Chaotic nature of holographic QCD

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In this paper, we bring together two topics in the holographic correspondence—quantum chaos and quark-gluon plasma (QGP). We establish that the first relativistic correction to drag force experienced by a charge carrier moving through a thermal medium (for example, a quark in QGP) at a constant velocity is fixed by the butterfly velocity. Moreover, we show that this result is robust against stringy corrections and anisotropy. For the jet quenching parameter, we find that it is related to the butterfly velocity along the momentum broadening direction and temperature. This opens a way to the reconstruction of butterfly velocity of quark-gluon plasma and other strongly coupled systems experimentally from rather simple observables.

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I. INTRODUCTION

Many properties of strongly interacting quantum systems are known to carry large imprint of universality. To explain this universality is a challenging and intriguing problem. A possible way to address it that has successfully been adopted in the past is to search for universal relations between seemingly different and unrelated physical quantities. Holographic correspondence is a versatile and powerful tool to reveal these relations. The broad range of the holography applications varies from the studies of the heavy-ions collisions and thermal QCD [1–3] to condensed matter theory [4,5] and quantum information realm [6]. The discussion about the relation between the AdS/CFT correspondence and real physical systems largely started from papers [7,8] establishing the universal result concerning the viscosity to entropy ratio η/s in holographic quantum systems

$$\text{KPSS viscosity relation:}\ \frac{\eta}{s}\sim\frac{1}{4\pi}\frac{\hbar}{k_B}. \tag{1}$$

where "KPSS" stands for Kovtun-Son-Starinets-Policatro. After the experiments at RHIC (for references and review see [1]), many probes amenable to the holographic description have been introduced and studied in the theory of strongly interacting systems and QGP in particular. In this work, we focus mainly on the drag force [9,10] and jet quenching parameter [11]. Both are related to the energy and momentum loss for projectiles moving in a strongly interacting quantum system (quark-gluon plasma). However, their origin and properties are slightly different, and we would like to stress the following points

(i) Drag force is associated with the momentum loss of a single quark (charge carrier) moving in the strongly interacting medium.
(ii) The jet quenching coefficient $\hat{q}$ plays the role of the collective transport coefficient (sometimes $\hat{q}$ is called a jet transport coefficient) describing the momentum broadening in a thermal medium.[1] This coefficient is essential for the description of a radiative parton energy loss. It is important to notice that the jet quenching probes very different scales of medium simultaneously [12].
(iii) Drag force also can be related to the transport coefficient, namely conductivity (see [13]) for the small charge carriers.

Recently, the transport properties of quantum systems have been related to quantum chaos [14,15]. The motivation for this takes its roots in [16], where it was suggested that some velocity v could determine the diffusion constant.

$$\text{Hartnoll bound:}\ D\sim\frac{\hbar v^2}{k_B T}. \tag{2}$$

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[1]The momentum broadening is defined as the probability of momentum increase by the hard parton after propagating through a medium.

In [14], this statement has been clarified for holographic theories with particle-hole symmetry and transformed into the relation between charge diffusion,[2] butterfly velocity v_B [17], and the temperature

$$\text{Blake relation:}, \qquad D_c = C \cdot \frac{v_B^2}{2\pi T}, \tag{3}$$

where the constant C depends only on the details of the infrared theory, namely $C = d_\theta/\Delta_\chi$. Here d_θ is the effective spatial dimensionality of the fixed point and Δ_χ is the scaling dimension of the susceptibility. In the paper [15] Blake, Davison and Sachdev (BDS) extended this relation to a more general class of theories and thermal diffusion constant D_T

$$\text{BDS relation:}\ D_T \sim v_B^2 \tau_L, \tag{4}$$

where $\tau_L = (2\pi T)^{-1}$ is the Lyapunov time.

Taking these relations as a prototypical example, we aim to establish a connection between drag force, jet quenching, and butterfly velocity.

In this paper we argue that the drag force and jet quenching parameter also has the imprint of universality analogous to (1)–(3). We show how they are related to the butterfly velocity and temperature. First of all, we obtain that in the holographic systems for small velocities v of charge carrier (quark), the properly normalized momentum loss dp_σ/dt is fixed by the butterfly velocity up to a first "relativistic," i.e., v^3 term

$$dp_\sigma/dt = -v - \mathcal{B} \cdot v^3 + \cdots, \qquad \mathcal{B} = \frac{1}{(d-1)v_B^2}, \tag{5}$$

where we denote the normalization factor by σ. One may consider this identity as the microscopic manifestation of the relations between diffusion and butterfly velocity. It is worth noticing that, in principle, this relation allows to measure butterfly velocity in a straightforward manner in experimental setups with charge carriers. The precision experiments in terms of this proposal is under question, however, at least it could me used to obtain the order of magnitude for v_B. Moreover, we provide evidence that this result is robust against higher-derivative corrections on the gravity side and anisotropy.

To reveal a similar universal relation for $\hat{q}$ one should remember that it is (especially sensitive) to anisotropy—we always have to specify two directions (direction of momentum broadening and direction where parton moves). Taking this into account, the relation similar to (2) and (3) can also be written down. We argue that the jet quenching can be expressed as

$$\hat{q}_y = \mathcal{A}\left(\frac{v_b^{(x)}}{v_b^{(y)}}\right)^2 T\sigma_x \tag{6}$$

where $v_B^{(x)}$ and $v_B^{(y)}$ are the butterfly velocities along the directions x and y, T is the temperature and σ_x is the coefficient defining the leading order drag force coefficient acting on the projectiles along direction x (5). Another interpretation of σ is the "string tension" calculated from the asymptotic of spatial Wilson loops [18].

The organization of this paper is as follows. First, we obtain (5) and discuss it, then turn to the jet quenching parameter and derive relation (6). In the Supplemental Material [19] we provide all necessary details of calculations.

II. THE CHAOTIC ORIGIN OF THE DRAG FORCE

Our main focus is on the $d+1$-dimensional metrics of the form

$$ds^2 = -g_{tt}dt^2 + g_{uu}du^2 + g_{ii}dx^i dx^i, \quad i = 1, \ldots, d-1, \tag{7}$$

where we assume g_{ij} to be diagonal, and the horizon located at $u = u_h$ fixes the temperature and entropy density in dual theory

$$s = \frac{\sqrt{\det g}}{4G_N}\bigg|_{u=u_h}, \qquad T = \frac{\sqrt{(g_{tt})'(g^{uu})'}}{4\pi}\bigg|_{u=u_h}, \tag{8}$$

where G_N is a gravitational constant.

Consider a heavy particle (quark or charge carrier) moving in the strongly interacting thermal medium with the temperature T at constant velocity v. According to holographic duality, the bulk description of this particle is given by a classical string hanging from the asymptotic AdS boundary. The particle worldline $x = v \cdot t$ fixes the boundary condition for this string, and this leads us to the string ansatz for the world sheet $x(t, u)$ of the form

$$x(t, u) = vt + \xi(u). \tag{9}$$

As the particle moves through the medium, it experiences momentum loss dp/dt due to the drag force $F = dp/dt$. The calculation of this drag force is well known [9,10] and the derivation details can be found in the Supplemental Material [19]. As a result, one can get that the string dynamic depends on the special bulk point u_c fixed by the condition

$$(g_{tt} - g_{xx}v^2)|_{u=u_c} = 0. \tag{10}$$

The momentum loss is defined by u_c as

[2] A similar relation takes place for shear viscosity and diffusion.

$$\frac{dp_x}{dt} = -\frac{v}{2\pi\alpha'} g_{xx}|_{u=u_c}, \quad (11)$$

where α' is the inverse string tension $T_f^{-1} = 2\pi\alpha'$. The solution of Eq. (10) can be found as a series

$$u_c = u_h - \frac{g_{xx}(u_h)}{g'_{tt}(u_h)} v^2 + \cdots, \quad (12)$$

leading to the expression for the drag force

$$F = -\frac{v}{2\pi\alpha'} g_{xx}(u_h) - \frac{v^3}{2\pi\alpha'} \left(\frac{g'_{xx}}{g'_{tt}} \cdot g_{xx} \right) \Big|_{u=u_h} + \cdots \quad (13)$$

Now turn to the chaotic properties of the holographic dual described by (7). There are different characteristics of quantum system relevant to quantum chaos recently proposed to be calculated [17,20–22] from the exponential growth of Hermitian operators commutators

$$\langle [\mathcal{O}_x(t_w), \mathcal{O}_y(0)]^2 \rangle_\beta \sim e^{\lambda_L (t_w - \tau_* - |x-y|/v_B)}. \quad (14)$$

Here scrambling time τ_* is the time of the chaos onset, butterfly velocity v_B defines the effective light cone constraining the spatial chaos spreading and Lyapunov exponent λ_L is related to the chaotic features of time evolution. In holographic correspondence one can calculate[3] τ_*, λ_L and v_B for the quantum system dual to (7) in terms of metric components values at the horizon u_h

$$\lambda_L = 2\pi T, \qquad v_B^2 = \frac{g'_{tt}(u_h)}{g'_{xx}(u_h)(d-1)}. \quad (15)$$

Combining (8), (15), (13) and expanding for small v we obtain, that momentum loss dp/dt normalized by the leading order coefficient σ depends only on the butterfly velocity at the first subleading order in v

$$\frac{dp_\sigma}{dt} = \frac{1}{\sigma}\frac{dp}{dt} = -v - v^3 \cdot \frac{1}{(d-1)v_B^2} + \cdots,. \quad (16)$$

where

$$\sigma = (2\pi\alpha')^{-1} g_{xx}(u_h), \quad (17)$$

and $p_\sigma = p/\sigma$ is the normalized momentum. A few comments are in order now

(i) We propose that using this formula, one can determine the butterfly velocity of the strongly interacting quantum system in the quite general experimental setup and for a wide range of quantum systems where the measurement of momentum loss is possible. Following [14,15] we assume that the systems where (16) take place are holographic theories, which we usually are understood as the strong coupling systems with a large number of freedom degrees. Also, the system is supposed to be thermal and with well-defined butterfly velocity where (15) makes sense. The presence of heavy (quasi)particles seems a natural requirement as well. One should take care when considering possible experiments with the *hypothetical* direct drag force measurement. The exact description of the experimental setup with such measurements is out of the scope of our paper. Also, it is worth noticing that for example in holographic phenomenological QGP studies the drag force is estimated indirectly through the nuclear modification factor R_{AA} [23]. In general, it is not clear how accurately we can estimate the butterfly velocity using (16). The discussion here and formula (16) seems to be not enough for general real-world applications and we leave necessary absent details for a future research. It would be interesting to understand our proposal beyond thermality, where the butterfly velocity is not so well defined. Typical setup without exponential growth of OTOC and the canonical notion of butterfly velocity corresponds to the extremal black holes (see [24] as example). The experimental setups and protocols allowing the measurement of butterfly velocity have been widely discussed previously for example in [25–28]. These protocols are quite exotic for general systems and in general OTOC measurement may require, for example, the inversion of time evolution. The drag force is relatively simple observable and it would be interesting to understand whether it could be applied for direct butterfly velocity measurements. If so, our proposal could be applied to a quite broad range of quantum systems (at least those with the massive charge carriers or (quasi)particles).

(ii) In the particular case of QGP the coefficient σ can be interpreted as a so-called "spatial string tension" defined from rectangular spatial Wilson loop asymptotic responsible for chromomagnetic fluctuations (see [18] and [29–31] for detailed discussion). Also σ can be related to spatial D_s and momentum diffusion D_p of heavy quarks as $T = D_s\sigma$, $D_p = T\sigma$. However, we would like to stress that one can avoid this interpretation and consider σ as just the leading order drag force coefficient.[4]

(iii) An important issue in the identities like (1) or (4) is the robustness of such results against stringy corrections and anisotropy. It is known that KPSS

[3]See Supplemental Material [19] for derivation of the butterfly velocity and references.

[4]These D_s and D_p are different from the charge and energy diffusion described in Blake and BDS relations.

bound (1) is violated by stringy corrections and anisotropy while BDS identity is robust. In the Supplemental Material [19] we provide evidence that our proposal is robust against stringy corrections. It is also robust against the inclusion of anisotropy with a very mild modification. Namely, we derive that the butterfly velocity $v_B^{(i)}$ along the spatial direction i depends on the first and second drag force coefficients along different direction $F_i = -\sigma_i v - \mathcal{B}_i v^3$ as

$$v_B^{(i)} = \sqrt{\frac{\sigma_i^{-1}}{\sum_{i=1}^{d-1} \mathcal{B}_i \sigma_i^{-2}}}. \tag{18}$$

(iv) While the simple and robust form of expressions (16) and (18) is quite unusual, in general, the existence of such relation between drag force and butterfly velocity is expected from the following kinetic theory and perturbative quantum field theory argument. The drag force can be evaluated and essentially depends on the matrix elements describing 2-2 scattering from the projectile $\mathcal{M}$ (the example for hot QGP and QED see [32–34]). On the other hand, as was shown in [35] some OTOC reformulations also lead to the connection with kinetic equations allowing us to express the Lyapunov exponent as the function of $\mathcal{M}$. At the moment a similar result for the butterfly velocity v_B is unknown. However, the Bethe-Salpether equations describing it could be cast in the kineticlike form indicating a similar relation as for the Lyapunov exponent. It would be interesting to get the precise form of such relation and a derivation of (16) on the QFT side.

III. JET QUENCHING AND BUTTERFLY VELOCITY

Another interesting quantity called a *jet quenching parameter* $\hat{q}$ also characterizes the energy and momentum loss of projectiles in a strongly interacting medium. In the studies of heavy-ions collisions and thermal QCD, the jet-quenching phenomena are called the disappearance or suppression of the bunch of hadrons resulting from the fragmentation of a parton after strong interaction leading to momentum loss in the dense medium (quark-gluon plasma). In general, the details of the mechanism of a jet energy loss depend on the medium properties. The jet quenching parameter can be defined in the perturbative framework and considered as a kind of transport coefficient. Also, it allows a nonperturbative definition in terms of adjoint lightlike Wilson loop [11] useful in the gauge/gravity duality

$$\langle W^A(\mathcal{C})\rangle \approx \exp\left[-\frac{1}{4\sqrt{2}}\hat{q}L^- L^2\right]. \tag{19}$$

Here L_- corresponds to the distance between the lightlike parts of the contour $\mathcal{C}$ and L between the transversal one with $L_- \gg L$. From the holographic viewpoint, this Wilson loop can be calculated by the string hanging from the lightlike contour on the boundary, and we leave all details of calculations in the Supplemental Material [19].

In general, we have in mind a dual metric of the form (7) and in particular, the geometries similar to the background with hyperscaling violation to make parallels with [14]. However, there are some possible subtleties related to the divergences stemming from UV structure of this kind of theory. It is worth stressing that while the stringy jet-quenching calculation is typically considered for a five-dimensional gravitational background, we assume an arbitrary space-time dimension. Also, we do not discuss the gravitational action that gives our metric as a solution and just takes quite a general form of ansatz because the jet-quenching formula depends only on metric details. Of course, one could meet some restrictions on the metric coefficients like in the hyperscaling violating theories.

We restrict our attention to d-dimensional metric (20) of the form

$$ds^2 = -\frac{f(z)}{z^{2\nu}}dt^2 + \frac{dz^2}{z^{2\nu_z} f(z)} + \frac{dx^2}{z^{2\nu}} + \frac{dy^2}{z^{2\nu_y}} + \sum_\alpha \frac{dx_\alpha^2}{z^{2\nu_\alpha}}, \tag{20}$$

and assume that the parton is moving along direction x while the momentum broadening occurs along y. We refer the reader to the Supplemental Material [19] for the computational details and derivation of the formula for jet quenching parameter [36][5] which has the form

$$\hat{q}_y = \frac{\sqrt{2}}{\pi\alpha'}\left(\int_0^{u_h} \frac{1}{g_{yy}}\sqrt{\frac{g_{uu}}{g_{--}}}du\right)^{-1}, \quad g_{--} = \frac{g_{xx} - g_{tt}}{2}, \tag{21}$$

for a general class of anisotropic metrics (7). The jet quenching parameter temperature dependence corresponding to the isotropic background with $\nu = \nu_t = \nu_z = \nu_y = \nu_\alpha = 1$ derived first in [11] has the form

$$q = \mathcal{B}_0 \cdot T^3, \tag{22}$$

where $\mathcal{B}_0$ is some constant. The butterfly velocity for this choice of parameters is temperature-independent. Thus it is not clear whether the jet quenching is related to it. One should notice that from the very beginning, jet quenching is intrinsically sensitive to anisotropy (see [36–40] for holographic studies of jet quenching parameter in the presence of anisotropy). We have to specify two directions for jet quenching in contrast to the drag force and conductivity. To reveal some nontrivial relation between $\hat{q}$ and chaotic characteristics, we focus on the jet quenching for some

[5]Notice the sign in the definition of g_{tt} component.

particular but still quite general anisotropic metric with $\nu_t = \nu_x = \nu$. We consider two different butterfly velocities $v_B^{(x)}$ and $v_B^{(y)}$ associated with spatial directions x and y respectively.[6] For metric (20) they have the form

$$v_B^{(x)} = \sqrt{\frac{a}{2(\sum_i \nu_i)}}, \qquad v_B^{(y)} = \sqrt{\frac{a}{2(\sum_i \nu_i)} z_h^{\nu_y - \nu}}, \tag{23}$$

where $\sum_i$ is the summation over all spatial ν_i. For our choice of anisotropic exponents the jet quenching parameter depends on z_h as

$$q = \frac{1}{2\pi\alpha'} \mathcal{B} \cdot z_h^{-2\nu_y + \nu_z - \nu - 1}, \tag{24}$$

where $\mathcal{B}$ depends only on exponents ν_x, ν, ν_z and spacetime dimension a (see the Supplemental Material [19] for the explicit form of $\mathcal{B}$). We are looking for the relation between the jet quenching parameter and butterfly velocity supplemented with the additional characteristic depending on the inverse string tension. Equation (24) is combined with the temperature of the metric (20)

$$T = \frac{a z_h^{\nu_z - \nu - 1}}{4\pi}, \tag{25}$$

and butterfly velocity $v_B^{(x)}$ and $v_B^{(y)}$ defined by (23) results in the relation

$$\hat{q}_y = \mathcal{A} \left(\frac{v_b^{(x)}}{v_b^{(y)}} \right)^2 T \sigma_x, \tag{26}$$

between the jet quenching parameter, the butterfly velocities $v_B^{(x)}$, $v_B^{(y)}$, and the leading order drag force coefficient (17). The constant $\mathcal{A}$ depends only on the dimension a and the infrared exponents ν as it should be. For the isotropic case, i.e., for $v_b^{(x)} = v_b^{(x)}$ this formula implies that $\hat{q}_y$ does not depend on the butterfly velocity. Notice, however, that anisotropy is important in the context of QGP, especially at early stages of its formation. The parameter σ_x can be interpreted as the string tension calculated from the asymptotic behavior of the spatial Wilson loop. Nevertheless it seems more natural here to consider σ_x in terms of the leading order drag force acting on the projectiles.

As we mentioned before, the jet quenching probes very different scales of the system: an initial fragmentation of parton which is weakly coupled and late-time interaction of jets with a thermal medium which needs nonperturbative description [12]. The relation (26) involves the thermodynamics (temperature T) and late time drag force coefficient σ, which defines the dynamics of slowly moving projectile. As we have shown before on the intermediate nonperturbative scales, the drag force acting on the projectile is described by the butterfly velocity which is also present in (26).

[6] A brief review concerning the derivation of the anisotropic butterfly velocities can be found in the Supplemental Material [19].

IV. RELATION BETWEEN BUTTERFLY VELOCITY AND JET QUENCHING VIA DRAG FORCE

As we already mentioned the jet quenching parameter is the multiscale probe, intrinsically sensitive to the physical setup under investigation. When we introduce $\hat{q}$ in terms of lightlike Wilson loop we also suggest (as was stressed in [11]) that it could be considered as the fundamental definition inherent to the nonperturbative regime of strongly coupled theory. However, before the introduction of such a quite universal definition especially well designed for AdS/CFT correspondence many efforts have been made to estimate $\hat{q}$ from various natural viewpoints. The approach especially interesting to us is based on the estimates of the relation between drag force and $\hat{q}$ in different regimes. A brief review can be found in [41] as well as an alternative holographic calculation of $\hat{q}$ in terms of dragged string fluctuations. As a summary of different approaches and setups, one can state the dependence

$$-\frac{dp}{dt} \sim \Upsilon \cdot \lambda \hat{q} \tag{27}$$

where Υ is the constant which rely on the derivation method, assuming that parton travels through medium of thickness λ. For example, based on the uncertainty principle in [42] the inequality

$$-\frac{dp}{dt} < \frac{1}{2} \hat{q} \lambda \tag{28}$$

has been proposed. Also, in [41] it was argued that this inequality tends to saturation for holographic theories and large enough v. This lead us to a rough estimate

$$\text{Gubser/uncertainty based estimate:}\ \Upsilon = 1/2. \tag{29}$$

From the analysis of radiative energy by Baier, Dokshitzer, Mueller and Schiff (BDMPS) in [43] the alternative value of Υ is

$$\text{BDMPS:}\ \Upsilon = \frac{\alpha_s N_c}{8}, \tag{30}$$

where N_c is the number of colors and α_s is 't Hooft coupling. While it should be perceived with the natural amount of caution we are left with the expression

$$\frac{q}{\sigma} \approx v\frac{1+\mathcal{B}v^2}{\lambda\Upsilon}, \qquad \mathcal{B} = \frac{1}{(d-1)v_B^2}, \qquad (31)$$

obtained by combination of (27) with (16). The parameter Υ is setup-dependent and we provided two different versions of it above. To summarize this section let us stress a few moments

(i) Serving as complementary consideration to the Wilson loop-based derivation outlined in the previous section, formulas (31) have a restricted range of applicability (defined by the method of derivation of Υ). We would like to stress, that the jet quenching parameter from the previous section and the ones considered in this section are different. The jet quenching parameter in the previous section is defined in terms of lightlike Wilson loop nonperturbatively and in a model-independent manner (as it was stressed in [11]). In [41] the jet quenching is related to the two-point function of force fluctuations acting on heavy quarks. In general, the calculations in [11] are related with the light quarks, while trailing string fluctuation calculations in [41] are associated with the heavy quarks.

(ii) We listed at least three setups where this relation fits quite well: the BDMPS approach, dragging string derivation by Gubser, and the quantum uncertainty approach.

V. DISCUSSION

In summary, we have obtained two relations between probes in strongly coupled quantum theory and butterfly velocity v_B. Both results are obtained in the framework of holographic correspondence. First, we have shown that the subleading (i.e., first "relativistic") coefficient in the drag force is fixed by butterfly velocity v_B. This leads us to the possibility of measuring butterfly velocity by experimental study of the velocity dependence for momentum loss of charge carriers in strongly coupled theories. Second, analogous to the charge diffusion constant [14], the jet quenching coefficient is defined by anisotropic butterfly velocity, temperature, and leading order drag force coefficient up to some constant. The jet quenching results are obtained for quite general theory dual to a metric similar to anisotropic hyperscaling geometry. The presented results indicate that these quantities in strongly coupled theories and, particularly, quark-gluon plasma are governed by butterfly velocity and thermodynamic quantities.

Let us briefly discuss possible future extensions of this work

(i) It would be interesting to extend the understanding of the relation between drag force and butterfly velocity in different directions, such as the corrections caused by quark mass or other drag force proposals [23] (for review of quark dynamic holographic description see [44]). Also, it is interesting to consider the anisotropic background where particle moves in an arbitrary direction and study a similar relation for the butterfly velocity and Wilson loops [30,31,45].

(ii) Another prospective question is to consider the phenomenological implications of our identities using the known backgrounds reproducing experimental results concerning drag force and jet quenching (see for example [23,46]).

(iii) Finally, the intriguing direction to study is the relation of chaos to other probes of QGP, including hot wind [47], glueball spectrum [48] or particles production multiplicity [49,50]. An interesting proposal revealing some relation between Lyapunov exponent and QCD recently appeared [51–54], and it would be very interesting to find the connection between our proposal and described in these papers.

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