Letter

$f(R)$ gravity after the detection of the orbital precession of the S2 star around the Galactic Center massive black hole

Ivan De Martino,[1,2,*] Riccardo della Monica,[1,†] and Mariafelicia De Laurentis[3,4,‡]

[1]*Universidad de Salamanca, Facultad de Ciencias, Física Teórica, Salamanca, Plaza de la Merced s/n. 37008, Spain*

[2]*Dipartimento di Fisica, Università di Torino, Via P. Giuria 1, I-10125 Torino, Italy*

[3]*Dipartimento di Fisica, Universitá di Napoli "Federico II", Complesso Universitario di Monte S. Angelo, Edificio G, Via Cinthia, I-80126 Napoli, Italy*

[4]*INFN Sezione di Napoli, Complesso Universitario di Monte S. Angelo, Edificio G, Via Cinthia, I-80126 Napoli, Italy*

(Received 27 April 2020; revised 4 October 2021; accepted 8 October 2021; published 16 November 2021)

The GRAVITY Collaboration achieved a remarkable detection of the orbital precession of the S2 star around the Galactic Center supermassive black hole, providing yet another proof of the validity of general relativity. The departure from the Schwarzschild precession is encoded in the parameter f_{SP}, which multiplies the predicted general relativistic precession. This parameter results in $f_{\mathrm{SP}} = 1.10 \pm 0.19$, which is consistent with general relativity ($f_{\mathrm{SP}} = 1$) at 1σ level. Nevertheless, this parameter may also hide an effect of the modified theories of gravity. Thus, we consider the orbital precession due to the Yukawa-like gravitational potential arising in the weak field limit of $f(R)$-gravity, and we use the current bound on the f_{SP} to constrain the strength and the scale length of the Yukawa-like potential. No deviations from general relativity are revealed at the scale of $\lambda < 6300$ AU with the strength of the Yukawa potential restricted to $\delta = -0.01^{+0.61}_{-0.14}$.

DOI: 10.1103/PhysRevD.104.L101502

I. INTRODUCTION

Extensions of general relativity (GR) are a compelling choice for providing an explanation to the ongoing accelerated expansion of the Universe, as well as the formation of self-gravitating systems, without resorting to exotic and still unknown fluids such as dark matter and dark energy (for comprehensive reviews see [1,2] and [3,4], respectively). These extensions can be obtained by replacing the Hilbert-Einstein action with a more general Lagrangian, which may include higher-order curvature invariants, such as R^2, $R_{\mu\nu}R^{\mu\nu}$, $R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}$, $R\Box R$, or $R\Box^k R$, and minimally or nonminimally coupled terms between scalar fields and geometry, such as $\phi^2 R$ [5–7]. It is argued that extended theories of gravity must reproduce GR in their weak field limit [8]. Nevertheless, these theories represent a large collection of models that can be developed on the basis of the curvature invariants considered, and of the coupling with matter (for a comprehensive review see for instance [5,6]).

One appealing theory is $f(R)$-gravity, where the Hilbert-Einstein action is replaced with a more general function of the Ricci scalar [9]. Indeed, the cosmological constant may be naturally explained as the effect of the higher order curvature terms in the field equations, with the first attempt of this type dated back in the 1980s [10], and followed by other successful ones [11–17]. Another fascinating feature of $f(R)$-gravity is that an R^2-term gives rise to a Yukawa-like correction to the Newtonian gravitational potential in the weak field limit [18]. These corrections may affect the astrophysical scales of galaxies and galaxy clusters (see for instance Ref. [2] and references therein). However, there is no statistical evidence favoring $f(R)$-gravity over GR [19,20] and additionally, GR has been collecting a huge amount of successful probes over the previous decades [21]. It is important to remember, among others, the direct detection of gravitational waves from a binary black hole merger [22], and the subsequent direct detection of gravitational waves from a binary neutron star merger [23]. While the first event served to show the effective existence of a fundamental pillar of GR, the second event, that was accompanied by an electromagnetic emission with a time delay of ~1.7 s with respect to the merger time, was later associated with GRB 170817A [24], allowing a probe of the equivalence principle and Lorentz invariance [23], and also excluded several alternatives to theories of gravity [25–29].

*ivan.demartino@usal.es
†rdellamonica@usal.es
‡mariafelicia.delaurentis@unina.it

Recently, Hees *et al.* (2017) [30] demonstrated the effectiveness of short-period stars orbiting around the supermassive black hole to constrain the strength and the scale length of the Yukawa potential due to the fifth force. They strongly restricted the parameter space using astrometric and spectroscopic measurement of the S2 star, obtaining an upper limit on the periastron advance of $\sim 10^{-3}$ rad/yr.

Finally, the GRAVITY Collaboration made a pioneering detection of the orbital precession of the star S2 orbiting a compact and variable x-ray, infrared, and radio source (Sgr A*) at the center of the Milky Way [31–33]. Sgr A* is supposed to be the closest supermassive black hole. It is surrounded by a cluster of stars orbiting around it, whose characteristics, such as distribution and kinematics of stars, have been obtained through radio and infrared observations [33]. The analysis of the orbital motion of those stars served to further confirm GR. The detection of the orbital precession of the S2 star of about $\delta\phi \approx 12'$ per orbital revolution has been used to constrain the parameter $f_{\rm SP}$, which multiplies the general relativistic precession and encodes any departure from the Schwarzschild metric or from GR. Its best fit value is $f_{\rm SP} = 1.10 \pm 0.19$ [34], where $f_{\rm SP} = 1$ leads to GR, and $f_{\rm SP} = 0$ reduces to Newtonian theory.

Here we use the results obtained by De Laurentis, de Martino, and Lazkoz [35], which computed analytically the precession of a pointlike star orbiting a massive and compact object under the Yukawa-like gravitational potential arising in $f(R)$-gravity, and obtain the first constraint on the strength of the gravitational potential from the aforementioned pioneering observations, as well as an upper limit on the graviton mass.

II. BACKGROUND EQUATIONS FROM *F(R)*-GRAVITY

The $f(R)$-gravity field equations are obtained by varying the action with respect to the metric. The main steps are the same, as in the case of the variation of the Einstein-Hilbert action, but there are also some important differences. The resulting field equations include terms containing derivatives of fourth order in the metric, and therefore are more complicated than GR field equations. The latter are second-order partial differential equations and are recovered as the special case when we set $f(R) = R$ (for more details see [6] and references therein). Generally speaking, the $f(R)$-Lagrangian must be specified to allow practical applications and constraints on the model. One way around that is to require that it is an analytic Taylor-expandable function. In such a case, one can straightforwardly perform the post-Newtonian limit and obtain the solution of field equations. Considering a general spherically symmetric metric, De Laurentis, de Martino, and Lazkoz obtained [35]

$$ds^2 = [1 + \Phi(r)]dt^2 - [1 - \Phi(r)]dr^2 - r^2 d\Omega, \quad (1)$$

where $d\Omega$ is the solid angle, and

$$\Phi(r) = -\frac{2GM}{(1+\delta)rc^2}\left(1 + \delta e^{-\frac{r}{\lambda}}\right) \quad (2)$$

is the Yukawa-like modification of the Newtonian gravitational potential. Here G is the Newton gravitational constant, M is the source mass, δ is a parameter of the theory (Newton's potential is recovered when it is turned off), and it modulates the strength of the Yukawa-like potential. Finally, λ is a scale length which naturally arises in higher order theories of gravity [6].

Relativistic equations of motion for massive particles can be obtained from the geodesic equations for timelike geodesics of the metric in Eq. (1):

$$\frac{d^2x^\mu}{ds^2} + \Gamma^\mu_{\nu\rho}\frac{dx^\nu}{ds}\frac{dx^\rho}{ds} = 0. \quad (3)$$

These equations provide differential equations for the four space-time components $\{t(s), r(s), \theta(s), \phi(s)\}$, where s is an affine parameter (the proper time of the star, in our case), that can be numerically integrated once the initial conditions are specified. In order to calculate the periastron shift of a pointlike particle of mass m around a massive object of mass M, one may assume the spherically symmetric metric in Eq. (1) and also that $m \ll M$ to further simplify the problem. Since the metric is symmetric about $\theta = \pi/2$, any geodesic that begins moving in that plane will remain there indefinitely (the plane is totally geodesic). Therefore, the coordinate system may be oriented so that the orbit of the particle lies in that plane, and fixes the θ coordinate to be $\pi/2$.

Using the equations of the geodesics, De Laurentis, de Martino, and Lazkoz [35,36] computed the equation of orbital precession as

$$\Delta\phi_{\rm Yukawa} = \frac{\Delta\phi_{GR}}{(\delta+1)}\left(1 + \frac{2\delta G^2M^2}{3a^2c^3(1-e^2)^2} - \frac{2\pi\delta G^2M^2}{ac^4(1-e^2)\lambda} - \frac{3\delta GM}{ac^4(1-e^2)} - \frac{\delta G^2M^2}{6c^4(\delta+1)\lambda^2} + \frac{\delta GM}{3\lambda c^2}\right), \quad (4)$$

where a is the semimajor axis and e is the eccentricity. The GR contribution to the periastron advance is

$$\Delta\phi_{\rm GR} = \frac{6\pi GM}{ac^2(1-e^2)}. \quad (5)$$

Additionally, De Laurentis, de Martino, and Lazkoz [35,36] showed, as an example case, that in the binary system composed by the S2 star and the supermassive black hole Sgr A*, differences in the orbital precession between GR and $f(R)$-gravity exist, but for a reasonable range of parameters, do not exceed 10%.

III. DATA AND DATA MODELLING

The stellar cluster in the Galactic Center of the Milky Way, orbiting around a central compact object, is the most recent test bench for GR. There are multiple pieces of evidence that such a compact object is a supermassive black hole of mass $M \approx 4 \times 10^6\ M_\odot$ [32], some examples being the stellar and/or gas kinematics [33,37–39]. This allows us to approximate the orbiting stars to massive pointlike objects, and also allows us to use Eq. (5) to predict the orbital precession of a star in GR and Eq. (4) to predict the orbital precession in the Yukawa-like potential of Eq. (2).

After an observational campaign lasting about two decades [40], the GRAVITY Collaboration has been able to measure the orbital precession of the S2 star. They used the equation of motion at first-order expansion in the post-Newtonian limit [41,42], and parametrized the departure from the Schwarzschild metric by introducing an *ad hoc* factor $f_{\rm SP}$ as follows:

$$\Delta\phi_{\rm per\ orbit} = f_{\rm SP} \times \Delta\phi_{\rm GR}. \tag{6}$$

This factor may include the effect related to the spin of the black hole as well as the departure from GR. Remarkably, $f_{\rm SP} = 1.10 \pm 0.19$ [34], recovering GR within 1σ.

The analysis uses measurements of the positions and spectra of the star S2 collected throughout several years, and includes 118 measurements obtained with the Very Large Telescope (VLT) infrared camera NACO between 2002 and 2019.7 of the position, 75 NACO and 54 GRAVITY measurements from 2003.3 to 2019.7 and from 2016.7 to 2019.7, respectively, of the direct S2-Sgr A* separation with rms uncertainties of 1.7 and 0.65 mas [43], respectively, 92 spectroscopic measurements of the 2.167 μm HI and the 2.11 μm HeI lines between 2003.3 and 2019.45 with the spectrometer SINFONI at the VLT [44], 2 NACO spectroscopic measurements from 2003, and 3 Keck-NIRC2 spectroscopic measurements between 2000 and 2002 [45]. For more details we refer to Refs. [34,43,44,46,47]. The data have been processed with a Monte Carlo Markov chain algorithm, and yield to constrain the orbital parameters (see Table E.1 in [34]) and the orbital precession.

Here, we use Eq. (4) to predict the orbital motion and the precession of the S2 star in the Yukawa-like potential of Eq. (2), and fit the results into the data. More details on the data and the data analysis are given in the Supplemental Material (SM) [48]. First, we computed the equations of motion (3) starting from the metric given in (1). Then, initial conditions $\{t(0), r(0), \theta(0), \phi(0)\}$ and numerical values of the parameters $\{\delta, \lambda\}$ were set to integrate the aforementioned equations numerically. Since GRAVITY data are not publicly available, we use the dataset published in [49]. This dataset includes 145 astrometric measurements [50–52] of the position of S2 relative to the Galactic Centre (GC) infrared reference system, [53] and 44 spectroscopic measurements [31,54,55], which provide radial velocity estimates for S2 in the local standard of rest (LSR).

The predicted positions and velocities of S2 must be corrected for some effects before being compared with the data. Here, we correct our predicted orbits for the Rømer delay and the frequency shift due to relativistic Doppler effect. Other relativistic effects could potentially modify the astrometric positions of the observed star, like the Shapiro delay, the Lense-Thirring effect on both the orbit and the photon (in the case of a rotating black hole), or the gravitational lensing of the light rays emitted by the star. Nevertheless, they are not detectable with the present sensitivity [56].

Finally, the orbit is fully determined once the parameters $(M_\bullet, R_\bullet, T, t_p, a, e, i, \Omega, \omega, x_0, v_{x,0}, y_0, v_{y,0}, v_{\rm LSR}, \delta, \lambda)$ have been assigned. The first two parameters, $M_\bullet$ and $R_\bullet$, describe the mass and the distance from Earth of the source of the gravitational potential in which the star moves. The seven Keplerian elements provide the initial conditions for the numerical integration of the geodesic equations, and with the Thiele-Innes elements necessary to project the resulting orbit in the observer's reference frame. Five additional parameters, $(x_0, v_{x,0}, y_0, v_{y,0}, v_{\rm LSR})$, take into account the zero-point offset and drift of the reference frame with respect to the mass centroid, and the parameters (δ, λ) select a particular metric (1) for $f(R)$-gravity. This results in a total of 16 parameters whose priors are given in Table I of SM, and whose posterior distributions are sampled with a Markov chain Monte Carlo (MCMC) algorithm. We estimated our log-likelihood as

$$\log\mathcal{L} = \log\mathcal{L}_{\rm Pos} + \log\mathcal{L}_{\rm VR} + \log\mathcal{L}_{\rm Pre}, \tag{7}$$

where $\log\mathcal{L}_{\rm Pos}$ is the likelihood of the positional data

$$\log\mathcal{L}_{\rm Pos} = -\sum_i \frac{(x^i_{\rm obs} - x^i_{\rm orb})^2}{2(\kappa\sigma^i_{x,\rm obs})^2} - \frac{(y^i_{\rm obs} - y^i_{\rm orb})^2}{2(\kappa\sigma^i_{y,\rm obs})^2}, \tag{8}$$

TABLE I. Best fit values for the $f(R)$-gravity parameters using only positional and radial velocity data from [49] (column 2) and using the additional measurement of the orbital precession from [34] (column 3).

Parameter	Fit w/o precession	Fit with precession
δ	$\gtrsim -0,07$	$-0.01^{+0.61}_{-0.14}$
λ (AU)	$\lambda \gtrsim 9540$	$\gtrsim 6300$

$\log \mathcal{L}_{\mathrm{VR}}$ is the likelihood of the radial velocities

$$\log \mathcal{L}_{\mathrm{VR}} = -\sum_i \frac{(\mathrm{VR}^i_{\mathrm{obs}} - \mathrm{VR}^i_{\mathrm{orb}})^2}{2(\kappa\sigma^i_{\mathrm{VR,obs}})^2}, \tag{9}$$

and $\log \mathcal{L}_{\mathrm{Pre}}$ is the log-likelihood of the orbital precession given by

$$\log \mathcal{L}_{\mathrm{Pre}} = -\frac{(f_{\mathrm{SP,obs}} - f_{\mathrm{SP,th}})^2}{2(\kappa\sigma_{f_{\mathrm{SP,obs}}})^2}, \tag{10}$$

where $f_{\mathrm{SP,th}} \equiv \Delta\phi_{\mathrm{Yukawa}}/\Delta\phi_{\mathrm{GR}}$ and κ is an auxiliary parameter that either takes the value $\kappa = 1$, when $\mathcal{L}_{\mathrm{Pre}}$ is set to 0 (i.e., when the precession is not taken into account in our analysis), or $\kappa = \sqrt{2}$, otherwise. This is done in order not to double-count astrometric and radial velocity data points that we implicitly assume appear twice in the log-likelihood when considering the measurement of the orbital precession (this has been done using the same dataset as we did).

IV. RESULTS AND DISCUSSIONS

We carried out two MCMC analyses. In the first run, we only used orbital positions and velocities, while we introduced the precession measurement in a second run. Here, we focus on the impact of our results on $f(R)$-gravity (we remand to SM for the full details). Fig. 1 depicts the 68%, 95%, and 99% confidence intervals. The upper and lower panels illustrate the results obtained, excluding and including the measurement of the orbital precession, i.e., results in the upper panel are obtained using only measurements of position and velocities given in [49], while in the lower panels results were obtained including the measurement of the orbital precession by GRAVITY Collaboration [34]. The constraints on the $f(R)$-gravity parameters $\{\delta, \lambda\}$ are shown in Table I.

The additional information about the orbital precession of the S2 star provided much tighter constraints on both δ and λ, resulting in a narrower confidence region on the (δ, λ) plane (see Fig. 1). Indeed, while the analysis without the precession was not able to place an upper limit on neither δ nor λ, in the latter analysis we were able to fully constrain the parameter δ, taking advantage of the greater constraining power of the orbital precession. Our analysis, thus, provides the first constraint on the strength of the Yukawa-like potential at the Galactic Centre: $\delta = -0.01^{+0.61}_{-0.14}$. While looking at the two dimensional contours, we only obtain a lower bound on the scale length of the Yukawa-like potential: $\lambda \gtrsim 6300$ AU at 1σ. This is rather expected, because this parameter is better constrained on a larger astrophysical scale [2], and further confirms the results in [35,36]. The 95% confidence contours from our analysis are fully consistent with the exclusion regions on the fifth force determined by Hees

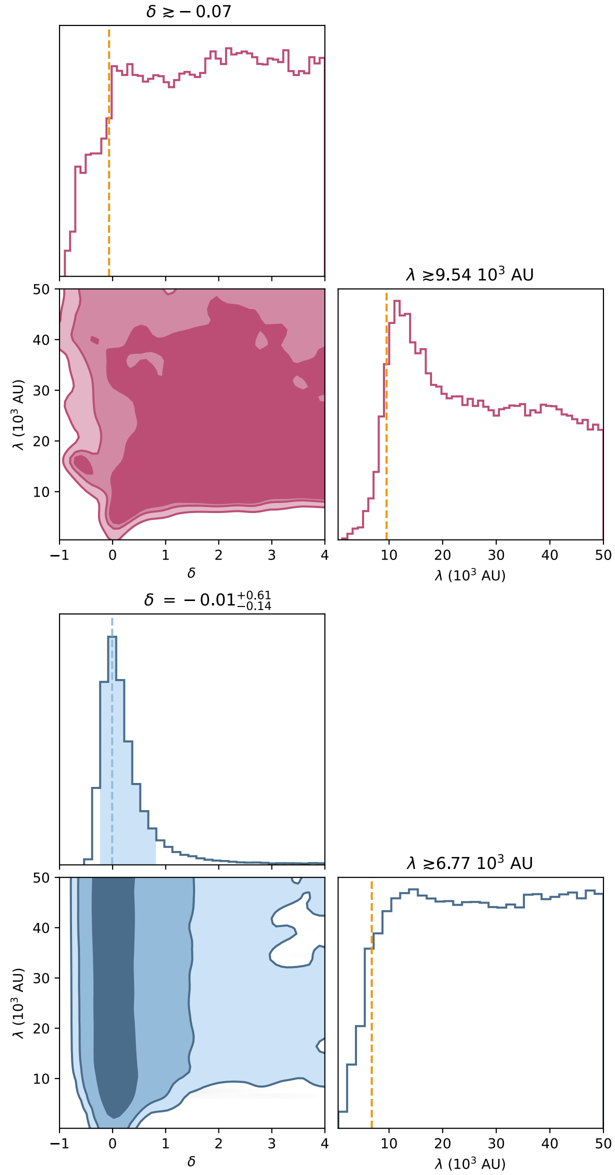

FIG. 1. 68%, 95%, and 99% confidence levels of the posterior probability density distributions for the two $f(R)$-gravity parameters $\{\delta, \lambda\}$. Top: posterior distributions are based only on data from [49]. Bottom: data include also the measurement of the precession provided by GRAVITY Collaboration [34]. This is an inset of the whole corner plot presented in Fig. 5 in the SM.

et al. (2017) [30] in the region of the parameter space that we have analyzed ($\lambda > 100$ AU).

We compared our results with existing constraints on $f(R)$-gravity coming from analyses at both astrophysical and cosmological scales. Specifically, note the following:

(i) Many constraints are available at scales of the Solar System for several different $f(R)$-Lagrangians, e.g., [57,58], but they are not directly comparable with our results. On the other hand, in [59], the orbit of the S2 star is used to constrain the Yukawa-like potential, and strongly positive values of δ are favoured. It is not clear whether in [59], all observational and relativistic effects were taken into account. Moreover, orbits were predicted using Newton's law instead of integrating the geodesic equations. On the contrary, we do not detect any deviation from GR; we take into account all observational and relativistic effects, and we compute the orbits using the geodesic equations. Additionally, we converted our results in the constraints on first and second derivatives of the $f(R)$ Lagrangian. Thus, we obtain $f'(R) = 0.98^{+0.26}_{-0.13}$. On the other hand, the upper limit on the scale length represents improvements of a factor ~ 100 with respect to similar analysis in [60].

(ii) Our results contrast with the ones obtained for elliptical galaxies [61] that pointed out a severe departure from GR constraining $\delta \sim -0.8$ and $\lambda \geq 10$ kpc. Also, their errors are at a level of 10%, making them impossible to reconcile with GR, while we do not detect any departure from GR.

(iii) Our results reach the same precision as the results obtained using a cluster of galaxies [62], but contrast with [63], where authors find a value of f'_0 not compatible with unity and a values of f''_0 weakly compatible with zero (which would mean GR). On the other hand, it is more difficult to directly compare our results with other constraints at the scale of galaxy clusters since the $f(R)$-Lagrangian is different, e.g., Hu-Sawicki model, R^n, among others.

It is worth noting that there are also other constraints based on Parametrized Post-Newtonian (PPN) parameters and pulsar timing, which are at least 5–6 orders of magnitude more accurate in the parameter δ than our results [64]. Nevertheless, our analysis is fully complementary to the other ones and can potentially reach, in the near future, the required accuracy to be competitive with PPN constraints.

V. CONCLUSIONS

The study of the star cluster orbiting around Sgr A* serves to improve our knowledge on the supermassive black hole at the center of our Galaxy, and to probe GR. Knowledge of complete orbits helps to improve the modeling of the black hole itself, which is invisible in the infrared band, measuring both spin and mass of Sgr A*. Currently, the stars' motion can be modeled using Newtonian physics and Kepler's laws with a high degree of accuracy, but a more detailed analysis reveals new deviations from Newtonian motion [34,44]. The ever-changing motion of the S2 star provides another piece of evidence toward Einstein's theory. The rosette effect of a star around a supermassive black hole, known as Schwarzschild precession, has been measured for the first time [34], testing GR in a new regime where gravity is stronger than in the Solar System (and even in binary pulsar systems, which provide some of the best strong-gravity tests right now) [65–67]. The S2 star has been studied for decades [31–34,40,43,44,46,47,68], and its unusual orbit was actually one of the first compelling pieces of evidence that there is a supermassive black hole at the center of the Milky Way [40]. Since it is the closest approaching star to Sgr A*, it plays an important role for testing gravitational theories.

Here, we have computed the orbital precession in the Yukawa-like gravitational potential arising in $f(R)$-gravity, which is given in Eq. (4). Following the same approach as the GRAVITY Collaboration [34], we have a set of 14 parameters that fully describe the orbital motion (they are accurately described in the SM). These parameters also serve to account for observational effects, such as the offset and drift of the reference frame and the Rømer time delay, and relativistic effects, such as the gravitational and Doppler redshift. Then we have two additional parameters, i.e., the strength and the scale length of the gravitational potential, δ and λ respectively, which determine a possible departure from GR. Using the remarkable measurement of the precession of the S2 star [34], we have constrained the strength $\delta = -0.01^{+0.61}_{-0.14}$, while setting a lower limit on the scale length $\lambda > 6300$ AU, as shown in Fig. 1. The latter is fully consistent with the prediction and sensitivity analysis made in Hees *et al.* [30], which means no deviations from GR are measured up to this scale.

Indeed, other stars and the interstellar medium that populates the Galactic centre affect observations [69]. Additionally, other problems are related to Earth's atmosphere [70], turbulence, and absorption or refraction effects [71]. Nevertheless, at infrared wavelengths, photons may pass through the dust clouds unimpeded and the motion of individual stars may be detected [33,72]. The outcomes of these observations include the measurement of the mass of the Milky Way's central black hole: approximately 4 million times the mass of the Sun [32,68]. The next horizon, quite literally, should come from the Event Horizon Telescope [73,74], a separate effort now straining to resolve space-time around the Milky Way's central black hole. Joint observations in different bands, i.e., radio and infrared, are the only avenue towards the detection of effects related to higher order theories of gravity. Finally, we note that the future is particularly promising, with higher precision radio observatories, such as SKA [75] and next-generation Event Horizon Telescope [76], and the next generation of telescopes like the Thirty Meter Telescope, which will offer greatly improved statistics for improving our constraints.

ACKNOWLEDGMENTS

I. D. M. acknowledges support from Ayuda IJCI2018-036198-I funded by MCIN/AEI/10.13039/501100011033 and: FSE "El FSE invierte en tu futuro" o "Financiado por la Unión Europea" "NextGenerationEU"/PRTR. I. D. M. also acknowledges support from the grant "The Milky Way and Dwarf Weights with SpaceScales," funded by University of Torino and Compagnia di S. Paolo

(UniTO-CSP), from Junta de Castilla y Len (SA096P20), and from the Spanish Ministerio de Ciencia, Innovacin y Universidades and FEDER (PGC2018-096038-B-I00). R. D. M. acknowledges support from Consejeria de Educacin de la Junta de Castilla y Len and from the European Social Fund. M. D. L. acknowledges INFN Sez. di Napoli (Iniziative SpecificaTEONGRAV and QGSKY).

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