Letter

# Quarkyonic effective field theory, quark-nucleon duality, and ghosts

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We present a field theoretical description of quarkyonic matter consisting of quark, nucleon, and ghost fields coupling to mesonic degrees of freedom. The ghosts are present to cancel overcounting of nucleon states that are Pauli blocked by the quark Fermi sea. Such a theory becomes an effective field theory of nucleons at low baryon density and as such will reproduce nucleonic matter phenomenology. This theory can accommodate chiral symmetry restoration and the dynamical generation of a shell of nucleons at the Fermi surface. It is valid for finite temperature and density. In such a theory, quark-nucleon duality is accomplished by inclusion of ghost fields so that the nucleons extra degrees of freedom, that are beyond those of quarks, are compensated by the ghost fields.

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## I. INTRODUCTION: ESSENTIAL INGREDIENTS OF QUARKYONIC MATTER

The concept of quarkyonic matter [1] was introduced to explain a remarkable feature of QCD in the limit of a large number of colors, $N_c$ [2,3]. In this limit, fermion loops are suppressed by a factor of $1/N_c$. This means that deconfinement at finite density cannot occur until a quark chemical potential reaches a value $\mu_Q \sim \sqrt{N_c}\Lambda_{\mathrm{QCD}}$. This scale can be parametrically large compared to the QCD scale. For degrees of freedom deep within a Fermi sea, interactions are controlled by exchange interactions, which are at a hard momentum scale, and the effect of these interactions should be phenomenologically accounted for by an effectively deconfined quark Fermi sea. At the Fermi surface, interactions at small angles and small momentum transfers are allowed because there are states not Fermi blocked above the Fermi surface, and one can scatter into these states. The effects of confinement are therefore important near the Fermi surface, and the particle degrees of freedom in this region should be thought of as confined nucleons and mesons.

At finite temperature $T \leq \Lambda_{\mathrm{QCD}}$, for baryon chemical potentials less than the nucleon mass $M_N$, nucleonic degrees of freedom are exponentially suppressed in large $N_c$ by a factor of $e^{-\beta(M_N-\mu_N)}$, where $\mu_N$ is the nucleon chemical potential, and $\beta$ is the inverse of the temperature. This means that there are three distinct regions of strongly interacting matter at finite temperature and density. There is a low temperature and density phase with no baryons that is confined. There is a high density and low temperature phase that is quarkyonic. There is a phase at high temperature, or low temperature and ultrahigh density that is deconfined.

The picture of zero temperature high density quarkyonic matter is thought of as a Fermi sea of quarks surrounded by a Fermi shell of nucleons. There are now models constructed that have this feature arising dynamically [4–6]. At low densities it is energetically favorable to have nucleons. To see this is very simple. The pressure of a nonrelativistic nucleon gas is

$$P^N = \kappa N^N_{\mathrm{d.o.f.}} \frac{(k_F^N)^5}{M_N}, \tag{1}$$

where $\kappa$ is a numerical factor, which is equal to 4 for an isospin symmetric system, and the nucleon Fermi momentum is defined as $k_F^N = \sqrt{\mu_N^2 - M_N^2}$, where $\mu_N$ is the nucleon chemical potential. For a nonrelativistic gas of quarks,

$$P^Q = \kappa N_c N^Q_{\mathrm{d.o.f.}} \frac{(k_F^Q)^5}{M_Q}, \tag{2}$$

where $k_F^Q = \sqrt{\mu_Q^2 - M_Q^2}$. For the quarks, the chemical potential is $\mu_Q = \mu_N/N_c$, and in the additive quark-nucleon model, the constituent quark mass is $M_Q = M_N/N_c$. This means that $k_F^N = N_c k_F^Q$ and that the ratio of pressure of quarks to that of baryons is

$$\frac{P^N}{P^Q} = N_c^3. \tag{3}$$

Therefore, when the system is a dilute gas of quarks, and the interaction energy is weak, then the system is entirely in

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nucleons. At nuclear matter densities $k_F^N \sim \Lambda_{\rm QCD}$, so that the associated quark Fermi momenta are quite small. One can also verify that the preferred phase at low density is nucleonic by computing the energy per baryon for the same baryon number density of quarks and of nucleons. One finds that the energy per baryons of a free gas of constituent quarks always exceed that of nucleons.

As the density increases, hard-core nucleon interactions may make the nucleon phase disfavored. This is because hard-core nucleon interactions are of strength $N_c$, and the Fermi energy of a nucleon can shift by of order of the nucleon mass. This would naively need to be associated with a huge shift in the density of the baryons, $\rho^N \sim (k_F^N)^3 \sim M_N^3 \sim N_c^3\Lambda_{\rm QCD}^3$. This huge increase in density is not realized in quarkyonic matter because the nucleons sit on a shell of decreasing thickness as the Fermi energy of the nucleons increase. The density of the nucleons should be expected to saturate at the density of matter corresponding to the hard cores of nucleons, which is of order $\rho_{\rm hard\ core} \equiv n_0 \sim \Lambda_{\rm QCD}^3$. Nucleonic matter sits on a shell and generates a density of nuclear matter that is approaching the hard-core density. As this density is approached, the baryon number density increases by increasing the quark density. Until the quark Fermi momentum is of order $k_F^Q \sim \Lambda_{\rm QCD}$, this increase associated with the quarks is quite small. As the baryon density associated with quarks slowly increases, the quark and nucleon Fermi energies rapidly rise until there is a quark Fermi sea with a Fermi energy of order $E_Q^F \sim \Lambda_{\rm QCD}$, and the nucleons become relativistic in the Fermi shell with $E_N^F \sim N_c\Lambda_{\rm QCD} \sim M_N$. Different from the models that exhibit a first-order phase transition, quarkyonic matter generates a soft equation of state for small densities, while the rapid increase in the Fermi energies at a slowly varying density leads to a hard equation of state with sound velocities of order one at moderate densities. This is particularly suitable for the neutron star phenomenology [4,5,7].

The central problem to deal with in constructing a field theoretical description of quarkyonic matter is that one needs to have both nucleonic and quark degrees of freedom [8,9]. An effective field theoretical description of nucleons can only describe matter near the Fermi surface. The nucleonic degrees of freedom should not be important inside the Fermi sea of quarks as a consequence of the Pauli blocking. Inside a Fermi sea, the low momentum states are occupied by quarks. A nucleon is composed of quarks and therefore cannot propagate when its momentum is $k^N < N_c k^Q$.

It is useful to have a field theoretical description of quarkyonic matter that is also valid at finite temperature [10]. This involves including pion degrees of freedom and, at least phenomenologically, the effect of meson nucleon interactions. Such a theory should reduce to a theory of nucleons and mesons at low densities and evolve to quarks at high density and temperature. It should allow for the possibility of quarkyonic matter.

It is the purpose of this paper to outline a possible solution to this problem. The central issue that needs to be resolved is the duality between nucleonic and quark descriptions. The nucleon can be thought of as a nucleonic state or as an ensemble of quarks. This means that if quarks occupy low momentum states, then the quarks composing a nucleon cannot occupy these same states. Therefore, if we have a field theoretical model, then we can have a field that corresponds to a nucleon, and a field that corresponds to quarks, so long as it is constrained so that the quark fields associated with these states do not overlap the same states as are occupied by the quarks. This can be accomplished by an unconstrained nucleon field, an unconstrained quark field, and a negative metric nucleon ghost field that fill precisely the same state as the quarks, and whose only purpose is to cancel away the degrees of freedom of the unconstrained nucleon field, whose quark states occupy states already occupied by quarks.

In the paper below, we first argue how such a theory is constructed for free quarks and nucleons. We then argue how such a theory might be generalized to include interactions with meson fields or in a model where nucleon interactions may be phenomenologically accounted for by an excluded volume. Such a theory may provide a model where one can simultaneously study the onset of quarkyonic matter and the restoration of chiral symmetry.

## II. GHOST AND REMOVING UNPHYSICAL STATES

If there is a Fermi sea of quarks with a chemical potential $\mu_Q$, then the nucleon cannot overlap a color singlet state with the quantum number of the nucleon which is composed of quarks. Such states cannot exist in the quark Fermi sea up to a chemical potential $\mu_G \sim N_c\mu_Q$. The density of such states is

$$\rho^G = \frac{1}{1+e^{\beta(N_C E_Q - \mu_G)}} - \frac{1}{1+e^{\beta(N_C E_Q + \mu_G)}}. \tag{4}$$

In the additive quark model, $E_N = N_c E_Q$, and if we also think about this color singlet state embedded in the quark Fermi sea as a nucleon, then the energy of this $N_c$ quark state should be thought of as a nucleon energy. Therefore,

$$\rho^G = \frac{1}{1+e^{\beta(E_N - \mu_G)}} - \frac{1}{1+e^{\beta(E_N + \mu_G)}}. \tag{5}$$

For a noninteracting gas of quarks and nucleons, the density of quarks is

$$\rho^Q = \frac{1}{1+e^{\beta(E_Q - \mu_Q)}} - \frac{1}{1+e^{\beta(E_Q + \mu_Q)}}, \tag{6}$$

and the density of nucleons, constrained not to propagate in the quark Fermi sea, and ignoring what will turn out to be very small contributions from antinucleons and antighosts is

$$\rho^N_{\rm const} = \rho^n = \frac{1}{1+e^{\beta(E_N-\mu_N)}} - \frac{1}{1+e^{\beta(E_N-\mu_G)}}. \qquad (7)$$

This equation is almost trivial since it places the nucleon in a shell of momenta above that of the quark sea, where they will not be Pauli blocked. In the above equations, the contributions from antinucleons and antighosts can almost always be ignored because they are suppressed by a factor of $e^{-M_N/T}$, which is small at the temperatures usually considered.

We now wish to generalize this description to a field theoretical model. We have an unconstrained nucleon field with chemical potential $\mu_N$ and mass $M_N$, a ghost nucleon field with chemical potential $\mu_G \sim N_c\mu_Q$ and mass $M_N$, and quark field with mass $m_Q = M_N/N_c$ and chemical potential $\mu_Q$. The nucleon field will be denoted by $N$, the quark field will be $Q$, and the ghost field will be $G$. The ghost field will have the same Lorentz structure as the nucleonic field. It will satisfy antiperiodic boundary conditions in imaginary time, like the nucleons. It will, however, be a commuting and not an anticommuting field. In a path integral, it will be represented by a $c$-number integration variable rather than a Grassmann algebra variable. The action for such a theory in Euclidean time is

$$\begin{aligned} S_E = \int_0^\beta dt \int_V d^3x \Big\{ & \bar{N}\left(\frac{1}{i}\gamma\cdot\partial - i\mu_N\gamma^0 + M_N\right)N \\ & + \bar{G}\left(\frac{1}{i}\gamma\cdot\partial - i\mu_G\gamma^0 + M_N\right)G \\ & + \bar{Q}\left(\frac{1}{i}\gamma\cdot\partial - i\gamma^0\mu_Q + M_Q\right)Q\Big\}. \end{aligned} \qquad (8)$$

It is useful to define the propagator

$$S(\mu_N, M) = \frac{1}{\frac{1}{i}\gamma\cdot\partial - i\mu_N\gamma^0 + M_N}. \qquad (9)$$

Now if we integrate over a Grassmann variable, then the path integral for the partition function will give

$$Z_N = \det{}^{-1} S(\mu_N, M_N), \qquad (10)$$

where the integration over the ghost $c$-number field yields,

$$Z_G = \det S(\mu_G, M_N). \qquad (11)$$

The formula for the grand potential is obtained from

$$\begin{aligned} \Omega = g\frac{1}{\beta V}\mathrm{Tr}\{ & \ln(S(\mu_N, M_N)) + N_c \ln(S(\mu_Q, M_Q)) \\ & - \ln(S(\mu_G, M_N))\}. \end{aligned} \qquad (12)$$

The factor of $N_c$ for the quarks comes from the fact that there are $N_c$ quark fields. If our quarks and nucleons are isodoublets, then the degeneracy factor is $g = 2$. As expected, the ghost contribution to the action has the opposite sign from that of the nucleons and is present to cancel out precisely the contribution of modes of the nucleon, where the quark states of the nucleon are already occupied by quark states. It is straightforward to evaluate these determinants of the various propagators by standard methods of diagonalizing in momentum space and performing a contour integral representation for the Matsubara frequency sum. The grand potential is obtained as

$$\begin{aligned} \Omega = -gT\int \frac{d^3p}{(2\pi)^3}\{ & \ln[1+e^{-\beta(E_N(\vec{p})-\mu_N)}] \\ & - \ln[1+e^{-\beta(E_N(\vec{p})-\mu_G)}] \\ & + N_c \ln[1+e^{-\beta(E_Q(\vec{p})-\mu_Q)}] + (\mu\to-\mu)\}, \end{aligned} \qquad (13)$$

where one can notice that the ghosts are present to subtract the contribution of the pressure of the nucleons due to Pauli blocking. Since the entropy and number density follow by the ordinary thermodynamic relations term by term in the expression above, except for an overall minus sign for the ghost contribution, all of the expressions for the pressure, energy density, entropy, and number density are simply the nucleon and quark contributions minus that of the ghost nucleons.

There is a contribution from the vacuum that we ignore in the above expression since it is $\mu$ and $T$ independent. Moreover, the contributions from the ghosts and nucleons cancel in the vacuum contribution leaving only the effects of quarks. While this is trivial in the free theory, when interactions are included, one may need to include the vacuum contribution when finding the proper minimum with respect to expectation values of fields. In the interacting vacuum, the effects of nucleons and ghosts still cancel completely since for each nucleon loop there is a contribution from a ghost loop of the opposite sign. The vacuum energy is entirely determined by the quarks and their interactions. This removes any problem associated with over counting when one includes both quarks and ghosts in a theory.

When interactions are included, one can explore various theories to see if one can obtain a reasonable shell structure for quarkyonic matter. As one can find in Eq. (13), the pressure is obtained in terms of the chemical potentials $\mu_i$, which determine the quasiparticle number densities $n_i$. Since this quarkyonic matter concept is the theory about the quasifree quarks and confined quarks (baryon) at the given total baryon number density, the pressure itself is not always the suitable quantity for determination of the matter configurations. For example, if presumed chemical potentials are given, then the number densities of the particles (thickness of the shell structure) and subsequent bulk properties can be obtained by extremization of the pressure. On the other hand, if the total baryon number density is given and the chemical potential depends on the particle density [$\mu_i(n_i)$] via the interaction Lagrangian, then the shell-like configuration can be determined by extremization of the free energy density ($F/V \equiv f = \epsilon - Ts = -p + \mu n$, where $\epsilon$ and $s$ denotes the energy and entropy density, respectively).

## III. DETERMINING THE PARAMETERS OF THE QUARKYONIC THEORY

The theory we have considered so far is free field theory. Interactions can be included in various ways. For example, we could imagine a field theory with nucleons and quarks interacting with mesons. The ghost and the nucleons would need to be constrained to have identical interactions with mesons in order to guarantee the precise cancellation of ghosts and nucleons in the region of occupied quark states. For the quarks, there are various possibilities. One can imagine a theory of quarks interacting with a pion and sigma field in order to generate phenomenological quark masses. Alternatively, one can treat the quarks fully in QCD interacting with a gluon field. The issue one would need to deal with ultimately is how to treat the interactions of the quarks with the nucleons. In an effective field theory with pions and sigma meson, this can be done; however, within QCD this is less easy to treat when vector mesons are brought into the problem. For now, we leave the problem of a specific effective theory for quark and nucleon interactions as an open problem.

Nevertheless, we see that our theory with a quark, ghost, and nucleon chemical potential can be defined. We need to ask how the three chemical potentials are determined. We first need a relationship between the chemical potential of ghosts and quarks. The simplest possibility is for a constituent quark model, where $N_c M_Q = M_N$. In this case the choice $\mu_G = N_c \mu_Q$ is reasonable. If there are small deviations from constituent quark scaling, $\Delta M = M_Q - M_N/N_c$, then we clearly want to begin having ghost states only when the first quark states appear. In this case, one should shift the chemical potentials by their threshold masses,

$$\mu_G = N_c(\mu_Q - \Delta M). \tag{14}$$

Another case is when a vector mean field is present. In this case, the effect of a vector mean field is to shift the energy by a constant and does not affect wave functions. If we have no mean field, then the above criteria of $N_c$ scaling should account for the orthogonality of quark and nucleon wave functions when the quark states are occupied. Shifting the energies by a constant, the mean field energy per nucleon or quark, should not affect this orthogonality. Therefore, the relationship between chemical potential, with the effect of the mean field constant shift in energies subtracted away, should satisfy $N_c$ scaling. Inclusion of different kinds of interactions would require a more exhaustive analysis, and it is work for the near future that will be reported elsewhere.

Although it will involve some careful thinking to establish the relationship between ghost chemical potential and that of quarks in various theoretical models, one can assume that some relationship is properly determined. Then the theory has only two independent parameters: the quark and nucleon chemical potentials, or alternatively the nucleon density $n_N$, and the quark baryon density $n_Q^B = n_Q/N_c$ because the ghost density is determined by that of the quarks. In terms of the total baryon density, $n_B = n_N - n_G(n_Q^B) + n_Q^B$, the configuration that minimizes the energy density is

$$d\epsilon = \frac{\partial\epsilon}{\partial n_N} dn_N - \frac{\partial\epsilon}{\partial n_G(n_Q^B)} dn_G(n_Q^B) + \frac{\partial\epsilon}{\partial n_Q^B} dn_Q^B. \tag{15}$$

Using $dn_B = dn_N + (1 - dn_G(n_Q^B)/dn_Q^B) dn_Q^B$ and the relations

$$\frac{\partial}{\partial n_i}\epsilon = \mu_i, \tag{16}$$

at zero temperature, we can find the proper extremization condition for the energy density at fixed total baryon density,

$$d\epsilon = -\mu_N\left(1 - \frac{dn_G(n_Q^B)}{dn_Q^B}\right) dn_Q^B + \left(\mu_Q^B - \mu_G\frac{dn_G(n_Q^B)}{dn_Q^B}\right) dn_Q^B = 0, \tag{17}$$

which leads to following relation:

$$\mu_N = (\mu_N - \mu_G)\frac{dn_G(n_Q^B)}{dn_Q^B} + \mu_Q^B, \tag{18}$$

where $\mu_Q^B \simeq N_c \mu_Q$ and $dn_G(n_Q^B)/dn_Q^B \simeq N_c^3$ in the weakly interacting quasiquark sea approximation.[1] Under this extremization constraint, the variation of the energy density can be expressed as

[1]In the most simple case where $M_Q = M_N/N_c$ and no interactions are considered, we have $\mu_Q^B = N_c \mu_Q$ and $dn_G(n_Q^B)/dn_Q^B = N_c^3$.

$$d\epsilon = \mu_N dn_N + \left(\mu_Q^B + (\mu_N - \mu_G)\frac{dn_G(n_Q^B)}{dn_Q^B}\right)dn_Q^B - \mu_N dn_G(n_Q^B) = \mu_N dn_B, \tag{19}$$

and this implies that there is a relationship between the chemical potential of baryons and nucleons,

$$\mu_B = \frac{d\epsilon}{dn_B} = \mu_N. \tag{20}$$

This relation will allow the study of all the thermodynamics of the system, as well as the determination of the thickness of the shell $\Delta = k_F^N - k_F^G$. The minimization of the energy density as described here is a general procedure, which is independent of the interactions that may be included on both nucleons and quark sectors. However, it is worth to mention that the ghost Fermi momentum $k_F^G$ may depend on the interactions included in the quark sector and on the confinement mechanism of the constituent quarks.

## IV. VARIOUS MODELS

It is useful to put this note into the context of previous computations. In Ref. [4], a shell-like structure was proposed for quarkyonic matter. The thickness of the shell was determined by fixing the nucleon density associated with the shell. No attempt was made to determine the thickness of the shell from an underlying theory. In Ref. [5], the thickness of the shell was determined in an excluded volume model of nuclear matter [11,12]. In such a theory, the thickness of the shell is dynamically determined. The problem with this theory is that the quarks and nucleons are treated as freely interacting quasiparticles, with the nucleon propagating freely in a volume that does not include the nuclear core size. Such a model is difficult to tie to more first principles theories of nuclear matter and quarks, and it is difficult to compute effects, due to chiral symmetry restoration. While the model demonstrates that the underlying effect of nuclear repulsion may be responsible for the formation of a nucleon shell, an underlying field theoretical treatment is lacking.

In Ref. [10], the effects of finite temperature are attempted to be included by a method similar to what is used in this paper, except that it is assumed that the underlying quark distribution fully occupies the phase space below some value of momentum. As such, thermal fluctuations of the contributions of quarks cannot be included, and the applicability of the theory is only for very low temperatures. There is not a connection with an underlying field theoretical description that will naturally allow thermal fluctuations of the quarks.

In Ref. [13], an attempt was made to provide an underlying field theoretical treatment of quarkyonic matter. This was done in the context of specific models of nucleon and quark interactions, and the considerations were specific to those models. A momentum scale that separates quarks and nucleons was proposed, but there was no attempt to determine this momentum scale.

The AdS-CFT correspondence was used in Ref. [14] to construct a string theory model of quarkyonic matter. This paper is closest in spirit to what we do here. It is argued that a separation of a quarkyonic phase, a nucleon phase, and a quark phase arises dynamically and that in the quarkyonic phase the various fraction of quarks and nucleons can be dynamically determined. These considerations generalize to finite temperature. A phase diagram in the $\mu_B - T$ plane was found. The computations are in the context of string theory, and what we do in this paper is set up this same problem dynamically within field theory.

## V. SUMMARY AND CONCLUSIONS

The quarkyonic model above allows for a dynamical study of the formation of a Fermi shell of nucleons. One simply computes the pressure, requires that the total baryon number is fixed, and then searches for an extremum as a function of the quark chemical potential. It is known that in an excluded volume model, such an extremum exists, and there is good reason to expect this in generic field theory models of nucleon interactions. Nevertheless, this needs to be carefully investigated.

There are many theoretical questions which may now be addressed. How does chiral symmetry restoration affect the formation of quarkyonic matter? How does one include the effects of nucleon interactions to make a viable model of nuclear matter that properly matches on to known physics at nuclear matter density and below? How does quarkyonic matter appear in the $T, \mu_B$ plane? How does confinement appear within this phase diagram? What form of nucleonic interactions can generate a phenomenologically successful theory of quarkyonic matter? Answers to these questions are currently being pursued.

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