_{3A} \} + ie Q_B f_B \frac{q_\alpha p_\nu}{q^2}. \end{aligned} \quad (2.3)$$

Here,  $Q_B \equiv Q_{\bar{B}_u} = Q_b - Q_u$  is the electric charge of the  $\bar{B}_u$  meson, and  $f_B > 0$  is defined according to

<sup>1</sup>Appendix A provides the relations between the amplitudes containing  $B_q$  and  $\bar{B}_q$  mesons.

$$\langle 0 | \bar{u} \gamma_\nu \gamma_5 b | \bar{B}_u(p) \rangle = i f_B p_\nu. \quad (2.4)$$

The last term in (2.3) is just the longitudinal contact term mentioned above. Let us briefly recall the standard way this term is obtained (see Ref. [25] for details): we calculate  $q_\alpha T_{\alpha\nu}^5$ , represent  $q_\alpha e^{iqx} = -i \frac{\partial}{\partial x^\alpha} e^{iqx}$ , and perform parts integration, moving the derivative to the  $T$  product. Making use of the conservation of the electromagnetic current  $\partial^\alpha j_\alpha^{e.m.} = 0$ , the only nonzero contribution comes from the differentiation of the  $\theta$  functions defining the  $T$  product, leading to the equal-time commutator. In the end, we obtain [ $\hat{Q}$  is the time-independent electric charge operator,  $\hat{Q} = \int d^3x j_0(x^0, \vec{x})$ ]

$$q_\alpha T_{\alpha\nu}^5 = -\langle 0 | [\hat{Q}, \bar{u} \gamma_\nu \gamma_5 b(0)] | B(p) \rangle = i Q_B f_B p_\nu. \quad (2.5)$$

This relation does not determine the longitudinal Lorentz structure in the unique way; one can, e.g., choose this structure in the form  $p_\alpha p_\nu / pq$  [10,11] or in the form  $q_\alpha p_\nu / q^2$  [13]. However, only the latter form, which is implemented in Eq. (2.3), corresponds to the longitudinal part in the form of a contact term.<sup>2</sup> Obviously, different choices of the longitudinal part lead to redefinitions of the form factors  $f_{iA}$  in the transverse part of the amplitude [27]. The choice of the longitudinal structure in the form of a contact term  $q_\alpha p_\nu / q^2$  is suggested by the structure of the quark electromagnetic vertex and is preferable with respect to the analytic properties of the form factors  $f_{iA}$  [11].

The projectors in Eq. (2.3) contain kinematical singularities at  $q^2 = 0$ . These singularities, however, should not be the singularities of the physical amplitude, as the spectrum of physical states does not contain a massless vector particle in the  $q^2$  channel; recall that the absence of massless vector mesons is a fundamental feature of QCD. Therefore, as the consequence of gauge invariance and the property of the spectrum of hadrons in QCD, we obtain the following relations between the form factors at  $q^2 = 0$ :

$$[f_{1A}(q'^2, q^2) + f_{3A}(q'^2, q^2)]_{q^2=0} = 0, \quad (2.6)$$

$$[q' q f_{2A}(q'^2, q^2)]_{q^2=0} = Q_B f_B. \quad (2.7)$$

To implement these constraints at  $q^2 = 0$ , we write down dispersion representations for the form factors  $f_{1A}$ ,  $f_{2A}$ , and  $f_{3A}$  in the variable  $q^2$  with one subtraction and determine the subtraction terms to satisfy (2.6) and (2.7). Such representations have the following form:

<sup>2</sup>By definition, a contact term is a quantity represented by a  $\delta$  function and its derivatives in configuration space; therefore,  $q_\alpha p_\nu / q^2$  is a contact term, whereas  $p_\alpha p_\nu / pq$  is not a contact term according to the standard definition. For further details, we refer to Ref. [26].

$$f_{1A}(q'^2, q^2) = \xi(q'^2) + q^2 \int \frac{ds}{\pi s(s - q^2)} \rho_{1A}(q'^2, s), \quad (2.8)$$

$$f_{2A}(q'^2, q^2) = \frac{2Q_B f_B}{M_B^2 - q'^2} + q^2 \int \frac{ds}{\pi s(s - q^2)} \rho_{2A}(q'^2, s), \quad (2.9)$$

$$f_{3A}(q'^2, q^2) = -\xi(q'^2) + q^2 \int \frac{ds}{\pi s(s - q^2)} \rho_{3A}(q'^2, s). \quad (2.10)$$

The form factor  $\xi(q'^2)$  is related to the form factor of the  $B \rightarrow \gamma l' \nu'$  transition, and for the spectral densities  $\rho_{Ai}(q'^2, s)$ , we will construct phenomenological

expressions based on the contributions of the light vector resonances  $\rho_0$  and  $\omega$ .

Next, we should add the bremsstrahlung contribution (i.e., the photon emitted from the lepton  $l'$  in the final state) that in the limit of a massless lepton  $m_{l'} = 0$  reads

$$\begin{aligned} A_{\text{Brems}}(B \rightarrow \gamma^* l' \nu') \\ = ie Q_l \frac{G_F}{\sqrt{2}} V_{ub} \bar{l}' \gamma_\nu (1 - \gamma_5) \nu' \epsilon_\alpha^*(q) f_B(-g_{\alpha\nu}), \quad Q_l = Q_B. \end{aligned} \quad (2.11)$$

The axial part of weak-transition amplitude  $B \rightarrow \gamma^* l' \nu'$  then takes the form

$$\begin{aligned} A_{\text{axial}}(B \rightarrow \gamma^* l' \nu') = ie \frac{G_F}{\sqrt{2}} V_{ub} \bar{l}' \gamma_\nu (1 - \gamma_5) \nu' \epsilon_\alpha^*(q) \left\{ \left( g_{\alpha\nu} - \frac{q_\alpha q_\nu}{q^2} \right) q' q f_{1A} + \left( q'_\alpha - \frac{q' q}{q^2} q_\alpha \right) [p_\nu f_{2A} + q_\nu f_{3A}] + Q_B f_B \frac{q_\alpha p_\nu}{q^2} \right\} \\ + ie Q_B \frac{G_F}{\sqrt{2}} V_{ub} \bar{l}' \gamma_\nu (1 - \gamma_5) \nu' \epsilon_\alpha^*(q) f_B(-g_{\alpha\nu}), \end{aligned} \quad (2.12)$$

the last term being the bremsstrahlung contribution (2.11).

The amplitude may be simplified by taking into account that  $q^\alpha \epsilon_\alpha^*(q) = 0$ , yielding

$$\begin{aligned} A_{\text{axial}}(B \rightarrow \gamma^* l' \nu') = ie \frac{G_F}{\sqrt{2}} V_{ub} \bar{l}' \gamma_\nu (1 - \gamma_5) \nu' \epsilon_\alpha^*(q) \\ \times \left\{ (g_{\alpha\nu} q' q - q'_\alpha q_\nu) \left[ f_{1A} - \frac{Q_B f_B}{q' q} \right] + q'_\alpha q_\nu \left[ f_{2A} + f_{3A} + f_{1A} - \frac{Q_B f_B}{q' q} \right] + q'_\alpha q'_\nu f_{2A} \right\}. \end{aligned} \quad (2.13)$$

Introducing dimensionless form factors  $F_{1A,2A}$  and  $F'_{2A}$ ,

$$\begin{aligned} \frac{F_{1A}(q'^2, q^2)}{M_B} &= f_{1A}(q'^2, q^2) - \frac{Q_B f_B}{q' q}, \\ \frac{F_{2A}(q'^2, q^2)}{M_B} &= f_{2A}(q'^2, q^2) + f_{3A}(q'^2, q^2) + f_{1A}(q'^2, q^2) - \frac{Q_B f_B}{q' q}, \\ \frac{F'_{2A}(q'^2, q^2)}{M_B} &= f_{2A}(q'^2, q^2), \end{aligned} \quad (2.14)$$

we find the final expression for the contribution of the axial-vector part of the quark current,  $\bar{u} \gamma_\nu \gamma_5 b$ , to the amplitude

$$A_{\text{axial}}(B \rightarrow \gamma^* l' \nu') = ie \frac{G_F}{\sqrt{2}} V_{ub} \bar{l}' \gamma_\nu (1 - \gamma_5) \nu' \epsilon_\alpha^*(q) \left\{ (g_{\alpha\nu} q' q - q'_\alpha q_\nu) \frac{F_{1A}}{M_B} + q'_\alpha q_\nu \frac{F_{2A}}{M_B} + q'_\alpha q'_\nu \frac{F'_{2A}}{M_B} \right\}. \quad (2.15)$$

The contribution of the Lorentz structure  $q'_\alpha q'_\nu$  in (2.15) is proportional to  $m_{l'}^2$  but generates the most singular,  $\sim m_{l'}^2/(q^2)^2$ , contribution to the differential decay rate and the enhanced,  $\sim m_{l'}^2/m_l^2$ , contribution to the integrated decay rate; see Sec. IV. The Lorentz structure  $q'_\alpha q'_\nu$  can be neglected in most of the cases, except for the case  $m_{l'} \ll m_l$ .

Notice that, as follows from Eqs. (2.6) and (2.7), the form factors  $F_{2A}$  and  $F'_{2A}$  for the real photon in the final state satisfy the conditions (see also Ref. [28])

$$F_{2A}(q'^2, q^2 = 0) = 0, \quad (2.16)$$

$$F'_{2A}(q^2, q^2 = 0) = \frac{2Q_B f_B M_B}{M_B^2 - q^2}. \quad (2.17)$$

The conditions (2.16) are (2.17) are crucial, as they determine the behavior of the differential distributions in  $B \rightarrow ll'l\nu$  at small  $q^2$ . For our parametrization of the amplitude in the form (2.3), the condition (2.16) comes out as a direct consequence of Eqs. (2.6) and (2.7). Physics, of course, does not depend on the parametrization of the amplitude, but we find the parametrization (2.3) particularly convenient for the analysis of  $B$  decays.

### III. $B \rightarrow \gamma l' \nu'$ TRANSITION

We now illustrate the way the well-known formulas for the amplitude and the differential distribution in the leptonic radiative  $B$  decay,  $B \rightarrow \gamma l' \nu'$ , emerge.

Since, as the consequence of gauge invariance,  $F_{2A}(q^2, q^2 = 0) = 0$ , only the form factors  $F_{1A}$  and  $F_V$  contribute to the amplitude for the real photon and the massless lepton in the final state, and one finds the  $B \rightarrow \gamma l' \nu'$  amplitude,

$$A(B \rightarrow \gamma l' \nu') = ie \frac{G_F}{\sqrt{2}} V_{ub} \bar{l}' \gamma_\nu (1 - \gamma_5) \nu' \varepsilon_\alpha^*(q) \times \left\{ (g_{av} q' q - q'_\alpha q_\nu) \frac{F_A(q'^2)}{M_B} + i \varepsilon_{\nu\alpha q' q} \frac{F_V(q'^2)}{M_B} \right\}, \quad (3.1)$$

where  $F_A(q'^2) \equiv F_{1A}(q'^2, q^2 = 0)$  and  $F_V(q'^2) \equiv F_V(q'^2, q^2 = 0)$ .

The differential decay rate (for the massless lepton  $m_{l'} = 0$ ) takes a simple form:

$$\frac{d\Gamma(B \rightarrow \gamma l' \nu')}{dE_\gamma} = \frac{G_F^2 V_{ub}^2}{48\pi^2} M_B^4 \alpha_{e.m.} x_\gamma^3 (1 - x_\gamma) (|F_A|^2 + |F_V|^2), \quad (3.2)$$

$$x_\gamma = 2E_\gamma / M_B,$$

where  $M_B E_\gamma = pq = q'q$  and  $E_\gamma$  is the photon energy in the  $B$ -meson rest frame.

### IV. $B^- \rightarrow l^+ l^- l' \bar{\nu}'$ TRANSITION

The amplitude of the  $B \rightarrow ll'l\nu'$  transition is readily obtained from the amplitudes of the  $B \rightarrow \gamma^* l' \nu'$  transitions induced by the vector and the axial quark current by performing the replacement

$$\varepsilon_\alpha^*(q) \rightarrow -e \frac{\bar{l}' \gamma_\alpha l}{q^2}, \quad (4.1)$$

so we obtain

$$A(B \rightarrow ll'l\nu') = ie^2 \frac{G_F}{\sqrt{2}} V_{ub} \cdot \bar{l}' \gamma_\alpha l \cdot \bar{l}' \gamma_\nu (1 - \gamma_5) \nu' \frac{1}{q^2} \times \left\{ (g_{av} q' q - q'_\alpha q_\nu) \frac{F_{1A}}{M_B} + q'_\alpha q_\nu \frac{F_{2A}}{M_B} + q'_\alpha q'_\nu \frac{F'_{2A}}{M_B} + i \varepsilon_{\nu\alpha q' q} \frac{F_V}{M_B} \right\}. \quad (4.2)$$

By summing over the lepton polarizations and integrating over the phase space of the  $l^+ l^-$  pair and the  $l' \nu'$  pair, one obtains the explicit analytic expression for the double differential distribution. Neglecting term proportional to the lepton masses  $m'_l$  and  $m_l$ , one obtains (see also Ref. [8])<sup>3</sup>

$$|A|_0^2 = \frac{G_F^2}{2} V_{ub}^2 \frac{48}{3} \frac{e^4}{(q^2)^2} \frac{1}{M_B^2} [2F_V F_V^* \bar{\lambda} q^2 q'^2 + F_{1A} F_{1A}^* \{2(q'q)^2 + q^2 q'^2\} q^2 q'^2 + (F_{1A} F_{2A}^* + F_{1A}^* F_{2A}) \bar{\lambda} q^2 q'^2 + F_{2A} F_{2A}^* \bar{\lambda}^2], \quad (4.3)$$

with  $\bar{\lambda} \equiv (q'q)^2 - q^2 q'^2 = \frac{1}{4} \lambda(M_B^2, q^2, q'^2)$ , where

$$\lambda(a, b, c) = (a - b - c)^2 - 4bc$$

is the triangle function. The factors 4/3 and 8/3 in Eq. (4.3) result from the summation over polarizations of massless leptons coming from the electromagnetic vertex and from the weak vertex, respectively. It is noteworthy that the expression in large square brackets behaves at small  $q^2$  as  $\propto q^2$  because of the constraint  $F_2(q^2, q^2 = 0) = 0$ .

More singular terms  $\sim 1/q^4$  arise when one considers the effects of nonzero lepton masses. Most of them can be safely neglected except for the contribution proportional to  $m_l'^2$ :

$$|A|_{m_l'^2}^2 = \frac{G_F^2}{2} V_{ub}^2 \frac{4}{3} \frac{e^4}{(q^2)^2} \frac{1}{M_B^2} [|F'_{2A}|^2 4m_l'^2 (q^2 - 4m_l'^2) \bar{\lambda}]. \quad (4.4)$$

This term is negligible compared to Eq. (4.3) in the full kinematical  $B$ -decay region except for a close vicinity of the end point  $q^2 = 4m_l'^2$  in the case  $m_l \ll m_l'$ . The term (4.4) leads to a singular  $\sim m_l'^2/q^4$  contribution to the differential decay rate and the  $\sim m_l'^2/m_l^2$  contribution to the integrated decay rate. The latter turns out to dominate the integrated decay rate in the case  $m_l \ll m_l'$ . It is noteworthy that the relevant form factor at  $q^2 = 0$  is fixed by the Ward identity

<sup>3</sup>Our form factors  $f_{1A,2A,3A}$  are related to the form factors used in Ref. [8] as  $f_{1A} = F_1/q'q$ ,  $f_{2A} = (Q_B f_B - q^2 F_4)/q'q$ ,  $f_{3A} = (-F_1 - q^2 F_6 + q^2 F_4)/q'q$ . Our form factors  $F_{1A,2A}$  are related to the form factors of Ref. [8] as  $F_{A\perp} = \frac{q'q}{pq} F_{1A}$ ,  $\tilde{F}_{A\parallel} = -F_{2A} + \frac{q^2}{pq} F_{1A}$ , and  $F_{A\parallel} = -F_{2A} - F_{1A} \frac{4q^2 q'^2}{\lambda}$ .

and contains only well-known parameters, Eq. (2.17). Finally, we write

$$|A|^2 = |A|_0^2 + |A|_{m_l'}^2 + \dots, \quad (4.5)$$

where the dots stand for those terms proportional to the lepton masses,  $m_l$  and  $m_l'$ , which may be safely neglected in the full kinematical region.

The double differential distribution then takes the form (we display separately all numerical factors according to the definition of the differential distribution):

$$\begin{aligned} \frac{d^2\Gamma(B \rightarrow ll'l'\nu)}{dq^2 dq'^2} &= \frac{(2\pi)^4}{2M_B} \frac{1}{(2\pi)^{12}} \frac{\pi\lambda^{1/2}(q^2, m_l^2, m_l'^2)}{2q^2} \\ &\times \frac{\pi\lambda^{1/2}(q^2, m_l'^2, 0)\pi\lambda^{1/2}(M_B^2, q^2, q'^2)}{2q'^2} |A|^2. \end{aligned} \quad (4.6)$$

The kinematical constraints on the variables  $q^2$  and  $q'^2$  come from the  $\lambda$  functions in Eq. (4.3) and read

$$4m_l^2 \leq q^2, \quad m_l'^2 \leq q'^2, \quad \sqrt{q^2} + \sqrt{q'^2} \leq M_B. \quad (4.7)$$

First, let us notice that, because of the gauge-invariance constraint (2.16), one finds the behavior  $|A|_0^2 \propto 1/q^2$  and not  $\propto 1/(q^2)^2$ , as may be naively obtained when the gauge-invariance constraint is not taken into account. Such terms in  $|A|^2$  lead to a mild logarithmic dependence of the integrated decay rate on  $m_l$ . Second, there are terms  $\propto m_l'^2/(q^2)^2$  in  $|A|^2$ , which emerge from  $|A|_{m_l'}^2$ ; these terms are, however, essential only in a specific case  $m_l \ll m_l'$ . They lead to the low- $q^2$  enhancement of the decay rate as  $m_l'^2/m_l^2$ .

We emphasize that the double differential distribution  $d^2\Gamma(B \rightarrow l^+l^-l'\nu)/dq^2 dq'^2$  is easily calculable due to the fact that the leptons emitted from the electromagnetic vertex and the lepton emitted from the weak vertex have different flavors; no exchange diagrams emerge in this case, and one obtains the explicit analytic expression for the double differential distribution.

## V. FORM FACTORS AND THE DIFFERENTIAL DISTRIBUTIONS

### A. Modeling the form factors

The form factors  $F_{1A,2A}(q^2, q'^2)$  are obtained from the form factors  $f_{1A,2A,3A}$ , using for the latter the  $q^2$ -dispersion representation with one subtraction at  $q^2 = 0$ , Eqs. (2.8)–(2.10). The subtraction procedure allows us to incorporate the constraints imposed by gauge invariance.

Similarly, for the form factor  $F_V(q^2, q'^2)$ , a single-subtracted dispersion representation in  $q^2$  is used: the form

factors  $F_A(0, q'^2)$  and  $F_V(0, q'^2)$  should be equal to each other at the leading order of the double  $1/E_\gamma$  ( $2M_B E_\gamma = M_B^2 - q'^2$ ) and  $1/M_B$  expansions in QCD [15]. To satisfy this requirement, we make a subtraction in  $F_V(q^2, q'^2)$  at  $q^2 = 0$  and add a subtraction term  $F_V(q^2)$ .

Furthermore, we assume that the spectral densities are saturated by the light vector-meson resonances  $\rho^0$  and  $\omega$  in the  $q^2$  channel, and since these resonances emerge in the physical region of the  $B$ -decay of interest, we take into account the finite-width effects of these resonances. In the end, we come to the following expressions:

$$\begin{aligned} F_{1A}(q^2, q'^2) &= F_A(q^2) - \frac{Q_b f_B M_B}{q' q} \\ &- q^2 \sum_{V=\rho^0, \omega} \left( \frac{1}{M_V^2} \frac{2M_B(M_B + M_V)}{M_B^2 - M_V^2 - q'^2} \right. \\ &\times \left. \frac{M_V f_V}{M_V^2 - q^2 - i\Gamma_V(q^2)M_V} A_1^{B \rightarrow V}(q'^2) \right), \end{aligned} \quad (5.1)$$

$$\begin{aligned} F_{2A}(q^2, q'^2) &= -q^2 M_B \sum_{V=\rho^0, \omega} \frac{1}{M_V^2 M_V^2 - q^2 - i\Gamma_V(q^2)M_V} \\ &\times \left[ \frac{M_B + M_V}{M_B^2 - M_V^2 - q'^2} A_1^{B \rightarrow V}(q'^2) - \frac{A_2^{B \rightarrow V}(q'^2)}{(M_B + M_V)} \right] \\ &+ Q_b f_B \left( \frac{2M_B}{M_B^2 - q'^2} - \frac{2M_B}{M_B^2 - q^2 - q'^2} \right), \end{aligned} \quad (5.2)$$

$$\begin{aligned} F_V(q^2, q'^2) &= F_V(q^2) - q^2 M_B \\ &\times \sum_{V=\rho^0, \omega} \left( \frac{1}{M_V^2 M_V^2 - q^2 - i\Gamma_V(q^2)M_V} \frac{2V^{B \rightarrow V}(q'^2)}{M_B + M_V} \right). \end{aligned} \quad (5.3)$$

Let us discuss the expressions above:

- (i) The form factors  $F_A(q^2)$  and  $F_V(q^2)$  describe the  $B \rightarrow \gamma l' l'$  transition; they emerge as subtraction terms at  $q^2 = 0$  in the  $q^2$ -dispersion representations for the form factors  $F_{1A, V}(q^2, q'^2)$ . The form factors  $F_A(q^2)$  and  $F_V(q^2)$  are equal to each other at the leading order of the double  $1/E_\gamma$  ( $2M_B E_\gamma = M_B^2 - q'^2$ ) and  $1/M_B$  expansions in QCD [15] but differ in the subleading orders [17,18,20]:

$$\begin{aligned} F_A(q^2) &= -\frac{Q_u f_B M_B}{2E_\gamma \lambda_B} + \frac{Q_b f_B M_B}{2E_\gamma m_b} \\ &+ O(Q_u f_B M_B/E_\gamma^2), \end{aligned} \quad (5.4)$$

$$\begin{aligned} F_V(q^2) &= -\frac{Q_u f_B M_B}{2E_\gamma \lambda_B} - \frac{Q_b f_B M_B}{2E_\gamma m_b} \\ &+ O(Q_u f_B M_B/E_\gamma^2). \end{aligned} \quad (5.5)$$

The magnitude of the form factors  $F_A(q^2)$  and  $F_V(q^2)$  is determined to a large extent by the parameter  $\lambda_B$ , the inverse moment of the  $B$ -meson light-cone distribution amplitude  $\phi_B$  [15]. The value of  $\lambda_B$  presently has a large uncertainty: for instance, Ref. [18] makes use of  $\lambda_B(1 \text{ GeV}) = 0.35 \text{ GeV}$ ; the sum-rule estimate of Ref. [16] led to  $\lambda_B(1 \text{ GeV}) = 0.57 \text{ GeV}$ ; Ref. [29] obtained  $\lambda_B(1 \text{ GeV}) = 0.46 \pm 0.11 \text{ GeV}$ ; a recent next-to-leading-order analysis of Ref. [24] reported  $\lambda_B(1 \text{ GeV}) = 0.36 \pm 0.11 \text{ GeV}$ ; the results of calculating  $F_A(q^2)$  and  $F_V(q^2)$  [17] using the dispersion approach [30] correspond to a relatively large value  $\lambda_B(1 \text{ GeV}) = 0.657 \text{ GeV}$ . Obviously, the uncertainty in the parameter  $\lambda_B$  dominates the uncertainty in the differential distributions  $d\Gamma(B \rightarrow ll'l')/dq^2$  at small  $q^2$ .

In Ref. [19], the form factors  $F_A(q^2)$  and  $F_V(q^2)$  have been calculated in a broad range  $0 < q^2 < 25 \text{ GeV}^2$  using the dispersion approach of Ref. [30]. It was found that the monopole form (5.4) and (5.5) describes the results of our calculation for  $0 < q^2 < 15 \text{ GeV}^2$  with a few percent accuracy, whereas at  $q^2 \sim 25 \text{ GeV}^2$ , the monopole formula overestimates the calculated form factors by  $\sim 20\%$ . Nevertheless, taking into account a large uncertainty in the present knowledge of the parameter  $\lambda_B$ , we find it eligible to use the monopole form (5.4) and (5.5) in the full kinematically allowed region of  $q^2$  and consider the variation of  $\lambda_B$  in the range  $\lambda_B(1 \text{ GeV}) = (0.5 \pm 0.15) \text{ GeV}$ .

- (ii) In the region  $0.4 \leq q^2(\text{GeV}^2) \leq 0.9$ , where light vector meson resonances show up in the differential distributions, the form factors of interest cannot be calculated using perturbative QCD [9]. To calculate the form factors  $F_{1A,2A,V}(q^2, q^2)$  in this region of  $q^2$  and for any  $q^2$  appropriate for the four-lepton decay, we make use of the dispersion representations and assume [6,19] that they may be saturated by the intermediate  $\rho^0$  and  $\omega$  states in the  $q^2$  channel.<sup>4</sup> Since the light neutral vector mesons lie in the physical decay region of  $q^2$ , it is necessary to take into account their finite  $q^2$ -dependent width  $\Gamma_V(q^2)$ . For a relatively broad  $\rho$  meson, the function  $\Gamma_V(q^2)$  takes into account the effects of the  $2\pi$  intermediate states; the appropriate formulas are given in Ref. [31]. In practical calculations, we use a simplified expression

<sup>4</sup>We would like to note that no relative phase between the  $\rho^0$  and  $\omega$  contributions to the form factors  $F_i(q^2, q^2)$  as proposed in Ref. [6] may emerge; these form factors contain a sum over the intermediate states  $|V\rangle\langle V|$ , so even if one introduces arbitrary complex phases in the states  $|V\rangle$ , these phases appear both in the decay constants  $f_V$  and the  $B \rightarrow V$  weak form factors such that they finally drop out from  $F_i(q^2, q^2)$ .

TABLE I. Meson parameters entering the expressions for the form factors, Eqs. (5.1)–(5.3). Data from Refs. [34,35].

$f_B$ (MeV)	$\sqrt{2}f_\rho^0$ (MeV)	$3\sqrt{2}f_\omega$ (MeV)	$\Gamma_{\rho^0}$ (MeV)	$\Gamma_\omega$ (MeV)
190	216	190	150	8.49

TABLE II. Selected theoretical predictions for the weak form factors describing  $B$  decays into light vector mesons. The form factors from Refs. [32,36,37] are expected to have a 10%–15% uncertainty. To obtain the form factors for  $B^- \rightarrow \rho^0$  and  $B^- \rightarrow \omega$  decays, the numbers given in this table should be multiplied by the isotopic factor  $1/\sqrt{2}$ .

Ref.	$A_1^{B \rightarrow \rho}(0)$	$A_1^{B \rightarrow \omega}(0)$	$A_2^{B \rightarrow \rho}(0)$	$A_2^{B \rightarrow \omega}(0)$	$V^{B \rightarrow \rho}(0)$	$V^{B \rightarrow \omega}(0)$
[32]	0.26	...	0.24	...	0.31	...
[36]	0.24	0.22	0.22	0.20	0.32	0.29
[37]	0.26	...	0.24	...	0.28	...
[33]	$0.22 \pm 0.1$	...	$0.19 \pm 0.11$	...	$0.27 \pm 0.14$	...

which takes into account the correct threshold behavior of the  $\rho \rightarrow \pi\pi$  phase space:  $\Gamma_{\rho^0}(q^2) = \theta(q^2 - 4m_\pi^2)(1 - 4m_\pi^2/q^2)^{3/2}/(1 - 4m_\pi^2/M_\rho^2)^{3/2}\Gamma_{\rho^0}$ . For a narrow  $\omega$  meson, we take an approximation of constant width. (This approximation is not fully theoretically clean; the imaginary part of the propagator of the vector meson should vanish below the threshold in the corresponding decay channel. This means that  $\Gamma_V(q^2)$  should vanish below the corresponding light-meson threshold. But for a narrow  $\omega$  meson, the effect is tiny). Table I gives the meson parameters entering the form factors Eqs. (5.1)–(5.3).

Notice that  $\Gamma_V(q^2)$  takes into account the contribution of continuum of light pseudoscalar mesons; in this way, we effectively take into account the contribution of hadron continuum to the spectral densities of the form factors  $F_{1A,2A,V}(q^2, q^2)$  [31]. In the end, one finds that the nonresonance  $q^2$  region,  $q^2 \geq 1.0 \text{ GeV}^2$ , gives a small contribution to the decay width of the  $B \rightarrow ll'l'$  decay. This agrees with the expectations of the analysis of Ref. [8].

- (iii) The contribution of the light vector mesons  $V = \rho^0, \omega$  to the form factors  $F_{1A,2A,V}(q^2, q^2)$  is *unambiguous* (cf. Ref. [8]) and are expressed via the form factors  $A_1^{B \rightarrow V}(q^2)$ ,  $A_2^{B \rightarrow V}(q^2)$ , and  $V^{B \rightarrow V}(q^2)$  describing the weak decay  $B \rightarrow V$ . In spite of many efforts to calculate these form factors in a broad kinematical decay region  $0 < q^2 < M_B^2$ , our knowledge of these quantities is not very accurate. Table II presents some selected results for the relevant form factors: although the central values of the form factors at  $q^2 = 0$  from different approaches are in reasonable agreement with each

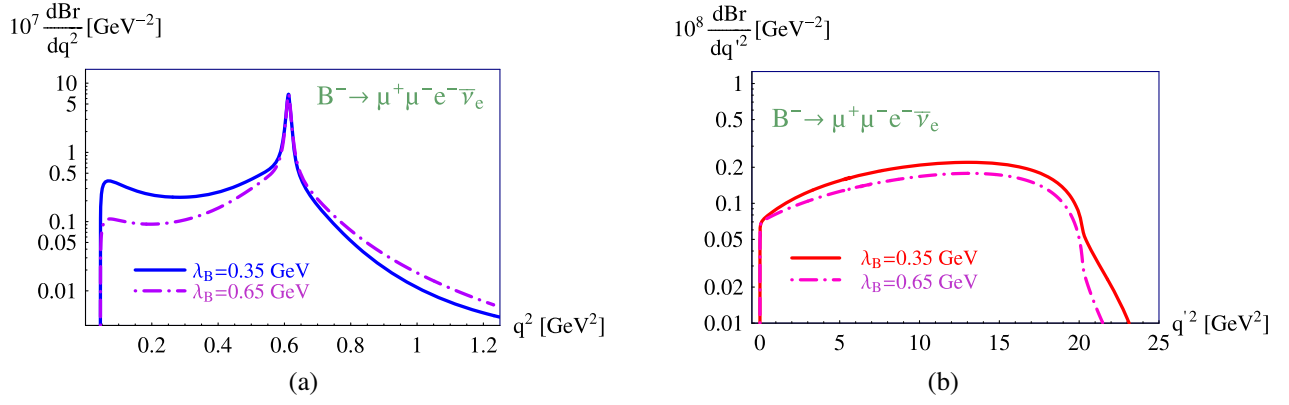


FIG. 2. The differential distributions (a)  $d\Gamma(B \rightarrow \mu^+\mu^-e\nu_e)/dq^2$  and (b)  $d\Gamma(B \rightarrow \mu^+\mu^-e\nu_e)/dq^2$ , for the weak transition  $B \rightarrow V$  form factors from Ref. [32]. Solid lines corresponds to  $\lambda_B = 0.35$  GeV, and dashed lines correspond to  $\lambda_B = 0.65$  GeV.

other, the uncertainties vary from an “educated guess” of 10% for Ref. [32] to almost 50% in Ref. [33]. The uncertainties in these form factors, along with the uncertainty in the parameter  $\lambda_B$ , is the second main source of the uncertainty in the theoretical predictions for  $B \rightarrow l^+l^-\nu\nu'$  decays. Appendix B summarizes the necessary parametrizations of the form factors used in our numerical estimates.

### B. Differential distributions

With the analytic expressions for the form factors, Eqs. (5.1)–(5.3) at hand, Eqs. (4.6) give the differential distributions in  $B \rightarrow \mu^+\mu^-e\nu_e$  decays, Fig. 2. Here, we use  $V_{ub} = 0.004$  and  $\tau_{B^-} = 1.63810^{-12}$  s. The plots show the impact of the parameter  $\lambda_B$  on the differential distributions  $d\Gamma/dq^2$  and  $d\Gamma/dq^2$ .

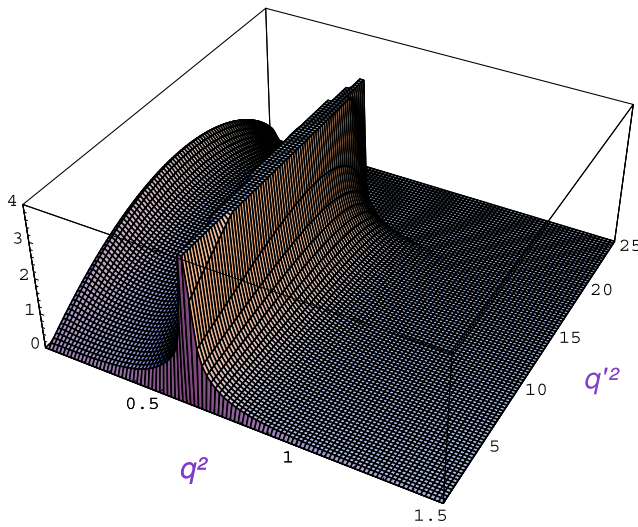


FIG. 3. The double differential distribution  $d^2\Gamma(B \rightarrow \mu^+\mu^-e\nu_e)/dq^2dq'^2$  calculated for  $\lambda_B = 0.65$  GeV and the form factors from Ref. [32].

Figure 3 shows the double differential distributions calculated for  $\lambda_B = 0.65$  GeV and the form factors from Ref. [32].

## VI. CONCLUSIONS

Our results are summarized below:

- (1) Gauge invariance provides essential constraints on the amplitude of Eq. (2.3):

$$T_{\alpha\nu}^5 = i \int dx e^{iqx} \langle 0 | T \{ J_\alpha^{e.m.}(x), \bar{u}\gamma_\nu\gamma_5 b(0) \} | \bar{B}_u(p) \rangle. \quad (6.1)$$

We emphasize that for a consistent analysis of the  $B \rightarrow l^+l^-\nu\nu'$  amplitude it is necessary to start with the amplitude (6.1) and properly parametrize this amplitude, taking into account all constraints imposed by the electromagnetic gauge invariance and analyticity. Taking into account these constraints leads to

$$F_{2A}(q^2, q^2 = 0) = 0, F'_{2A}(q^2, q^2 = 0) = \frac{2Q_B f_B M_B}{M_B^2 - q^2}.$$

These relations determine the behavior of the differential decay rate  $d^2\Gamma(B \rightarrow ll'\nu\nu')/dq^2dq'^2$  at small  $q^2$  [see Eqs. (4.3) and (4.4)].

- (2) For the form factors  $F_{V,1A,2A}(q^2, q^2)$ , describing the amplitude of  $B \rightarrow ll'\nu\nu'$  decay, we obtained dispersion representations in  $q^2$  with one subtraction. This allows us to properly take into account both the constraints imposed by gauge invariance at small  $q^2$  and the contributions of vector mesons ( $V$ ); the latter involve the weak form factors describing  $B \rightarrow V$  decays. Assuming that the light vector mesons  $\rho^0$  and  $\omega$  saturate the spectral densities, we obtained analytic representations for the form factors in a broad range of  $q^2$  and  $q'^2$ .



TABLE III. The branching ratios of the  $B \rightarrow l^+ l^- l' \nu$  integrated over the specific  $q^2$  ranges for the form factors from Ref. [32] and  $\lambda_B = 0.5$  GeV. For  $B \rightarrow e^+ e^- e \nu_e$  and  $B \rightarrow \mu^+ \mu^- \mu \nu_\mu$ , the results for  $\Gamma(B \rightarrow ll'l'\nu)|_{m_l=m_l}$  are given.

Mode	$q^2 = [4m_e^2, 4m_\mu^2]$	$q^2 = [4m_\mu^2, 0.4 \text{ GeV}^2]$	$q^2 = [0.4 \text{ GeV}^2, 1 \text{ GeV}^2]$	$q^2 = [1 \text{ GeV}^2, q_{\text{max}}^2]$	Total
$e^+ e^- \mu \nu_\mu$	$6.05 \times 10^{-7}$	$6.72 \times 10^{-9}$	$2.51 \times 10^{-8}$	$4.14 \times 10^{-10}$	$6.38 \times 10^{-7}$
$\mu^+ \mu^- e \nu_e$	...	$5.42 \times 10^{-9}$	$2.42 \times 10^{-8}$	$4.10 \times 10^{-10}$	$3.01 \times 10^{-8}$
$\mu^+ \mu^- \mu \nu_\mu$	...	$5.41 \times 10^{-9}$	$2.41 \times 10^{-8}$	$4.07 \times 10^{-10}$	$3.00 \times 10^{-8}$
$e^+ e^- e \nu_e$	$1.96 \times 10^{-8}$	$6.81 \times 10^{-9}$	$2.52 \times 10^{-8}$	$4.17 \times 10^{-10}$	$5.21 \times 10^{-8}$

Our assumption may seem oversimplified in comparison with a sophisticated analysis of the form factors presented in Ref. [8]. Moreover, our spectral representations saturated by merely light vector mesons do not reproduce the correct  $q^2$  behavior and overshoot the form factors at large values of  $q^2$ , where the form factors may be calculated using operator product expansion [8,9]; this means that our form factors do not produce realistic differential distributions at large values of  $q^2$ .

Nevertheless, our approach has a certain advantage compared to that of Ref. [8]: making use of the once-subtracted  $q^2$ -dispersion representations allows us to take properly into account both the gauge-invariance constraints and the resonance contributions to the form factors; the latter may be calculated unambiguously and are found to be nonzero (see Eq. (2.16)). On the other hand, in Ref. [8], the resonance contributions to  $F_{2A}$  are omitted in order to satisfy the gauge-invariance constraints.

A proper description of the resonance region of  $q^2$  is crucial as it produces the bulk of the  $B \rightarrow \mu^+ \mu^- l \nu$  cross section and nearly a half of the  $B \rightarrow e^+ e^- l \nu$  cross section (the other half comes from the region of small  $q^2$ ). So, from the point of view of obtaining numerical predictions, we find it eligible to trade the proper description of the region of  $0 < q^2 < 1 \text{ GeV}^2$  against overestimating the contribution of the region of large  $q^2$  which anyway, even with our overshoot form factors, contributes at less than a percent level.

- (3) We derived an explicit analytic expression for the differential distributions  $d\Gamma/dq^2 dq'^2$  in  $B \rightarrow ll'l'\nu$  decays including the  $O(m_l^2/q^4)$  terms which provide the most singular behavior of the differential distribution at small  $q^2$ . We then obtained numerical predictions for the differential and the integrated branching ratios of the  $B \rightarrow l^+ l^- l' \nu$  decays.

To illustrate the lepton-mass effects, Table III presents the numerical results for various decay modes. For the modes with identical particles in the final state, instead of the full decay rate that includes the exchange diagrams, Table III shows the quantity  $\Gamma(B \rightarrow ll'l'\nu)|_{m_l=m_l}$ . The full results for the identical leptons in the final state are discussed in the next item.

- (4) We now present the numerical results including the estimated uncertainties. The uncertainties in our predictions come from the two main sources:

- (i) First is the uncertainty in the parameter  $\lambda_B$ , which governs the behavior of the  $q^2$ -differential distributions at small  $q^2 \leq 0.4 \text{ GeV}^2$  but has an impact on the  $q^2$  distributions in the broad range of  $q^2$ . We allow the parameter  $\lambda_B$  to vary in the range  $\lambda_B = 0.35\text{--}0.65 \text{ GeV}$  (the lower values of this range has been advocated in several analyses [15,18,24], whereas the upper values of  $\lambda_B$  are obtained in explicit model calculations [16,17]).
- (ii) Second is the uncertainties in the  $B \rightarrow \omega, \rho$  weak form factors  $V, A_1, A_2$ , which mainly govern the differential distributions in the region of  $q^2 = (0.4 - 0.9) \text{ GeV}^2$ . To obtain the numerical estimates, we use as the basic scenario the form factors calculated in Ref. [32] and, in order to estimate the uncertainties, allow a 15% uncertainty on these form factors. We take into account a 10% suppression of the  $B \rightarrow \omega$  form factors compared to the corresponding  $B \rightarrow \rho$  form factors according to Ref. [36].

Taking into account these uncertainties, we obtain the following estimates:

$$\text{Br}(B \rightarrow \mu^+ \mu^- e \nu_e) = (3.01_{-0.19}^{+0.53} |_{\lambda_B} \pm 0.82 |_{\text{weak ffs}}) 10^{-8}, \quad (6.2)$$

$$\text{Br}(B \rightarrow e^+ e^- \mu \nu_\mu) = (6.38_{-0.12}^{+0.31} |_{\lambda_B} \pm 0.08 |_{\text{weak ffs}}) 10^{-7}. \quad (6.3)$$

We emphasize that the full integrated rate  $\text{Br}(B \rightarrow e^+ e^- \mu \nu_\mu)$  is an order of magnitude larger than  $\text{Br}(B \rightarrow \mu^+ \mu^- e \nu_e)$ . The former is fully dominated by the region  $4m_e^2 < q^2 < 4m_\mu^2$ , where the distribution contains an enhancement factor  $(f_B/M_B)^2 (m_\mu/m_e)^2$  due to the  $O(m_\mu^2)$  terms in the amplitude (1.5).

For the decay  $B \rightarrow l^+ l^- l^+ \nu_l$  ( $l = \mu, e$ ) with identical positive-charged leptons in the final state, the amplitude is given by the sum of direct and exchange diagrams,  $A = A_{\text{dir}} + A_{\text{exchange}}$ , and the phase space includes a factor

1/2 because of the presence of the identical particles in the final state. The phase-space integration of both  $|A_{\text{dir}}|^2$  and  $|A_{\text{exchange}}|^2$  leads to the same result,  $\Gamma(B \rightarrow ll'l')|_{m_l=m_l'}$ , and one can write (see, e.g., Ref. [8]),

$$\Gamma(B \rightarrow ll'l) = \Gamma(B \rightarrow ll'l')|_{m_l=m_l'} + \Gamma_{\text{interference}}(B \rightarrow ll'l), \quad (6.4)$$

where  $\Gamma_{\text{interference}}(B \rightarrow ll'l)$  is the phase-space integral of  $(A_{\text{dir}}A_{\text{exchange}}^* + A_{\text{dir}}^*A_{\text{exchange}})$ . The interference term should be calculated numerically as the integral over the phase space. A simple analytic result similar to Eqs. (4.3) and (4.6) cannot be obtained. We have performed a numerical calculation of the branching fraction (6.4) and found that the interference branching fraction leads to a very mild increase of the integrated branching fraction  $\text{Br}(B \rightarrow ll'l')$  at the level of less than 1% (our detailed results for the differential distributions for this case will be presented in Ref. [38]). We report

$$\text{Br}(B^+ \rightarrow \mu^+\mu^-\mu^+\bar{\nu}_\mu) = (3.02_{-0.25}^{+0.45}|_{\lambda_b} \pm 0.62|_{\text{weak ffs}})10^{-8}. \quad (6.5)$$

This estimate agrees with the result of Ref. [8] and is only marginally compatible with the upper limits obtained by the LHCb Collaboration [4]  $\text{Br}(B^+ \rightarrow \mu^+\mu^-\mu^+\bar{\nu}_\mu) \leq 1.6 \times 10^{-8}$ . Recall, however, that the experimental upper bound applies certain kinematical cuts, whereas our result corresponds to the branching fraction integrated over the full allowed region of the lepton momenta.

For electrons in the final state, we find

$$\text{Br}(B^+ \rightarrow e^+e^-e^+\bar{\nu}_e) = (5.26_{-1.05}^{+2.60}|_{\lambda_b} \pm 0.70|_{\text{weak ffs}})10^{-8}. \quad (6.6)$$

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## APPENDIX A: RELATIONS BETWEEN THE $B_q$ AND $\bar{B}_q$ AMPLITUDES

Here, we derive the relations between the amplitudes of  $B_q$  and  $\bar{B}_q$  mesons. Such relations are obtained by applying charge conjugation.

The fermion field transforms under charge conjugation  $\hat{C}$  ( $\hat{C}^2 = 1$ ,  $\hat{C}^{-1} = \hat{C}$ ) as [39]

$$\hat{C}\psi\hat{C} = \eta_c C\bar{\psi}^T, \quad (A1)$$

$$\hat{C}\bar{\psi}\hat{C} = \eta_c^* \psi^T (C^T)^{-1}, \quad (A2)$$

$|\eta_c| = 1$ , where the charge-conjugation matrix  $C$  is defined by the relation

$$C\gamma_\mu^T C^{-1} = -\gamma_\mu \quad (A3)$$

and has the following properties:  $C^T = -C$ ,  $C^{-1} = -C$ ,  $C^2 = -1$ . In the Dirac representation of the  $\gamma$  matrices, one can choose  $C = i\gamma_0\gamma_2$ , leading to

$$C\gamma_5^T C = -\gamma_5, \quad (A4)$$

$$C(\gamma_\mu\gamma_5)^T C = -\gamma_\mu\gamma_5, \quad (A5)$$

$$C(\sigma_{\mu\nu})^T C = \sigma_{\mu\nu}, \quad (A6)$$

$$C(\sigma_{\mu\nu}\gamma_5)^T C = \sigma_{\mu\nu}\gamma_5. \quad (A7)$$

Making use of these relations, one obtains the expression for charge conjugation of bilinear currents (of anticommuting) fermion operators,

$$\hat{C}(\bar{\psi}_1 \mathcal{O} \psi_2) \hat{C} = -\bar{\psi}_2 (C \mathcal{O}^T C) \psi_1, \quad (A8)$$

leading to

$$\hat{C}(\bar{\psi}_1 \psi_2) \hat{C} = \bar{\psi}_2 \psi_1, \quad (A9)$$

$$\hat{C}(\bar{\psi}_1 \gamma_5 \psi_2) \hat{C} = -\bar{\psi}_2 \gamma_5 \psi_1, \quad (A10)$$

$$\hat{C}(\bar{\psi}_1 \gamma_\mu \psi_2) \hat{C} = -\bar{\psi}_2 \gamma_\mu \psi_1, \quad (A11)$$

$$\hat{C}(\bar{\psi}_1 \gamma_\mu \gamma_5 \psi_2) \hat{C} = \bar{\psi}_2 \gamma_\mu \gamma_5 \psi_1, \quad (A12)$$

$$\hat{C}(\bar{\psi}_1 \sigma_{\mu\nu} \psi_2) \hat{C} = -\bar{\psi}_2 \sigma_{\mu\nu} \psi_1, \quad (A13)$$

$$\hat{C}(\bar{\psi}_1 \sigma_{\mu\nu} \gamma_5 \psi_2) \hat{C} = -\bar{\psi}_2 \sigma_{\mu\nu} \gamma_5 \psi_1. \quad (A14)$$

The  $C$ -conjugate states are related to each other as follows (no arbitrary phase is implied):

$$\hat{C}|B_q(p)\rangle = |\bar{B}_q(p)\rangle. \quad (A15)$$

The QCD vacuum state is  $C$  invariant,  $\hat{C}|0\rangle = |0\rangle$ . So, if we are going to consider QCD effects in the amplitudes, we can apply  $C$  conjugation and relate to each other the amplitudes

$$\langle 0 | \bar{q} \gamma_\mu \gamma_5 b | \bar{B}_q(p) \rangle = i \bar{f}_B p_\mu, \quad (A16)$$

$$\langle 0 | \bar{b} \gamma_\mu \gamma_5 q | B_q(p) \rangle = i f_B p_\mu \quad (A17)$$

and obtain the relation  $f_B = \bar{f}_B$ . Similar relations may be obtained for more complicated amplitudes such as

$$\bar{T}_\alpha^\mathcal{O}(p, q) \equiv i \int dx e^{iqx} \langle 0 | T \{ J_\alpha^{\text{e.m.}}(x), \bar{q}(0) \mathcal{O} b(0) \} | \bar{B}_q(p) \rangle, \quad (\text{A18})$$

$$T_\alpha^\mathcal{O}(p, q) \equiv i \int dx e^{iqx} \langle 0 | T \{ J_\alpha^{\text{e.m.}}(x), \bar{b}(0) \mathcal{O} q(0) \} | B_q(p) \rangle. \quad (\text{A19})$$

The parametrizations of these amplitudes are given via the same form factors but with appropriate sign adjustments between (A18) and (A19). For instance, for the relation between the amplitudes (we use in this Appendix the notation  $\bar{T}$  for the amplitudes containing  $\bar{B}_q$  meson in the initial state)

$$\bar{T}_{\alpha\mu} = i \int dx e^{iqx} \langle 0 | T \{ J_\alpha^{\text{e.m.}}(x), \bar{q}(0) \gamma_\mu b(0) \} | \bar{B}_q(p) \rangle, \quad (\text{A20})$$

$$\bar{T}_{\alpha\mu}^5 = i \int dx e^{iqx} \langle 0 | T \{ J_\alpha^{\text{e.m.}}(x), \bar{q}(0) \gamma_\mu \gamma_5 b(0) \} | \bar{B}_q(p) \rangle, \quad (\text{A21})$$

$$\bar{T}_{\alpha,\mu\nu} = i \int dx e^{iqx} \langle 0 | T \{ J_\alpha^{\text{e.m.}}(x), \bar{q}(0) \sigma_{\mu\nu} b(0) \} | \bar{B}_q(p) \rangle, \quad (\text{A22})$$

$$\bar{T}_{\alpha,\mu\nu}^5 = i \int dx e^{iqx} \langle 0 | T \{ J_\alpha^{\text{e.m.}}(x), \bar{q}(0) \sigma_{\mu\nu} \gamma_5 b(0) \} | \bar{B}_q(p) \rangle \quad (\text{A23})$$

and the corresponding amplitudes  $T_{\alpha\mu}$ ,  $T_{\alpha\mu}^5$ ,  $T_{\alpha,\mu\nu}$ , and  $T_{\alpha,\mu\nu}^5$  as defined according to Eqs. (A18) and (A19), we obtain

$$\bar{T}_{\alpha\mu}(p, q) = T_{\alpha\mu}(p, q), \quad (\text{A24})$$

$$\bar{T}_{\alpha\mu}^5(p, q) = -T_{\alpha\mu}^5(p, q), \quad (\text{A25})$$

$$\bar{T}_{\alpha\mu\nu}(p, q) = T_{\alpha\mu\nu}(p, q), \quad (\text{A26})$$

$$\bar{T}_{\alpha\mu\nu}^5(p, q) = T_{\alpha\mu\nu}^5(p, q). \quad (\text{A27})$$

In conclusion, the amplitudes (A18) and (A19) are related to each other by charge conjugation.

## APPENDIX B: PARAMETRIZATIONS OF THE FORM FACTORS

### 1. $F_{V,A}(q^2)$

In our numerical estimates, we use the following parametrizations for the form factors ( $Q_u = 2/3$ ,  $Q_b = -1/3$ ):

$$F_V(q^2) = -Q_u \frac{M_B^2}{M_B^2 - q^2} \frac{f_B}{\lambda_B} - Q_b \frac{M_B^2}{M_B^2 - q^2} \frac{f_B}{m_b}, \quad (\text{B1})$$

$$F_A(q^2) = -Q_u \frac{M_B^2}{M_B^2 - q^2} \frac{f_B}{\lambda_B} + Q_b \frac{M_B^2}{M_B^2 - q^2} \frac{f_B}{m_b}, \quad (\text{B2})$$

with  $f_B = 190$  MeV and  $m_b = 5$  GeV. The parameter  $\lambda_B$  varies in the range  $\lambda_B = 0.35\text{--}0.65$  GeV.

### 2. $V(q^2), A_1(q^2), A_2(q^2)$

All the form factors are parametrized as follows:

$$F_i(q^2) = \frac{F_i(0)}{(1 - \sigma_0^{(i)} r)(1 - \sigma_1^{(i)} r + \sigma_2^{(i)} r^2)}, \quad r \equiv q^2/M_R^2. \quad (\text{B3})$$

For the basic scenario of Melikhov and Stech [32], the parameters are given below, and  $M_R = M_{B^*} = 5.32$  GeV. Note that all tables give  $F_i(0)$  for the  $B \rightarrow \rho^+$  transition (for  $B \rightarrow \rho^0$  transition,  $F_i(0)$  should be multiplied by isotopic factor  $1/\sqrt{2}$ ):

- (i) For  $B \rightarrow \omega$  transition, a reduction of the form factors at zero by 10% compared to  $B \rightarrow \rho^0$  was applied following the estimates of Ref. [36]. The  $q^2$  dependence is taken the same as for  $B \rightarrow \rho$ .
- (ii) To estimate the uncertainty in the predictions for the rates, the range of the form factors from Ref. [33] was used (see Table II).

	$V^{B \rightarrow \rho^+}(0)$	$A_1^{B \rightarrow \rho^+}(0)$	$A_2^{B \rightarrow \rho^+}(0)$
$F(0)$	0.31	0.26	0.24
$\sigma_0$	1	0	0
$\sigma_1$	0.59	0.73	1.4
$\sigma_2$	0	0.10	0.50

### 3. Resonance $q^2$ -dependent width

$f_B$ (MeV)	$\sqrt{2}f_\rho^0$ (MeV)	$3\sqrt{2}f_\omega$ (MeV)	$\Gamma_{\rho^0}$ (MeV)	$\Gamma_\omega$ (MeV)
190	216	190	150	8.49

For a relatively broad  $\rho^0$  meson, the function  $\Gamma_V(q^2)$  takes into account the effects of the  $2\pi$  intermediate states; the appropriate formulas are given in Ref. [31]. In practical calculations, we use a simplified expression, which takes into account the correct threshold behavior of the  $\rho \rightarrow \pi\pi$  phase space:

$$\Gamma_{\rho^0}(q^2) = \theta(q^2 - 4m_\pi^2) (1 - 4m_\pi^2/q^2)^{3/2} / (1 - 4m_\pi^2/M_\rho^2)^{3/2} \Gamma_{\rho^0}. \quad (\text{B4})$$

For a narrow  $\omega$  meson, we take an approximation of constant width.

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