

Entangled quantum Unruh Otto engine is more efficient

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We propose a relativistic quantum Otto cycle between an entangled state of two qubits and their composite excited (or ground) state whose efficiency can be greater than the usual single qubit quantum Otto engine. The hot and cold reservoirs are constructed by providing uniform accelerations to these qubits along with the interaction between the background field and individual qubits. The efficiency, as measured from one of the qubits' frame, not only depends on the energy gap of the states but also the relative acceleration between them. For lower acceleration of our observer's qubit compared to the other one, the cycle is more efficient than the single qubit quantum Otto engine. Furthermore, a complete protocol to construct such a cycle is being provided.

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The combination of quantum mechanics (QM) and relativity [appropriately defined through the language of quantum field theory (QFT)] brings many surprising results for us, like Hawking [1,2] and Unruh [3,4] effects and it is expected to bring more in the future. Along with the efforts of combining QM and relativity, continuous efforts have been made to study the foundations of the thermodynamics of the quantum systems [5]. If a quantum system, (e.g., qubits which we will be considering here) has Hamiltonian $H_0(t)$ and is described by density operator $\rho(t)$, the energy at any instant is determined by the expectation value $\langle E(t) \rangle = \text{Tr}[\rho(t)H_0(t)]$. The variation of it from time t_i to t_f yields the definitions of heat transfer and work done on the system as [5]

$$\langle Q \rangle = \int_{t_i}^{t_f} \text{Tr} \left(\frac{d\rho(t)}{dt} H_0 \right) dt, \quad (1)$$

and

$$\langle W \rangle = \int_{t_i}^{t_f} \text{Tr} \left(\rho(t) \frac{dH_0}{dt} \right) dt, \quad (2)$$

respectively. Within the area of thermodynamics of quantum systems, intense focus is devoted to quantum heat engines. Although the concept was introduced in the year 1959 [6], a lot of attention and interest have imparted after works by Kieu [7,8] who proposed the single-qubit *quantum Otto engine* (QOE). Similar to the *classical Otto engine* (COE), QOE consists of two adiabatic and two isochoric stages. The efficiency of this engine is given by

$$\eta_0 = 1 - \frac{\omega_1}{\omega_2}. \quad (3)$$

In contrast to COE, production of finite work in QOE is possible provided we have the condition $\omega_1 T_H > \omega_2 T_C$, where T_H (T_C) is the temperature of hot (cold) reservoir and ω_1 (ω_2) is the energy gap before (after) adiabatic process.

Recently in a relativistic setup, a QOE has been introduced [9] in the light of the Unruh effect [called as *Unruh quantum Otto engine* (UQOE)]. In the adiabatic stages, the qubit is moving with a constant velocity. The hot and cold reservoirs for the other two stages of the cyclic process are being mimicked by allowing this qubit to uniformly accelerate in addition to the interaction with the background massless real scalar field. The pointlike qubit is taken to be a monopole. Its behavior is identical to QOE. The efficiency is again η_0 , and the condition for producing work also comes out to be similar. The same analysis has been extended in [10] for different background quantum fields and interactions. Although the efficiency remains unchanged, the work output comes out to be different for different background fields and interactions. The UQOE has also been constructed between the degenerate states [11]. Another important phenomenon in QM is the quantum entanglement. People have found that the entanglement between two relativistic accelerating detectors has a profound impact on the Unruh effect [12–15]. On the other side, the entanglement between two states is relativistically observer-dependent [16–18]—an accelerated observer will see the degradation of entanglement. A similar conclusion has also been drawn with localized field modes by trapping them within a cavity [19]. Relativistic effects on quantum teleportation with respect to a nonuniformly moving frame have been investigated as well, and it has been observed that the upper bound to the optimal fidelity is degraded due to this motion [20]. The equivalence principle lays a

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profound path for understanding several features of gravity through uniformly accelerated frames. Therefore, understanding the accelerated frame can enlighten few aspects of gravity. Nevertheless, a direct encounter of gravity has also been investigated in few instances. For example, it has been shown that the gravity induces decoherence of stationary matter superposition states [21,22] (also see [23]). Moreover, entanglement harvesting between two causally disconnected detectors is possible under a relativistic setup [24–29]. Within this relativistic setup, harvesting of coherence in the initial state of the scalar field is possible as well [30,31]. These findings provide robust evidences of new understandings of our physical systems when combining quantum mechanics with relativity in terms of QFT language.

Here, combining entanglement and relativity, we propose an Otto engine composed of two qubits. We construct the Otto cycle between their composite entangled and the excited (or ground) states. The steps of the cycle are similar to those of UQOE (we refer to [9] for details of the four stages of UQOE cycle), and therefore we name it as *entangled Unruh quantum Otto engine* (EUQOE). The description of these stages is discussed later. Surprisingly, the efficiency η_E , measured from the frame of one of the qubits, can be larger than η_0 by tuning the accelerations of the two qubits. If the engine is initially in the maximally entangled state and our frame of reference is not accelerating, then $\eta_E = 2\eta_0$. We find that this enhancement arises as a result of both entanglement and relativity. A protocol for building a EUQOE with $\eta_E > \eta_0$ is being provided at the end. We also show in our protocol that in any case, η_E is less than unity.

Our system is composed of double two-level identical qubits (sometimes we call them detectors). Their proper times are denoted by τ_1 and τ_2 , respectively. The total internal free Hamiltonian of the detectors, following [12,32], is chosen to be

$$H_0 = \frac{\omega(\tau)}{2} \left(\frac{d\tau_1}{d\tau} S_1^z \otimes \mathbb{I}_2 + \frac{d\tau_2}{d\tau} \mathbb{I}_1 \otimes S_2^z \right), \quad (4)$$

where $S_j^z = (|e_j\rangle\langle e_j| - |g_j\rangle\langle g_j|)$ with $|g_j\rangle$ and $|e_j\rangle$ represent ground and excited states of j th detector, respectively. ω is the energy difference between the levels of each qubit. Here, $|g_j\rangle$ has energy $-\omega/2$ while $|e_j\rangle$ corresponds to energy eigenvalue $\omega/2$ and \mathbb{I}_j denotes identity operator for j th detector. τ denotes the proper time of the detector from which all the measurements will be done. In other words, τ is the proper time for the observer, who is analyzing the process. For our present purpose, we choose this to be $\tau = \tau_1$ as everything will be measured from the first detector's frame. Hence, we will substitute $d\tau_1/d\tau = 1$. On the other hand τ_1 and τ_2 are not the same when the two detectors move with different velocities or accelerations; i.e., in a general case $d\tau_2/d\tau \neq 1$. They are shown to be related

through the relation $\tau_2 = \alpha\tau_1$, where α is the Lorentz factor corresponding to the relative velocity between them during their constant velocity motions while α is the ratio of accelerations of the detectors during their uniform accelerated motions (see [14] for details. We have also presented an analysis in the Supplemental Material [33]). If the qubits are moving with different velocities or accelerations then $\alpha \neq 1$.

Here, the two-qubit system has the following four normalized eigenstates:

$$\begin{aligned} E_e &= \omega, & |e\rangle &= |e_1\rangle|e_2\rangle; \\ E_s &= 0, & |s\rangle &= \frac{1}{\sqrt{2}}(|e_1\rangle|g_2\rangle + |g_1\rangle|e_2\rangle); \\ E_a &= 0, & |a\rangle &= \frac{1}{\sqrt{2}}(|e_1\rangle|g_2\rangle - |g_1\rangle|e_2\rangle); \\ E_g &= -\omega, & |g\rangle &= |g_1\rangle|g_2\rangle. \end{aligned} \quad (5)$$

We consider each detector as a monopole and it interacts with the background massless real scalar field through linear coupling of the form $\mu_i\kappa(\tau_i)m_i\Phi(X_i(\tau_i))$ (i.e., Unruh De-Witt type) [34]. Here $\Phi(X)$ is the *massless real scalar field* and $\kappa(\tau_i)$ denotes the switching function which decides whether the interaction is on or off. This is unity during the interaction and vanishes otherwise. m_i represents the monopole operator for the i th detector, given by

$$m_i = |g_i\rangle\langle e_i| + |e_i\rangle\langle g_i|, \quad (6)$$

and τ_i is the proper time of this detector. If our observer's proper time is τ then the interaction term for the evolution process can be represented as $\int d\tau_i\mu_i\kappa(\tau_i)m_i\Phi(X_i(\tau_i))$ where, since τ_i is a function of τ , we have to use $d\tau_i = (d\tau_i/d\tau)d\tau$. Therefore, the total interaction Hamiltonian for the two qubits-massless real scalar field system is chosen as

$$H_{\text{int}} = \sum_{i=1,2} \mu_i\kappa(\tau_i)m_i\Phi(X_i(\tau_i)) \frac{d\tau_i}{d\tau}. \quad (7)$$

Similar argument has been considered in [35,36] for the derivative coupling case (for monopole coupling cases see [15,26,27,29]). As the detectors are identical, we keep the coupling constants as $\mu_1 = \mu_2 = \mu$. The above introduction of the scalar field Φ brings the role of QFT in our analysis, which we shall encounter later.

This two-qubits system has four eigenstates [see Eq. (5)]. So several Otto cycles can be constructed between any of the two eigenstates. It is already known from earlier analysis [15] that if the detectors are in uniform acceleration then the transition only from symmetric (i.e., $|s\rangle$) or antisymmetric (i.e., $|a\rangle$) to excited (i.e., $|e\rangle$) or ground (i.e., $|g\rangle$) states is possible whereas from $|g\rangle$ to $|e\rangle$ is not possible. Moreover, the transition rate for $|s\rangle \rightarrow |g\rangle$ is same as that of $|s\rangle \rightarrow |e\rangle$. Similarly, transition rates between $|a\rangle \rightarrow |g\rangle$ and

$|a\rangle \rightarrow |e\rangle$ are also equal. Motivated by these facts we choose our Otto cycle between $|s\rangle$ and $|e\rangle$ (or $|g\rangle$). The other choice can be between $|a\rangle$ and $|e\rangle$ (or $|g\rangle$). Note that $|s\rangle$ and $|a\rangle$ are maximally entangled states. To keep our discussion general, we choose our Otto cycle between the entangled state

$$|\chi\rangle = b_1|e_1\rangle|g_2\rangle + b_2|g_1\rangle|e_2\rangle, \quad (8)$$

and $|e\rangle$ (consideration of $|g\rangle$ does not change the final conclusion). The coefficients b_1 and b_2 satisfy $|b_1|^2 + |b_2|^2 = 1$ in order to make $|\chi\rangle$ a normalized state. Here the values of b_1 and b_2 can be chosen to be $b_1 = 1/\sqrt{2} = b_2$ for $|s\rangle$ and $b_1 = 1/\sqrt{2}$, $b_2 = -1/\sqrt{2}$ for $|a\rangle$. For the moment we keep them in general form and at the end we will consider these specific values. Therefore our initial density matrix for the two-qubits system is

$$\rho_{A_0} = p|e\rangle\langle e| + q|\chi\rangle\langle\chi|, \quad (9)$$

where, $q = 1 - p$. Initially, the energy gap between $|e\rangle$ and $|\chi\rangle$ is $\omega = \omega_1$. For the two-qubits plus the scalar field system we choose the density matrix as $\rho_0^I = \rho_{A_0} \otimes \rho_{f_0}$, where $\rho_{f_0} = |0_M\rangle\langle 0_M|$ by considering the field initially is in Minkowski vacuum $|0_M\rangle$. In our analysis, since we are interested only to qubits system, all the relevant quantities will be evaluated by taking trace over field states.

Let us now describe the four stages of our EUQOE. As mentioned earlier, we aim to calculate work and heat in the frame of detector 1. For this, we require the duration of interaction in terms of proper time τ_1 . This duration is calculated by analysing the kinematics of the entire process in reference to a fixed inertial frame associated with Minkowski coordinates (T, X) . The steps in the cycle are similar to those proposed in UQOE [9] for the single-qubit system. (1) The first phase is the adiabatic phase, where no heat is exchanged with the environment. In this phase, two detectors move with the constant velocities $-v_1$ and $-v_2$, respectively, and the switching function in Eq. (7) is chosen to be $\kappa(\tau_i) = 0$. The velocities are measured with reference to the fixed inertial frame as mentioned above. The time intervals for this stage are $\Delta\tau_1^1$ and $\Delta\tau_2^1$. In this process, ω_1 changes to ω_2 at the cost of work being done on the system. (2) Once the energy difference changes to ω_2 , the qubits start to accelerate uniformly with proper accelerations a_{H1} and a_{H2} , respectively. In this stage, the interaction with the background scalar field is on; i.e., $\kappa(\tau_i) = 1$. This process continues till they reach the velocities $+v_1$ and $+v_2$ from $-v_1$ and $-v_2$. The time duration of this stage with respect to individual detectors are $\Delta\tau_1^2$ and $\Delta\tau_2^2$, respectively. The general relations between the Minkowski coordinates and the proper time of a uniformly accelerated detector, moving along the Minkowski X -axis, are

$$T = \frac{1}{a} \sinh(a\tau); \quad X = \frac{1}{a} \cosh(a\tau). \quad (10)$$

From (10) we find the proper time τ at which the velocity is v as $v = \tanh(a\tau)$. Therefore the time at which the velocity is $-v$ is $-\tau$. So the total duration for changing from $-v$ to v is 2τ . This yields

$$\Delta\tau_1^2 = \frac{2}{a_{H1}} \operatorname{arctanh}(v_1); \quad \Delta\tau_2^2 = \frac{2}{a_{H2}} \operatorname{arctanh}(v_2). \quad (11)$$

The relation between these time intervals during the accelerated stage, following [33], is

$$a_{H1}\Delta\tau_1^2 = a_{H2}\Delta\tau_2^2. \quad (12)$$

This implies that we can take the velocities of both the detectors to be same; i.e., $v_1 = v_2 = v$ (say). Therefore in stage 1 they are moving with the same velocity $-v$ and at the end of stage 2 their velocity changes to $+v$. Since in this stage they are at different proper accelerations, the durations of interaction with the background scalar field are different. Due to their accelerations, the interaction with scalar fields yields transition between the energy levels in the individual detectors and each qubit feels the Minkowski vacuum as thermal bath [3,4]. Our observer (the frame of first qubit) will then see himself in a thermal bath and then the Otto engine will absorb heat from the environment. (3) In this stage, the detectors stop accelerating and they move with constant velocity $+v$. The interaction is also turned off. Now, again this is an adiabatic phase with time durations $\Delta\tau_1^3$ and $\Delta\tau_2^3$, where ω_2 changes to ω_1 . Similar to stage 1, the engine will perform work without any heat exchange with the environment. (4) Finally, in the last part, similar to the second step, both the detectors decelerate with a_{C1} and a_{C2} from velocity $+v$ to $-v$. We again switch on the interaction between the detectors and the scalar field for durations $\Delta\tau_1^4$ and $\Delta\tau_2^4$. The two detectors will see cold reservoirs, and due to similar reasons, as discussed in stage 2, they will reach the same velocity but with different durations. Here our observer will find the background as a cold bath, and hence the Otto engine will reject heat to the environment. For future purpose denote $\alpha_{a_H} = a_{H1}/a_{H2}$ and $\alpha_{a_C} = a_{C1}/a_{C2}$.

In step 2, the density matrix will change from $\rho_{A_0} \rightarrow \rho_{A_0} + \delta\rho^H$. In step 4, it will change from $\rho_{A_0} + \delta\rho^H \rightarrow \rho_{A_0} + \delta\rho^H + \delta\rho^C$. Hence, for the cyclic process, we impose a constraint:

$$\delta\rho^H + \delta\rho^C = 0. \quad (13)$$

H and C stand for heating and cooling, respectively. Using the definitions (1) and (2) we will now find the amount of work and heat transfer in each stage. To calculate them, we need to know how the density operator changes with time.

The results, till second-order perturbation, are derived using the standard procedure [33].

In the adiabatic process (i.e., in stage 1 and stage 3), the work is done on or done by the engine without any heat exchange with the environment. The quantum definition of work is given by (2). The internal free Hamiltonian for the two qubit system is (4), which for our chosen basis (i.e., $\{|e_1, e_2\rangle, |e_1, g_2\rangle, |g_1, e_2\rangle, |g_1, g_2\rangle\}$), has a matrix representation as follows:

$$H_0 = \frac{\omega}{2} \begin{pmatrix} (1 + \alpha) & 0 & 0 & 0 \\ 0 & (1 - \alpha) & 0 & 0 \\ 0 & 0 & (-1 + \alpha) & 0 \\ 0 & 0 & 0 & (-1 - \alpha) \end{pmatrix} \equiv \omega h_\alpha. \quad (14)$$

h_α denotes the matrix whose explicit form has been obtained through $\tau_2 = \alpha\tau_1$ [33] with the choice $\tau = \tau_1$. We put α in the subscript which will be denoted as α_v during the motions with constant velocity while it will be α_a during the motions with uniform accelerations. In stages, 1 and 3, both the detectors move with a constant velocity and only the separation between the energy levels changes. Whereas, the population of the energy levels remains the same. So ω is a function of time while other quantities remain fixed. Hence, work done in stage 1 is

$$\langle W \rangle_1 = (\omega_2 - \omega_1) \text{Tr}(\rho_{A_0} h_{\alpha_v}), \quad (15)$$

and the same in stage 3 is

$$\langle W \rangle_3 = (\omega_1 - \omega_2) \text{Tr}((\rho_{A_0} + \delta\rho^H) h_{\alpha_v}). \quad (16)$$

The total work done on the system is then

$$\langle W \rangle_{\text{tot}} = \langle W \rangle_1 + \langle W \rangle_3 = (\omega_1 - \omega_2) \text{Tr}(\delta\rho^H h_{\alpha_v}), \quad (17)$$

which depends on the change of density matrix during the heating process in stage 2.

Heat exchange between the system and the environment takes place in stages 2 and 4 as there is population change of the energy levels. The quantum heat is defined by (1). Since here H_0 is constant of time, no work is done on or by the system. In stage 2, the amount of heat absorbed by the engine is

$$\langle Q \rangle_2 = \omega_2 \text{Tr}(\delta\rho^H h_{\alpha_H}), \quad (18)$$

and the heat transferred to the environment in stage 4 turns out to be

$$\langle Q \rangle_4 = \omega_1 \text{Tr}(\delta\rho^C h_{\alpha_C}) = -\omega_1 \text{Tr}(\delta\rho^H h_{\alpha_C}). \quad (19)$$

In the last step (13) has been used. Therefore net heat absorbed by our engine is

$$\langle Q \rangle_{\text{tot}} = \langle Q \rangle_2 + \langle Q \rangle_4 = \omega_2 \text{Tr}(\delta\rho^H h_{\alpha_H}) - \omega_1 \text{Tr}(\delta\rho^H h_{\alpha_C}). \quad (20)$$

An important difference between the single-qubit system and our present two-qubits system is as follows. Usually, for the cyclic process the condition to be imposed is given by (13). Note that it appears both in single-qubit and in our system. But in present engine we have additional constraint to be imposed in order to maintain the conservation of energy in the cyclic process; i.e., $\langle W \rangle_{\text{tot}} + \langle Q \rangle_{\text{tot}} = 0$. This is trivially satisfied in single qubit system, but here we need to impose the following relation

$$\omega_2 \text{Tr}(\delta\rho^H h_{\alpha_H}) - \omega_1 \text{Tr}(\delta\rho^H h_{\alpha_C}) = (\omega_2 - \omega_1) \text{Tr}(\delta\rho^H h_{\alpha_v}). \quad (21)$$

We will comeback to these conditions later.

The efficiency of a cyclic system is the ratio between the total work done by the system and heat absorbed by the same system; i.e., $\eta_E = -\langle W \rangle_{\text{tot}} / \langle Q \rangle_2$. Therefore, for our Otto cycle it turns out to be

$$\eta_E = \left(1 - \frac{\omega_1}{\omega_2}\right) \frac{\text{Tr}(\delta\rho^H h_{\alpha_v})}{\text{Tr}(\delta\rho^H h_{\alpha_H})} = \eta_0 \frac{\text{Tr}(\delta\rho^H h_{\alpha_v})}{\text{Tr}(\delta\rho^H h_{\alpha_H})}, \quad (22)$$

where η_0 is given by (3). This is quite a general expression that depends on the chosen states to construct the Otto cycle. For our case, this is given by (9). Now we aim to find η_E for our specific configuration.

To evaluate (22) it is now sufficient to calculate $\text{Tr}(\delta\rho^\alpha h_{\alpha'})$ in which at the end we will choose $\alpha' = \alpha_v = 1$ and $\alpha' = \alpha_{aH}$ with $\alpha = \alpha_{aH}$ to get explicit forms of numerator and denominator. As we are interested only in the two-qubits system, this term appeared after tracing over field states. Therefore, the trace operation in this term (denoted by ‘‘Tr’’ here), which is to be evaluated to calculate the efficiency, is over the basis of two-qubits system. During the heating stage, ρ is changing and velocity changes from $-v$ at $-\tau$ to v at τ . Therefore we choose the interval of τ_1 for interaction as $[-\tau_a, \tau_a]$. With this we find [33] for $|\chi\rangle = |s\rangle$ and $p = 0$ (i.e., initially the system is in $|\chi\rangle$ and so $q = 1$)

$$\begin{aligned} \text{Tr}(\delta\rho^\alpha h_{\alpha'}) &= i\mu^2 \alpha(1 + \alpha') [e^{i\omega(\alpha-1)\tau_a} + e^{-i\omega(\alpha-1)\tau_a}] \\ &\times \int_{-\tau_a}^{\tau_a} \int_{-\tau_a}^{\tau_a} \sin(\omega(\alpha\tau'_1 - \tau''_1)) \\ &\times G_{12}(\tau'_1, \tau'_2(\tau'_1)) d\tau'_1 d\tau'_2. \end{aligned} \quad (23)$$

In the above we denote $G_{ij}(\tau'_i, \tau'_j) = \langle \Phi_i(\tau'_i) \Phi_j(\tau'_j) \rangle$ which is the positive frequency Wightman function for real massless scalar field. The final form, as we are measuring from the frame of first detector, is achieved by expressing second detector’s proper time in terms of that of the first one. Then (22) gives us the measure of efficiency as

$$\eta_E = \eta_0 \left(\frac{2}{1 + \alpha_{a_H}} \right). \quad (24)$$

The same can also be obtained for choosing antisymmetric entangled state as the initial one, i.e., for $|\chi\rangle = |a\rangle$.

The result (24) implies that the efficiency can be greater than η_0 if one chooses $\alpha_{a_H} < 1$; i.e., if $a_{H_1} < a_{H_2}$; or in other words the second qubit must accelerate more than the first one. It implies that the second detector must be closer to $X = \pm T$ null surface than the other one. By increasing the difference between the accelerations with the above condition, we can have a more efficient Otto engine. Otherwise, the efficiency of our entangled system is less than that of the single-qubit system. Suppose, in the second stage, the first detector is not accelerating (i.e., it is either at rest or in uniform velocity) while the second one is accelerating. In this case we have $\alpha_{a_H} = (a_{H_1}/a_{H_2}) = 0$. Note that although $a_{H_1} = 0$, still the whole system will see the background as a thermal bath since the other qubit is in acceleration and the initial state is an entangled one. Then the efficiency is given by $\eta_E = 2\eta_0$.

In the analytical analysis done so far, we have seen that different variables affect the process. They appear in $\delta\rho^H$ and hence finally appear in the expression for efficiency. We now aim to provide a protocol that will allow one to find an Otto cycle with $\eta_E > \eta_0$ for appropriate choices of various variables. We first list the variables we have to investigate and the conditions those are required. The variables are α_{a_H} , τ_a , α_{a_C} and the conditions to satisfy are as follows. (i) Cyclic condition (13) along with the energy conservation, given by (21): A *sufficient* (not necessary) condition for simultaneous satisfaction of these two is $\omega_2 h_{\alpha_{a_H}} - \omega_1 h_{\alpha_{a_C}} = (\omega_2 - \omega_1) h_{\alpha_v}$; i.e.,

$$\frac{\alpha_{a_H} \omega_2 - \omega_2 + \omega_1}{\omega_1} = \alpha_{a_C}, \quad (25)$$

with $\alpha_{a_C} > 0$. (ii) In order to have positive work done by the system and also positivity of heat absorbed and heat transferred, impose $\text{Tr}(\delta\rho^H h_{\alpha'}) > 0$ with $\alpha' = \alpha_v$, $\alpha' = \alpha_{a_H}$ and $\alpha' = \alpha_{a_C}$ for these three different quantities. Second condition together with (21) then implies that $(\omega_2 - \omega_1) \times \text{Tr}(\delta\rho^H h_{\alpha_v}) < \omega_2 \text{Tr}(\delta\rho^H h_{\alpha_{a_H}})$. Then, since $\eta_0 < 1$, by (22) we have $\eta_E < 1$. As $\alpha_{a_C} > 0$, (25) implies $\eta_0 < \alpha_{a_H}$. Now, since we want $\eta_E > \eta_0$, then according to earlier discussion one has $\alpha_{a_H} < 1$. Therefore $0 < \eta_0 < 1$ implies that for our process we need to choose α_{a_H} such that

$$0 < \eta_0 < \alpha_{a_H} < 1. \quad (26)$$

Consequently if we choose α_{a_H} from Eq. (26) then from Eq (25) we get that,

$$0 < \alpha_{a_C} < \alpha_{a_H}. \quad (27)$$

The argument is as follows. Let $\alpha_{a_C} > \alpha_{a_H}$, then (25) implies $\alpha_{a_H} > 1$ which violates (26). Hence this is not possible, so we have at most $\alpha_{a_C} \leq \alpha_{a_H}$. If $\alpha_{a_C} = \alpha_{a_H}$, then (25) implies they must be equal to unity, which is again in contrary to (26). Therefore, we must have (27). Let us come back to the second condition again. For the present case one finds

$$\text{Tr}(\delta\rho^H h_{\alpha'}) = \pm 2(1 + \alpha') I_1. \quad (28)$$

Positive sign is for $|s\rangle$ while the negative sign is for $|a\rangle$. In general I_1 is a function of α_{a_H} , ω_2 and τ_a (for explicit expressions in (1 + 1) and (1 + 3) spacetime dimensions, refer to [33]). Now for a choice of α_{a_H} , ω_2 , satisfying (26) and (27), we have to choose τ_a , the half time duration of acceleration of first detector during heating, such that the cyclic condition along with condition (ii) is being satisfied. If one chooses τ_a such that $I_1 > 0$, then initial state must be $|s\rangle$ in order to satisfy condition (ii). Whereas if $I_1 < 0$ for a τ_a , then one needs to consider $|\chi\rangle = |a\rangle$. This describes a complete protocol to build a cycle for $\eta_E > \eta_0$ with $\eta_E < 1$.

To summarize, here we proposed a quantum Otto engine using two qubits in the lights of quantum entanglement and relativity. We found that entanglement and relativity together might be useful to get enhanced efficiency. The efficiency becomes double to that of QOE when the qubits initially are in a maximally entangled state, and the frame of measurement is not accelerating during the heating process. If the acceleration of the frame of reference is less than that of the other qubit, then one has $\eta_E > \eta_0$. We also provided a detailed protocol to choose the available parameters in order to obtain enhancement of efficiency.

This analysis shows a possibility of building a QOE with greater efficiency and in turn may suggest an indirect approach for verifying the Unruh effect. Therefore, if an experimental setup for EUQOE can be built, then by measuring the efficiency, one may investigate the properties of Unruh radiation. Moreover, the dependence of efficiency on the accelerations of the qubits can be helpful to set up an apparatus for indirect experimental verification of the Unruh effect. The investigation in this direction is kept as a future goal.

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