Quantum discrete levels of the Universe from the early trans-Planckian vacuum to the late dark energy

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We go forward in completing the Standard Model of the Universe back in time with Planckian and trans-Planckian physics before inflation in agreement with observations, classical-quantum gravity duality, and quantum space-time. The quantum vacuum energy bends the space-time and produces a constant curvature de Sitter background. We link the de Sitter Universe and the cosmological constant to the (classical and quantum) harmonic oscillator. We find the quantum discrete cosmological levels: size, time, vacuum energy, Hubble constant, and gravitational (Gibbons-Hawking) entropy and temperature from the very early trans-Planckian vacuum to the classical vacuum energy today. For each level $n = 0, 1, 2, \ldots$, the two post- and pre-(trans)-Planckian phases are covered: In the post-Planckian Universe $t_{\text{planck}} \equiv t_P \leq t \leq 10^{61} t_P$, the levels (in Planck units) are Hubble constant $H_n = 1/\sqrt{(2n+1)}$, vacuum energy $\Lambda_n = 1/(2n+1)$, and entropy $S_n = (2n+1)$. As n increases, radius, mass, and S_n increase, H_n and Λ_n decrease, and *consistently* the Universe *classicalizes*. In the pre-Planckian (trans-Planckian) phase $10^{-61} t_P \leq t \leq t_P$, the quantum levels are $H_{Qn} = \sqrt{(2n+1)}$, $\Lambda_{Qn} = (2n+1)$, and $S_{Qn} = 1/(2n+1)$, Q denoting quantum. The n levels cover *all* scales from the far past highest excited trans-Planckian level $n = 10^{122}$ with finite curvature, $\Lambda_Q = 10^{122}$, and minimum entropy $S_Q = 10^{-122}$; n decreases till the Planck level ($n = 0$) with $H_{\text{planck}} = 1 = \Lambda_{\text{planck}} = S_{\text{planck}}$ and enters the post-Planckian phase, e.g., $n = 1, 2, \ldots$, $n_{\text{inflation}} = 10^{12}, \ldots$, $n_{\text{cmb}} = 10^{114}, \ldots$, $n_{\text{reoin}} = 10^{118}, \ldots$, and $n_{\text{today}} = 10^{122}$, with the most classical value $H_{\text{today}} = 10^{-61}$, $\Lambda_{\text{today}} = 10^{-122}$, and $S_{\text{today}} = 10^{122}$. We implement the Snyder-Yang algebra in this context, yielding a consistent group-theory realization of quantum discrete de Sitter space-time, classical-quantum gravity duality symmetry, and a clarifying unifying picture.

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I. INTRODUCTION AND RESULTS

Planckian and trans-Planckian energies are theoretically allowed and physically motivated, too; the Universe and its very early stages have all the quantum conditions for such extreme quantum gravitational regimes and energies, and black hole interiors, too. The *truly* quantum gravity *domain* is not reduced to be fixed at the Planck scale or the neighborhoods of it but extends deep beyond the Planck scale in the highly quantum trans-Planckian range.

In this paper, we go forward in completing the Standard Model of the Universe back in time with Planckian and trans-Planckian physics before inflation, in agreement with observations, classical-quantum gravity duality, and quantum space-time in this context.

Quantum theory is more complete than classical theory and tells us what value a classical observable should have. The classical-quantum (or wave-particle) duality is a robust and universal concept (it does not depend on the nature or number of space-time dimensions, compactified or not, nor on particular space-time geometries, topologies, or symmetries, nor on other *a priori* conditions). Moreover, the quantum trans-Planckian eras in the far past Universe determine the post-Planckian eras, e.g., the inflation and the cosmological vacuum energy until dark energy today, namely, the evolution from the quantum very early phases to the semiclassical and classical phases and the arrow of time as determined by gravitational entropy.

The complete Universe is composed of two main phases, the Planck scale being the *transition scale*: the quantum pre-Planckian or trans-Planckian phase $0 < 10^{-61} t_P \leq t \leq t_P$ and the semiclassical and mostly classical post-Planckian Universe $t_P \leq t \leq t_{\text{today}} = 10^{61} t_P$, t_P being the Planck time. The pre-Planckian era could be tested indirectly through its post-Planckian observables, e.g., primordial graviton signals, inflation, and the cosmic microwave background (CMB) till dark energy today. This framework provides, in particular, the gravitational entropy and

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temperature (classical, semiclassical, and quantum) in different cosmological regimes and eras [1,2], in particular, the Gibbons-Hawking entropy and temperature. Interesting too (and related) are the classical and quantum cosmological vacuum energy values (Λ, Λ_Q) dual of each other: For instance, the quantum Λ_Q obtained from the classical-quantum (or wave-particle) duality approach turns out to be the saddle point obtained from the quantum gravity path integral Euclidean approach, which action is the well-known Gibbons-Hawking de Sitter entropy, showing the consistency of the results [1,2].

The huge difference between the observed value of the cosmological *classical* vacuum energy Λ *today* and the *theoretically* evaluated value of the *quantum* particle physics vacuum Λ_Q must correctly and physically be like that, because the two values correspond to two huge different physical vacua and eras. The observed Λ value today corresponds to the classical, large, and dilute (mostly *empty*) Universe today (termed voids and supervoids in cosmological observations and termed vacuum space-time in classical gravitation), and this is consistent with the very low observed Λ vacuum value (10^{-122} in Planck units), while the computed quantum value Λ_Q corresponds to the quantum, small, and highly dense energetic Universe in its far (trans-Planckian) past, and this is consistent with its extremely high, trans-Planckian, value (10^{122} in Planck units). As is well known, the theoretical value $\Lambda_Q \simeq 10^{122}$ is clearly trans-Planckian; this value corresponds to and fits correctly the value of Λ_Q in the far past trans-Planckian era and its physical properties: quantum size and time 10^{-61}, quantum (Gibbons-Hawking) temperature 10^{61}, and entropy 10^{-122}. Consistently too, the trans-Planckian era provides the quantum precursor of inflation from which the known classical and semiclassical inflation era, and its CMB observables and quantum corrections are recovered in agreement with the set of well-established cosmological observations.

Starting from quantum theory to reach the Planck scale and trans-Planckian domain (instead of starting from classical gravity by quantizing general relativity) reveals successful novel results, *"quantum relativity"* and quantum space-time structure [1–3]. Beyond the classical-quantum duality of the space-time, the space-time coordinates can be promoted to quantum noncommuting operators. Comparison to the harmonic oscillator and global phase space is enlightening. The hyperbolic quantum space-time structure generates the *quantum light cone*. The classical space-time null generators $X = \pm T$ *disappear* at the quantum level due to the relevant $[X, T]$ commutator, which is *always* nonzero, and a *new* quantum vacuum region beyond the Planck scale emerges.

In this paper, we analyze the new vacuum quantum region inside the Planck-scale hyperbolas which delimitate the quantum light cone. The effect of the zero-point (vacuum) quantum energy bends the space-time and produces a constant curvature de Sitter background. We find the quantum discrete levels in the cosmological vacuum trans-Planckian region and in the post-Planckian one. The quantum light cone is generated by the quantum Planck hyperbolas $X^2 - T^2 = \pm[X, T]$ due to the quantum uncertainty $\Delta X \Delta T$ or nonzero commutator $[X, T]$, the classical light cone generators $X = \pm T$ being a particular case of it. This generalizes the classical known space-time structure and reduces to it in the classical case (zero quantum commutators). In higher D space-time dimensions, the quantum noncommuting space and time coordinates (X, T) and the transverse commuting spatial coordinates $X_{\perp j}$ generate the quantum two-sheet hyperboloid $X^2 - T^2 + X_{\perp j} X_{\perp}^j = \pm 1$; $j = 2, \ldots (D-2)$.

Interestingly enough, the quantum space-time structure turns out to be *discretized* in quantum hyperbolic levels. For times and lengths larger than the Planck time and length (t_P, l_P), the levels are $(X_n^2, T_n^2) = (2n+1)$, $n = 0, 1, 2, \ldots$ (in Planck units), $(X_n,\ T_n)$ and the *mass levels* being $\sqrt{(2n+1)}$. The discrete allowed levels from the quantum Planck scale $(X_n, T_n) = 1$ $(n = 0)$ and the quantum levels (low n) until the quasiclassical and classical ones (intermediate and large n) tend asymptotically (very large n) to a continuum classical space-time. In the trans-Planckian domain—times and lengths smaller than the Planck scale—the (X_n, T_n) levels are $1/(2n+1)^{1/2}$, the most high n being the more excited quantum and trans-Planckian ones.

For each level $n = 0, 1, 2, \ldots$, the two post- and pre-(trans-)Planckian phases are covered: In the post-Planckian Universe $t_P \equiv t_{\text{planck}} < t \leq t_{\text{today}} = 10^{61} t_P$, the levels (in Planck units) for the Hubble constant H_n, vacuum energy Λ_n, and gravitational (Gibbons-Hawking) entropy S_n are, respectively,

$$H_n = 1/\sqrt{(2n+1)}, \qquad \Lambda_n = 1/(2n+1),$$
$$S_n = (2n+1), \quad n = 0, 1, 2, \ldots. \tag{1.1}$$

As n increases, the radius and mass increase, H_n and Λ_n decrease, S_n increases, and *consistently* the universe *classicalizes*. In the pre-Planckian (trans-Planckian) phase $10^{-61} t_P \leq t \leq t_P$, the quantum trans-Planckian levels (Q denoting quantum) are

$$H_{Qn} = \sqrt{(2n+1)}, \qquad \Lambda_{Qn} = (2n+1),$$
$$S_{Qn} = 1/(2n+1), \quad n = 0, 1, 2, \ldots. \tag{1.2}$$

The scalar curvature levels in the respective phases are $R_{Qn} = (2n+1)$ and $R_n = 1/(2n+1)$. The n levels cover *all* scales from the remote past highly excited trans-Planckian level $n = 10^{122}$ with maximum curvature $R_Q = 10^{122}$, vacuum $\Lambda_Q = 10^{122}$, and minimum entropy $S_Q = 10^{-122}$; n decreases, passing the Planck level $n = 0$—$H_{\text{planck}} = 1 = \Lambda_{\text{planck}} = S_{\text{planck}}$—and enters the post-Planckian phase—$n = 1, 2, \ldots,\quad n_{\text{infl}} = 10^{12}, \ldots,$

$n_{\rm cmb} = 10^{114}, \ldots, \ n_{\rm reion} = 10^{118}, \ldots, \ n_{\rm today} = 10^{122}$—with the most classical value $H_{\rm today} = 10^{-61}$, $\Lambda_{\rm today} = 10^{-122}$, and $S_{\rm today} = 10^{122}$.

The space-time (the arena of events) in the quantum domain is described by *a quantum algebra* of space-time position and momenta. We implement the *Snyder-Yang* algebra in the cosmological context, thus yielding a consistent group-theory realization of quantum discrete de Sitter space-time, classical-quantum gravity duality, and its symmetry with a clarifying unifying picture: Our complete (classical and quantum) length $L_{QH}(l_P, L_H) = L_Q + L_H = l_P(L_H/l_P + l_P/L_H)$, where L_H is the classical Universe radius and $L_Q = l_P^2/L_H$ is its quantum size (the Compton length), turns out to be the appropriate length for the two-parameter Snyder-Yang algebra, thus providing a quantum operator realization of the complete de Sitter Universe including the quantum trans-Planckian and classical late de Sitter phases.

This paper is organized as follows: In Sec. II, we describe the Standard Model of the Universe extended back in time before inflation, thus covering its different phases: classical, semiclassical, and quantum—Planckian and trans-Planckian—domains and their properties including the gravitational entropy and temperature. Section III summarizes our arguments, clarifying the cosmological constant problem as a vacuum energy and the gravitational entropy precisely covering the quantum (trans-Planckian and Planckian), the semiclassical, and the classical gravitational regimes. Sections VI–X include genuinely new material and the new results of the current manuscript: In Secs. IV and V, we describe the classical, quantum dual, and complete de Sitter Universe covering the different de Sitter regimes. Sections VI and VII show the link of the de Sitter Universe and the cosmological constant to the harmonic oscillator. Section VIII shows the link of the space-time structure to the phase space (classical and quantum) harmonic oscillator and describes the quantum space-time discrete levels. In Secs. VIII and IX, we find the quantum discrete levels of the Universe: size, time, vacuum energy, Hubble constant, entropy, and their properties from the very early trans-Planckian phase to dark energy today. In Sec. X, we describe the Snyder-Yang algebra as a group-theory realization of quantum discrete de Sitter space-time and of classical-quantum gravity duality symmetry. Section XI provides discussion and clarification on the results of this approach, in particular, the time discrete levels and those of the vacuum energy, varying cosmological constant, and the cosmic evolution in the context of quantum field theory (QFT). Section XII summarizes remarks and conclusions and the clarifying unifying picture we obtained.

II. THE STANDARD MODEL OF THE UNIVERSE BEFORE INFLATION

The set of robust cosmological data (cosmic microwave background, large-scale structure and deep galaxy surveys, supernovae observations, measurements of the Hubble-Lemaître constant, and other data) support the standard (concordance) model of the Universe and place de Sitter (and quasi–de Sitter) stages as a real part of it [4–10]. Moreover, the physical classical, semiclassical, and quantum Planckian and trans-Planckian de Sitter regimes are particularly important for several reasons:

(i) the classical, present time accelerated expansion of the Universe and its associated dark energy or cosmological constant in the era today: classical cosmological de Sitter regime;

(ii) the semiclassical early accelerated expansion of the Universe and its associated inflation era: semiclassical cosmological de Sitter (or quasi–de Sitter) regime (classical general relativity plus quantum field fluctuations);

(iii) the quantum, very early stage preceding the inflation era: Planckian and super-Planckian quantum era. Besides its high conceptual and fundamental physics interest, this era could be of realistic cosmological interest for the test of quantum theory itself at such extreme scales, as well as for the search of gravitational wave signals from quantum gravity for e-LISA [11], for instance, after the success of LIGO [12,13]. In addition, this quantum stage should be relevant in providing quantum precursors and consistent initial states for the semiclassical (fast-roll and slow-roll) inflation and their imprint on the observable primordial fluctuation spectra, for instance. Moreover, a novel result is that this quantum era allows a clarification of dark energy as the vacuum cosmological energy or cosmological constant.

(iv) de Sitter is a simple and smooth constant curvature vacuum background without any physical singularity; it is maximally symmetric and can be described as a hyperboloid embedded in Minkowski space-time with one more spatial dimension. Its radius, curvature, and equivalent density are described in terms of only one physical parameter: the cosmological constant.

The lack of a complete theory of quantum gravity (in field and in string theory) does not preclude exploring and describing quantum Planckian and trans-Planckian regimes. Instead of going from classical gravity to quantum gravity by quantizing general relativity (it is not our aim here to review it), we start from quantum physics and its foundational milestone—the classical-quantum (wave-particle) duality—and extend it to include gravity and the Planck-scale domain, namely, wave-particle-gravity duality (or classical-quantum gravity duality) [1,14]. As a consequence, the different gravity regimes are covered: classical, semiclassical, and quantum, together with the Planckian and trans-Planckian domain and the elementary particle mass range as well. This duality is *universal*, as the wave-particle duality does not rely on the number of space-time dimensions (compactified or not) nor on any symmetry, isometry, nor any other *a priori* condition. It includes the known classical-quantum duality as a special case and allows a general clarification from which physical understanding and cosmological results can be extracted. This is not an assumed or conjectured duality.

The Standard Model of the Universe extended to earlier trans-Planckian eras. The gravitational history of the Universe before the inflation era and the current picture can be extended by including the quantum precursor phase within the Standard Model of the Universe in agreement with observations. Quantum physics is more complete than classical physics and contains it as a particular case: It adds a new quantum Planckian and trans-Planckian phase of the Universe from the Planck time t_P until the extreme past $10^{-61} t_P$, which is an upper bound for the origin of the Universe, with energy $H_Q = 10^{61} h_P$; in a similar manner, the present age is a lower bound to the (unknown) future age.

The classical large dilute Universe today and the highly dense very early quantum trans-Planckian Universe are classical-quantum duals of each other in the precise meaning of the classical-quantum duality. This means the following: The classical Universe today U_Λ is clearly characterized by the set of physical gravitational magnitudes or observables (age or size, mass, density, temperature, and entropy) $\equiv (L_\Lambda, M_\Lambda, \rho_\Lambda, T_\Lambda, S_\Lambda)$:

$$U_\Lambda = (L_\Lambda, M_\Lambda, \rho_\Lambda, T_\Lambda, S_\Lambda). \tag{2.1}$$

The highly dense very early quantum Universe U_Q is characterized by the corresponding set of quantum dual physical quantities $(L_Q, M_Q, \rho_Q, T_Q, S_Q)$ in the precise meaning of the classical-quantum duality:

$$U_Q = (L_Q, M_Q, \rho_Q, T_Q, S_Q), \tag{2.2}$$

$$U_Q = \frac{u_P^2}{U_\Lambda}, \qquad u_P = (l_P, m_P, \rho_P, t_P, s_P), \tag{2.3}$$

u_P standing for the corresponding quantities at the fundamental constant Planck scale, the *crossing scale* between the two main (classical and quantum) gravity domains. The classical U_Λ and quantum U_Q Universe eras or regimes (classical and semiclassical eras of the known Universe and its quantum Planckian and trans-Planckian very early phases) satisfy Eqs. (2.1)–(2.3). The *total* Universe $U_{Q\Lambda}$ is composed by their classical or semiclassical and quantum phases:

$$U_{Q\Lambda} = (U_Q + U_\Lambda + u_P). \tag{2.4}$$

Subscript Λ—or, equivalently, H for Hubble Lemaître—stands for the classical magnitudes, Q for quantum, and P for the fundamental Planck-scale constant values.

In particular, the quantum dual de Sitter Universe U_Q is generated from the classical de Sitter Universe U_Λ through Eqs. (2.1)–(2.4): *classical-quantum de Sitter duality.* The *total* (classical plus quantum dual) de Sitter Universe $U_{Q\Lambda}$ endows automatically a *classical-quantum de Sitter symmetry.* This includes, in particular, the classical, quantum, and total de Sitter temperatures and entropies and allows one to characterize in a complete and precise way the different classical, semiclassical, quantum Planckian, and super-Planckian de Sitter regimes. H stands for the classical Hubble-Lemaître constant or its equivalent $\Lambda = 3(H/c)^2$. H_Q (or Λ_Q) stands for quantum dual and $Q\Lambda$ (or QH) for the total or complete quantities including both.

The size of the Universe is the gravitational length $L_\Lambda = \sqrt{3/\Lambda}$ in the classical regime, it is the quantum Compton length L_Q in the quantum dual regime (which is the full quantum Planckian and super-Planckian regime), and it is the Planck length l_P at the fundamental Planck scale: the *crossing scale*. The *total* (or complete) size $L_{Q\Lambda}$ is the sum of the two components. Similarly, the horizon acceleration (surface gravity) K_Λ of the Universe in its classical gravity regime becomes the quantum acceleration K_Q in the quantum dual gravity regime. The temperature T_Λ, measure of the classical gravitational length or mass, becomes the quantum temperature T_Q (measure of the quantum size or Compton length) in the quantum regime. Consistently, the Gibbons-Hawking temperature is *precisely* the quantum temperature T_Q. Similarly, the classical or semiclassical gravitational area or entropy S_Λ (Gibbons-Hawking entropy) has its quantum dual S_Q in the quantum gravity (Planckian and trans-Planckian) regime. In Secs. III and VIII, we discuss the concept of gravitational entropy and its expressions in the different gravity regimes. The concept of gravitational entropy is *the same* for any of the gravity regimes: area$/4l_P^2$ in units of k_B. For a classical object of size L_Λ, this is the classical area $A_\Lambda = 4\pi L_\Lambda^2$. For a quantum object of quantum size L_Q, this is the area $A_Q = 4\pi L_Q^2$:

$$A_\Lambda = a_P \left(\frac{L_\Lambda}{\lambda_P}\right)^2, \quad A_Q = a_P \left(\frac{\lambda_P}{L_\Lambda}\right)^2 = \frac{a_P^2}{A_\Lambda}, \quad a_P = 4\pi l_P^2, \tag{2.5}$$

a_P being the Planck area. The corresponding gravitational entropies S_Λ and S_Q are, respectively,

$$S_\Lambda = \frac{\kappa_B}{4} \frac{A_\Lambda}{l_P^2}, \qquad S_Q = \frac{\kappa_B}{4} \frac{A_Q}{l_P^2}, \tag{2.6}$$

and the total (classical and quantum) gravitational entropy $S_{Q\Lambda}$ is given by

$$S_{Q\Lambda} = 2\left[s_P + \frac{1}{2}(S_\Lambda + S_Q)\right], \qquad s_P = \frac{\kappa_B}{4}\frac{a_P}{l_P^2} = \pi\kappa_B, \tag{2.7}$$

s_P being the Planck entropy.

III. CLASSICAL, SEMICLASSICAL, AND QUANTUM VACUUM ENERGY OF THE UNIVERSE

The classical Universe today U_Λ is precisely a *classical dilute gravity vacuum dominated by voids and supervoids* as shown by observations [15–17] whose observed ρ_Λ or Λ value today [6–10] is *precisely* the classical dual of its quantum precursor values ρ_Q and Λ_Q in the quantum very early precursor vacuum U_Q as determined by Eqs. (2.1) and (2.2). The high density ρ_Q and cosmological constant Λ_Q are precisely the quantum particle physics trans-Planckian value 10^{122}. This is precisely expressed by Eqs. (2.1) and (2.2) applied to this case:

$$\Lambda = 3H^2 = \lambda_P\left(\frac{H}{h_P}\right)^2 = \lambda_P\left(\frac{l_P}{L_H}\right)^2 = (2.846 \pm 0.076)\times 10^{-122} m_P^2, \tag{3.1}$$

$$\Lambda_Q = 3H_Q^2 = \lambda_P\left(\frac{h_P}{H}\right)^2 = \lambda_P\left(\frac{L_H}{l_P}\right)^2 = (0.3516 \pm 0.094)\times 10^{122} h_P^2, \tag{3.2}$$

$$\Lambda_Q = \frac{\lambda_P^2}{\Lambda}, \qquad \lambda_P = 3h_P^2. \tag{3.3}$$

The quantum dual value Λ_Q is *precisely* the quantum vacuum value $\rho_Q = 10^{122}\rho_P$ obtained from particle physics:

$$\rho_Q = \rho_P\left(\frac{\Lambda_Q}{\lambda_P}\right) = \frac{\rho_P^2}{\rho_\Lambda} = 10^{122}\rho_P. \tag{3.4}$$

In the last rhs of Eqs. (3.1)–(3.3), the data from Refs. [6–10] have been used, which we also *link to the gravitational entropy and temperature of the Universe*. The *complete* total vacuum energy density $\rho_{Q\Lambda}$ or $\Lambda_{Q\Lambda}$ is the sum of its classical and quantum components (corresponding to the classical era today and its quantum Planckian and trans-Planckian precursor):

$$\Lambda_{Q\Lambda} = \lambda_P\left(\frac{\Lambda}{\lambda_P} + \frac{\lambda_P}{\Lambda} + 1\right) = \lambda_P(10^{-122} + 10^{+122} + 1). \tag{3.5}$$

The observed Λ or ρ_Λ today is the *classical gravity vacuum* value in the classical Universe U_Λ *today*. Such an observed value must be consistent in such a way because of the *large classical* size of the Universe today $L_\Lambda = \sqrt{3/\Lambda}$ and of the empty or vacuum dilute state today dominated by *voids and supervoids* as shown by the set of large structure observations [15–17]. This is one main physical reason for such a *low* Λ value at the present age today $10^{61}t_P$. Its precursor value and density Λ_Q and ρ_Q have a high super-Planckian value precisely because this is a high density very early *quantum cosmological vacuum* in the extreme past $10^{-61}t_P$ of the quantum trans-Planckian precursor phase U_Q.

The quantum vacuum density $\Lambda_Q = \rho_Q = 10^{122}$ (in Planck units) in the precursor trans-Planckian phase U_Q at $10^{-61}t_P$ (the extreme past) became the classical vacuum density $\Lambda = \rho_\Lambda = 10^{-122}$ in the classical Universe U_Λ today at $10^{61}t_P$. The trans-Planckian value is consistent in such a way because is an extreme quantum gravity (trans-Planckian) vacuum in the extreme quantum past $10^{-61}t_P$ with minimal entropy $S_Q = 10^{-122} = \Lambda = \rho_\Lambda$. Equations (3.1)–(3.5) concisely *explain why* the classical gravitational vacuum Λ or ρ_Λ *coincides* with such an observed *low value* 10^{-122} in Planck units and *why* their corresponding quantum gravity precursor vacuum has such an extremely *high* trans-Planckian *value* 10^{122}. The classical gravitational entropy S_Λ today has *precisely* such a high value:

$$S_\Lambda = s_P\left(\frac{\rho_Q}{\rho_P}\right) = s_P\left(\frac{\lambda_P}{\Lambda}\right) = s_P \times 10^{+122}, \tag{3.6}$$

$$S_Q = s_P\left(\frac{\rho_\Lambda}{\rho_P}\right) = s_P\left(\frac{\Lambda}{\lambda_P}\right) = s_P \times 10^{-122}. \tag{3.7}$$

The *total* $Q\Lambda$ (classical and quantum) gravitational entropy $S_{Q\Lambda}$ derives from the general expression

$$S_{Q\Lambda} = (A_{Q\Lambda}/4l_{l_P}^2)k_B,$$

where the total area

$$A_{Q\Lambda} = 4\pi L_{Q\Lambda}^2 = 4\pi(L_Q + L_\Lambda)^2$$

is expressed as $A_{Q\Lambda} = A_Q + A_\Lambda + 2a_P$. Recall that $L_Q = l_P^2/L_\Lambda$ and $a_P = 4\pi l_P^2$.

As a consequence,

$$S_{Q\Lambda} = 2s_P + S_\Lambda + S_Q = 2s_P\left[1 + \frac{1}{2}(10^{+122} + 10^{-122})\right], \tag{3.8}$$

s_P being the Planck entropy. The *total* $Q\Lambda$ gravitational entropy turns out to be the sum of the three components as it must be: classical (subscript Λ), quantum (subscript Q), and Planck value (subscript P) corresponding to the three gravity regimes. The term $2s_P$ arises from the duality between the quantum and classical lengths L_Q and L_Λ across the Planck scale. The factor 2 reflects the complete $Q\Lambda$ covering, the Planck scale being the bordering or crossing scale common to the two (classical and quantum) Q and Λ domains.

The gravitational entropy S_Λ of the present time large *classical Universe* is a very *huge number*, consistent with the fact that the Universe today contains a very huge amount of information. Moreover, to reach such a huge size and entropy today 10^{+122}, the Universe in its very beginning should have been in a hugely energetic initial vacuum 10^{+122}.

A whole picture.—Overall, a consistent unifying picture of the gravitational cosmic history through *its vacuum energy* does emerge from the extreme past quantum trans-Planckian, Planckian, and post-Planckian phases—semiclassical (inflation) and classical phases today and their relevant physical magnitudes: size, age, gravitational entropy, and temperature, all in terms of the vacuum energy. This sheds light on inflation and dark energy. The whole duration (of the trans-Planckian plus post-Planckian eras) is precisely $10^{-61} \leq t \leq 10^{+61}$ (in Planck units $t_P = 10^{-44}$ s). That is to say, *each* time component *naturally* dominates in each phase: classical time component 10^{+61} in the classical era and quantum Planck time t_P in the quantum preceding era. The present time of the Universe at $10^{+61} t_P$ is *a lower bound* for the future (if any) age of the Universe; the remote past quantum precursor equal to $10^{-61} t_P$ is an *upper bound* for the origin of the Universe. The known classical and semiclassical inflation era which occurred at about $10^{+6} t_P$, $H = 10^{-6} h_P$ has a preceding quantum era at $10^{-6} t_P$, $H = 10^{6} h_P$ which is, in fact, a semiquantum era ("low H" with respect to the extreme past trans-Planckian state $H = 10^{61} h_P$) and, similarly, for any of the other known eras in the classical post-Planckian Universe: They have a corresponding quantum precursor era in the trans-Planckian phase. This appears to be the way in which the Universe has evolved.

The *total or complete (classical plus quantum)* physical quantities are invariant under the classical-quantum duality: $H \leftrightarrow Q$ (or $\Lambda \leftrightarrow Q$) as it must be: This means physically that (i) what occurred in the quantum phase before t_P *determines* through Eqs. (2.1)–(2.4) what occurred in the classical phase after t_P, and (ii) what occurred in the quantum phase before the Planck time t_P is the *same observable* which occurred after t_P but in a different physical state in the precise meaning of Eqs. (2.1)–(2.4). That is to say, the quantum quantities in the phase before t_P are the *quantum precursors* of the classical and semiclassical quantities after t_P. As the wave-particle duality at the basis of quantum mechanics, the wave-particle-gravity duality is reflected in all cosmological eras and its associated quantities, temperatures, and entropies. Cosmological evolution goes from a quantum trans-Planckian vacuum energy phase to a semiclassical accelerated era (de Sitter inflation) and then to the classical known eras until the present classical de Sitter phase. The classical-quantum or wave-particle-gravity duality specifically manifests in this evolution, between the different gravity regimes, and could be viewed as a mapping between asymptotic (in and out) states characterized by sets U_Q and U_Λ and, thus, as a scattering-matrix description.

IV. CLASSICAL AND QUANTUM DUAL DE SITTER UNIVERSES

de Sitter space-time in D space-time dimensions is the hyperboloid embedded in $(D+1)$-dimensional Minkowski space-time:

$$X^2 - T^2 + X_j X^j + Z^2 = L_H^2, \quad j = 2, 3, \ldots (D-2). \tag{4.1}$$

L_H is the classical radius or characteristic length of the de Sitter Universe. The scalar curvature R is constant. Classically,

$$L_H = c/H, \qquad R = H^2 D(D-1) = \frac{2D}{(D-2)}\Lambda,$$
$$\Lambda = \frac{H^2}{2}(D-1)(D-2).$$

A mass M_H can be associated to L_H or H, such that ($D = 4$ for simplicity)

$$L_H = GM_H/c^2 \equiv L_G, \qquad M_H = c^3/(GH). \tag{4.2}$$

The corresponding quantum dual magnitudes L_Q and M_Q are, respectively,

$$L_Q = \frac{\hbar}{M_H c} = \frac{\hbar G H}{c^3} = \frac{l_P^2}{L_H}, \quad M_Q = \frac{\hbar H}{c^2} = \frac{m_P^2}{M_H}, \tag{4.3}$$

$$\text{i.e.,} \qquad L_Q = \frac{l_P^2}{L_H}, \qquad M_Q = \frac{m_P^2}{M_H}, \tag{4.4}$$

l_P and m_P being the Planck length and Planck mass, respectively:

$$l_P = \sqrt{\hbar G/c^3}, \qquad m_P = \sqrt{c\hbar/G}. \tag{4.5}$$

The quantum dual Hubble constant H_Q and the quantum curvature R_Q are, respectively,

$$H_Q = h_P^2/H, \qquad R_Q = r_P^2/R, \qquad \Lambda_Q = \lambda_P^2/\Lambda, \tag{4.6}$$

where h_P, r_P, and λ_P are the Planck-scale values of the Hubble constant, scalar curvature, and cosmological constant, respectively:

$$h_P = c/l_P, \quad r_P = h_P^2 D(D-1), \quad \lambda_P = \frac{h_P^2}{2}(D-1)(D-2), \tag{4.7}$$

$$h_P = c^2\sqrt{c/\hbar G}, \qquad r_P = 12h_P^2 = 4\lambda_P,$$
$$\lambda_P = 3(c^5/\hbar G)(D=4). \tag{4.8}$$

V. TOTAL DE SITTER UNIVERSE AND ITS DUALITY SYMMETRY

The classical and quantum lengths L_H and L_Q can be extended to a more complete length L_{QH} which contains both the Q and H lengths:

$$L_{QH} = (L_H + L_Q) = l_P\left(\frac{L_H}{l_P} + \frac{l_P}{L_H}\right), \tag{5.1}$$

and we have then

$$X^2 - T^2 + X_j X^j + Z^2 = L_{QH}^2$$
$$= 2l_P^2\left[1 + \frac{1}{2}\left[\left(\frac{L_H}{l_P}\right)^2 + \left(\frac{l_P}{L_H}\right)^2\right]\right] \tag{5.2}$$

with $j = 2, 3, \ldots(D-2)$. Z is the extra coordinate for the embedding of de Sitter space-time in Minkowski space-time.

Equation (5.2) quantum generalizes de Sitter space-time including the classical, semiclassical, and quantum Planckian and trans-Planckian de Sitter regimes as well. It contains two nonzero lengths (L_H, L_Q) or two relevant scales (H, l_P), enlarging the possibilities for the space-time phases.

(i) For $L_H \gg l_P$, i.e., $L_Q \ll L_H$, Eq. (5.2) yields the classical de Sitter space-time. For intermediate L_H values between l_P and L_Q, it yields the semiclassical de Sitter space-time.

(ii) For $L_H = l_P$, i.e., $L_Q = l_P = L_{QH}$, Eq. (5.2) yields the Planck-scale de Sitter hyperboloid.

(iii) For $L_H \ll l_P$, i.e., $L_Q \gg L_H$, it yields the highly quantum de Sitter regime, deep inside the Planck domain.

$H = c/L_H$ is (c^{-1}) times the surface gravity (or gravity acceleration) of the classical de Sitter space-time. Similarly, $H_Q = c/L_Q$ and $H_{QH} = c/L_{QH}$ are the surface gravity in the quantum and whole QH de Sitter phases, respectively. Similarly, Eqs. (5.1) and (4.2)–(4.4) yield for the mass

$$M_{QH} = (M_H + M_Q) = m_P\left(\frac{M_H}{m_P} + \frac{m_P}{M_H}\right), \tag{5.3}$$

$$\frac{M_{QH}}{m_P} = m_P\left(\frac{L_H}{l_P} + \frac{l_P}{L_H}\right) = \frac{L_{QH}}{l_P}. \tag{5.4}$$

M_{QH}/m_P and L_{QH}/l_P both have the same expression with respect to their respective Planck values.

A. The complete QH Hubble constant H_{QH}, curvature R_{QH}, and Λ_{QH}

The total (classical and quantum) QH Hubble constant H_{QH}, curvature R_{QH}, and Λ_{QH} follow from the QH de Sitter length L_{QH} [Eq. (5.1)]:

$$H_{QH} = \frac{c}{L_{QH}}, \qquad R_{QH} = H_{QH}^2 D(D-1),$$
$$\Lambda_{QH} = \frac{H_{QH}^2}{2}(D-1)(D-2), \tag{5.5}$$

where from Eqs. (5.1) and (4.6)

$$H_{QH} = \frac{H}{[1 + (l_P H/c)^2]}, \qquad H_{QH}/h_P = \frac{(H/h_P)}{[1 + (H/h_P)^2]},$$
$$h_P = c/l_P, \tag{5.6}$$

which exhibit the *symmetry* of H_{QH}/h_P under $(H/h_P) \to (h_P/H)$, i.e., under $H \to H_Q = (h_P^2/H)$:

$$H_{QH}(H/h_P) = H_{QH}(h_P/H). \tag{5.7}$$

The classical H and quantum H_Q are classical-quantum duals of each other through the Planck scale h_P, but the total H_{QH} is *invariant*. And similarly, for the total quantum curvature R_{QH} and cosmological constant Λ_{QH} [Eq. (5.5)],

$$R_{QH}(H/h_P) = R_{QH}(h_P/H),$$
$$\Lambda_{QH}(H/h_P) = \Lambda_{QH}(h_P/H), \tag{5.8}$$

where

$$R_{QH} = \frac{R_H}{[1 + R_H/r_P]^2} = \frac{R_Q}{[1 + R_Q/r_P]^2}, \qquad r_P = 12h_P^2, \tag{5.9}$$

$$\Lambda_{QH} = \frac{\Lambda_H}{[1 + \Lambda_H/\lambda_P]^2} = \frac{\Lambda_Q}{[1 + \Lambda_Q/\lambda_P]^2}, \qquad \lambda_P = 3h_P^2. \tag{5.10}$$

The classical $H/h_P \ll 1$, quantum $H/h_P \gg 1$, and Planck $H/h_P = 1$ regimes are clearly exhibited in the QH expressions Eqs. (5.5) and (5.6):

$$H_{QH(H\ll h_P)} = H[1 - (H/h_P)^2] + O(H/h_P)^4, \tag{5.11}$$

$$H_{QH}(H = h_P) = \frac{h_P}{2}, \qquad h_P = c/l_P, \tag{5.12}$$

$$H_{QH(H\gg h_P)} = (h_P^2/H)[1 - (h_P/H)^2] + O(h_P/H)^4. \tag{5.13}$$

The three above equations show, respectively, the three different de Sitter phases:

(i) the classical gravity de Sitter Universe (with lower curvature than the Planck scale r_P) *outside* the Planck domain ($l_P < L_H < \infty$),
(ii) the Planck curvature de Sitter state ($R_H = r_P$, $L_H = l_P$), and
(iii) the highly quantum or high curvature ($R_H \gg r_P$) de Sitter phase *inside* the quantum gravity Planck domain ($0 < L_H \leq l_P$).

It is natural here to define the dimensionless magnitudes:

$$\mathcal{L} \equiv L_{QH}/l_P, \qquad \mathcal{M} \equiv M_{QH}/m_P, \qquad \mathcal{H} \equiv H_{QG}/h_P,$$
$$l \equiv L_H/l_P, \qquad h \equiv H/h_P = l^{-1}, \tag{5.14}$$

in terms of which Eqs. (5.1), (5.3), and (5.6) and their duality symmetry Eqs. (5.7) and (5.8) simply read

$$\mathcal{L} = \left(l + \frac{1}{l}\right) = \mathcal{M}, \qquad \mathcal{H} = \frac{1}{(l + \frac{1}{l})} = \mathcal{L}^{-1}, \tag{5.15}$$

$$\mathcal{L}(l^{-1}) = \mathcal{L}(l), \qquad \mathcal{M}(l^{-1}) = \mathcal{M}(l), \tag{5.16}$$

$$\mathcal{H}(l^{-1}) = \mathcal{H}(l), \qquad \mathcal{R}(l^{-1}) = \mathcal{R}(l), \qquad \mathbf{\Lambda}(l^{-1}) = \mathbf{\Lambda}(l). \tag{5.17}$$

The QH magnitudes are complete variables covering both classical and quantum, Planckian and trans-Planckian, domains. Similarly, for the classical, quantum, and QH de Sitter densities (ρ_H, ρ_Q, and ρ_{QH}), ρ_P being the Planck density scale,

$$\rho_H = \rho_P (H/h_P)^2 = \rho_P (\Lambda/\lambda_P),$$
$$\rho_P = 3h_P^2/8\pi G, \qquad \lambda_P = 3h_P^2/c^4, \tag{5.18}$$

$$\rho_Q = \rho_P (H_Q/h_P)^2 = \rho_P (\Lambda_Q/\lambda_P) = \rho_P^2/\rho_H$$
$$= \rho_P (h_P/H)^2 = \rho_P (\lambda_P/\Lambda), \tag{5.19}$$

$$\rho_{HQ} = \rho_H + \rho_Q = \rho_P (H_{HQ}/h_P)^2 = \rho_P (\Lambda_{HQ}/\lambda_P), \tag{5.20}$$

from which it follows that

$$\rho_{HQ} = \frac{\rho_H}{[1 + \rho_H/\rho_P]^2} = \frac{\rho_Q}{[1 + \rho_Q/\rho_P]^2}, \tag{5.21}$$

which satisfies

$$\rho_{HQ}(\rho_H) = \rho_{HQ}(\rho_Q) = \rho_{HQ}(\rho_P^2/\rho_H).$$

For small and high densities with respect to the Planck density ρ_P, the QH density ρ_{QH} behaves as

$$\rho_{QH}(\rho_H \ll \rho_P) = \rho_H [1 - 2(\rho_H/\rho_P)] + O(\rho_H/\rho_P)^2, \tag{5.22}$$

$$\rho_{QH}(\rho_H = \rho_Q = \rho_P) = \frac{1}{4}\rho_P: \qquad \text{(Planck-scale density)}, \tag{5.23}$$

$$\rho_{QH}(\rho_H \gg \rho_P) = \rho_Q [1 - 2(\rho_P/\rho_H)] + O(\rho_P/\rho_H)^2, \tag{5.24}$$

corresponding to the classical and semiclassical de Sitter regime (and its quantum corrections), Planck-scale de Sitter state, and highly quantum trans-Planckian de Sitter density, respectively. The complete QH de Sitter magnitudes (L_{QH}, H_{QH}, and M_{QH}) [and their constant Planck-scale values (l_P, h_P, and m_P) depending only on $(c, \hbar, G)$] allow one to characterize in a precise way the classical, semiclassical, Planckian, and quantum (super-Planckian) de Sitter regimes.

(i) $L_{QH} = L_{QH}(L_H, L_Q) \equiv L_{QH}(H, \hbar)$ yields the *whole* (classical and semiclassical, Planck-scale, and quantum (super-Planckian) de Sitter Universe.
(ii) $L_{QH} = L_H = L_Q$ yields the Planckian de Sitter state (Planck length de Sitter radius, Planckian vacuum density, and Planckian scalar curvature): $L_H = l_P$, $H = h_P$, $\lambda_P = 3h_P^2$, $R = r_P = 4\lambda_P$, and $l_P = \sqrt{(\hbar G/c^3)}$.
(iii) $L_{QH} = L_H \gg L_Q$, i.e., $L_H \gg l_P$ and $H \ll h_P$, yields the classical de Sitter space-time.
(iv) $L_{QH} = L_Q \gg L_H$, i.e., $L_H \ll l_P$ and $H \gg h_P$ (high curvature $R \gg r_P = 4\Lambda_P$), yields a full quantum gravity trans-Planckian de Sitter phase (inside the Planck domain $0 < L_H \leq l_P$).
(v) $L_{QH} \gg L_Q$, i.e., $L_{QH} \to \infty$ for $L_H \to \infty$, i.e., $H \to 0$, i.e., $\Lambda \to 0$ (zero curvature), yields consistently the classical Minkowski space-time, equivalent to the limit $L_Q \to 0$, i.e., $l_P \to 0$ ($\hbar \to 0$).

The three de Sitter regimes are characterized in a complete and precise way.

(i) *Classical and semiclassical de Sitter regimes* (inflation and, more generally, the whole known—classical and semiclassical—Universe is within this regime): $l_p < L_H < \infty$, i.e., $0 < L_Q < l_P$, $0 < H < h_P$, and $m_P < M_H < \infty$.
(ii) *Planck-scale de Sitter state with Planck curvature and Planck radius:* $L_H = l_P$, $L_Q = l_P$, $H = h_P = c/l_P$, and $M_H = m_P$.
(iii) *Quantum Planckian and trans-Planckian de Sitter regimes:* $0 < L_H \leq l_P$, i.e., $l_P \leq L_Q < \infty$, $h_P \leq H < \infty$, and $0 < M_H < m_P$.

VI. DE SITTER UNIVERSE AND THE HARMONIC OSCILLATOR

As is known, the Einstein equations in the presence of a constant vacuum energy (cosmological constant) are

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{6.1}$$

and the energy-momentum tensor corresponding to the vacuum energy density ρ and pressure p is

$$T_{\mu\nu} = p g_{\mu\nu} = -\rho g_{\mu\nu} \qquad (p = w\rho, \quad w \equiv -1), \tag{6.2}$$

the vacuum energy being equivalent to a cosmological constant: $\rho_\Lambda = \Lambda c^4/(8\pi G)$.

As known, de Sitter space-time has *constant* scalar space-time curvature:

$$R = 12H^2 = 4\Lambda, \qquad \Lambda = 3H^2 (D = 4).$$

We restrict to $D = 4$. Recall that the energy-momentum tensor for massive particles of density ρ plus vacuum constant energy (or cosmological constant) Λ is

$$T^\mu_\nu = \rho_\Lambda \delta^\mu_\nu + \rho \delta^{\mu 0}\delta_{\nu 0}, \qquad T \equiv T^\mu_\mu = 4\rho_\Lambda + \rho, \tag{6.3}$$

where we neglected the pressure to better illustrate our purpose. The corresponding Einstein equations are

$$R^\mu_\nu = 8\pi G\left(T^\mu_\nu - \frac{\delta^\mu_\nu}{2} T\right), \qquad 0 \le \mu, \quad \nu \le 3, \tag{6.4}$$

and for nonrelativistic matter its pressure is neglected with respect to its rest mass.

In the weak field limit

$$g_{00} = 1 + 2V, \qquad g_{ik} = -\delta_{ik}, \qquad R^0_0 = \nabla^2 V,$$

V being the gravitational potential, Einstein's Eqs. (6.4) become

$$\nabla^2 V = 4\pi G\rho - 8\pi G\rho_\Lambda, \tag{6.5}$$

$$V(\vec{X}) = V_\rho(X) - \frac{4\pi G\rho_\Lambda}{3} X^2. \tag{6.6}$$

For a distribution of rest particles of mass m, $\rho(\vec{X}) = m\sum_i \delta(\vec{X} - \vec{X}_i)$, the gravitational potential $V(\vec{X})$, gravitational field $\mathcal{G}$, and potential energy $\mathcal{U}$ of the system are, respectively,

$$V(\vec{X}) = V(\vec{X})_m - \frac{4\pi G\rho_\Lambda}{3} X^2,$$
$$V(\vec{X})_\rho \equiv V(\vec{X})_m = -G\sum_i \frac{m}{|\vec{X} - \vec{X}_i|},$$
$$\vec{\mathcal{G}}(\vec{X}) = -\nabla V(\vec{X}) = \vec{\mathcal{G}}_m + \frac{8\pi G\rho_\Lambda}{3}\vec{X}, \tag{6.7}$$

$$\mathcal{U} = \mathcal{U}_m - \frac{4\pi G\rho_\Lambda}{3} m \sum_i X_i^2. \tag{6.8}$$

Therefore, the Hamiltonian is equal to

$$\frac{P_i P^i}{m^2} + \mathcal{U} = \frac{P_i P^i}{m^2} - \frac{4\pi G\rho_\Lambda}{3} m X_i^2. \tag{6.9}$$

For a relativistic perfect fluid with $T^{\mu\nu} = (p+\rho)u^\mu u^\nu + p g^{\mu\nu}$ and continuity equation $D_\nu T^{\mu\nu} = 0$, the 00 component of the Einstein equations yields a relativistic Poisson equation similar to Eq. (6.5) sourced with the addition of the fluid pressure p to the density and to the Λ term which remains unchanged. In this case, the potential is coupled to the Euler fluid equations, which linearized perturbations for each component can be reduced to an equation of the form $\ddot{\delta} + f\delta = 0$, f depending of the unperturbed background fluid components and on the Λ term, which has always opposite sign to the other component terms. Our interest in this paper not being in the structure formation and evolution but in the vacuum Λ energy, we will not discuss more here on this case.

The cosmological constant energy contribution to the potential energy $\mathcal{U}$ Eq. (6.9) decreases for increasing values of the particle distances r_i to the center of mass. The gravitational effect of the vacuum zero-point energy or cosmological constant push particles outward, and, equivalently, the last term of the gravitational field Eq. (6.7) points outward (the repulsive cosmological constant effect). The Hamiltonian Eq. (6.9) is like that of a harmonic oscillator for a particle of mass m and oscillator constant $\omega^2 m$. We analyze it in Sec. VII below.

The nonrelativistic particle motion and the relativistic geodesics both exhibit the same runaway behavior. The nonrelativistic approximation reflects well the relativistic particle motion in the de Sitter space-time and its connection to the harmonic oscillator. In the relativistic situation, g_{00} determined by the Einstein equations for the de Sitter metric entails the harmonic oscillator potential, e.g.,

$$d/dr(r g_{00}) = 1 - (8\pi G\rho_{00} + \Lambda)r^2.$$

Parametrization of the de Sitter hyperboloid in terms of the coordinates (t, r, θ, ϕ):

$$T = H^{-1}(1 - H^2 r^2)^{1/2}\sinh Ht;$$
$$X = H^{-1}(1 - H^2 r^2)^{1/2}\cosh Ht,$$

$$X_2 = r\cos\theta; \qquad X_3 = r\sin\phi\cos\theta; \qquad Z = r\sin\phi,$$

yields

$$ds^2 = -(1 - H^2 r^2)dt^2 + (1 - H^2 r^2)^{-1}dr^2 + r^2 d\Omega^2$$

and $X^2 - T^2 = H^{-2}(1 - H^2 r^2)$, containing the (inverted) harmonic oscillator potential.

Our purpose in this paper is to show within a minimal setting the essential features relating de Sitter space-time, e.g., the cosmological constant or vacuum energy density, to the harmonic oscillator. We are interested in the vacuum energy density, and so we do not include all other particle interactions. Self-gravitation interaction among the particles is described by the parameter (Gm^2N), while the interaction with Λ is through the parameter (Λm). Their quotient, namely, $\eta = (\text{vacuum energy}/\text{mass}) = \Lambda/(mGN)$, N being the number of particles, determines the condition on whether one dominates over the other, and clearly our interest in this paper is in the regimes where the vacuum energy Λ dominates over the self-particle interactions, i.e., $\eta \leq 1$, which is the condition for interactions to be neglected. Of course, virialization occurs too for larger η for self-gravitating particles in the presence of Λ.

In a QFT description, particle production from the vacuum or inflation driven by Λ are within this situation of Λ we consider, in the pre-Planckian and in the post-Planckian eras. Particle interactions at the Planck scale considered mainly in the context of particle physics, perturbatively and nonperturbatively, as string collisions or as point particle QFT [18–21] yield to the conclusion that the resulting collisional and particle interacting effect can be well described by the field felt by one particle in the effective gravitational curved background produced by all the others. Here, we consider the (nonperturbative) curved space-time background from the beginning. This can be, thus, viewed as the effective background field felt by one particle produced by the particle interactions of all the others.

In the early Planckian and trans-Planckian phases, namely, the nearest possible ones to the Universe origin, it is natural to consider the Λ or vacuum dominance background as we consider which is also motivated by a description of the origin of the Universe "from nothing." Indeed, the Λ or vacuum dominance background in the early phases could be considered as formed as a condensate from such particle interactions. In summary, the effective result of such particle interactions is to produce the curved background.

VII. THE HARMONIC OSCILLATOR AND THE COSMOLOGICAL CONSTANT

For simplicity and physical insight, we consider the case of just one particle; Eqs. (6.5) and (6.6) yield

$$\ddot{\vec{X}} = \frac{\Lambda}{3}\vec{X}. \tag{7.1}$$

This is an *harmonic oscillator* equation with imaginary frequency and *oscillator constant* κ_{oscill}:

$$\ddot{\vec{X}} = -\kappa_{\text{oscill}}\vec{X}, \qquad \kappa_{\text{oscill}} = \omega^2 m, \qquad \omega = \sqrt{\frac{\Lambda}{3m}}, \tag{7.2}$$

with the solution

$$\vec{X}(t) = \vec{X}(0)\cosh Ht + \frac{1}{H}\dot{\vec{X}}(0)\sinh Ht, \tag{7.3}$$

where

$$H \equiv \sqrt{\Lambda/3}.$$

The particle runs away exponentially fast in time. The Hubble constant H^2 is the constant of the oscillator

$$\kappa_{\text{osc}} = H^2, \qquad H = \omega\sqrt{m}, \tag{7.4}$$

the oscillator length l_{osc} being

$$l_{\text{osc}} = \sqrt{3/\Lambda}, \qquad H = c/l_{\text{osc}} = \kappa \equiv \text{surface gravity}.$$

The length of the oscillator is the Hubble radius, and the Hubble constant is the surface gravity of the Universe (similar to the black hole surface gravity, the inverse of the black hole radius).

The nonrelativistic or weak field Newtonian results reproduce very well the full space-time relativistic effects in the presence of the cosmological constant. The exact solution of the Einstein equations for the energy-momentum tensor Eq. (6.3) with $\rho = 0$ is the de Sitter Universe. It must be stressed that the nonrelativistic trajectories Eq. (7.3) exhibit the same exponential runaway behavior of the exact relativistic geodesics in de Sitter space-time. The nonrelativistic approximation keeps the essential features of the particle motion in de Sitter space-time [22–24].

The description of de Sitter space-time as an (inverted) harmonic oscillator appears either in a relativistic or in a nonrelativistic consideration. This stems from its geometrical hyperbolic description: $-T^2 + X^2 + X_i^2 + Z^2 = L^2$ as a hyperboloid embedded in a flat Minkowski space-time with one more spatial dimension. The propagation equations of particles, waves, fields, and strings in de Sitter space-time all reflect the de Sitter space-time connection to the inverted harmonic oscillator; namely, in all these cases, a term of the form and sign of the inverted oscillator [Eqs. (6.9) and (7.1)] does appear; e.g., see [24–27]. In particular, in several regimes, e.g., asymptotically for $t \to \pm\infty$ or for Λ dominance with respect to other field parameters (as masses and couplings), the propagation equations in de Sitter space-time (cosmic time, for instance) reduce to

$$\ddot{\chi} - \nu^2 H^2 \chi = 0, \qquad \nu^2 \equiv \nu^2(m^2, H^2, \xi).$$

In general, $\nu^2 \equiv \nu^2(m^2, H^2, \xi)$, and its sign depends on the relationship between H, m, and the couplings ξ. For instance, for inflation, H^2 dominates over m^2 and the couplings, and ν^2 is positive. Squeezed states are characteristic of this propagation, e.g., for quantum fields; see for example Refs. [25,26].

We summarize in the following our main results allowing one to describe de Sitter (and anti–de Sitter) space-time as a classical and quantum harmonic oscillator.

(i) The motion of a particle in a harmonic oscillator potential corresponds to the particle motion in the nonrelativistic limit of a constant curvature space-time. The harmonic oscillator with an imaginary frequency, namely, the inverted oscillator for $\Lambda > 0$, corresponds to de Sitter space-time; the real frequency normal oscillator $\Lambda < 0$ describes anti–de Sitter space-time; and the free motion is flat Minkowski space-time $\Lambda = 0$.

(ii) The constant of the oscillator is the cosmological constant, as shown by Eq. (7.2), which is the Hubble constant H^2 or surface gravity squared [Eq. (7.4)].

(iii) For the *classical* harmonic oscillator, the phase space is the classical one, and the algebra of the (X, P) variables or (X, T) variables is commuting. The classical Hamiltonian is $2H_{\text{osc}} = X^2 + P^2$ or $2H_{\text{inv-osc}} = X^2 - P^2$ for the inverted oscillator, in light-cone variables $2UV = 2VU$. The light-cone structure $X^2 - T^2$ is the classical known one; there is no difference with the Minkowski light-cone structure of special relativity. Upon the identification $P = T$, the classical commuting (X, T) variables of Minkowski space-time and its invariant distance $s^2 = X^2 - T^2$ correspond to a classical phase space (X, P) and Hamiltonian $s^2 = 2H_{\text{inv-osc}} = X^2 - P^2$, which is the (inverted) harmonic oscillator Hamiltonian.

(iv) The nonrelativistic approximation describes very well the essential properties of the constant curvature—de Sitter or anti–de Sitter—geometries and captures its physics. Thus, the classical nonrelativistic de Sitter invariant space-time, or the anti–de Sitter space-time, and the Minkowski Poincaré-invariant space-time all three describe special relativity. We see that this reaches from another approach and motivation, the fact that a constant curvature space-time describes special relativity, as in Refs. [28,29], or the so-called "triple relativity" $\Lambda > 0$, $\Lambda < 0$, and $\Lambda = 0$.

(v) For the *quantum* harmonic oscillator, the quantum zero-point energy bends the light hyperbolic cone generators $X^2 - T^2 = 1$, and, therefore, the space-time is curved: de Sitter (or anti–de Sitter) space-time. And, as is known, the nonrelativistic and relativistic de Sitter space-times are very similar.

(vi) Upon the identification $T = P$, the quantum noncommuting coordinates (X, T) of Minkowski space-time and its distance $s^2 = X^2 - T^2$ are the noncommuting phase space (X, P) and quadratic form $2H_{\text{inv-osc}} = X^2 - P^2 \to s^2$, which is the harmonic oscillator Hamiltonian. And this is also the Hamiltonian of a particle in a constant curvature (cosmological constant) de Sitter or anti–de Sitter space (in its nonrelativistic limit).

(vii) Explicitly, the (a, a^+) creation and annihilation operators are the light-cone-type quantum coordinates of the phase space (X, P): $a = (X + iP)/\sqrt{2}$ and $a^+ = (X - iP)/\sqrt{2}$. The temporal variable T in the space-time configuration (X, T) is like the momentum in phase space (X, P). The identification $P = T$ yields

$$X = (a + a^+)/\sqrt{2}, \qquad T = (a - a^+)/i\sqrt{2}, \qquad [a, a^+] = 1, \tag{7.5}$$

$$2X^2 = [(2a^+a + 1) + (a^2 + a^{+2})],$$
$$2T^2 = [(2a^+a + 1) - (a^2 + a^{+2})]$$

with the algebra

$$H_{\text{osc}} = (X^2 + T^2) = (2a^+a + 1),$$
$$H_{\text{inv-osc}} = (X^2 - T^2) = (a^2 + a^{+2}),$$
$$[X, T] = i, \qquad [H_{\text{inv-osc}}, X] = 2iT, \qquad [H_{\text{inv-osc}}, T] = 2iX, \tag{7.6}$$

$a^+a = N$ being the number operator.

(viii) In other words, the nonrelativistic cosmological constant (de Sitter or anti–de Sitter) space-time, the harmonic oscillator phase space, and Minkowski space-time are in correspondence one to another. The line element in Minkowski space-time in D space-time dimensions $s^2 = X^2 - T^2 + X_j^2$ is equal to the (nonrelativistic) harmonic oscillator Hamiltonian $2H_{\text{inv-osc}} = X^2 - P^2 + X_j^2$. Thus, there are three possibilities for special relativity. The interesting point in our studies is that the *quantum* harmonic oscillator algebra describes the *quantum noncommuting space-time* structure.

(ix) Upon the identification $T = P$, the de Sitter hyperboloid Eq. (4.1) yields

$$X^2 - P^2 + X_j^2 + Z^2 = L_{QH}^2, \quad j = 2, 3, \ldots (D-2), \tag{7.7}$$

corresponding to a (inverted) harmonic oscillator (X, P) embedded in a Minkowski space of $(D - 2 + 2) = D$ spatial dimensions, i.e., a Minkowski space-time of $(D + 1)$ space-time dimensions.

VIII. QUANTUM DISCRETE LEVELS OF THE UNIVERSE

Let us go beyond the classical-quantum duality of the space-time recently discussed and promote the space-time coordinates to quantum noncommuting operators. As we have seen, comparison to the harmonic oscillator (X, P)

variables and global phase space is enlightening: The hyperbolic phase space here $(X, P = T)$ describes the hyperbolic quantum space-time structure and generates the quantum light cone. The classical Minkowski space-time null generators $X = \pm T$ *disappear* at the quantum level due to the relevant $[X, T]$ commutator which is *always* nonzero. A new quantum Planck-scale vacuum region emerges. In the case of the Rindler and Schwarzschild-Kruskal space-time structures, the four Kruskal regions merge inside a single quantum Planck-scale region [1,3].

The quantum space-time structure consists of discrete levels of odd numbers:

$$X_n^2 = (2n+1), \qquad T_n^2 = (2n+1)(\text{in Planck units}), \quad n = 0, 1, 2\ldots, \tag{8.1}$$

(X_n, T_n) and the *mass levels* being $\sqrt{(2n+1)}$, $n = 0, 1, 2\ldots$.

The Planck-scale level $(X, T)(n = 0) = 1$ is the fundamental $(n = 0)$ level from which the space-time levels (X_n, T_n) go to the quantum (low n) levels and to the semiclassical and classical (large n) levels. Asymptotically, for very large n, the space-time becomes continuum.

In terms of variables $(x_{n\pm}, t_{n\pm})$, covering only one—the pre-Planckian or the post-Planckian phase—the space-time discrete levels read

$$x_{n\pm} = [\sqrt{2n+1} \pm \sqrt{2n}], \tag{8.2}$$

$$t_{n\pm} = [\sqrt{2n+1} \pm \sqrt{(2n+1)+1/2}], \tag{8.3}$$

$$x_{n=0}(+) = x_{n=0}(-) = 1 : \text{Planck scale.}$$

The low-n, intermediate, and large-n levels describe, respectively, the quantum, semiclassical, and classical behaviors; interestingly enough, the $(\pm)$ branches consistently reflect the classical-quantum duality properties.

(X_n, T_n) and (x_n, t_n) are given in Planck (length and time) units. In terms of the global quantum gravity dimensionless length $\mathcal{L} = L_{QH}/l_P$ and mass $\mathcal{M} = M_{QH}/m_P$, Eqs. (5.14) or the local ones $x = m/m_p$ translate into the discrete mass levels:

$$\mathcal{L}_n = \sqrt{(2n+1)} = \mathcal{M}_n, \quad n = 0, 1, 2, \ldots, \tag{8.4}$$

$$L_{QHn\gg1} = l_P\left[\sqrt{2n} + \frac{1}{2\sqrt{2n}} + O(1/n^{3/2})\right], \tag{8.5}$$

$$M_{QHn\gg1} = m_P\left[\sqrt{2n} + \frac{1}{2\sqrt{2n}} + O(1/n^{3/2})\right]. \tag{8.6}$$

The above equations for L_{QHn} and M_{QHn} yield the levels for $L_{Hn\pm}$ and $M_{Hn\pm}$, respectively:

$$L_{Hn\pm} = \left[L_{QHn} \pm \sqrt{L_{QHn}^2 - l_P^2}\right], \tag{8.7}$$

$$M_{Hn\pm} = \left[M_{QHn} \pm \sqrt{M_{QHn}^2 - m_P^2}\right]. \tag{8.8}$$

The condition $L_{QHn} \geq l_P$, $M_{QHn} \geq m_P$ consistently corresponds to the whole spectrum $n \geq 0$, the lowest level $n = 0$ being the Planck mass and length:

$$L_{Hn\pm} = l_P[\sqrt{2n+1} \pm \sqrt{2n}] \quad \text{for all } n = 0, 1, 2, \ldots, \tag{8.9}$$

$$M_{Hn\pm} = m_P[\sqrt{2n+1} \pm \sqrt{2n}] \quad \text{for all } n = 0, 1, 2, \ldots. \tag{8.10}$$

The mass and radius of the Universe M_H and L_H have discrete levels $L_{Hn\pm}$ and $M_{Hn\pm}$, respectively, from the most fundamental one $(n = 0)$, going to the semiclassical (intermediate n), to the classical ones (large n) which yield a continuum classical Universe as it must be. This is clearly seen from the mass level $M_{Hn\pm}$ expressions (and similarly for the radius levels). Explicitly,

$$M_{H(n=0)+} = M_{H(n=0)-} = M_{QH(n=0)} = m_P, \quad n = 0 : \text{Planck mass,} \tag{8.11}$$

$$M_{Hn+} = m_P\left[2\sqrt{2n} - \frac{1}{2\sqrt{2n}} + O(1/n^{3/2})\right], \quad \text{large } n : \text{branch}(+) : \text{masses} > m_P, \tag{8.12}$$

$$M_{Hn-} = \frac{m_P}{2\sqrt{2n}} + O(1/n^{3/2}), \quad \text{large } n : \text{branch}(-) : \text{masses} < m_P. \tag{8.13}$$

Large n levels are semiclassical, tending toward a classical continuum space-time. Low n are quantum, the lowest mode $(n = 0)$ being the Planck scale. Two dual $(\pm)$ branches are present in the local variables $(\sqrt{2n+1} \pm \sqrt{2n})$ reflecting the duality of the large- and small-n behaviors and covering the *whole* spectrum: from the largest cosmological masses and scales in branch $(+)$ to the quantum smallest masses and scales in branch $(-)$ passing by the Planck mass and length.

IX. QUANTUM DISCRETE LEVELS OF THE HUBBLE CONSTANT

Equations (8.4) yield the (dimensionless) quantum levels for the total, Hubble constant, vacuum energy, and constant curvature:

$$\mathcal{H}_n = \frac{1}{\sqrt{(2n+1)}}, \qquad \mathbf{\Lambda}_n = \frac{1}{(2n+1)},$$
$$\mathcal{R}_n = \frac{1}{(2n+1)}, \quad n = 0, 1, 2, \ldots, \tag{9.1}$$

$$n = 0{:}\, \mathcal{H}_0 = 1, \qquad \mathbf{\Lambda}_0 = 1, \qquad \mathcal{R}_0 = 1{:}$$
$$\text{Planck scale (dimensionless)}, \tag{9.2}$$

$$H_{QH(n=0)} = \frac{c}{l_P} = h_P, \qquad \Lambda_{QH(n=0)} = \lambda_P,$$
$$R_{QH(n=0)} = 4\lambda_P{:} \quad \text{Planck-scale values.} \tag{9.3}$$

And for the gravitational entropy,

$$S_n = (2n+1) \,\text{in Planck units}\, s_P = 4\pi.$$

The lowest $n = 0$ level corresponds to the fundamental Planck-scale values $(h_P, \lambda_P, 4\lambda_P, s_P)$ for the Hubble constant, cosmological constant, constant curvature, and gravitational entropy, respectively. Let us analyze now the implications of these results and the general picture which they arise.

In the post-Planckian Universe $t_P \le t \le t_{\text{today}} = 10^{61} t_P$, we see that the physical magnitudes as the Hubble radius, vacuum energy density, constant curvature, and entropy start at the Planck scale: the zero level ($n = 0$). As n increases, the Universe radius, mass, and entropy increase; the Hubble constant, curvature, and vacuum energy *consistently decrease*; and the Universe *classicalizes*. The decreasing with n of these quantities is given by Eq. (9.1) and for large n, H_n, Λ_n, and $\mathcal{R}_n$ *classicalize* as

$$\mathcal{H}_{n\gg1} = \frac{c}{l_P\sqrt{2n}}\left[1 - O\left(\frac{1}{2n}\right)\right] \ll 1, \tag{9.4}$$

$$\mathbf{\Lambda}_{n\gg1} = \frac{3c^2}{l_P^2(2n)}\left[1 - O\left(\frac{1}{2n}\right)\right] \ll 1, \tag{9.5}$$

$$\mathcal{R}_{n\gg1} = \frac{12c^2}{l_P^2(2n)}\left[1 - O\left(\frac{1}{2n}\right)\right] \ll 1, \tag{9.6}$$

precisely accounting for the low classical values of H and Λ *in the Universe today, which is a classical, large, and dilute Universe*. The present Universe values $H_{\text{today}} = 10^{-61}$ and $\rho_\Lambda = 10^{-122}$ correspond to a large n level $n = 10^{122} \equiv n_{\text{today}}$.

More generally, in the post-Planckian Universe $t_P \le t \le t_{\text{today}} = 10^{61} t_P$, Eq. (9.1) yields the quantum n levels:

$$n = \frac{1}{2}(H_n^{-2} - 1){:} \quad t_{(n=0)} = t_P \le t_n \le t_{n\,\text{today}} = 10^{61} t_P. \tag{9.7}$$

Thus, the more characteristic evolution values from the Planck time t_P till today:

$$h_P, \ldots, H_{\text{inf}}, \ldots, H_{\text{cmb}}, \ldots, H_{\text{reion}}, \ldots, H_{\text{today}} \tag{9.8}$$

corresponds to the n levels:

$$n = 0, 1, 2, \ldots n_{\text{inf}} = 10^{12}, \ldots n_{\text{cmb}} = 10^{114}, \ldots n_{\text{reoin}} = 10^{118}, \ldots n_{\text{today}} = 10^{122} \tag{9.9}$$

and the *discrete* H_n, Λ_n, and S_n values:

$$H_n = 1, 0.577, \ldots H_{n,\text{inf}} = 10^{-6}, \ldots H_{n,\text{cmb}} = 10^{-57}, \ldots H_{n,\text{reoin}} = 10^{-59}, \ldots H_{n,\text{today}} = 10^{-61}, \tag{9.10}$$

$$\Lambda_n = 1, 0.333, \ldots \Lambda_{n,\text{inf}} = 10^{-12}, \ldots \Lambda_{n,\text{cmb}} = 10^{-114}, \ldots \Lambda_{n,\text{reoin}} = 10^{-118}, \ldots \Lambda_{n,\text{today}} = 10^{-122}, \tag{9.11}$$

$$S_n = 1, 3, \ldots S_{n,inf} = 10^{12}, \ldots S_{n,cmb} = 10^{114}, \ldots S_{n,reoin} = 10^{118}, \ldots S_{n,today} = 10^{122}. \tag{9.12}$$

In the pre-Planckian or precursor phase, namely, the trans-Planckian phase:

$$10^{-61} t_P \le t_n \le t_P (n = 0), \tag{9.13}$$

the quantum n levels for H_{Qn}, Λ_{Qn}, and S_{Qn} [Eqs. (4.6)] are, respectively,

$$H_{Qn} = \sqrt{2n+1}, \qquad \Lambda_{Qn} = (2n+1), \qquad S_{Qn} = \frac{1}{(2n+1)}, \quad n = 0, 1, 2, \ldots. \tag{9.14}$$

Thus,

$$n = \frac{1}{2}(H_{Qn}^2 - 1), \qquad 10^{-61} t_P \le t_n \le t_P \quad (n = 0), \tag{9.15}$$

and the more characteristic values in this phase, namely,

$$h_P, \ldots H_{Q\,\mathrm{inf}}, \ldots H_{Q\mathrm{cmb}}, \ldots H_{Q\mathrm{reion}}, \ldots H_{Q\mathrm{today}} \equiv H_{\mathrm{far\,past}}, \tag{9.16}$$

correspond to the n-level values:

$$n = 0, 1, \ldots n_{Q\,\mathrm{inf}} = 10^{12}, \ldots n_{Q\mathrm{cmb}} = 10^{114}, \ldots n_{Q\mathrm{reoin}} = 10^{118}, \ldots n_{Q\mathrm{today}} \equiv n_{\mathrm{far\,past}} = 10^{122}. \tag{9.17}$$

And the H_{Qn}, Λ_{Qn}, and S_{Qn} levels have the values

$$H_{Qn} = 1, 1.732, \ldots H_{Q\,\mathrm{inf}} = 10^{6}, \ldots H_{Q\mathrm{cmb}} = 10^{57}, \ldots H_{Q\mathrm{reoin}} = 10^{59}, \ldots H_{Q\mathrm{today}} = 10^{61}, \tag{9.18}$$

$$\Lambda_{Qn} = 1, 3, \ldots \Lambda_{Q\,\mathrm{inf}} = 10^{12}, \ldots \Lambda_{Q\mathrm{cmb}} = 10^{114}, \ldots \Lambda_{Q\mathrm{reoin}} = 10^{118}, \ldots \Lambda_{Q\mathrm{today}} = 10^{122}, \tag{9.19}$$

$$S_{Qn} = 1, 0.333, \ldots S_{Q\,\mathrm{inf}} = 10^{-12}, \ldots S_{Q\mathrm{cmb}} = 10^{-114}, \ldots S_{Q\mathrm{reoin}} = 10^{-118}, \ldots S_{Q\mathrm{today}} = 10^{-122}. \tag{9.20}$$

The whole picture is described at the end of Sec. X including both the pre-Planckian and post-Planckian phases and the complete discrete spectrum of levels from the far past to today level. The Universe pre-Planckian phase, namely, the quantum precursor phase, is the setting of the physically meaningful quantum trans-Planckian energies. In the post-Planckian (semiclassical and classical) eras, *no* trans-Planckian energies are present: Only mathematically or artificially (nonphysical) trans-Planckian energies could be generated in the present Universe. This is a direct consequence of the classical-quantum gravity dual relations Eqs. (2.2)–(2.4), which apply to any physical relevant magnitude, and in the respective domains as discussed in Sec. II and Refs. [1–3]. The trans-Planckian energy domain remains in the phase of the Universe totally before the Planck time t_P, e.g., in a totally quantum gravity domain, while the energies in the Universe after the Planck time turn out smaller than the Planck energy (semiclassical or semiquantum gravity and classical gravity) as determined by the classical-quantum gravity duality relations. However, signals or observables from the quantum precursor phase are present in the classical and semiclassical Universe, the most known being inflation and the present dark (vacuum) energy.

Consistently, the pre-Planckian phase covering $10^{-61} t_P \leq t \leq t_P$ provides too the two dual $(+)$ and $(-)$ branches, as it must be:

$$H_{n\pm} = h_P[\sqrt{2n+1} \pm \sqrt{2n}], \quad n = 0, 1, 2 \ldots, \tag{9.21}$$

$$H_{n=0} = h_P: \quad \text{Planck-scale value}, \tag{9.22}$$

$$H_{n+,n\gg1} = h_P\left[2\sqrt{2n} - \frac{1}{2\sqrt{2n}} + O(1/n^{3/2})\right] \gg 1, \quad \text{large } n: \text{branch}(+), \tag{9.23}$$

$$H_{n-,n\gg1} = \frac{h_P}{2\sqrt{2n}} + O(1/n^{3/2}) \ll 1, \quad \text{large } n: \text{branch}(-). \tag{9.24}$$

And for the Universe radius levels L_{Hn}:

$$L_{H(n=0)+} = L_{H(n=0)-} = L_{QH(n=0)} = l_P, \quad n = 0: \text{Planck length}, \tag{9.25}$$

$$L_{Hn+,n\gg1} = l_P\left[2\sqrt{2n} - \frac{1}{2\sqrt{2n}} + O(1/n^{3/2})\right] \gg 1, \quad \text{large } n: \text{branch}(+), \tag{9.26}$$

$$L_{Hn-,n\gg1} = \frac{l_P}{2\sqrt{2n}} + O(1/n^{3/2}) \ll 1, \quad \text{large } n: \text{branch}(-). \tag{9.27}$$

The same expressions hold for the mass levels $M_{Hn}(\pm)$; the vacuum levels $\Lambda_n(\pm)$ and the gravitational entropy $S_n(\pm)$ levels follow from them.

The quantum levels cover *all* the range of scales from the largest cosmological scales and time $10^{61} t_P$ today to the smallest one $10^{-61} l_P$ in the extreme past $10^{-61} t_P$ of the precursor or trans-Planckian phase, passing through the Planck scale (l_P, t_P), covering the two phases: post- and pre-Planckian phases, respectively. The quantum mass levels are associated to the quantum space-time structure. Quantum mass levels here cover *all* masses $10^{-61} m_P \leq M_n \leq 10^{61} m_P$ of the Universe phases. The two *dual mass* branches $(\pm)$ correspond to the larger and smaller masses with respect to the Planck mass m_P, respectively; they cover the *whole mass range* from the Planck mass in branch $(+)$ until the largest cosmological masses and from the smallest masses in branch $(-)$, the pre-Planckian phase, until near the Planck mass. As n increases, masses in the branch $(+)$ *increase* (as $2\sqrt{2n}$). Masses in the branch $(-)$, the very quantum one, *decrease* in the large n behavior, precisely as $1/(2\sqrt{2n})$; large n are very excited levels in this branch, *consistently* with the fact that this branch is the dual of branch $(+)$.

X. THE SNYDER-YANG ALGEBRA AND QUANTUM DE SITTER SPACE-TIME

The space-time coordinates in the Planckian and super-Planckian domain are no longer commuting, but they obey nonzero commutation relations: The concept of space-time is replaced by a quantum algebra. The classical space-time is recovered from the quantum algebra as a particular case in which the quantum space and time coordinate operators become the classical space-time continuum coordinates (c numbers) with all commutators vanishing and the discrete spectrum becomes the classical continuum space-time.

Here, the quantum space-time description is reached *directly* from the quantum noncommuting space-time coordinates and not through the quantization procedure of the classical gravitational field. This is so because the gravity field is itself a classical concept which loses meaning at the Planck scale. The space-time (the arena of events) is a classical concept which is more direct to extend to, or to replace by, *a quantum algebra* of space-time position and momenta:

$$[X_i, X_j] = iM_{ij}.$$

The Snyder algebra is a Lorentz-covariant deformation of the Heisenberg algebra, where the position operators are noncommuting and have discrete spectra [30] soon extended by Yang [31] to include one more length parameter. It describes a noncommutative discrete space-time compatible with Lorentz-Poincaré symmetry. The discrete position spectra, representations of the algebra, imply a discrete space description of space.

- (i) The Snyder algebra is precisely a description of a 4D constant curvature space of momenta; this corresponds to a de Sitter hyperboloid embedded in a 5D Minkowski momentum space. In the space of 5D momenta p_A, this includes precisely the motion of a particle of mass m and momentum on the de Sitter momentum hyperboloid $\eta^{AB} p_A p_B = m^2$.
- (ii) In geometric terms, the Snyder quantized space-time is a projective geometry approach to the phase space or momentum de Sitter space in which the space-time coordinates are identified with the 4-translation generators of the $SO(1,4)$ de Sitter group (and are, therefore, noncommutative) and with other operators as the angular momentum in $SO(1,3)$.
- (iii) In projective or Beltrami coordinates, the Euclid, Riemann, and Lobachevsky spaces [32] corresponding to zero, positive, and negative spatial curvature, respectively, are upon Wick rotation the Minkowski, de Sitter, and anti–de Sitter space-times with the invariance groups $ISO(1,3)$, $SO(1,4)$, and $SO(2,3)$ respectively.

In D dimensions, the Lorentz-covariant Snyder-Yang quantum algebra follows from the Inonu-Wigner [33] group contraction of the $SO(D-1,1)$ algebra with the generators:

$$\Sigma_{AB} = i(q_A \partial_{q_B} - q_B \partial_{q_A}). \tag{10.1}$$

Σ_{AB} live on the $(D+2)$ parameter space q_A (hyperboloid) which satisfies

$$-q_0^2 + q_1^2 + \cdots + q_{D-1}^2 + q_a^2 + q_b^2 = L^2, \tag{10.2}$$

$$A = (\mu, a, b); (\mu = 1, 2, \ldots D);$$
$$(a, b) \text{ being extra space dimensions, and } q_0 \equiv q_D. \tag{10.3}$$

The D-dimensional operators $(X_\mu, P_\mu, M_{\mu\nu})$—space-time operator X_μ, momentum operator P_μ, angular momentum operators $M_{\mu\nu}$, and the completing operator N_{ab}—are all defined by the generators $\Sigma_{\mu a}$ [Eq. (10.1)] as follows:

$$X_\mu \equiv l_P \Sigma_{\mu a}, \qquad P_\mu \equiv (\hbar/L)\Sigma\mu b,$$
$$M_{\mu\nu} \equiv \hbar \Sigma_{\mu\nu}, \qquad N_{ab} \equiv (l_P/L)\Sigma_{ab}. \tag{10.4}$$

This set of operators $(X_\mu, P_\mu, M_{\mu\nu}, N)$ satisfies the contracted algebra of $SO(D+1,1)$, namely, the quantum Yang-Snyder space-time algebra:

$$[X_\mu, X_\nu] = -i(l_P^2/\hbar)M_{\mu\nu}, \qquad [P_\mu, P_\nu] = -i(\hbar/L^2)M_{\mu\nu}, \tag{10.5}$$

$$[X_\mu, P_\nu] = -i\hbar N \delta_{\mu\nu}, \qquad [X_\mu, N] = i(l_P^2/\hbar)P_\mu,$$
$$[P_\mu, N] = -i(\hbar/L^2)X_\mu. \tag{10.6}$$

And the operators $M_{\mu\nu}$ satisfy angular-momentum-type relations:

$$[M_\mu, M_\nu] = -i(l_P^2/\hbar)M_{\mu\nu}. \tag{10.7}$$

A. Classical-quantum duality in the Snyder-Yang algebra

The Snyder-Yang algebra contains two parameters (a, L): small-scale parameter a and large-scale parameter L which in our context are naturally the Planck length l_P and the Universe radius L_H. Our complete (classical and quantum) radius L_{QH} [Eq. (5.1)] *contains intrinsically both lengths*, the classical length L_H and its quantum dual L_Q (Compton radius of the Universe) and provides a basis for a framework naturally free of infrared and ultraviolet divergences:

$$a \equiv l_P, \qquad L \equiv L_{QH} = L_H + L_Q = l_P\left(\frac{L_H}{l_P} + \frac{l_P}{L_H}\right). \tag{10.8}$$

We see that the Snyder-Yang algebra with the complete length $L_{QH}(l_P, L_H)$ as a parameter provides a quantum operator realization of the complete (classical and quantum) de Sitter Universe, including the quantum early and classical late de Sitter phases duals of each other. This provides further description of the pre-Planckian and post-Planckian de Sitter phases, within a group-theory realization of the quantum discrete de Sitter space-time and of classical-quantum gravity duality.

Finally, let us mention as an example of the different classical and quantum de Sitter phases the cosmological vacuum energy, the most direct candidate to the dark energy today [4–10], for which the observed value is

$$\rho_\Lambda = \Omega_\Lambda \rho_c = 3.28 \times 10^{-11}\ (\mathrm{eV})^4 = (2.39\ \mathrm{meV})^4, \quad \mathrm{m\,eV} = 10^{-3}\ \mathrm{eV}, \tag{10.9}$$

corresponding to $h = 0.73$, $\Omega_\Lambda = 0.76$, and $H = 1.558 \times 10^{-33}$ eV. The CMB data yield the values [10]

$$H = 67.4 \pm 0.5\ \mathrm{Km\,sec^{-1}\,Mpc^{-1}}, \quad \Omega_\Lambda h^2 = 0.0224 \pm 10^{-4} \tag{10.10}$$

and

$$\Omega_\Lambda = 0.6847 \pm 0.0073, \qquad \Omega_\Lambda h^2 = 0.3107 \pm 0.0082, \tag{10.11}$$

which implies for the cosmological vacuum *today*:

$$\Lambda = (4.24 \pm 0.11) \times 10^{-66}\ (\mathrm{eV})^2 = (2.846 \pm 0.076) \times 10^{-122} m_P^2. \tag{10.12}$$

The density ρ_Λ associated to Λ [Eq. (10.9)] is precisely

$$\rho_\Lambda = \Lambda/8\pi G = \rho_P(\Lambda/\lambda_P), \tag{10.13}$$

where the Planck-scale values ρ_P and λ_P are $\rho_P = \lambda_P/8\pi G$ and $\lambda_P = 3h_P^2$, respectively. The quantum vacuum value expected from microscopic particle physics is evaluated to be $\Lambda_Q \approx 10^{122}$.

B. Crossing the Planck scale

The two values (Λ, Λ_Q) refer to the same concept of vacuum energy, but they are in two huge different vacuum states and two huge different cosmological epochs: classical state and classical dilute epoch today for Λ observed today with the most classical levels and quantum state and quantum very early epoch with the most excited levels for the quantum mechanical trans-Planckian value Λ_Q. The classical value today $\Lambda = 3H^2$ corresponds to the classical Universe today of classical rate H and classical cosmological radius $L_H = c/H$. The quantum mechanical value $\Lambda_Q = 3H_Q^2$ corresponds to the early quantum Universe of quantum rate H_Q and quantum radius $L_Q = l_P^2/L_H = \hbar/M_H c$, which is exactly the quantum dual of the classical horizon radius L_H: L_Q is *precisely* the quantum Compton length of the Universe for the gravitational mass $M_H = L_H c^2/G$.

C. Two extremely different physical conditions and gravity regimes

This is a realistic, clear, and precise illustration of the *physical classical-quantum duality* between the two extreme Universe scales and gravity regimes or phases *through the Planck scale*: the dilute state and horizon size of the Universe today on the one largest known side and the trans-Planckian scales and highest density state on the smallest side: The size, mass, and their associated time (Hubble rate) and vacuum energy density (Λ, ρ_Λ) of the Universe *today* are truly *classical*, while its extreme past at $10^{-61} t_P = 10^{-105}$ s deep inside the trans-Planckian domain of extremely small size and high vacuum density value (Λ_Q, ρ_Q) are truly *quantum and trans-Planckian*. This manifests the *classical-quantum or wave-particle duality* between the classical macroscopic (cosmological) gravity physical phase and the quantum microscopic particle physics and trans-Planckian phase through the *crossing* of the Planck scale, Planck scale duality in short.

D. A unifying picture

Starting from the earliest past quantum era from $10^{-61} t_P$ to t_P, with the quantum excited level $n = 10^{122}$, the entropy S_{Qn} increases in discrete levels $s_P/(2n+1)$ from its extreme small value $S_Q = 10^{-122} s_P$ at the earliest time $10^{-61} t_P$ till, for instance, its quantum inflation value $10^{-12} s_P$ $(n_{Q\mathrm{infl}} = 10^{12})$, at time $10^{-6} t_P$, to its Planck value $(n = 0)$: $S_Q = s_P = \pi\kappa_B$ at the Planck time t_P, the *crossing scale*, after which it goes to its semiclassical and classical levels $(2n+1)s_P$, e.g., inflationary value $S_{\Lambda\mathrm{inflation}} = 10^{12} s_P$ $(n = 10^{12})$ at the classical inflationary stage at $10^6 t_P$, and it follows *increasing and classicalizes* till the most classical level today $n = 10^{122}$: $S_\Lambda = 10^{122} s_P$ at the present time $10^{61} t_P$. And as far as the Universe will continue expanding its horizon as $l_P\sqrt{(2n+1)}$, $S_{\Lambda n}$ will continue increasing as $(2n+1)$.

The *total* $Q\Lambda$ gravitational entropy (for the whole history) is the sum of the three values above discussed corresponding to the three regimes: classical Λ, quantum Q, and Planck values (subscript P). In the past remote and more quantum (Q) eras: $10^{-61} t_P \le t \le t_P$, the Planck entropy value $(n = 0)$: $s_P = \pi\kappa_B$ dominates S_Q. In the classical eras $t_P \le t \le 10^{61} t_P$, the entropy value today $(n = 10^{+122})$: $S_\Lambda = 10^{+122} s_P$ dominates.

The whole picture is depicted in Fig. 1, where Λ refers to the cosmological constant (or associated Hubble-Lemaître constant H) in the classical gravity phase. Q means

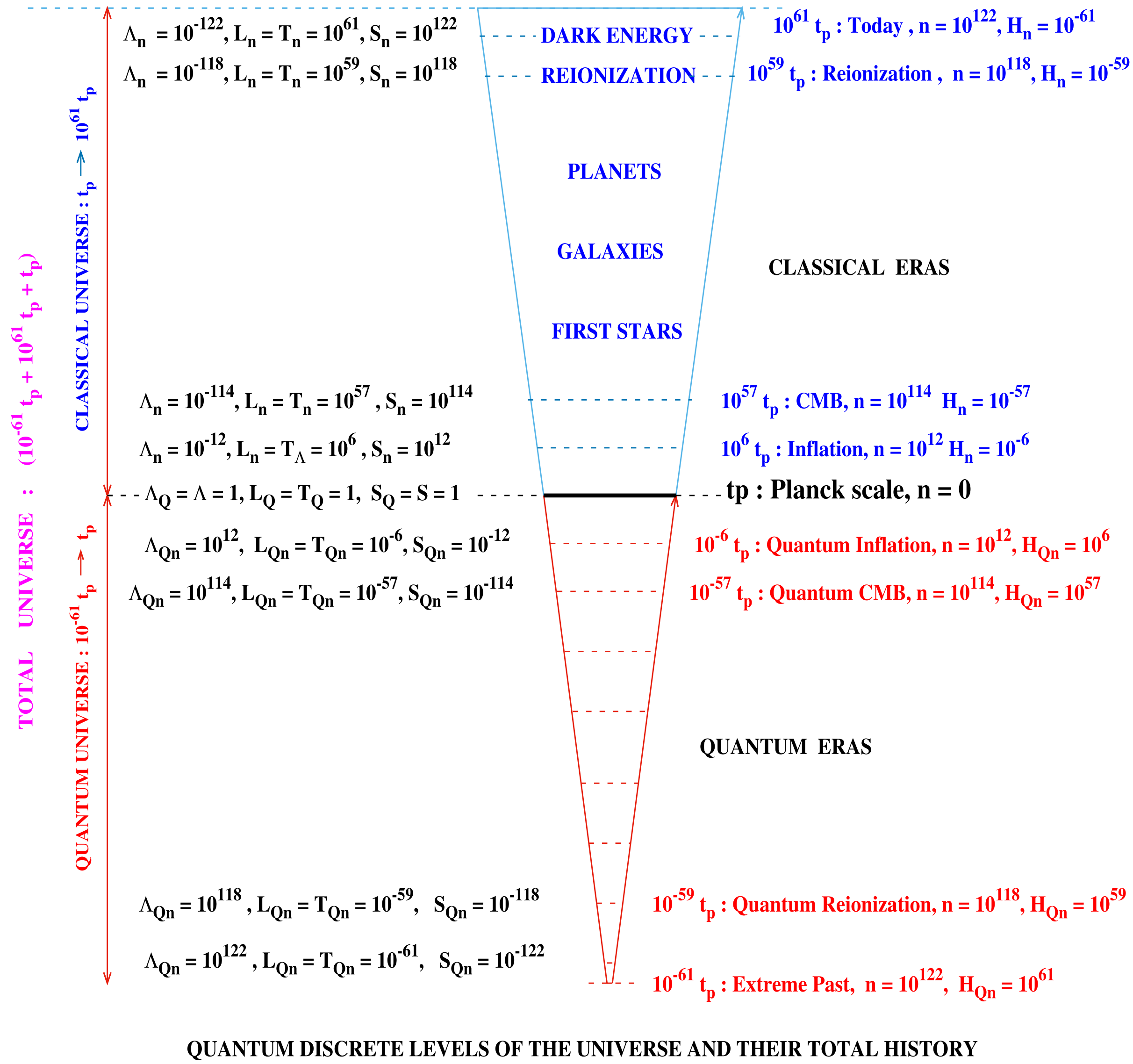

FIG. 1. The quantum discrete levels of the Universe from its early trans-Planckian era to dark energy today. In the pre-Planckian (trans-Planckian) phase $10^{-61}t_P \le t \le t_P \equiv t_{\text{planck}}$, the quantum levels are (in Planck units) $H_{Qn} = \sqrt{(2n+1)}$, $\Lambda_{Qn} = (2n+1)$, $S_{Qn} = 1/(2n+1)$, $n = 0, 1, 2, \ldots$, Q denoting quantum. The n levels cover *all* scales from the past highest excited trans-Planckian level $n = 10^{122}$, passing the Planck level $(n = 0)$ and entering the post-Planckian phase, e.g., $n = 1, 2, \ldots, n_{\text{inflation}} = 10^{12}, \ldots, n_{\text{cmb}} = 10^{114}, \ldots, n_{\text{reoin}} = 10^{118}, \ldots, n_{\text{today}} = 10^{122}$. In the post-Planckian Universe $t_P \le t \le 10^{61}t_P$, levels are $H_n = 1/\sqrt{(2n+1)}$, $\Lambda_n = 1/(2n+1)$, and $S_n = (2n+1)$: As n increases, radius, mass, and S_n increase and *consistently* the Universe *classicalizes*. See the text at the end of Sec. X.

quantum, P means Planck scale, and Planck's units, natural to the system, greatly simplify the history. (The complete history is a theory of pure numbers.) Each stage is characterized by the set of main physical gravitational quantities (Λ, density ρ_Λ, size L_Λ, and gravitational entropy S_Λ). In the quantum trans-Planckian phase, levels are labeled with the subscript Q. Total means the whole history including the two phases or regimes. The present age of the Universe 10^{61} (with $\Lambda = \rho_\Lambda = 10^{-122} = 1/S_\Lambda$) is a *lower bound* to the future Universe age and similarly for the present entropy level S_Λ. The past 10^{-61} (with $\Lambda_Q = 10^{122} = \rho_Q = 1/S_Q$) is an *upper bound* to the extreme past (origin) of the Universe and quantum initial entropy (arrow of time). [Similarly, the values given in Fig. 1 (in Planck units) for the CMB are the classical CMB age (3.8×10^5 yr $= 10^{57}t_P$) and the set of gravitational properties of the Universe at this age and their corresponding precursors in the quantum preceding era at $10^{-57}t_P$. S_Λ constitute also an upper bound to the entropy of the CMB photon radiation.]

Shown in Fig. 1 are the quantum discrete levels of the Universe from its early trans-Planckian era to classical vacuum energy today (dark energy), namely, the Standard Model of the Universe completed back in time with quantum physics in terms of its vacuum history. The Universe is composed of two main phases: after and before the Planck scale (Planck time t_P and Planck units). The complete history goes from $10^{-61} t_P$ to $10^{61} t_P$: In the pre-Planckian (trans-Planckian) phase $10^{-61} t_P \leq t \leq t_P \equiv t_{\text{planck}}$, the quantum levels are $H_{Qn} = \sqrt{(2n+1)}$, $\Lambda_{Qn} = (2n+1)$, and $S_{Qn} = 1/(2n+1)$, $n = 0, 1, 2, \ldots$, Q denoting quantum. The n levels cover *all* scales starting from the past highest excited trans-Planckian level $n = 10^{122}$ with *finite curvature* $R_Q = 10^{122}$, $\Lambda_Q = 10^{122}$, and minimum entropy $S_Q = 10^{-122}$, as n decreases: S_{Qn} increases and (H_{Qn}, Λ_{Qn}) decrease, passing the Planck level ($n = 0$): $H_{\text{planck}} = 1 = \Lambda_{\text{planck}} = S_{\text{planck}}$ and entering the post-Planckian phase, e.g., $n = 1, 2, \ldots, n_{\text{inflation}} = 10^{12}, \ldots, n_{\text{cmb}} = 10^{114}, \ldots, n_{\text{reoin}} = 10^{118}, \ldots, n_{\text{today}} = 10^{122}$. In the post-Planckian Universe $t_P \leq t \leq 10^{61} t_P$, the levels are $H_n = 1/\sqrt{(2n+1)}$, $\Lambda_n = 1/(2n+1)$, and $S_n = (2n+1)$: As n increases, the radius, mass, and S_n increase, (H_n, Λ_n) decrease, and *consistently* the universe *classicalizes*. The present age of the Universe $10^{61} t_P$ with its most classical value $H_{\text{today}} = 10^{-61}$, $\Lambda_{\text{today}} = 10^{-122} = 1/S_{\text{today}}$ is a *lower bound* to the future Universe age and similarly for the present entropy level S_n. The far past $10^{-61} t_P$ (with $\Lambda_Q = 10^{122} = 1/S_Q$) is an *upper bound* to the extreme known past ("origin") of the Universe and quantum initial entropy (arrow of time).

XI. DISCUSSION AND CLARIFICATIONS

A full quantum cosmology dynamics including the full trans-Planckian quantum phase and the discrete time levels is not yet fully accomplished, but that does not mean that this approach does not allow time dependence dynamics, on the contrary. Classical-quantum gravity duality here, quantum discrete levels in cosmology, and its connection to the observational values are a first (nonperturbative) step toward the completion of a quantum cosmology dynamics including the trans-Planckian domain.

Recall that, in the foundation of quantum theory, the quantum dynamical equations were written well after that classical-quantum duality, wave-particle duality, and its implied quantum uncertainty were formulated, and they were also motivated by experimental results.

As stated in Sec. IX, after Eqs (9.18)–(9.20), a direct consequence of the classical-quantum gravity duality (CQGD) relations, e.g., Eqs (2.2)–(2.4) and Refs. [1–3], is that in the post-Planckian (semiclassical and classical) eras, no trans-Planckian energies are present: Only mathematically or artificially (nonphysical) trans-Planckian energies could be generated in the present Universe. The trans-Planckian energy domain necessarily remains in the phase totally before the Planck time t_P, i.e., in a totally quantum gravity domain.

A recent trans-Planckian censorship conjecture (TCC) in string theory, e.g., Ref. [34], says that no (low-energy) effective field theory emerging from superstring theory could lead to a regime where fluctuation modes which were initially trans-Planckian ever exit the Hubble radius. Or, equivalently, no modes with four-momenta higher than the Planck scale can enter in the low-energy effective action [thereby promoting the role of the Planck mass scale as an ultraviolet (UV) momentum cutoff]. One could say, therefore, that a part of such TCC finds support in the CQG duality, even if we have not formulated CQG duality for such a TCC at all (and TCC was formulated later than CQGD). On the other hand, the CQG duality refers to an ultimate finite quantum theory of gravity. In the CQG duality, the Planck scale is not an ultimate UV cutoff but a transition scale [between the dual quantum gravity (trans-Planckian) and the classical gravity (non-trans-Planckian)]. The CQG duality implies a precursor new phase (a whole domain) before the Planck time, smaller than the Planck size, which is full trans-Planckian, and the post-Planckian Universe is necessarily non-trans-Planckian.

The classical-quantum gravity duality here and its quantum levels imply a varying vacuum energy, i.e., a varying Λ_n at each discrete level n, and yield support to it: H_n varies and Λ_n varies, too, even if, of course, this variation is mild. After all, inflation also needs such a vacuum or Λ variation. It is known that what it is called Hubble constant is, in fact, a Hubble rate; it should be admitted also that what is called Λ cosmological constant would be, in fact, a Λ rate.

Recall that the purpose of this approach is the extension of the semiclassical and classical cosmological phases to the quantum Planckian and trans-Planckian domain, mainly the de Sitter (and quasi–de Sitter) early and late phases, as these are relevant in this problem. By no means is this to disregard the other expanding intermediate phases, although this paper is not particularly devoted to them.

In the introduction, Eq. (1.1) refers to the magnitudes in the post-Planckian (classical and semiclassical) Universe. Equation (1.2) (and the subscript Q) refers to the magnitudes in the pre-Planckian (quantum trans-Planckian and Planckian) Universe. $n = 0$ yields the Planckian (zero-point) level (in Planck units). The magnitudes at the Planck scale (the crossing scale) as explained in the paper are just a constant: Their corresponding expressions (subscript P) are totally in terms of the Planck constant, as in Eq. (2.3) and, for example, in the case of the gravitational entropy Eq. (2.7) [Eq. (3.8)]. When we refer to the total or "complete" magnitudes (also when we stand them with the subscript total or QG), we refer to the total (QG) magnitudes which are the sum of the magnitudes in the three regimes: Q, G, and P, and, automatically, the additive constant magnitude in terms of the Planck constant does appear (m_P or other Planck constant magnitude according to the physical magnitude considered), as, for example, in

Eqs (2.5), (2.6), (3.5), and (3.8), e.g., Λ_P, ρ_P, and similarly in the other magnitudes. This applies to all physical magnitudes, as described in Secs. II–V.

The discrete levels refer to the quantum space-time levels and in the post-Planckian time eras become continuous for the classical and semiclassical gravity, but this by no means at all that the QFT of all propagating fields and their interactions (not considered here) became classical: Semiclassical gravity is precisely QFT in curved continuous space-time. In the post-Planckian epochs, the discrete levels become classical for the space-time and gravity, but the genuine QFT of different matter and spins is perfectly valid, which is precisely what semiclassical or semiquantum (non-Planckian) gravity does mean. In semiclassical or semiquantum gravity (e.g., without the QG variables, without the trans-Planckian domain), quantum fields propagate in the classical or semiclassical continuous curved space-time and background fields. Here we are not considering all the different quantum interactions and contributions to the vacuum, although it is possible to consider and separate them, but this is not the purpose of this paper.

This approach provides the results of QFT in curved space-time in its own range of validity. Semiclassical gravity, e.g., QFT, in curved space-times and an effective theory of gravity are valid in the post-Planckian time nonquantum gravity Universe. In particular, in the semiclassical or semiquantum gravity and classical gravity regimes, e.g., $H \ll h_P$, $L \ll l_P$, and $\rho \ll \rho_P$, the expressions from the QG variables provide the series in powers of $(H/m_P)^2$ plus the constant Planckian terms, h_P, l_P, or ρ_P, respectively, according to the physical magnitude considered: Equations (5.6) for H_{HQ}, Eqs. (5.10) for $\Lambda_{\Lambda Q}$, and Eq. (5.21) for ρ_{HQ} of this paper. The additive constant Planck terms are always present.

This approach does not replace QFT, but it brings an extension to it; the QG variables (and classical-quantum gravity duality) are a first step to such (nonperturbative) extension. The classical-quantum gravity duality, thus, appears as a guiding property toward the construction of a complete theory beyond Planck scale. A full complete quantum gravity theory would be, thus, a finite theory (which is more than a renormalizable theory): In the sense of the physical renormalization idea, the renormalization procedure applies for the noncomplete theories, because they are valid in their own limited range of validity, and this is so because such known QFTs are not complete at the Planck scale and beyond it. The extension of QFT in this case is by using the complete QG variables which provide a finite theory. This is so because of the classical-quantum gravity duality through the Planck scale. Or the nonzero $[X, T]$ space-time commutators due to the related quantum uncertainty, which generate the quantum light cone, and the zero distance or UV singularities are smeared out or eliminated. (And this is not just extending a cutoff beyond the Planck constant value.) The whole quantum dynamics linking the total evolution from the early trans-Planckian phase to the present phase is not yet fully known.

Well after the Planck time, during inflation and after it, and in the late Universe, the semiclassical gravity description does apply, e.g., QFT in curved space-time and its backreaction. References [35–38] well account for such a description. But is important to keep in mind that such QFT gravity and its renormalization is an effective theory; e.g., it is not the final theory: It does not include the Planckian and trans-Planckian domain, and in this sense such QFT even if very useful cannot be considered a full quantum cosmology theory. Behind and beyond such effective QFT theory and its renormalized effects, it should be a more complete quantum theory from which such an effective theory is a sector or approximation.

In Ref. [35], we computed dark energy as the vacuum energy from QFT in the expanding FLRW universe and find a dark energy equation of state $p = w(z)\rho$ in which $w(z) < -1$ asymptotically, reaching the value -1 from below. Of course, the time dependence comes from the expanding background, and the quantum effects (after renormalization) depend on time. Also, in Refs. [39–41], we computed the various quantum-type corrections to inflation and to the local renormalized magnitudes (e.g., effective potential, correlation functions, and energy-momentum tensor) from which we found the quantum corrections to the power inflationary spectra.

The present nonperturbative approach with the QG variables extends to the quantum higher trans-Planckian phase and goes beyond the perturbative corrections in the power spectra. The QG extension and its H_{QG} variables are a first step to cover nonperturbatively both the non-Planckian and the quantum (Planckian and trans-Planckian) gravity domains. They extend (and, in particular, are in agreement with) the QFT perturbative results of the classical and semiclassical (non-Planckian) domains which express themselves as a series in powers of H^2. For instance, the quantum QG extension of inflation we computed in Ref. [3] and further discuss here below [Eqs (11.1) and (11.2)] agrees, in particular, with the quantum corrections to inflation computed in the framework of perturbative QFT in the classical and semiclassical (e.g., non-Planckian) cosmological domains [39–42]. We discuss the effective field theory of inflation in the Ginzburg-Landau approach, which is a powerful theoretical scheme for predictions, confrontation to observations, and analysis of real data; see, e.g., Refs. [39]–, and references therein. This covers a wide class of inflation models (small field or symmetry-breaking families, as well as large field or non-symmetry-breaking inflation), not just one model. The (QG) extended inflationary power spectra are given by

$$[\Delta^S_{k,QH}] = \frac{[\Delta^S_{k,H}]}{[1 + (H/h_P)^2](1 - \delta\epsilon_{QH})^{1/2}}, \tag{11.1}$$

$$[\Delta^T_{k,QH}] = \frac{[\Delta^T_{k,H}]}{[1+(H/h_P)^2]}. \quad (11.2)$$

QH stands for the total inflation phase including the known classical and semiclassical inflation and its precursor: the quantum inflation era (in the Planckian and trans-Planckian) phase. $[\Delta^S_{k,H}]$ and $[\Delta^T_{k,H}]$ are the known standard spectra of scalar curvature and tensor perturbations, respectively, in classical H inflation, and $\delta\epsilon_{QH}$ is the first-order QH slow-roll parameter which contains, in particular, the classical known slow-roll parameter ϵ_H. The total QH spectra contain both the standard known spectra of the classical and semiclassical inflation including its quantum corrections of the order of $(H/h_P)^2 = 10^{-12}$ in the classical and semiclassical gravity phase $H = 10^{-6}h_P$, at $t = 10^6 t_P$ (or $10^{-5}M_P$ for the reduced Planck mass $M_P = m_P/\sqrt{8\pi}$) and their quantum dual spectra in the quantum precursor inflation era $H_Q = 10^6 h_P$ at $t = 10^{-6}t_P$.

This description gives support to dynamical vacuum energy, e.g., dynamical dark energy, as computed from QFT in a classical FLRW expanding universe [35] and the running vacuum [36–38]. The vacuum expectation value of the renormalized energy-momentum tensor of quantum fields in a FLRW space-time (see Refs. [35–38]) yields the vacuum energy density and pressure, e.g., the dark energy equation of state $p = w(z)\rho$. The results can be expressed in terms of different parametrizations, but the different magnitudes for the vacuum density, vacuum pressure, and dark energy equation of state, as well as the inflation fluctuation spectra, all yield to quantum terms which can be recast as a series in powers of H^2.

Vacuum dominance at the trans-Planckian era (e.g., the Universe arises from vacuum) implies that this is the de Sitter or quasi–de Sitter phase in the very earliest stage. In the quantum trans-Planckian era, from the most initial higher excited levels, namely, $n_{\max}$ with the smallest entropies $S_{Qn} = 1/(2n_{\max}+1)$ (Sec. IX), the decreasing or deexcitation of the levels through $n_{\max}, n_{\max}-1, \ldots, 1$, till $n = 0$ (Planck scale), yields the decreasing of H_{Qn} and Λ_{Qn}, the increasing of the size L_{Qn}, and the increasing of the entropy S_{Qn} passing by its Planck value at $n = 0$ and entering the semiclassical and classical phase $S_n = (2n+1)$, L_n, H_n, and Λ_n till its most high values in the late Universe. The time levels t_n and associated physical magnitudes are accounting for evolution. These are not the full dynamical wave function equations, but the discrete space-time levels and the associated physical magnitude levels $H(t_n)$, $\Lambda(t_n)$, $n = 0, 1, 2, \ldots$, consistently account for evolution.

At each level n, the physical magnitudes, e.g., $(L_n, H_n, \Lambda_n, S_n)$, take the corresponding value at that time level t_n. In the post-Planckian time era $t > t_P$, e.g., $(n = 0)$—that is, energies smaller than the Planck energy—the levels $n = 1, 2, \ldots$ yield increasing times t_n and increasing sizes L_n, together with increasing entropy S_n and smaller Hubble values H_n and Λ_n, and the system becomes more and more classical. In such classical and semiclassical evolution, the known QFT semiclassical gravity and classical gravity regimes hold, as well as the known QFT in curved space-time dynamics, its back-reaction effects, and its quantum corrections.

The connection between the cosmological constant and the (inverted) harmonic oscillator is derived from the Einstein equations in Secs. VI and VII. The space-time discrete n levels and their expressions X_n and T_n [Eqs. (8.1)] are not a "hypothesis" nor a "conjecture" but derived expressions. They are general and apply to black holes, too, as derived in Ref. [2], also supported by Refs. [1,3]. (X_n, T_n) is the notation for the (space, time) coordinate levels. The expressions for X_n and T_n are given by Eq. (8.1) and the line below it.

This paper does not treat the cosmic coincidence problem. As is known, a part of the so-called cosmological constant problem is connected with the mismatch between the quantum vacuum particle physics estimated value (10^{122}) and the low observed value (10^{-122}) in Planck units. That is not the total CC problem, but we are mainly interested here in the problem connected with the Planckian and trans-Planckian (that is to say, fully quantum gravity) domains. The problem is to know too which is (or are) the main particle(s) associated to this vacuum and the detection of such particles, as well as the whole evolution and, of course, to test such quantum evolution with the most complete cosmological dataset. This is important, too, in clarifying or resolving the present H_0 problem or tension, as discussed in Refs. [36–38] with the running vacuum energy.

The cosmological constant (CC) is not really constant along the cosmic history in this approach. This is a difference with a rigid CC but not with dark energy as a vacuum energy. Here the cosmological term appears to be like a "running" vacuum energy with the value of n as $1/(2n+1)$ (in Planck units) and ultimately with t. We are not using a renormalization group approach; n is a space-time level. $n = 0$ corresponds to the Planckian constant (or zero-point) level. It could be thought by analogy as an effective running, although we have not used nor thought n in this way. In addition, and independently, in Ref. [35] we have found varying time vacuum energy from QFT in a curved expanding FLRW universe, and we have not used analogy with "running," but such time varying could be compared to, or interpreted as, a running.

In the process of the classicalization as n increases and $t_n = (2n+1)^{1/2}$ increases, the huge value of the initial Λ_n diminishes as $\Lambda_n = 1/(2n+1)$, and when n is huge, say, $n = 10^{2x}$ with large $x \gg 1$, the Λ_n value is 10^{-2x} smaller than the highly quantum initial one and, hence, in the desired range of the classical measurement at time $t_n = (2n+1)^{1/2} = 10^x$. This is coherently accompanied by the decreasing Hubble constant $H_n = 1/(2n+1)^{1/2}$, the increasing size $L_n = (2n+1)^{1/2}$, and increasing gravitational entropy $S_n = (2n+1) = 10^{2x}$ from the early eras to the present time.

As explained in the above points and Sec. VIII, Eq. (8.1), and Sec. IX, $n = 0, 1, 2, \ldots$ is determined by the time levels $t_n = (2n+1)^{1/2}$ and its dual branch $t_{Qn} = 1/(2n+1)^{1/2}$, and conversely. The time, in particular, the present time 10^{61} (in Planck units t_P), determines $n = 10^{122}$, and, therefore, the values today for $\Lambda_n = 1/(2n+1)$ and $S_n = (2n+1)$ correspond to such n. That is to say, $n = 10^{122}$ is not an arbitrary choice. Of course, any other similar high n corresponding to a time near such an era, $n = 10^{100}$ say, explains as well the huge difference between the very early and late Λ_n values due to the classical-quantum duality relations between the trans-Planckian and the classical (late) eras.

On the other hand, if we would start from the extreme early past Universe, it is not known what is the most early past remote time, except that it should be a very small fraction of Planck time: If 10^{-x} is such an *a priori* unknown number, $x > 0$ to be determined, then, starting from $t_Q = 10^{-x} t_P$, the results of Secs. VIII and IX, e.g., $t_{Qn} = 1/(2n+1)^{1/2}$, yield the quantum level $n = 10^{2x}$, the most early quantum $\Lambda_{Qn} = 10^{2x}$, and most early quantum entropy (in Planck units) $S_{Qn} = 10^{-2x}$. The classical-quantum gravity duality relations yield then for the most late future phase observables $t_H = 10^x t_P$, $H = 10^{-x}$, $L_H = 10^x$, and $S_H = 10^{2x}$ for the (dimensionless) gravity entropy. In particular, today $t_H = 10^{61} t_P$ yields $x > 61$, $t_Q = 10^{-61} t_P$, and $S_Q = 10^{-122}$ as the upper bounds for the most early remote time and quantum gravitational entropy; $n = 10^{122}$ and $\Lambda_Q = 10^{122}$ (in Planck units) are, respectively, the lower bounds for the corresponding quantum level and energy. Such quantum huge energies and sizes $L_Q = 10^{-122} \ll l_P$ are truly typical of the trans-Planckian domain and appear in other quantum gravity problems, too (as black holes, for instance). Thus, the early quantum trans-Planckian magnitudes can be connected to the late time measurements through the classical-quantum duality relations. They provide the most stringent upper bounds to the most early remote time and early quantum entropy and the most stringent lower bounds for the most early quantum level and energy.

Let us comment now about the value of n at a given time of the cosmological expansion such that the value of Λ_n does not perturb any segment of the thermal history of the Universe, e.g., big bang nucleosynthesis (BBN) or other known period: Recall that $t_n = (2n+1)^{1/2}$, $n = 0, 1, 2, \ldots$, and $\Lambda_n = 1/(2n+1)$ hold for all the post-Planckian cosmic history: $t > t_P$, that is, in all classical and semiclassical eras, as well as all the other gravitational levels $L_n = (2n+1)^{1/2}$, $H_n = 1/(2n+1)^{1/2}$, and $S_n = (2n+1)$ in such eras. In the radiation and matter eras, that is, in the classical and/or semiclassical gravity regimes, the vacuum-dominated expressions for H_n, Λ_n, and S_n represent *the upper bounds* (maximum values) to the values of these magnitudes in such radiation and matter eras when the vacuum energy is not the dominant one, that are computed from the semiclassical and classical dynamics (QFT and Einstein equations). Recall that S_n is the gravitational cosmological entropy or Bekenstein-Gibbons-Hawking entropy, and, thus, this is always un *upper bound* to the entropies of the different content parts: radiation, or matter or other partial entropies. Therefore, these values will not alter the cosmic history, BBN, or other part. This does not excludes the existence of an early vacuum energy which could explain the H_0 tensions or even a recently discussed BBN tension [43], but we do not discuss such tensions here.

This is not an alternative description to standard cosmology but a quantum extension of it, and doing that the vacuum energy appears as time evolving. In the QFT description in curved FLRW space-time, vacuum energy turns out to be time dependent, even if such time variation is mild in the late Universe. Quantum discrete levels of the space-time are neither an alternative description to space-time but a more complete description of it. A time-evolving vacuum energy density is not only a prediction of QFT in curved space-time, but it may provide a better description to the cosmological data than merely the constant Λ vacuum in the so-called ΛCDM model; see, e.g., Refs. [36–38]. And as has been discussed in the literature, phenomenological models proposing a time-evolving Λ (and, hence, a dynamical ρ_Λ) help in alleviating the several cosmological parameter problems or tensions; see, e.g., Refs. [44,45] and particularly for H_0 Refs. [46,47].

The Standard Model of the Universe in fundamental grounds is based on general relativity and classical fields and quantum field theory for the description of matter, and, having dark matter and dark energy in its major components, this last described by a vacuum energy. The fact that the vacuum energy is time varying or time running is compatible with semiclassical gravity, general relativity, and QFT and is not an alternative to it; it is neither a modified gravity theory in the sense of “modified classical Newton gravity” nor modified classical Einstein nor other proposed alternative gravities.

We treat the area gravitational (Gibbons-Hawking [48] and Bekenstein [49]) entropy in terms of the relevant size of the system (or object) considered and its extension to the quantum gravity trans-Planckian domain. In the pre-Planckian time phase (fully quantum gravity trans-Planckian phase), the relevant appropriate size of the quantum system is the Compton length. In the post-Planckian Universe, the relevant size is the gravitational size, the Hubble horizon, the apparent horizon, as de Sitter gravitational entropy or Gibbons-Hawking entropy. The entropies in the two different phases are classical-quantum gravity duals of each other. The total gravitational entropy is the sum of the entropies in the three main gravity regimes: quantum gravity, Planckian, and classical and semiclassical gravity regimes [Eq. (2.7) of this paper]. The complete (or QG) variables entail precisely those three regimes and provide the additive constant, too, that is the pure Planckian scale term (a constant). This is discussed in Sec. II [Eq. (2.7)]. In Sec. II, the general physical magnitudes in each of the three gravity regimes are explained, e.g., Eqs. (2.1)–(2.3). Equation (2.4) gives the total QG gravitational magnitudes, sum of the quantum trans-Planckian (Q), classical, and Planckian ones. This paper is not discussing already known entropy aspects nor the entropy generation mechanisms or their thermodynamics, e.g., Ref. [50] and references therein. The total entropy in Ref. [50] refers to the sum of the area plus the volume entropy in the post-Planckian time Universe (e.g., in the semiclassical and classical gravity phases), but the quantum gravity trans-Planckian dual entropy in the pre-Planckian time phase is not included.

XII. CONCLUSIONS

We have accounted in the introduction and along the paper the main results and will not include all of them here. We summarize below with some conclusions and remarks.

(i) The Standard Model of the Universe is extended back in time with Planckian and trans-Planckian physics before inflation in agreement with observations, classical-quantum gravity duality, and quantum space-time. The quantum vacuum energy bends the space-time and produces a constant curvature de Sitter background. We find the quantum discrete cosmological levels: size, time, vacuum energy, Hubble constant, and gravitational (Gibbons-Hawking) entropy and temperature from the very early trans-Planckian vacuum to the classical vacuum energy today. The n levels cover *all* scales from the far past highest excited trans-Planckian level $n = 10^{122}$ with finite curvature, $\Lambda_Q = 10^{122}$ and minimum entropy $S_Q = 10^{-122}$; n decreases till the Planck level ($n = 0$) with $H_{\text{planck}} = 1 = \Lambda_{\text{planck}} = S_{\text{planck}}$ and enters the post-Planckian phase, e.g., $n = 1, 2, \ldots, n_{\text{inflation}} = 10^{12}, \ldots, n_{\text{cmb}} = 10^{114}, \ldots, n_{\text{reoin}} = 10^{118}, \ldots, n_{\text{today}} = 10^{122}$ with the most classical value $H_{\text{today}} = 10^{-61}$, $\Lambda_{\text{today}} = 10^{-122}$, and $S_{\text{today}} = 10^{122}$. We implement the Snyder-Yang algebra in this context, yielding a consistent group-theory realization of quantum discrete de Sitter space-time, classical-quantum gravity duality symmetry, and a clarifying unifying picture.

(ii) A picture for the de Sitter background and the Universe epochs emerges, for both its classical (post-Planckian) and quantum (pre-Planckian) regimes, depicted in Fig. 1. This is achieved by considering classical-quantum gravity duality, trans-Planckian physics, quantum space-time, and quantum algebra to describe it. Concepts such as the Hawking temperature and the usual (mass) temperature are precisely the same concept in the different classical gravity (post-Planckian) and quantum gravity regimes, respectively. Similarly, it holds for the Bekenstein-Gibbons and Hawking entropy. A unifying clarifying picture emerges in terms of the main physical gravitational intrinsic magnitudes of the Universe: age, size, mass, vacuum energy, temperature, and entropy, covering the relevant gravity regimes and cosmological stages—classical, semiclassical, quantum Planckian, and trans-Planckian eras. The total or global mass levels are $M_n = m_P\sqrt{2n+1}$ for all $n = 0, 1, 2, \ldots$. *Two* dual branches $m_{n\pm} = m_P[\sqrt{2n+1} \pm \sqrt{2n}]$ do appear for the usual mass variables, covering the *whole mass range*: from the Planck mass ($n = 0$) until the largest cosmological ones in the post-Planckian branch ($+$) and from the smallest masses till near the Planck mass in the pre-Planckian branch ($-$).

(iii) The quantum space-time structure arises from the relevant nonzero space-time commutator $[X, T]$ or nonzero quantum uncertainty $\Delta X \Delta T$. The *quantum light cone* due to the quantum nonzero uncertainty $[X, T]$ allows a *new quantum region* which is purely quantum vacuum or zero-point Planckian and trans-Planckian energy and constant curvature. The quantum de Sitter space-time is described through the relevant quantum noncommutative coordinates and the quantum hyperbolic structure. They generalize the classical de Sitter space-time and reduce to it in the classical zero quantum commutator coordinates. Interestingly enough, de Sitter space-time turns out to be *discretized* in quantum levels, e.g., $(X_n, T_n) = \sqrt{2n+1}$, $n = 0, 1, 2, \ldots$.

(iv) In the post-Planckian domain, the quantum de Sitter space-time extends in discrete levels from the Planck-scale level ($n = 0$) and the quantum (low n) levels to the quasiclassical and classical levels (intermediate and large n), tending asymptotically for the very large n to a classical continuum space-time. Consistently, these levels have larger gravitational (Gibbons-Hawking) entropy S_n, lower vacuum density Λ_n, and lower Hubble rate H_n. In the pre-Planckian trans-Planckian domain, quantum de Sitter extends from the Planck-scale level ($n = 0$) to the lengths and time smaller than the Planck scale, the quasiquantum trans-Planckian levels (small and medium n), until the deep extreme highly excited trans-Planckian levels (very large n) which are those of smaller entropy S_{Qn}, higher vacuum density Λ_{Qn}, and higher H_{Qn}.

(v) Cosmological evolution goes from the pre-Planckian or trans-Planckian quantum phase to the Planck scale and then to the post-Planckian Universe—semiclassical accelerated de Sitter era (field theory inflation)—then to the classical phase until the present *diluted* de Sitter era. This evolution between the different gravity regimes could be viewed as a mapping between asymptotic (in and out) states characterized by the sets U_Λ and U_Q and, thus, as a scattering-matrix description: the most early quantum trans-Planckian state in the remote past being the "in state" and the very late classical dilute state being the far future or today "out state."

(vi) The classical-quantum gravity duality relations are not "abstract" relations: Observational values allow one to verify them. In the item below, we include the implications of the trans-Planckian phase for inflation and its effects, which are testable by the CMB and gravitational-wave observations. Inflation and the fluctuations are derived with this model: It yields (i) the known (classical and semiclassical) inflation and its primordial scalar and tensor fluctuation spectra and their predicted numbers tested by the CMB and other cosmological observations, (ii) the quantum corrections and the numbers for these corrections which are in agreement with other independent computations of quantum corrections to inflation (as quantum inflaton decay and inflaton-fermion interactions, for instance), and (iii) the resummation for them. All them can be confronted or constrained by the CMB and large-scale structure data, as well as in the future by gravitational-wave observations in the primordial gravitational wave frequency range.

The classical-quantum gravity duality relations through the Planck scale are well-motivated ones; e.g., as shown in Ref. [1], they reduce, in particular, to the well-known classical-quantum (de Broglie, Compton) duality without gravity. They are supported by the dynamics of quantum fields and strings in curved space-times [14,51,52]. They are not purely conjectured relations or hypotheses.

(vii) QFT in an expanding universe allows one to consider particle creation, cosmological perturbations, inflation, and its nearly scale-invariant spectra. In this context, QFT in the complete (with H_{QH}) de Sitter space-time yields that the total or complete QH inflationary spectra turn out expressed by Eqs. (11.1) and (11.2) as shown in Ref. [3] and discussed in Sec. XI. The features the pre-Planckian phase and those the quantum discrete levels could imprint in the inflationary spectra deserve to be explored and are beyond the scope of the present paper. A full quantum description including the quantum space-time algebra, its discrete levels, and coherent states in the complete QH de Sitter group within a group-theory quantization approach deserve more investigation.

(viii) Inflation is part of the standard cosmological model and is supported by the CMB data of temperature and temperature-E polarization anisotropies. This points to $10^{-6} m_P$, (or $10^{-5} M_P$ for the reduced mass $M_P = m_P/\sqrt{8\pi}$) as the energy scale of inflation [41,42,53] safely below the Planck energy scale m_P of the onset of quantum gravity. This implies that known post-Planckian inflation is consistently in the *semiclassical gravity regime*. This, in turn, implies that the preceding phase of inflation corresponds to a Planckian and pre-Planckian quantum phase. Inflation being a de Sitter (or quasi–de Sitter) stage, it has a smooth space-time curvature *without any physical space-time singularity.*

(ix) Integrating the above results, and because the earliest stages of the Universe are de Sitter (or quasi–de Sitter) eras, it appears that there is *no singularity* at the Universe's origin. First, the so-called $t = 0$ Friedmann-Robertson-Walker mathematical singularity is *not* physical: It is the result of extrapolation of the purely classical (nonquantum) general relativity theory, *out of its domain of physical validity.* The Planck scale is not merely a useful system of units but a physically meaningful scale: The onset of quantum gravity, this scale precludes the extrapolation until zero time or length. This is precisely what is expected from quantum trans-Planckian physics in gravity: the smoothness of the classical gravitational singularities. Second, inflation (classical or quantum) in the very past ($10^6 t_P$ or $10^{-6} t_P$) is mainly a de Sitter or quasi–de Sitter smooth constant curvature era *without any curvature singularity.* Third, the extreme past (at $10^{-61} t_P$) is a trans-Planckian de Sitter state of high *bounded* trans-Planckian constant curvature and, therefore, *without singularity.* This paper is not devoted to the singularity issue, but our results here and the whole picture emerging from this paper and Ref. [2] indicate the trend and insight into the problem.

(x) Further couplings, interactions, and background fields can be added. The conceptual results here will not change by adding further couplings or interactions or further background fields to the background here. Of course, this is just a first input in the construction of a complete physical theory and understanding *in agreement with observations*. Besides its conceptual and fundamental physics interest, this framework reveals deep and useful clarification for relevant cosmological eras and its quantum precursors and for the cosmological vacuum. This could provide realist insights and science directions where to place the theoretical effort for cosmological missions and future surveys such as Euclid, DESI, WFIRST, LSST-Vera C. Rubin Observatory, and Simons Observatory, for instance [54–58] and for the searching of cosmological quantum gravitational signals for e-LISA [11], for instance, after the success of LIGO [12,13].

(xi) The exhibit of (c, G, h) helps in recognizing the different relevant scales and physical regimes. Even if a hypothetical underlying "theory of everything" could require only pure numbers (option three in Ref. [59]), physical touch at some level asks for the use of fundamental constants [60,61]. Here we used three fundamental constants (tension being c^2/G). It appears from our study here and in Ref. [1] that a complete quantum theory of gravity would be a theory of pure numbers.

(xii) We can similarly think in quantum string coordinates (collection of point oscillators) to describe the quantum space-time structure (which is different from strings propagating on a fixed space-time background). This yields similar results for the string expectation values X^2 and T^2 and other related operators and yields also a quantum *hyperbolic space-time width bending* for the characteristic lines and light-cone generators or for the space-time horizons [1,3,62]. In string theory, the width appears as due to the nonzero size (of the order of the Planck scale) of the quantum string. Moreover, the $\sqrt{n}$ quantization we found in this paper is like the string mass quantization $M_n = m_s\sqrt{n}$, $n = 0, 1, \ldots$, with the Planck mass m_P instead of the string mass m_s, that is to say, with the gravitational constant G/c^2 instead of the string constant α'.

(xiii) Our results on conceptual unification [52] and QFT and string quantization in a wide class of curved space-times, e.g., Refs. [14,52], support the classical-quantum gravity relations, irrespective of the number and nature of the space-time dimensions and of whether dimensions are or are not compactified. The classical-quantum gravity duality here does not require the existence of any isometry or symmetry in the curved background, neither any other *a priori* condition. Several types of relativistic operations, e.g., $L \to \alpha'/L$, appear in string theory due to the existence of the dimensional constant α' (for example, T duality, the duality symmetry between the winding and propagating modes in orbifold compactifications). However, the duality we are considering is the classical-quantum (or wave-particle) duality (de Broglie or Compton type) relating classical or semiclassical and quantum behaviors extended to include the quantum Planckian and trans-Planckian regime. The de Broglie $L_q = h/p$ or Compton $L_q = h/mc$ relation is not the expression of a symmetry transform between physically equivalent theories but a link, through h, of two different behaviors and regimes. This duality and our results on QFT and quantum strings in curved backgrounds inspired our classical-quantum gravity relations. In a similar spirit, $L_Q = l_{pl}^2/L_G$ and, more generically, for a general observable O: $O_Q = o_{\mathrm{Pl}}^2/O_G$ relates two different classical and quantum gravity regimes of nature through the Planck scale (the crossing scale). The complete (classical and quantum) magnitudes $O_{QG} = O_Q + O_G = o_{\mathrm{Pl}}(O_G/o_{\mathrm{Pl}} + o_{\mathrm{Pl}}/O_G)$ are invariant under the exchange $Q \leftrightarrow G$.

(xiv) The quantum uncertainty or noncommutativity among the space and time coordinates acts as a quantum dressing or quantum width for the quantum light cone or "dressed light cone." In a complete covering of the space-time causal regions, a whole quantum vacuum region emerges. In our comments in the item above on string theory, the width appears as due to the noncommuting quantum string coordinates and the nonzero size of the string. In the context of QFT gravity, perturbative corrections to the dispersion relation, e.g., for a scalar field near the light cone, is of the form $X^2 - T^2 = G/(30\pi)$ [63], again as a shifted or quantum-corrected light-cone relation. Recall that quantum backreaction effects, gravitational scattering near a event horizon structure, produces a quantum shift too (the shifted horizon) [64–66]. QFT in curved space-times and their backreaction using the complete QG variables (as QH and its associated variables) is a step to describe QFT effects including the trans-Planckian domain and to go beyond the literature in the field. A full QFT description including quantum space-time, its discrete levels, and coherent states of the QH de Sitter group within a group-theory approach quantization deserves investigation and is beyond the scope of this paper.

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