Letter

Novel 3D supersymmetric massive Yang-Mills theory

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We construct three-dimensional, $\mathcal{N} = 1$ off-shell supersymmetric massive Yang-Mills (YM) theory whose YM equation is "third-way" consistent. This means that the field equations of this model do not come from variation of a local action without additional fields, yet the gauge-covariant divergence of the YM equation still vanishes on shell. To achieve this, we modify the massive Majorana spinor equation so that its supersymmetry variation gives a modified YM equation whose bosonic part coincides with the third-way consistent pure massive YM model.

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I. INTRODUCTION

In the absence of couplings to other fields, tensors that constitute equations of motion of massless spin-1 and spin-2 fields are conserved (i.e., have vanishing covariant divergence), when they stem from an action. This ensures that they can be consistently coupled to conserved charge/energy currents. Usually, this conservation holds identically, by virtue of, e.g., Bianchi identities. In [1], a novel mechanism was discovered, in which conservation of the field equation only holds on shell, i.e., upon using the equations of motion for the spin-1 or spin-2 fields themselves. This was dubbed "third-way consistency" (see [2] for a review). Specifically, the theory of [1] is a three-dimensional (3D) pure gravity theory, in which Einstein's field equation is modified by adding an extra curvature squared interaction term. This extra term does not come from variation of a local action for the metric alone and as a consequence the covariant divergence of the modified field equation does not vanish identically. Nevertheless, the modified field equation still makes sense, since the covariant divergence of the additional term vanishes if one uses the field equation again. Other such 3D gravity models were obtained in [3,4] and third-way consistent 3D massive Yang-Mills (YM) theory [5], and interacting p-form theories in general dimensions [6] have been found as well.

The importance of third-way consistent gravity stems from the fact that it potentially forms a new class of unitary 3D gravity theories that can be used to study aspects of quantum gravity. Three-dimensional gravity theories, whose action contains higher-derivative/higher-curvature terms, such as topologically massive gravity [7,8] or new massive gravity [9], are often considered as toy models for gravity that, unlike 3D Einstein gravity, contain propagating (albeit massive) spin-2 degrees of freedom. Typically however, in the presence of a negative cosmological constant, these theories exhibit negative-energy black hole solutions, whenever the spectrum of perturbative degrees of freedom is ghost-free. This phenomenon is referred to as the "bulk-boundary clash" and it implies nonunitarity of the theories under consideration. Third-way consistent 3D gravity of [1] can evade this bulk-boundary clash and thus lead to unitary 3D gravity models.

Constructing supersymmetric versions of third-way consistent 3D gravities is clearly important if one wishes to ameliorate their ultraviolet behavior or to study their nonperturbative regime in a better-controlled setting. At present however, supersymmetric versions of third-way consistent theories have not been obtained yet. In this paper we will improve on this by supersymmetrizing the third-way consistent massive YM theory of [5]. The latter can be viewed as a deformation of the well-known 3D topologically massive Yang-Mills (TMYM) theory that was constructed a long time ago [7,8,10]. TMYM is a spin-1 analog of topologically massive gravity, and supersymmetric generalizations of it have been formulated; see, e.g., [11,12]. The model of [5] is obtained by adding an extra term to the field equation of TMYM, which is quadratic in the YM field strength and cannot be derived from an action that only involves the YM field.

Here, we will supersymmetrize the model of [5], by constructing a third-way consistent deformation of the equations of motion of supersymmetric TMYM, such that the resulting bosonic and fermionic equations are mapped

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to each other under supersymmetry. Since we will directly supersymmetrize the equation of motion of [5], we will work with an off-shell YM supermultiplet, for which the supersymmetry algebra closes without using the field equations, in order to avoid having to modify the supersymmetry transformation rules. In this paper, we will study the simplest possible choice, namely a single off-shell $\mathcal{N}=1$ YM supermultiplet which only has a Majorana spinor in addition to the YM gauge field.

II. A REVIEW OF 3D, $\mathcal{N}=1$ TOPOLOGICALLY MASSIVE SUPER-YANG-MILLS

The $\mathcal{N}=1$ supersymmetric version of the third-way consistent massive 3D YM theory of [5] that we construct in this paper corresponds to a deformation of $\mathcal{N}=1$ supersymmetric TMYM theory. We will therefore first review the latter here. This section also introduces the notation and conventions used in the rest of this paper (see also [13]).

We will consider YM theory for an arbitrary non-Abelian gauge group G with structure constants f^I_{JK} (with I, J, $K=1,\ldots,\dim(G)$). The off-shell $\mathcal{N}=1$ YM supermultiplet then consists of the gauge field A^I_μ and a Majorana spinor χ^I, both transforming in the adjoint of G. In our conventions, the mass dimensions of A^I_μ and χ^I are given by 1 and 3/2, respectively. The gauge-covariant field strength $F^I_{\mu\nu}$ and covariant derivative $D_\mu X^I$ of any object X^I in the adjoint representation of G are given by

$$F^I_{\mu\nu}=2\partial_{[\mu}A^I_{\nu]}+f^I_{JK}A^J_\mu A^K_\nu,$$
$$D_\mu X^I=\partial_\mu X^I+f^I_{JK}A^J_\mu X^K. \tag{1}$$

Note that the following Bianchi identity then holds: $D_{[\mu}F^I{}_{\nu\rho]}=0$.

The supersymmetry transformation rules of A^I_μ and χ^I are given by (see, e.g., [14])

$$\delta A^I_\mu=-\bar\epsilon\gamma_\mu\chi^I,\qquad \delta\chi^I=\frac{1}{8}\gamma^{\mu\nu}F^I_{\mu\nu}\epsilon, \tag{2}$$

where the Majorana spinor ϵ denotes the supersymmetry parameter. One can check that the supersymmetry algebra then indeed closes off shell on the fields A^I_μ and χ^I.

The equations of motion of $\mathcal{N}=1$ off-shell supersymmetric TMYM are given by

$$\psi^I=0\quad\text{and}\quad\xi^I_\mu=0\quad\text{with} \tag{3}$$

$$\psi^I\equiv\gamma^\mu D_\mu\chi^I+2\mu\chi^I\quad\text{and} \tag{4}$$

$$\xi^I_\mu\equiv D_\nu F^{\nu\mu I}+\mu\epsilon^{\mu\nu\rho}F^I_{\nu\rho}+2f^I_{KL}\bar\chi^K\gamma^\mu\chi^L, \tag{5}$$

where μ is a mass parameter. These equations of motion can be derived from an action whose explicit expression we refrain from giving here, as it will not be needed in this paper. Note that the bosonic equation of motion $\xi^I_\mu=0$ consists of two terms that only involve the spin-1 field A^I_μ and a third term that describes a coupling to a spin-1/2 source current $j^I_\mu=f^I_{JK}\bar\chi^J\gamma_\mu\chi^K$. The covariant divergence of j^I_μ vanishes upon using the fermionic equation of motion $\psi^I=0$. For consistency, the covariant divergence of the first two terms of ξ^I_μ should then also vanish and this holds identically.

The bosonic and fermionic equations of motion, given in (3), (4), and (5), transform into each other under the supersymmetry transformations (2), as

$$\delta\psi^I=\frac{1}{4}\xi^I_\mu\gamma^\mu\epsilon,\qquad \delta\xi^I_\mu=-\bar\epsilon\gamma_\mu{}^\nu D_\nu\psi^I. \tag{6}$$

We will use the form of the first of these transformation rules as our guiding principle in constructing the supersymmetric version of the model of [5] in the next section.

III. THIRD-WAY CONSISTENT 3D, $\mathcal{N}=1$ MASSIVE SUPER-YANG-MILLS

Before addressing its $\mathcal{N}=1$ supersymmetric version, let us first briefly review the third-way consistent massive YM theory constructed in [5]. Its source free-field equation for an arbitrary gauge group G is obtained by adding an extra term to the equation of motion of TMYM [itself obtained by setting χ^I to zero in (5)]:

$$\epsilon_\mu{}^{\nu\rho}D_\nu\tilde F^I_\rho+2\mu\tilde F^I_\mu+\frac{2}{m}\epsilon_\mu{}^{\nu\rho}f^I_{JK}\tilde F^J_\nu\tilde F^K_\rho=0, \tag{7}$$

where m is another mass parameter and we introduced the dual-field strength $\tilde F^I_\mu$ notation as

$$\tilde F^I_\mu=\frac{1}{2}\epsilon_\mu{}^{\nu\rho}F^I_{\nu\rho}\quad\Leftrightarrow\quad F^I_{\mu\nu}=-\epsilon_{\mu\nu}{}^\rho\tilde F^I_\rho. \tag{8}$$

The special cases $\mu=0$ and $\mu=2m$ of this model were studied earlier in [15,16], respectively.

One can see that (7) cannot be the Euler-Lagrange equation of a gauge-invariant local action for the YM vector field alone [5] and hence its left-hand side is not identically conserved. Indeed, if we apply a covariant derivative D^μ to the left-hand side of (7), the first two terms give zero identically whereas the interaction term does not. The covariant divergence of this interaction term is however found to vanish, if one uses (7) (along with the Jacobi identity) again, which is the essence of the third-way consistency mechanism. It is also possible to add a matter current $\mathcal{J}^I_\mu$ to (7) as

$$\epsilon_\mu{}^{\nu\rho}D_\nu\tilde F^I_\rho+2\mu\tilde F^I_\mu+\frac{2}{m}\epsilon_\mu{}^{\nu\rho}f^I_{JK}\tilde F^J_\nu\tilde F^K_\rho=\mathcal{J}^I_\mu, \tag{9}$$

provided that it satisfies

$$D_\mu\mathcal{J}^{\mu I}+\frac{4}{m}f^I_{JK}\tilde F^J_\mu\mathcal{J}^{\mu K}=0, \tag{10}$$

to maintain third-way consistency [5].

We will now discuss the $\mathcal{N}=1$ off-shell supersymmetric extension of the field equation (7). Since (7) does not come from an action without extra auxiliary fields, we will construct this extension by applying the Noether procedure at the level of equations of motion. The advantage of having off-shell supersymmetry is then that the supersymmetry variations (2) do not get modified during the procedure. Notice that (7) is simply a deformation of the equation defined by (5) when fermions are set to zero. Therefore, to achieve our goal we will start from the fermionic equation of motion (4) of the $\mathcal{N}=1$ supersymmetric TMYM theory and modify it so that its supersymmetry variation contains a term proportional to $\Xi^I_\mu\gamma^\mu\epsilon$ with the bosonic part of $\Xi^I_\mu=0$ given by (7). As in supersymmetric TMYM, the full $\Xi^I_\mu=0$ equation will be taken as the new bosonic field equation. We will then see that the fermionic and bosonic field equations obtained transform into each other under supersymmetry. In this case, the bosonic field equation will be of the form (9) and we will thus also have to check that the resulting fermionic current $\mathcal{J}^I_\mu$ satisfies the third-way consistency condition (10).

Note that (7) contains $\epsilon_\mu{}^{\nu\rho}f^I_{JK}\tilde{F}^J_\nu\tilde{F}^K_\rho$ as an additional term to (5), and the supersymmetry variation of the spinor (2) immediately suggests adding a term of the form $f^I_{JK}\tilde{F}^J_\mu\gamma^\mu\chi^K$ to the spinor field equation (4). However, the supersymmetry variation of this term gives, in addition to what we want, a term of the form $f^I_{JK}\bar{\chi}^J\gamma^\mu D_\mu\chi^K\epsilon$. Due to the modification of the spinor field equation this term is neither zero on shell, nor does it have the gamma-matrix structure that we want. Hence, we need to modify the spinor field equation further to cancel this term, either identically or on shell. The form of this unwanted piece indicates that on-shell cancellation can be achieved by adding an extra term that is cubic in χ^I. We thus consider the following modification of (4) as a candidate equation of motion of χ^I:

$$\Psi^I \equiv \gamma^\mu D_\mu\chi^I + 2\mu\chi^I + \frac{2}{m}f^I_{JK}\tilde{F}^J_\mu\gamma^\mu\chi^K + af^I_{JK}f^K_{MN}\bar{\chi}^M\gamma^\mu\chi^N\gamma_\mu\chi^J = 0, \tag{11}$$

where "a" is a constant to be determined. As mentioned above, we determine a by requiring that the supersymmetry transformation of Ψ^I is on-shell proportional to $\Xi^I_\mu\gamma^\mu\epsilon$. After repeatedly using Fierz, Jacobi, and gamma-matrix identities, we find that with the choice $a=\frac{16}{3m^2}$ one gets

$$\delta\Psi^I = \frac{1}{4}\Xi^I_\mu\gamma^\mu\epsilon - \frac{4}{m}f^I_{JK}\bar{\chi}^J\epsilon\Psi^K, \tag{12}$$

where

$$\begin{aligned}\Xi^I_\mu \equiv{}& D^\nu F^I_{\nu\mu} + \mu\epsilon_\mu{}^{\nu\rho}F^I_{\nu\rho} - \frac{1}{m}f^I_{JK}\epsilon^{\rho\sigma\nu}F^J_{\nu\mu}F^K_{\rho\sigma} \\ &+\left(2-\frac{16\mu}{m}\right)f^I_{JK}\bar{\chi}^J\gamma_\mu\chi^K + \frac{8}{m}\epsilon_\mu{}^{\nu\rho}f^I_{JK}\bar{\chi}^J\gamma_\nu D_\rho\chi^K \\ &+\frac{16}{m^2}f^I_{JK}f^K_{MN}\bar{\chi}^M\gamma^\nu\chi^N F^J_{\nu\mu} \\ &+\frac{32}{m^3}\epsilon_\mu{}^{\nu\rho}f^I_{KL}f^K_{JO}f^L_{MN}\bar{\chi}^J\gamma_\nu\chi^O\bar{\chi}^M\gamma_\rho\chi^N.\end{aligned} \tag{13}$$

A lengthy computation then shows that the supersymmetry variation of Ξ^I_μ is given by

$$\begin{aligned}\delta\Xi^I_\mu ={}& -\bar{\epsilon}\gamma_\mu{}^\nu D_\nu\Psi^I + \frac{4}{m}f^I_{JK}\bar{\epsilon}\chi^K\Xi^J_\mu \\ &-\frac{16}{m^2}f^I_{JK}f^K_{MN}\bar{\chi}^M\gamma^\nu\chi^N\bar{\epsilon}\gamma_{\mu\nu}\Psi^J.\end{aligned} \tag{14}$$

Together with (12), we thus see that Ψ^I and Ξ^I_μ transform into each other under supersymmetry. We can then propose $\Psi^I=0$ and $\Xi^I_\mu=0$ as a supersymmetric set of equations of motion. Since the bosonic part of $\Xi^I_\mu=0$, i.e., the first three terms in (13), coincides with that of the equation of motion of pure third-way consistent massive YM theory [5] given in (7), we see that $\Psi^I=0$ and $\Xi^I_\mu=0$ can be identified as the equations of motion of the $\mathcal{N}=1$ supersymmetric version of (7). Note however that in the presence of supersymmetry, the equation of motion (7) of the pure theory gets modified by extra terms that represent a coupling of the spin-1 gauge vector A^I_μ to the spin-1/2 gaugino χ^I.

Summarizing, we propose the following equations of motion for the third-way consistent 3D, $\mathcal{N}=1$ supersymmetric massive Yang-Mills theory:

$$\begin{aligned}&\epsilon_\mu{}^{\nu\rho}D_\nu\tilde{F}^I_\rho + 2\mu\tilde{F}^I_\mu + \frac{2}{m}\epsilon_\mu{}^{\nu\rho}f^I_{JK}\tilde{F}^J_\nu\tilde{F}^K_\rho = \mathcal{J}^I_\mu, \\ &\gamma^\mu D_\mu\chi^I + 2\mu\chi^I + \frac{2}{m}f^I_{JK}\tilde{F}^J_\mu\gamma^\mu\chi^K + \frac{16}{3m^2}f^I_{JK}f^K_{MN}\bar{\chi}^M\gamma^\mu\chi^N\gamma_\mu\chi^J = 0, \\ &\text{with}\quad \mathcal{J}^I_\mu = \left(\frac{16\mu}{m}-2\right)j^I_\mu + \frac{4}{m}\epsilon_\mu{}^{\nu\rho}D_\nu j^I_\rho + \frac{16}{m^2}\epsilon_\mu{}^{\nu\rho}f^I_{JK}\tilde{F}^J_\nu j^K_\rho - \frac{32}{m^3}\epsilon_\mu{}^{\nu\rho}f^I_{JK}j^J_\nu j^K_\rho, \\ &\text{where}\ j^I_\mu = f^I_{JK}\bar{\chi}^J\gamma_\mu\chi^K.\end{aligned} \tag{15}$$

We still need to verify that our YM equation is indeed third-way consistent. This will be done in the next subsection.

A. Third-way consistency

Note that our matter source $\mathcal{J}^I_\mu$ that appears in (15) is of the form

$$\mathcal{J}^I_\mu = c_1 j^I_\mu + c_2 \epsilon_\mu{}^{\nu\rho} D_\nu j^I_\rho + c_3 \epsilon_\mu{}^{\nu\rho} f^I_{JK} \tilde{F}^J_\nu j^K_\rho + c_4 \epsilon_\mu{}^{\nu\rho} f^I_{JK} j^J_\nu j^K_\rho, \tag{16}$$

where $c_1 = \frac{16\mu}{m} - 2$, $c_2 = \frac{4}{m}$, $c_3 = \frac{16}{m^2}$, $c_4 = -\frac{32}{m^3}$. To have a third-way consistent system, $\mathcal{J}^I_\mu$ should satisfy (10) as we discussed above. The form of the current $\mathcal{J}^I_\mu$ in terms of j^I_μ is the same as in [5]; however, unlike [5] we will not assume that j^I_μ is conserved. Using (16), one can derive that on shell the following holds:

$$D_\mu \mathcal{J}^{\mu I} + \frac{4}{m} f^I_{JK} \tilde{F}^{\mu J} \mathcal{J}^K_\mu = c_1 D_\mu j^{\mu I} + \left(c_2 - 2\mu c_3 + \frac{4}{m} c_1\right) f^I_{JK} \tilde{F}^J_\mu j^{\mu K} + (2c_4 + c_2 c_3) \epsilon^{\mu\nu\rho} f^I_{JK} D_\mu j^J_\nu j^K_\rho + \left(c_3^2 + \frac{8}{m} c_4\right) \epsilon^{\mu\nu\rho} f^I_{JK} f^J_{MN} \tilde{F}^M_\mu j^N_\nu j^K_\rho + \left(c_3 - \frac{4}{m} c_2\right) \epsilon^{\mu\nu\rho} f^I_{JK} D_\mu j^J_\nu \tilde{F}^K_\rho. \tag{17}$$

Note that in order to derive this result, we needed to use the bosonic equation of motion (9). For our $j^I_\mu = f^I_{JK} \bar{\chi}^J \gamma_\mu \chi^K$, we have that $D_\mu j^{\mu I} \neq 0$; instead, an explicit computation [using the fermionic equation of motion of (15)] gives that

$$D_\mu j^{\mu I} = -\frac{2}{m} f^I_{JK} \tilde{F}^{\mu J} j^K_\mu. \tag{18}$$

We can thus replace $D_\mu j^{\mu I}$ by $-\frac{2}{m} f^I_{JK} \tilde{F}^{\mu J} j^K_\mu$. Using this in (17), it is easy to see that the coefficients of all terms on the right-hand side of (17) vanish and hence the consistency condition (10) is satisfied for our model.

It was realized in [6] that the model constructed in [5] can be found starting from the flat connection equation $F^I_{\mu\nu} = 0$ and then shifting the connection A^M_μ with an arbitrary linear combination of $\tilde{F}^M_\mu$ and j^M_μ. After this, it is possible to add a multiple of $\tilde{F}^M_\mu$ and j^M_μ to this equation provided that either $D^\mu j^M_\mu = 0$ as in [5] or $D^\mu j^M_\mu$ is on-shell proportional to one of the terms on the right-hand side of (17) as in (18) without spoiling the third-way consistency. This also shows that $\mathcal{J}^I_\mu$ will always have the structure given in (16) in terms of j^I_μ. It is remarkable that this mechanism works for the spinor equation in our model as well. Indeed, a comparison of the spinor field equation of the topologically massive super-Yang-Mills theory (4) with ours (11) shows that the latter can be obtained from the former by shifting A^M_μ as

$$A^M_\mu \to A^M_\mu + \alpha \tilde{F}^M_\mu + \beta j^M_\mu, \tag{19}$$

where constants are fixed uniquely as $\alpha = \frac{2}{m}$ and $\beta = -\frac{16}{3m^2}$ by requiring the supersymmetry.

IV. CONCLUSION

The theory presented in (15), which gives the first example of a supersymmetric third-way consistent model, is the main result of this paper. Its equations of motion cannot be derived from a local action, with the field content considered in this paper. Note however that, in case $\mu \neq m$, an action for the bosonic part of our model can be constructed upon introducing an auxiliary vector field [5]. This action takes the form of the difference of two Chern-Simons terms with an interaction term. This suggests that (in case $\mu \neq m$) an action for our results that involves bosonic and fermionic auxiliary fields can be obtained by supersymmetrizing the bosonic action of [5]. It would be interesting to investigate this further. This could for instance be done by considering a superspace Chern-Simons action for two vector supermultiplets and studying whether a suitable interaction term can be written in superspace. Note that the fact that a possible action formulation should include an extra fermionic auxiliary field is already suggested by how the fermionic equation of motion appears in its own supersymmetry transformation [see (12)] [17]. There are also various extensions of our model to consider such as coupling with $\mathcal{N} = 1$ supersymmetric scalar or gravity multiplets and constructing $\mathcal{N} > 1$ supersymmetric versions. In [6] it was shown that one can obtain higher derivative extensions of [5] by shifting the connection (19) with further terms which are not necessarily conserved. It would be interesting to see whether a supersymmetric extension would still be possible for such deformations.

We hope that our construction will provide some insight for finding supersymmetric versions of the third-way consistent gravity [1,3,4] and p-form theories [6]. As we saw in (19) the extra terms that appear in our model can be understood as coming from shifting the gauge connection A^I_μ with bosonic and fermionic current 1-forms of the initial theory. We expect this to be a key feature of all such models. In 3D gravity examples [1,3,4] the shift occurs in the spin connection in their first-order formulation and we anticipate this to be supplemented with appropriate fermionic current terms in the supersymmetric case. Observe that the supersymmetry variation of our spinor field equation involves not only the YM field equation but also contains a term proportional to itself [see (12)] and a similar conclusion holds for the YM field equation [see (14)]. This could be a generic feature of third-way consistent supersymmetric theories.

The model of [5] exhibits a Higgs mechanism [15,16] and is also related to multi M2-branes of 11D supergravity [18]. Moreover, the presence of higher derivative terms in the bosonic third-way consistent models can be understood as spontaneous breaking of a local symmetry as was illustrated for [1] in [19]. It would be desirable to clarify these connections for our supersymmetric model too.

Finally, third-way consistent gravity models already found an important application as possible toy models for 3D quantum gravity [2]. The fact that we were able to supersymmetrize [5] can be taken as a further indication that generic third-way consistent models physically make sense (at least at the classical level). It could then be interesting to study whether the novel 3D YM theory considered here finds applications in low-dimensional (e.g., condensed- matter) physics.

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