

Rapid transition of G_{eff} at $z_t \simeq 0.01$ as a possible solution of the Hubble and growth tensions

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The mismatch in the value of the Hubble constant from low- and high-redshift observations may be recast as a discrepancy between the low- and high-redshift determinations of the luminosity of Type Ia supernovae, the latter featuring an absolute magnitude which is ≈ 0.2 mag lower. Here, we propose that a rapid transition in the value of the relative effective gravitational constant $\mu_G \equiv \frac{G_{\text{eff}}}{G_N}$ at $z_t \simeq 0.01$ could explain the lower luminosity (higher magnitude) of local supernovae, thus solving the H_0 crisis. In other words, here the tension is solved by featuring a transition at the perturbative rather than background level. A model that features $\mu_G = 1$ for $z \lesssim 0.01$ but $\mu_G \simeq 0.9$ for $z \gtrsim 0.01$ is trivially consistent with local gravitational constraints but would raise the Chandrasekhar mass and so decrease the absolute magnitude of type Ia supernovae at $z \gtrsim 0.01$ by the required value of ≈ 0.2 mag. Such a rapid transition of the effective gravitational constant would not only resolve the Hubble tension but it would also help resolve the growth tension as it would reduce the growth of density perturbations without affecting the Planck/ Λ CDM background expansion.

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I. INTRODUCTION

The mismatch in the determination of the Hubble constant using two different well-understood calibrators—the Cepheid-calibrated supernovae Ia (SnIa) and the CMB-calibrated sound horizon at last scattering—is perhaps the most important open problem of modern cosmology [1–3]. The latest CMB constraint from the Planck Collaboration [4], Table 2] is

$$H_0^{P18} = 67.36 \pm 0.54 \text{ km s}^{-1} \text{ Mpc}^{-1}, \quad (1)$$

while the latest local determination by SH0ES reads [5]:

$$H_0^{R20} = 73.2 \pm 1.3 \text{ km s}^{-1} \text{ Mpc}^{-1}. \quad (2)$$

This 9% discrepancy, corresponding to more than 4σ , is the so-called H_0 crisis.

II. MISMATCH IN THE SnIa ABSOLUTE MAGNITUDE

As pointed out in [6,7], it is useful to look at the H_0 crisis via the corresponding constraints on the absolute

magnitude M_B of SnIa. Propagating, via BAO measurements, the CMB constraint on the recombination sound horizon r_d to the SnIa absolute magnitude M_B using the parametric-free inverse distance ladder of [8], one obtains:

$$M_B^{P18} = -19.401 \pm 0.027 \text{ mag}, \quad (3)$$

while, using the demarginalization method of [9], the constraint of equation (2) implies:

$$M_B^{R20} = -19.244 \pm 0.037 \text{ mag}. \quad (4)$$

These two determinations are in tension at 3.4σ . Note that these values are relative to the calibration of the Pantheon supernova dataset [10]: supernovae Ia become standard candles only after standardization and the method used to fit supernova Ia light curves, and its parameters, can influence the inferred value of M_B .

The local constraint of equation (4) comes from the astrophysical properties of anchors, Cepheids and the white dwarfs responsible for the SnIa explosions. The CMB determination of Eq. (3) comes instead from the combined constraining power of CMB, BAO, and SnIa observations. More precisely, first, CMB and BAO constrain the luminosity distance-redshift relation $d_L(z)$ and so the distance modulus $\mu(z)$:

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$$\mu(z) \equiv m_B(z) - M_B = 5 \log_{10} \left[\frac{d_L(z)}{10 \text{ pc}} \right]. \quad (5)$$

Then, the Pantheon dataset, by constraining the SnIa apparent magnitudes m_B , produces a calibration of M_B . Here, one exploits the fact that SnIa are standard candles with an *a priori* unknown M_B .

Seen from this point of view, the H_0 crisis is more of an astrophysical problem than a cosmological one. Indeed, as the local calibration is performed via Cepheid stars at redshifts less than 0.01, it seems that a solution to this crisis could be the existence of a mechanism that changes the physics of SnIa so that they are dimmer at $z < 0.01$ as compared to their luminosity at higher z .

A simple phenomenological approach is to assume a transition in the value of the absolute magnitude M_B at a redshift $z_t \simeq 0.01$ [11]:

$$M_B(z) = \begin{cases} M_B^{\text{R20}} & \text{if } z \leq z_t \\ M_B^{\text{R20}} + \Delta M_B & \text{if } z > z_t \end{cases}, \quad (6)$$

where the needed gap in luminosity is approximately related to the corresponding gap in H_0 according to [see Eq. (5)]:

$$\Delta M_B \equiv M_B^{\text{P18}} - M_B^{\text{R20}} \approx 5 \log_{10} \frac{H_0^{\text{P18}}}{H_0^{\text{R20}}} \approx -0.2. \quad (7)$$

Figure 1 illustrates this scenario showing the inferred SnIa absolute magnitudes $M_{B,i} = m_{B,i} - \mu(z_i)$ of the binned Pantheon sample [10] for a Planck/ Λ CDM luminosity distance. The data are inconsistent with the determination of Eq. (4) but they become consistent if there is the M_B transition proposed in Eq. (6). In other words, a model

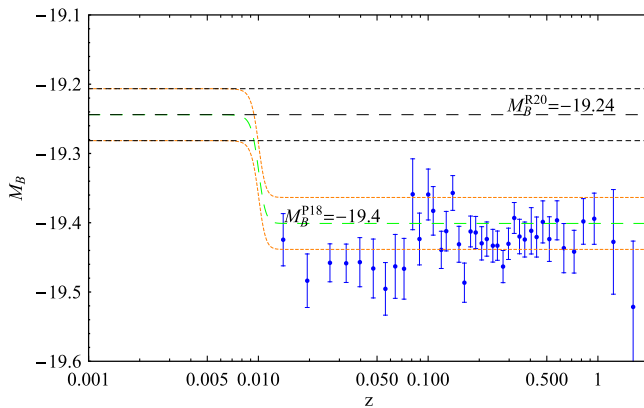


FIG. 1. The inferred SnIa absolute magnitudes $M_{B,i} = m_{B,i} - \mu(z_i)$ of the binned Pantheon sample [10] under the assumption of a Planck/ Λ CDM luminosity distance. The data are inconsistent with the M_B^{R20} of Eq. (4) but they become consistent if there is a transition in the absolute magnitude with amplitude $\Delta M_B \approx -0.2$. Note that in this model there is no transition in the Hubble function $H(z)$.

which features this transition and with a phenomenology similar to the one of the standard Λ CDM model would be compatible with the local Cepheid calibration, SnIa, BAO, and CMB. Note that here, contrary to the models discussed in [6,7,11–14], there is no transition in the Hubble function $H(z)$, which remains as in the standard Λ CDM model. As discussed in the introduction, this model addresses the H_0 tension directly at the level of the supernova absolute luminosity. In other words, differently from previous works, here the tension is solved by featuring a transition at the perturbative rather than background level.

III. THE GRAVITATIONAL TRANSITION MODEL AND THE H_0 CRISIS

As we will now argue, a sudden change in the value of the effective gravitational constant G_{eff} could induce the M_B transition of Eq. (6) and explain, in addition, the reduced growth rate of perturbations that is suggested by the $\sigma_8 - \Omega_{0m}$ tension.

By “effective gravitational constant” we refer to the strength of the gravitational interaction and not to the Planck mass related G_* which determines the cosmological expansion rate [15] (see [16] for a model with a time-dependent G_* that explains the accelerated expansion without a cosmological constant). In other words, we are considering a scenario in which perturbations are suppressed but the background expansion does not deviate from the one of the phenomenologically successful standard Λ CDM model, for which the low-redshift expansion rate $H(z)$ is of the form

$$H(z) = H_0^{\text{P18}} \sqrt{\Omega_{0m}(1+z)^3 + 1 - \Omega_{0m}}. \quad (8)$$

A SnIa explosion occurs when the mass of a white dwarf reaches the critical Chandrasekhar mass m_c by accreting matter from a companion. The constancy in time of this critical mass is the cornerstone on which the “SnIa standard candle” hypothesis is based. Even though SnIa at $z > 0.01$ seem consistent with the hypothesis of a constant M_B [17] and, therefore, a smooth variation of m_c is unlikely, a sudden transition of m_c at $z < 0.01$ can not be easily excluded.

The Chandrasekhar mass depends on both G_{eff} and the mass per electron m' according to [18]:

$$m_c \simeq \frac{3}{m'} \left(\frac{\hbar c}{G_{\text{eff}}} \right)^{3/2}. \quad (9)$$

In the standard model, this result is independent of the accretion history and of the white dwarf progenitor details [19] and leads to $m_c \simeq 1.4 M_\odot$. A possible sudden transition of a fundamental constant would induce a corresponding transition of both the Chandrasekhar mass and therefore the SnIa peak absolute luminosity L [20].

A reasonable hypothesis is that L increases as m_c increases or equivalently as G_{eff} (and/or m') decreases [21].

Assuming a fixed m' (and fine structure constant), this may be expressed via a power law dependence

$$L \sim G_{\text{eff}}^{-\alpha}, \quad (10)$$

where $\alpha > 0$ is of $O(1)$. The simplest hypothesis $L \sim m_c$ leads to $\alpha = 3/2$. This value of α will be assumed in the present analysis [21]. Since the SNIa absolute magnitude is connected with the absolute luminosity via

$$M_B - M_{0B} = -\frac{5}{2} \log_{10} \frac{L}{L_0}, \quad (11)$$

where the index $_0$ indicates the local (present) values, it is clear that a decrease of G_{eff} would lead to an increase of L and a decrease of M_B .

It then follows that the M_B transition of Eq. (6), and so the H_0 crisis, could be explained by the following evolution of the effective gravitational constant:

$$\mu_G(z) \equiv \frac{G_{\text{eff}}}{G_N} = \begin{cases} 1 \equiv \mu_G^< & \text{if } z \leq z_t \\ 1 + \Delta\mu_G \equiv \mu_G^> & \text{if } z > z_t \end{cases}, \quad (12)$$

where we considered the perturbation parameter μ_G (see, e.g., [4]), G_N is the locally measured Newton's constant, and the change in μ_G is connected to the change in the SNIa absolute magnitude via:

$$\Delta M_B = -\frac{5}{2} \log_{10} \frac{L^{\text{P18}}}{L^{\text{R20}}} = \frac{15}{4} \log_{10} \mu_G^>, \quad (13)$$

where L^{P18} and L^{R20} are the CMB-calibrated and Cepheid-calibrated SNIa luminosities, respectively, so that:

$$\Delta\mu_G = 10^{\frac{4}{15}\Delta M_B} - 1 \approx -0.12. \quad (14)$$

IV. ADDRESSING THE GROWTH TENSION

Such a $z > z_t$ suppression of the gravitational interaction is trivially consistent with local constraints on Newton's constant and it has the potential to explain the growth tension by decreasing the growth rate of cosmological matter fluctuations $\delta(z) \equiv \frac{\delta\rho}{\rho}(z)$ according to the dynamical linear growth equation:

$$\delta'' + \left[\frac{(H^2)'}{2H^2} - \frac{1}{1+z} \right] \delta' \approx \frac{3H_0^2}{2H^2} (1+z) \mu_G(z) \Omega_{0m} \delta, \quad (15)$$

where $'$ denotes derivative with respect to redshift z . In fact, a 12% reduction of μ_G would fit the same data with a 12% larger Ω_{0m} .

Dynamical probes of cosmological perturbations including cluster counts (CC) [22–25], weak lensing (WL)

[26–33] and redshift-space distortions (RSD) [34–38] consistently favor a lower value of the matter density parameter Ω_{0m} in the context of general relativity (GR). This implies weaker gravitational growth of perturbations than the growth indicated by GR in the context of a Planck18/ Λ CDM background geometry at about $2-3\sigma$ level [35,37,38]. This weakened growth is quantified by considering the parameter σ_8 , defined as the matter density rms fluctuations within spheres of radius $8h^{-1}$ Mpc at $z = 0$. The value of σ_8 is connected with the amplitude of the primordial fluctuations and is determined by the growth rate of cosmological fluctuations. A useful bias-free statistic probed by RSD data is the quantity $f\sigma_8$:

$$f\sigma_8(a) = \frac{\sigma_8}{\delta(a=1)} a\delta'(a, \Omega_{0m}, \mu_G), \quad (16)$$

where δ is obtained by solving equation (15). This theoretical prediction may now be used to constrain, via $f\sigma_8$ data, the parameters Ω_{0m} , σ_8 and $\mu_G(z)$ in the context of a specific background $H(z)$ and a parametrization $\mu_G(z)$. In the present analysis we assume the steplike transition of $\mu_G(z)$ proposed in Eq. (12) and either a Λ CDM background or a fixed dark energy equation of state w .

In the context of a Λ CDM background $H(z)$, RSD and WL data are well fit by [39–41]:

$$\Omega_{0m}^{\text{growth}} = (1 + \Delta\mu_G) \Omega_{0m}^{\text{P18}} = 0.256_{-0.031}^{+0.023}, \quad (17)$$

which, adopting $\Omega_{0m}^{\text{P18}} = 0.3153 \pm 0.0073$ [4], implies:

$$\Delta\mu_G = -0.19 \pm 0.09, \quad (18)$$

which is in good agreement with the value required to explain the M_B transition [see Eq. (14)], and is compatible with CMB [4] and gravitational wave [42] constraints. This approximate estimation of $\Delta\mu_G$ relies on the fact that in Eq. (15) the relevant parameter that drives the growth of perturbations is the product $\mu_G \times \Omega_{0m}$. The actual constraints from the full fit to the growth RSD data are presented in Fig. 2, where we show the $\sigma_8 - \Omega_{0m}$ likelihood contours (blue) from RSD data for Planck/ Λ CDM (upper panel), the μ_G -transition model ($z_t = 0.01$ and $\Delta\mu_G = -0.12$) that actually resolves the H_0 tension (middle panel), and w CDM with fixed dark energy equation of state parameter $w = -1.2$ that attempts to resolve the H_0 tension via a smooth deformation of $H(z)$ [41] (lower panel). We adopted the conservative robust $f\sigma_8$ dataset of [[43], Table 2], which is a subset of the up-to-date compilation presented in [40]. Superposed are the corresponding Planck/ Λ CDM CMB TT likelihood contours (red). The growth tension is resolved only in the case of the μ_G -transition model (middle panel), while the smooth $H(z)$ deformation approach (lower panel) worsens the growth

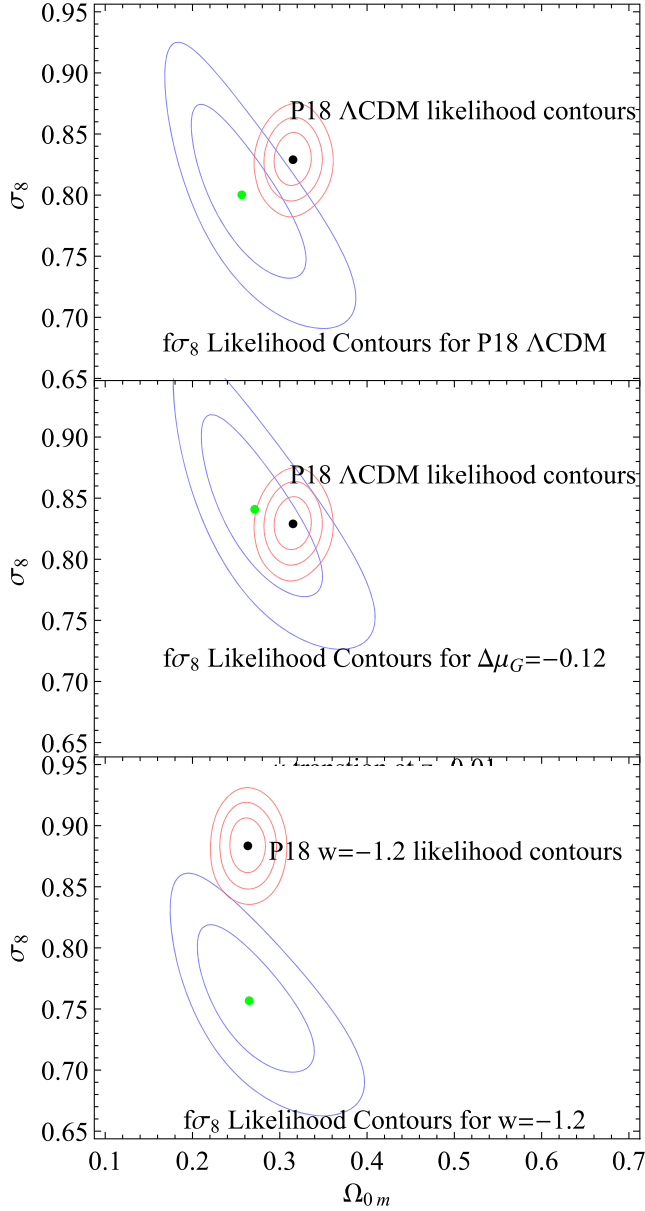


FIG. 2. The $\sigma_8 - \Omega_{0m}$ constraints (blue) from redshift space distortion data for Planck/ Λ CDM (upper panel), the μ -transition model ($z_t = 0.01$ and $\Delta\mu_G = -0.12$) that actually resolves the H_0 crisis (middle panel) and w CDM with $w = -1.2$ that attempts to resolve the H_0 tension via a smooth deformation of $H(z)$ (lower panel).

tension [41,44] seen in the upper panel in the context of Planck/ Λ CDM.

V. DISCUSSION

We have showed that a rapid 10% increase of the effective gravitational constant roughly 100–150 million years ago ($z_t \simeq 0.01$) can solve simultaneously the H_0 crisis and the $\sigma_8 - \Omega_{0m}$ growth tension. A physical model where such a transition could be realized is a scalar-tensor theory

with a steplike scalar field potential $V(\phi)$ and/or with an abrupt feature in the functional form of the nonminimal coupling function $F(\phi)$ [15,45,46]. In this theory the gravitational interactions are determined by $G_{\text{eff}} = \frac{1}{8\pi F} \frac{2F+4F_{,\phi}^2}{2F+3F_{,\phi}^2}$ while the background expansion is controlled by the Planck mass which corresponds to $G_* = \frac{1}{8\pi F}$. Alternatively, a quantum tunneling between two distinct degenerate vacua of a scalar tensor potential $V(\phi)$ would also induce such a gravitational phenomenological transition for an appropriate form of a nonminimal coupling $F(\phi)$.

One could test the μ_G transition model on cosmological and astrophysical scales. First, future surveys will tightly constrain the growth of perturbation at $0 < z < 2$ [47], producing constraints on μ_G that may rule out the model.

Second, low-redshift gravitational wave observations could not only rule out the mechanism here proposed but, if detected, map it through redshift. For example, gravitational waves of merging binary neutron stars carry information about the value of G_{eff} at the time the merger took place. Thus bounds can be placed on the variation of G_{eff} between the merger time and the present time [42] assuming that the actual masses of the neutron stars are consistent with the theoretically allowed range. Similarly, one could constrain G_{eff} via the gap in the mass spectrum of black holes due to the existence of pair-instability supernovae [48]. Alternatively, one may use standard sirens to measure the low- z luminosity distance and compare it with the corresponding luminosity distance obtained with SnIa standard candles where the Chandrasekhar mass and G_{eff} are assumed constant. Future standard siren gravitational wave measurements are expected to constrain (or detect) such variations of G_{eff} at a level of 1.5% [49]. Lastly, we would like to point out that the detection of the electromagnetic counterpart GRB170817A to the gravitational wave signal GW170817 [50] has led to important constraints on the propagation speed of GWs, which have ruled out a wide range of Horndeski's modified gravity theories that predict evolving G_{eff} . However, a wide range of scalar tensor theories, as the ones previously discussed, which also predict the cosmological evolution of G_{eff} for proper forms of the nonminimal coupling and the scalar field potential, have remained viable and consistent with these constraints [51–54].

The μ_G transition scenario may also be constrained via stellar constraints. Indeed, one expects the evolution of a star to be strongly dependent on the strength of the gravitational interaction. A variation of G_{eff} would indeed affect the hydrostatic equilibrium of the star and in particular its pressure profile and temperature. This, in turn, will then affect the nuclear reaction rates so that the lifetimes of the various stars will be modified [55]. From this point of view, it is interesting that an analysis of recent

Tully-Fisher data [56] has identified hints for a transition of magnitude and sign as required by the proposed mechanism albeit at somewhat more recent times: about 70 million years ago, corresponding to $z_t \simeq 0.005$. Also interesting is the fact that a recent analysis, which analyzes the luminosity of Cepheids in anchor galaxies and SnIa host galaxies, finds a systematic brightening of Cepheids at distances larger than about 20 Mpc [[57], Fig. 4]. This brightening would be enough to resolve the Hubble tension and the authors attribute it to variation of dust properties. This effect appears to be consistent with our approach and, in the context of the proposed mechanism, it would occur due to a fundamental physics transition about 70 million years ago.

Also earth paleontology data may lead to interesting constraints on the proposed mechanism. A crude approach to this problem, discussed in [55] and originally proposed in [58], indicates that the temperature on Earth varies according to $G^{2.25}$, assuming that the mass of the Sun remains constant and ignoring atmospheric and other climatological effects (green-house effects etc.). Within this approximation, if G had been smaller by 10% 100–150 million years ago, the temperature of the Earth would have been about 20% lower. Transitions in the temperature of the Earth of this magnitude are known to have taken place during the last 500 million years, and are attributed to climatological effects or to the meteorite that is believed to have led to the extinction of various species, including the dinosaurs, about 70 million years ago. Therefore, if atmospheric effect uncertainties are taken into account, a 10% gravitational transition cannot be easily *a priori* excluded.

Finally, an important assumption made in our analysis is that the SnIa luminosity increases with the Chandrasekhar mass. This is intuitively correct and consistent with a large part of the literature [18,21,59,60]. However, a recent analysis [61], using a semianalytical model to calculate SnIa light curves in nonstandard gravity has claimed that

the standardized peak luminosity, the quantity that is important for supernova cosmology, may actually decrease with the Chandrasekhar mass, that is, the α of Eq. (10) may have a different value. If this was the case then the proposed model would require stronger gravity to resolve the Hubble crisis and would thus be unable to resolve simultaneously the growth tension. Then, as discussed in, e.g., [62–64], SnIa progenitors are not necessarily Chandrasekhar-mass white dwarfs and a significant fraction can arise from sub-Chandrasekhar explosions. While this surely calls for a more detailed analysis of the dependence of the SnIa luminosity on G_{eff} , one must note that the Chandrasekhar mass scale is a fundamental reference scale that plays an important role in all SnIa explosions. However, a nontrivial relation between progenitors and SnIa could again imply that the relation of Eq. (10) could feature a different value of α . Lastly, it is important to study the covariance between the supernova calibrator and Hubble-flow supernovae within the proposed model, following the methodology introduced in [14]. The clarification of these important issues is therefore an interesting extension of the present analysis.

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