

Subprocesses $\rho(770, 1450) \rightarrow K\bar{K}$ for the three-body hadronic D meson decays

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We construct the theoretical framework for quasi-two-body D meson decays with the help of pion and kaon electromagnetic form factors and with which we study the contributions of the subprocesses $\rho(770, 1450) \rightarrow K\bar{K}$ for the three-body D decays within the flavor $SU(3)$ symmetry. Because of the limitations imposed by phase space and strong coupling, the contributions for the kaon pair from the virtual bound state $\rho(770)$ are channel dependent and generally small for the concerned three-body D decays, but some quasi-two-body processes could still be observed in the Dalitz plot analyses for related decays, such as $D^0 \rightarrow K^-\rho(770)^+ \rightarrow K^-K^+K_S^0$ and $D^+ \rightarrow K_S^0\rho(770)^+ \rightarrow K_S^0K^+K_S^0$, they are predicted to have the branching fractions $\mathcal{B} = (0.82 \pm 0.04) \times 10^{-4}$ and $\mathcal{B} = 0.47_{-0.03}^{+0.05} \times 10^{-4}$, which are $(1.86 \pm 0.16)\%$ and $(1.84_{-0.16}^{+0.21})\%$, respectively, of the total branching fractions for the corresponding three-body D decays. We find in this work that the normal subprocesses like $\rho(1450)^+ \rightarrow \pi^+\pi^0$ or $\rho(1450)^+ \rightarrow K^+\bar{K}^0$, which are bound by the masses of decaying initial states, will provide virtual contributions in some special decays.

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I. INTRODUCTION

Three-body hadronic D meson decays provide us a rich field to test the Standard Model and beyond. These decays are proceeded predominantly through the quasi-two-body processes [1–3]. Due to their small nonresonant components and abundant intermediate states, three-body D decays as well as their subprocesses were widely employed to study the properties and substructures of various resonant states [4–17], to analyze hadron-hadron interactions [18–24], and to extract information on the $\pi\pi$, $K\pi$, and KK S -wave amplitude in the low energy region [25–35]. In the experimental analyses for relevant decay amplitudes [26,36–42], Dalitz plot technique [43] was widely adopted in recent years. The corresponding expressions of the decay amplitudes are usually composed of the coherent sum of the resonant and nonresonant contributions within the isobar formalism [44–46]. For the precise and accurate Dalitz plot analyses, all reliable and necessary strong dynamical components should be present in the expressions of the decay amplitudes.

The contributions for the kaon pair in the final states of three-body D decays from the ρ family resonances, such as $\rho(1450)^\pm$ and $\rho(1700)^\pm$ for $K_S^0K^\pm$ in the decays $D^+ \rightarrow K_S^0K^+\pi^0$ and $D^0 \rightarrow K_S^0K^\pm\pi^\mp$, have been noticed by BESIII [47], LHCb [40], and CLEO [48] Collaborations. In addition, the subprocess $\rho(1450)^0 \rightarrow K^+K^-$ was found to contribute a surprising large fit fraction for the three-body decays $B^\pm \rightarrow \pi^\pm K^+K^-$ by LHCb Collaboration in Ref. [49]. The results of Ref. [50] indicate a sizable branching fraction, around 22%, of $D^0 \rightarrow K_S^0K^+K^-$ comes from the subprocesses $\rho(770, 1450, 1700)^+ \rightarrow K_S^0K^+$. For the quasi-two-body B meson decays, resonance contributions for the kaon pair originating from the intermediate states $\rho(770, 1450, 1700)$ have been specifically studied in Refs. [51–53]. In this paper, we shall extend our previous works [51–53] to the quasi-two-body D meson decays and concentrate on the contributions of $\rho(770, 1450) \rightarrow K\bar{K}$ for relevant processes.

The schematic diagram for the cascade decays $D \rightarrow hR \rightarrow hK\bar{K}$ is shown in Fig. 1. In the rest frame of the initial state, D meson decays into the intermediate state R and the bachelor h , and then R decays into its daughters, the kaon pair. When $q_{1,2}$ is the light quark u or d , as shown in the Feynman diagram Fig. 1(b), the intermediate states could be the resonances $\rho(770, 1450)$. The natural decay modes for $\rho(770) \rightarrow K\bar{K}$ are blocked because the pole mass for the resonant state $\rho(770)$ is below the threshold of the kaon pair. However, the virtual contribution [54–57]

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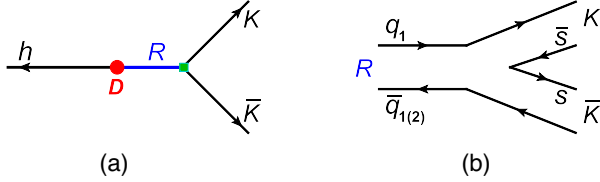


FIG. 1. Schematic view of the cascade decays $D \rightarrow hR \rightarrow hK\bar{K}$, where R stands for the intermediate states $\rho(770, 1450)$ decaying into $K\bar{K}$ in this work, and $q_{1,2}$ is the light quark u or d .

from the Breit-Wigner (BW) formula [58] tail of $\rho(770)$ was found to be indispensable for the production of the kaon pair in the processes of $\pi^- p \rightarrow K^- K^+ n$ and $\pi^+ n \rightarrow K^- K^+ p$ [59,60], $\bar{p} p \rightarrow K^+ K^- \pi^0$ [61,62], $e^+ e^- \rightarrow K^+ K^-$ [63–71], and $e^+ e^- \rightarrow K_S^0 K_L^0$ [72–77]. Besides, $\rho(770, 1450)^\pm$ are important intermediate states for $K^\pm K_S^0$ for the final states of hadronic τ decays [78–81]. With the kaon electromagnetic form factors studied in Refs. [65,66,68,81–86], including the ρ , ω , and ϕ families resonant states, we predicted the branching fractions for the charmless decays $B \rightarrow h\rho(1450) \rightarrow hK\bar{K}$ (with $h = \pi, K$) to be about a tenth of their corresponding quasi-two-body decays with $\rho(1450)$ decaying into a pion pair [51,52]. What's more, we found that the branching fraction of virtual contribution for $K\bar{K}$ from the BW tail of $\rho(770)$ is larger than the corresponding contribution from $\rho(1450)$ in a charmless quasi-two-body B decay [52].

Unlike hadronic B meson decays, for which the heavy quark expansion tools and factorization approaches have been successfully used for decades, the two-body and three-body hadronic D meson decays are challenging to be reliably described based on quantum chromodynamics (QCD) on the theoretical side because of the c -quark mass. In this context, model-independent ways such as the factorization-assisted topological-amplitude approach [87–89] and the topological-diagram approach [90–95] have been adopted in references for various hadronic D decays. In addition, starting from the weak effective Hamiltonian [96], the experimental data for the decays $D^0 \rightarrow K_S^0 K^+ K^-$ from BABAR [97] and $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ from BABAR [97] and Belle [98] were analyzed in [50,99], respectively, within the quasi-two-body factorization framework. While in [100], with the chiral unitary approach used to take into account the final state interaction, a model was developed to study the three-body $D^0 \rightarrow K^- \pi^+ \eta$ decay. In [101], the multimeson model based on chiral effective Lagrangians was proposed to describe the $D^+ \rightarrow K^- K^+ K^+$ decay. The decay $D^+ \rightarrow K^- \pi^+ \pi^+$ was studied in [102] utilizing dispersion theory. Flavor $SU(3)$ sum rules for $D \rightarrow PP$ and $D \rightarrow PV$ decay amplitudes were presented in [103], where P is a light pseudo-scalar, and V is a light vector. $SU(3)$ flavor symmetry relationship was used in Ref. [104] to analyze the ratios of

amplitudes and phases for $D^0 \rightarrow PV$ in the decays $D^0 \rightarrow \pi^+ \pi^- \pi^0$, $D^0 \rightarrow \pi^0 K^+ K^-$, and $D^0 \rightarrow K^- \pi^+ \pi^0$.

This paper is organized as follows: In Sec. II, we construct the theoretical framework for the quasi-two-body D meson decays with the help of the pion and kaon electromagnetic form factors, and we derive the relation between the branching fractions for the two-body and quasi-two-body D meson decays. In Sec. III, we present our numerical results for the contributions of the subprocesses $\rho(770, 1450) \rightarrow K\bar{K}$ for concerned D meson decays, along with some necessary discussions. A brief summary of this work is given in Sec. IV.

II. FRAMEWORK

For the cascade decays $D \rightarrow hR \rightarrow hh_1 h_2$, with $h_{1(2)}$ being a light pseudoscalar pion or kaon, the related effective weak Hamiltonian is written as [96]

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[\sum_{q=d,s} \lambda_q (C_1 O_1 + C_2 O_2) - \lambda_b \sum_{i=3}^6 C_i O_i - \lambda_b C_{8g} O_{8g} \right], \quad (1)$$

where $G_F = 1.1663787(6) \times 10^{-5} \text{ GeV}^{-2}$ [105] is the Fermi coupling constant, $\lambda_q = V_{cq}^* V_{uq}$ and $\lambda_b = V_{cb}^* V_{ub}$ stand for the product of the Cabbibo-Kobayashi-Maskawa (CKM) matrix elements, C 's are the Wilson coefficients at scale μ , the O_1 and O_2 are current-current operators, $O_3 - O_4$ are QCD penguin operators, and O_{8g} is chromomagnetic dipole operator. With Eq. (1) as the effective weak Hamiltonian, the decay amplitudes for two-body processes $D \rightarrow PV$ were described with typical topological diagram amplitudes $T_{P,V}$, $C_{P,V}$, $E_{P,V}$, and $A_{P,V}$, as well as additional penguin amplitudes in the factorization-assisted topological-amplitude approach [88] and the topological-diagram approach [91,93–95]. For the detailed discussions on these topological diagram amplitudes, one is referred to Refs. [88,91,93–95]. Take $D^0 \rightarrow K^- \rho(770, 1450)^+ \rightarrow K^- \pi^+ \pi^0$ and $D^0 \rightarrow K^- \rho(770, 1450)^+ \rightarrow K^- K^+ \bar{K}^0$ as the examples, now we construct the decay amplitudes for the decays $D \rightarrow hR \rightarrow hh_1 h_2$.

If the subprocesses $\rho(770, 1450) \rightarrow \pi\pi$ were shrunk to the mesons $\rho(770, 1450)$, the quasi-two-body processes $D^0 \rightarrow K^- \rho(770, 1450)^+ \rightarrow K^- \pi^+ \pi^0$ will become the two-body channels $D^0 \rightarrow K^- \rho(770, 1450)^+$. The decay amplitudes of $D^0 \rightarrow K^- \rho(770, 1450)^+$ are dominated by the color-favored tree amplitude T_P with $D^0 \rightarrow K^-$ transition, which is formulated as [88,91,93,94]

$$T_P = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \left[\frac{C_1}{3} + C_2 \right] f_\rho m_\rho \times F_1^{D \rightarrow K}(m_\rho^2) 2[\epsilon_\rho \cdot p_D], \quad (2)$$

where the subscript ρ stands for $\rho(770)$ or $\rho(1450)$, $F_1^{D \rightarrow K}$ is the form factor for $D^0 \rightarrow K^-$ transition and is parametrized as [106]

$$F_1^{D \rightarrow K}(Q^2) = \frac{0.78}{(1 - Q^2/2.11^2)(1 - 0.24Q^2/2.11^2)}, \quad (3)$$

ϵ_ρ is the polarization vector, and p_D is the momentum for D^0 . Beyond the amplitude T_P , there is a W -exchange nonfactorizable contribution from amplitude E_V , which is extracted from experimental data in the topological-diagram approach [91,93,94] and is parametrized as [88]

$$E_V = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} C_2 \chi_q^E e^{i\phi_q^E} f_D m_D [f_\rho / f_{\rho(770)}] \times [f_K / f_\pi] [\epsilon_\rho \cdot p_D] \quad (4)$$

in the factorization-assisted topological-amplitude approach, with the decay constants f_D , f_K , and f_π for the D^0 , kaon and pion, respectively. The $\chi_q^E = 0.25$ and $\phi_q^E = 1.73$ [88] are the parameters that characterize the strength and strong phase of the corresponding amplitude. Thus, one has the decay amplitudes $\mathcal{M} = T_P + E_V$ for the decays $D^0 \rightarrow K^- \rho(770, 1450)^+$. Utilizing the partial decay width [105],

$$\Gamma(D \rightarrow \rho K) = \frac{|\vec{p}|^3}{8\pi m_\rho^2} |\tilde{\mathcal{M}}|^2, \quad (5)$$

and the mean life of D^0 meson, one will achieve the branching fractions for the two-body decays $D^0 \rightarrow K^- \rho(770, 1450)^+$ with the relation $\mathcal{M} = \epsilon_\rho \cdot p_D \tilde{\mathcal{M}}$, where the magnitude of the momentum for ρ or K in the rest frame of D meson is

$$|\vec{p}| = \frac{1}{2m_D} \sqrt{[m_D^2 - (m_\rho + m_K)^2][m_D^2 - (m_\rho - m_K)^2]}, \quad (6)$$

and the m_i 's are the masses for relevant particles above with $i = \{D, \rho, K\}$.

The subprocesses $\rho(770, 1450)^+ \rightarrow \pi^+ \pi^0$ of the quasi-two-body decays $D^0 \rightarrow K^- \rho(770, 1450)^+ \rightarrow K^- \pi^+ \pi^0$ are associated with the pion electromagnetic form factor $F_\pi(s)$, with s the squared invariant mass for pion pair. The form factor for the pion has been measured with high precision in Refs. [107–113] by different collaborations. For the $\rho(770)$ and $\rho(1450)$ components, one has [83,85]

$$F_\pi^R(s) = c_R^\pi \text{BW}_R(s), \quad (7)$$

where R represents the resonance $\rho(770)$ or $\rho(1450)$, and the coefficient $c_R^\pi = f_R g_{R\pi\pi} / (\sqrt{2} m_R)$ [83] is determined by the decay constant f_R , coupling constant $g_{R\pi\pi}$, and the mass m_R . The BW formula for F_π has the form [114],

$$\text{BW}_R = \frac{m_R^2}{m_R^2 - s - im_R \Gamma_R(s)}, \quad (8)$$

and the s -dependent width is

$$\Gamma_R(s) = \Gamma_R \frac{m_R}{\sqrt{s}} \frac{|\vec{q}|^3}{|\vec{q}_0|^3} X^2(|\vec{q}| r_{\text{BW}}^R). \quad (9)$$

The Blatt-Weisskopf barrier factor for ρ family resonances is given by [115]

$$X(z) = \sqrt{\frac{1 + z_0^2}{1 + z^2}}, \quad (10)$$

with the barrier radius $r_{\text{BW}}^R = 1.5 \text{ GeV}^{-1}$ [116]. The magnitude of the momentum $|\vec{q}| = \frac{1}{2} \sqrt{s - 4m_\pi^2}$ in Eq. (9), and $|\vec{q}_0|$ is $|\vec{q}|$ at $s = m_R^2$.

By connecting subprocesses $\rho(770, 1450)^+ \rightarrow \pi^+ \pi^0$ and the two-body channels $D^0 \rightarrow K^- \rho(770, 1450)^+$ together, we get the quasi-two-body decay processes. The amplitudes for $D^0 \rightarrow K^- \rho(770, 1450)^+ \rightarrow K^- \pi^+ \pi^0$ are written as

$$\begin{aligned} & \langle (\pi^+ \pi^0)_\rho K^- | \mathcal{H}_{\text{eff}} | D^0 \rangle \\ &= \langle \pi^+ \pi^0 | \rho(770, 1450)^+ \rangle \frac{1}{m_R^2 - s - im_R \Gamma_R(s)} \\ & \quad \times \langle \rho(770, 1450)^+ K^- | \mathcal{H}_{\text{eff}} | D^0 \rangle \\ &= g_{\rho\pi\pi} \epsilon_\rho \cdot (p_{\pi^+} - p_{\pi^0}) \frac{1}{m_\rho^2 - s - im_\rho \Gamma_\rho(s)} \\ & \quad \times [T_P + E_V], \end{aligned} \quad (11)$$

where the p_{π^+} and p_{π^0} are the four momenta for π^+ and π^0 , respectively. Then, in the rest frame of the intermediate states, we have the expression of the differential branching fraction (\mathcal{B}) [105],

$$\frac{d\mathcal{B}}{\sqrt{s} d\sqrt{s}} = \tau_D \frac{|\vec{q}|^3 |\vec{p}_h|^3}{12\pi^3 m_D^3} |\mathcal{A}|^2, \quad (12)$$

by taking into account the Eq. (7), with τ_D as the mean lifetime for D meson. The magnitude of the momentum $|\vec{p}_h|$ for the state h is written as

$$|\vec{p}_h| = \frac{1}{2\sqrt{s}} \sqrt{[m_D^2 - (\sqrt{s} + m_h)^2][m_D^2 - (\sqrt{s} - m_h)^2]}, \quad (13)$$

with m_h as the mass for the bachelor final state, which is K^- for $D^0 \rightarrow K^- \rho(770, 1450)^+ \rightarrow K^- \pi^+ \pi^0$ decays. The decay amplitude \mathcal{A} in Eq. (12) is related to that of the corresponding two-body process with the relation,

$$A \approx \tilde{\mathcal{M}}/m_\rho, \quad (14)$$

and with the replacement $f_\rho \rightarrow F_\pi^\rho(s)$ for the resonant states.

The subprocesses $\rho(770, 1450)^+ \rightarrow K^+ \bar{K}^0$ in the decays $D^0 \rightarrow K^- \rho(770, 1450)^+ \rightarrow K^- K^+ \bar{K}^0$ are related to the kaon electromagnetic form factors. These form factors have been extensively studied in Refs. [81–85] on the theoretical side. The experimental information on kaon form factors comes from the measurements of the reactions $e^+ e^- \rightarrow K^+ K^-$ [65,66,71] and $e^+ e^- \rightarrow K^+ K^-(\gamma)$ [68]. Since $K\bar{K}$ is not an eigenstate of isospin, both isospin 0 and 1 resonant states such ϕ , ω , and ρ families need to be taken into the components of the form factors for the kaon [68]. When concerning only the contributions for $K^+ K^-$ and $K^0 \bar{K}^0$ from the resonances $\iota = \rho(770, 1450, 1700)$ and $\varsigma = \omega(782, 1420, 1650)$, one has [117]

$$\begin{aligned} F_{K^+ K^-}^u(s) &= F_{K^0 \bar{K}^0}^d(s) \\ &= \frac{1}{2} \sum_\iota c_\iota^K \text{BW}_\iota(s) + \frac{1}{2} \sum_\varsigma c_\varsigma^K \text{BW}_\varsigma(s), \end{aligned} \quad (15)$$

$$\begin{aligned} F_{K^+ K^-}^d(s) &= F_{K^0 \bar{K}^0}^u(s) \\ &= -\frac{1}{2} \sum_\iota c_\iota^K \text{BW}_\iota(s) + \frac{1}{2} \sum_\varsigma c_\varsigma^K \text{BW}_\varsigma(s). \end{aligned} \quad (16)$$

For the $K^+ \bar{K}^0$ and $K^0 K^-$ pairs, which have no contribution from the neutral states $\omega(782, 1420, 1650)$, one has [81–83]

$$F_{K^+ \bar{K}^0}(s) = F_{K^0 K^-}(s) = \sum_\iota c_\iota^K \text{BW}_\iota(s). \quad (17)$$

The relevant coefficients c_R^K 's (with $R = \iota, \varsigma$) in kaon form factors, which are proportional to their corresponding coupling constants $g_{RK\bar{K}}$, have been detailed discussed in our previous work [52], and we adopt the same values for them as in Ref. [52]. With the relations $g_{\rho(770)K\bar{K}} = g_{\omega(782)K\bar{K}} = \frac{1}{\sqrt{2}} g_{\phi(1020)K\bar{K}}$ and $g_{\rho(1450)K\bar{K}} \approx \frac{1}{2} g_{\rho(1450)\pi^+\pi^-}$ within flavor $SU(3)$ symmetry [83] and Eq. (11) with the replacements $\pi^+ \rightarrow K^+$ and $\pi^0 \rightarrow \bar{K}^0$, one could easily obtain the decay amplitudes for the cascade decays $D^0 \rightarrow \rho(770, 1450)^+ K^- \rightarrow K^+ \bar{K}^0 K^-$.

By neglecting the effect of the alteration of $m_\rho^2 \rightarrow s$ for $F_1^{D \rightarrow K}$ in Eq. (2), one can relate partial decay width expression the Eqs. (5) to (12) the differential branching fraction for quasi-two-body decays. From Eq. (5), we have

$$|\tilde{\mathcal{M}}/m_\rho|^2 = \mathcal{B}(D \rightarrow \rho K) \frac{1}{\tau_D} \frac{8\pi}{|\vec{p}|^3}. \quad (18)$$

The right-hand side of this equation can be undoubtedly transplanted to the differential branching fraction of Eq. (12), along with the replacements $f_\rho \rightarrow F_{\pi,K}^\rho(s)$ for

the quasi-two-body decays. Then we reach the branching fraction,

$$\mathcal{B}_{\text{Q2B}} \approx \mathcal{B}(D \rightarrow \rho K) \int ds \frac{|\vec{q}|^3}{3\pi^2 m_D^3} \left| \frac{\vec{p}_h}{\vec{p}} \right|^3 \left| \frac{F_{\pi,K}^\rho(s)}{f_\rho} \right|^2, \quad (19)$$

for the quasi-two-body decays with the subprocesses $\rho(770) \rightarrow \pi\pi$ or $\rho(770) \rightarrow K\bar{K}$. This formula for the branching fractions of the quasi-two-body decays with the corresponding two-body results is different from the narrow width approximation relation discussed in Ref. [118], which is not appropriate for the processes with the virtual bound state [54,55] decays like $\rho(770) \rightarrow K\bar{K}$. The integral part of Eq. (19) is approximately equal to the branching fraction for the normal subprocess such as $\rho(770) \rightarrow \pi\pi$ decay, but for the virtual processes like $\rho(770) \rightarrow K\bar{K}$, its integral result could be different for each decay channel.

III. RESULTS AND DISCUSSIONS

In the numerical calculation, we adopt the decay constant $f_\rho = 0.216 \pm 0.003$ GeV [119] for $\rho(770)$, and the mean lives $\tau_{D^\pm} = 1.040(7)$ ps, $\tau_{D^0} = 0.4101(15)$ ps, and $\tau_{D_s^\pm} = 0.504(4)$ ps [105] for $D_{(s)}$ mesons. For $\rho(1450)$, we employ $f_{\rho(1450)} = 0.185_{-0.035}^{+0.030}$ GeV [120] resulting from the data [121]. The masses for particles in relevant decay processes, the decay constants for pion and kaon, the full widths for resonances $\rho(770)$ and $\rho(1450)$ (in units of GeV), the Wolfenstein parameters for CKM matrix elements [105], and the decay constants f_D and f_{D_s} for $D_{(s)}$ [122] are presented in Table I.

Utilizing Eq. (12) as the differential branching fraction, we have the branching fraction,

$$\begin{aligned} \mathcal{B}(D^0 \rightarrow K^- \rho(770)^+ \rightarrow K^- \pi^+ \pi^0) \\ = (9.40 \pm 0.26 \pm 0.21)\%, \end{aligned} \quad (20)$$

with the amplitudes T_P and E_V in Eq. (2) and Eq. (4), respectively. The coefficient $c_\rho^\pi = 1.177$ for the pion form factor, with

$$g_{\rho(770)\pi\pi} = \sqrt{\frac{6\pi m_{\rho(770)}^2 \Gamma_{\rho(770)}}{|\vec{q}_0|^3}}, \quad (21)$$

has been adopted. The two errors of the result (20) are induced by the uncertainties of the decay constant $f_{\rho(770)}$ and the CKM matrix elements, respectively. The quasi-two-body branching fraction Eq. (20) is consistent with the result $\mathcal{B} = 9.6\%$ in Ref. [88] within the factorization-assisted topological-amplitude approach for the two-body decay $D^0 \rightarrow K^- \rho(770)^+ \rightarrow \pi\pi$ in view of $\mathcal{B}(\rho(770) \rightarrow \pi\pi) \sim 100\%$ [105], but it is slightly less than the data

TABLE I. Masses for relevant states, decay constants for pion and kaon, full widths of $\rho(770)$ and $\rho(1450)$ (in units of GeV), and the Wolfenstein parameters for CKM matrix elements from [105] and f_D and f_{D_s} in [122].

$m_{D^\pm} = 1.870$	$m_{D^0} = 1.865$	$m_{D_s^\pm} = 1.968$	$m_{\pi^\pm} = 0.140$
$m_{\pi^0} = 0.135$	$m_{K^\pm} = 0.494$	$m_{K^0} = 0.498$	$f_D = 0.212$
$f_{D_s} = 0.250$	$f_\pi = 0.130$	$f_K = 0.156$	
$m_{\rho(770)} = 0.775$			$\Gamma_{\rho(770)} = 0.149$
$m_{\rho(1450)} = 1.465 \pm 0.025$			$\Gamma_{\rho(1450)} = 0.400 \pm 0.060$
$\lambda = 0.22650 \pm 0.00048$			$A = 0.790^{+0.017}_{-0.012}$
$\bar{\rho} = 0.141^{+0.016}_{-0.017}$			$\bar{\eta} = 0.357 \pm 0.011$

$\mathcal{B} = (11.3 \pm 0.7)\%$ in *Review of Particle Physics* [105] averaged from [116,123]. While with the form factor $F_\pi(s)$ measured in [109] by the BABAR Collaboration with Gounaris-Sakurai (GS) model [124] for the ρ family resonances, the branching fraction will be enhanced to be $\mathcal{B} = (10.19 \pm 0.28 \pm 0.23)\%$ for the quasi-two-body decay $D^0 \rightarrow K^- \rho(770)^+ \rightarrow K^- \pi^+ \pi^0$.

Now, we switch to the subprocess $\rho(770)^+ \rightarrow K^+ \bar{K}^0$ for $D^0 \rightarrow K^- \rho(770)^+ \rightarrow K^- K^+ \bar{K}^0$ decay. Because of the pole mass of $\rho(770)$, we have only the virtual contribution for $K^+ \bar{K}^0$ from the resonance in the decay $D^0 \rightarrow K^- \rho(770)^+ \rightarrow K^- K^+ \bar{K}^0$. With the relation, $g_{\rho(770)K^+K^-} = g_{\omega(782)K^+K^-} = \frac{1}{\sqrt{2}} g_{\phi(1020)K^+K^-}$ [83] for the strong couplings within flavor $SU(3)$ symmetry, the coefficient $c_{\rho(770)}^K = 1.247 \pm 0.019$ was determined in Ref. [52], which is consistent with the results in [83,85,86] for the kaon electromagnetic form factors. Then it's trivial to obtain the branching fraction,

$$\begin{aligned} \mathcal{B}(D^0 \rightarrow K^- \rho(770)^+ \rightarrow K^- K^+ \bar{K}^0) \\ = (1.64 \pm 0.05 \pm 0.04 \pm 0.05) \times 10^{-4}, \end{aligned} \quad (22)$$

where the first two errors have the same sources as those in Eq. (20), and the third one comes from the uncertainty of coefficient $c_{\rho(770)}^K$. Considering the meson \bar{K}^0 decays half

TABLE II. Experimental data of the branching fractions from *Review of Particle Physics* [105] for the concerned three-body D meson decays.

Mode	Unit	\mathcal{B} [105]
$D^0 \rightarrow K_S^0 K^+ K^-$	10^{-3}	4.42 ± 0.32
$D^0 \rightarrow K_S^0 K^- \pi^+$	10^{-3}	3.3 ± 0.5
$D^0 \rightarrow K_S^0 K^+ \pi^-$	10^{-3}	2.17 ± 0.34
$D^0 \rightarrow K^+ K^- \pi^0$	10^{-3}	3.42 ± 0.14
$D^+ \rightarrow K^+ K_S^0 K_S^0$	10^{-3}	2.54 ± 0.13
$D^+ \rightarrow K^+ K^+ K^-$	10^{-5}	6.14 ± 0.11
$D^+ \rightarrow K_S^0 K^+ \pi^0$	10^{-3}	5.07 ± 0.30
$D^+ \rightarrow K^+ K^- \pi^+$	10^{-3}	9.68 ± 0.18
$D_s^+ \rightarrow K^+ K^+ K^-$	10^{-4}	2.16 ± 0.20
$D_s^+ \rightarrow K^+ K^- \pi^+$	%	5.39 ± 0.15
$D_s^+ \rightarrow K^+ K_S^0 \pi^0$	%	1.52 ± 0.22

into K_S^0 , one receives the contribution of the subprocess $\rho(770)^+ \rightarrow K^+ K_S^0$ to be about $(1.86 \pm 0.13 \pm 0.09)\%$ for the tree-body decay $D^0 \rightarrow K_S^0 K^+ K^-$ when comparing the result (22) with the corresponding data in Table II. The two errors come from the uncertainties of the data $(4.42 \pm 0.32) \times 10^{-3}$ for $D^0 \rightarrow K_S^0 K^+ K^-$ and the result in Eq. (22), respectively.

In Fig. 2, we show the differential branching fractions for the decays $D^0 \rightarrow K^- \rho(770)^+ \rightarrow K^- \pi^+ \pi^0$ and $D^0 \rightarrow K^- \rho(770)^+ \rightarrow K^- K^+ \bar{K}^0$. In the inset, the feature of $\rho(770)$ is clearly and fully presented through the curve with a peak at about $s = m_{\rho(770)}^2$ for the decay process $D^0 \rightarrow K^- \rho(770)^+ \rightarrow K^- \pi^+ \pi^0$. The obvious comparison is a bump at about $s = 1.2 \text{ GeV}^2$ for $D^0 \rightarrow K^- \rho(770)^+ \rightarrow K^- K^+ \bar{K}^0$, which should not but potentially could be claimed as a resonant state with quite a large decay width. The peak location of the bump for the subprocess $\rho(770) \rightarrow K\bar{K}$ of D meson decay like $D^0 \rightarrow K^- K^+ \bar{K}^0$ is distinctly different from it for the same subprocess in the three-body B meson decays that have been studied in Refs. [51,52], the former is closer to the pole mass of the resonance $\rho(770)$. This phenomenon is attributed to kinematic characteristics in the corresponding decay processes rather than the properties of the resonant states involved. Because the related three-body phase space of D meson decays is much smaller than that of B decays, the phase space factor $|\vec{p}_h|$ in Eq. (12) will drop more quickly in

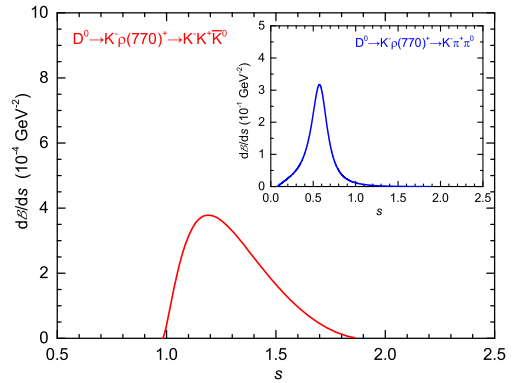


FIG. 2. The differential branching fractions for the quasi-two-body decays $D^0 \rightarrow K^- \rho(770)^+ \rightarrow K^- \pi^+ \pi^0$ (inset) and $D^0 \rightarrow K^- \rho(770)^+ \rightarrow K^- K^+ \bar{K}^0$.

TABLE III. Virtual contributions of the subprocess $\rho(770) \rightarrow K\bar{K}$ for the concerned three-body D decays.

2B modes	Data [105]	Q2B modes	Results
$D^0 \rightarrow K_S^0 \rho(770)^0$	$6.3_{-0.8}^{+0.6} \times 10^{-3}$	$D^0 \rightarrow K_S^0[\rho(770)^0 \rightarrow]K^+K^-$	$2.48_{-0.32-0.10}^{+0.24+0.10} \times 10^{-6}$
		$D^0 \rightarrow K_S^0[\rho(770)^0 \rightarrow]K^0\bar{K}^0$	$2.20_{-0.28-0.09}^{+0.21+0.09} \times 10^{-6}$
$D^0 \rightarrow K^-\rho(770)^+$	$(11.3 \pm 0.7)\%$	$D^0 \rightarrow K^-\rho(770)^+ \rightarrow]K^+\bar{K}^0$	$1.71_{-0.11-0.07}^{+0.11+0.07} \times 10^{-4}$
$D^0 \rightarrow \pi^+\rho(770)^-$	$(5.15 \pm 0.25) \times 10^{-3}$	$D^0 \rightarrow \pi^+[\rho(770)^- \rightarrow]K^-K^0$	$1.88_{-0.09-0.08}^{+0.09+0.08} \times 10^{-5}$
$D^0 \rightarrow \pi^-\rho(770)^+$	$(1.01 \pm 0.04)\%$	$D^0 \rightarrow \pi^-[\rho(770)^+ \rightarrow]K^+\bar{K}^0$	$3.69_{-0.15-0.15}^{+0.15+0.15} \times 10^{-5}$
$D^0 \rightarrow \pi^0\rho(770)^0$	$(3.86 \pm 0.23) \times 10^{-3}$	$D^0 \rightarrow \pi^0[\rho(770)^0 \rightarrow]K^+K^-$	$3.69_{-0.22-0.15}^{+0.22+0.15} \times 10^{-6}$
		$D^0 \rightarrow \pi^0[\rho(770)^0 \rightarrow]K^0\bar{K}^0$	$3.40_{-0.20-0.14}^{+0.20+0.14} \times 10^{-6}$
$D^+ \rightarrow K_S^0\rho(770)^+$	$6.14_{-0.35}^{+0.60}\%$	$D^+ \rightarrow K_S^0[\rho(770)^+ \rightarrow]K^+\bar{K}^0$	$9.35_{-0.53-0.39}^{+0.91+0.39} \times 10^{-5}$
$D^+ \rightarrow K^+\rho(770)^0$	$(1.9 \pm 0.5) \times 10^{-4}$	$D^+ \rightarrow K^+[\rho(770)^0 \rightarrow]K^+K^-$	$7.81_{-2.06-0.32}^{+2.06+0.32} \times 10^{-8}$
		$D^+ \rightarrow K^+[\rho(770)^0 \rightarrow]K^0\bar{K}^0$	$6.95_{-1.83-0.29}^{+1.83+0.29} \times 10^{-8}$
$D^+ \rightarrow \pi^+\rho(770)^0$	$(8.3 \pm 1.5) \times 10^{-4}$	$D^+ \rightarrow \pi^+[\rho(770)^0 \rightarrow]K^+K^-$	$7.99_{-1.44-0.33}^{+1.44+0.33} \times 10^{-7}$
		$D^+ \rightarrow \pi^+[\rho(770)^0 \rightarrow]K^0\bar{K}^0$	$7.36_{-1.33-0.30}^{+1.33+0.30} \times 10^{-7}$
$D_s^+ \rightarrow K^+\rho(770)^0$	$(2.5 \pm 0.4) \times 10^{-3}$	$D_s^+ \rightarrow K^+[\rho(770)^0 \rightarrow]K^+K^-$	$1.57_{-0.25-0.06}^{+0.25+0.06} \times 10^{-6}$
		$D_s^+ \rightarrow K^+[\rho(770)^0 \rightarrow]K^0\bar{K}^0$	$1.42_{-0.23-0.06}^{+0.23+0.06} \times 10^{-6}$

the D processes. This will also result in the ratio of the contributions from subprocesses $\rho(770) \rightarrow K\bar{K}$ to $\rho(770) \rightarrow \pi\pi$ to be channel dependent in the three-body D decays and much smaller than that for the three-body B decays, which will decline from one or two percent in B decays [52] to the level of thousandth or even smaller, as exhibited by the results in Table III.

The resonance $\rho(770)$ as a virtual bound state [54,55] in the decay $\rho(770) \rightarrow K\bar{K}$ will not completely present its properties in the concerned processes because of the phase space. Nevertheless, the quantum number of the involved resonance could be fixed from its decay daughters, for example, $K\bar{K}$, along with CKM matrix elements in the decay on theoretical side. The certain resonant source for the final states $K\bar{K}$ makes the cascade decay like $D^0 \rightarrow K^-\rho(770)^+ \rightarrow K^-K^+\bar{K}^0$ to be a quasi-two-body process, although the invariant mass region for the kaon pair is excluded from the region around pole mass of the intermediate state $\rho(770)$. What we want to stress here is that the nonresonant contribution in the three-body D or B meson decays should not include the specific known contribution from a certain determinate resonant state like $\rho(770)$ for $K\bar{K}$ in the experimental studies.

Different from the contribution of the subprocess $\rho(770) \rightarrow \pi\pi$ for the decays like $D^0 \rightarrow K^-\rho(770)^+$, the virtual contributions of the subprocesses $\rho(770) \rightarrow K\bar{K}$ for the concerned D meson decays are nearly unaffected by the full width of the resonance $\rho(770)$. Take the decay $D^0 \rightarrow K^-\rho(770)^+ \rightarrow K^-K^+\bar{K}^0$ as an example, we will have its branching fraction slightly changed from 1.638×10^{-4} to 1.640×10^{-4} when the $\Gamma_{\rho(770)}$ is altered from 0.149 GeV to zero. The best explanation for this is that the s -dependent width for the BW formula of the Eq. (8) fades into insignificance when the invariant mass square s , which starts from the threshold of kaon pair, is large enough, then

the BW expression for $\rho(770)$ is charged only by the coefficient c_ρ^K of the kaon form factors and the gap between the squared mass of the involved resonant state and the invariant mass square s for kaon pair. Thanks to the phase space factor $|\vec{q}|^3$ in Eq. (12), the portion of the contribution in the shadow of the large decay width $\Gamma_{\rho(770)}$ for the process $D^0 \rightarrow K^-\rho(770)^+ \rightarrow K^-K^+\bar{K}^0$ is the strongly suppressed.

The branching fraction of the decay $D^0 \rightarrow K^-\rho(770)^+ \rightarrow K^-K^+\bar{K}^0$ can also be achieved from the Eq. (19) with the help of the data for $D^0 \rightarrow K^-\rho(770)^+$. With the two-body branching fraction $\mathcal{B}(D^0 \rightarrow K^-\rho(770)^+) = (11.3 \pm 0.7)\%$ [105], one has $\mathcal{B} = (1.71 \pm 0.11 \pm 0.07) \times 10^{-4}$, as shown in Table III for the quasi-two-body decay $D^0 \rightarrow K^-\rho(770)^+ \rightarrow K^-K^+\bar{K}^0$. This result is also in agreement with the value in Eq. (22). The first error of these results for the quasi-two-body decays in Table III comes from the uncertainties of the corresponding data for the relevant two-body decays in the same table; the second one is induced by the uncertainties of the coefficient $c_{\rho(770)}^K = 1.247 \pm 0.019$ [52] and decay the constant $f_\rho = 0.216 \pm 0.003$ GeV [119].

The quasi-two-body branching fractions in Table III, which are derived from the experimental data for corresponding two-body decays with the Eq. (19), are generally very small compared with the corresponding experimental data in Table II, but some of them have the potential to be observed in the Dalitz plot analyses for related three-body D decays. In addition to $D^0 \rightarrow K^-\rho(770)^+ \rightarrow K^-K^+\bar{K}^0$ with $\bar{K}^0 \rightarrow K_S^0$, the subprocess $\rho(770)^+ \rightarrow K^+K_S^0$ will contribute about $(1.84 \pm 0.09_{-0.13}^{+0.19})\%$ of the total branching fraction for the $D^+ \rightarrow K^+K_S^0K_S^0$ decay, with two errors coming from the uncertainties of the data $\mathcal{B} = (2.54 \pm 0.13) \times 10^{-3}$ in Table II and the corresponding quasi-two-body result in Table III, while the quasi-two-body decays

$D^0 \rightarrow \pi^-[\rho(770)^+ \rightarrow]K^+K_S^0$ and $D_s^+ \rightarrow K^+[\rho(770)^0 \rightarrow]K^+K^-$ are predicted to provide $(0.85 \pm 0.13 \pm 0.05)\%$ and $(0.73 \pm 0.07 \pm 0.12)\%$, respectively, for the total branching fractions of their corresponding three-body decay processes shown in Table II.

Because of the phase space and the strong coupling, the virtual contribution of the subprocess $\rho(770)^0 \rightarrow K^+K^-$ for the three-body decay $D^0 \rightarrow K_S^0K^+K^-$ shown in Table III is very small, which is less than $1/10^3$ of the total branching fraction $(4.42 \pm 0.32) \times 10^{-3}$ in Table II for the corresponding three-body decay process. Actually, because of the suppression from the factor $1/2$ in Eqs. (15) and (16), the kaon form factors, the subprocesses $\rho(770)^0 \rightarrow K^+K^-$ and $\rho(770)^0 \rightarrow K^0\bar{K}^0$ will contribute to the smaller branching fractions compared with the $\rho(770)^\pm \rightarrow K^\pm K_S^0$. This partly causes the decays $D^0 \rightarrow K_S^0\rho(770)^0$, $D^0 \rightarrow \pi^0\rho(770)^0$, $D^+ \rightarrow K^+\rho(770)^0$, and $D^+ \rightarrow \pi^+\rho(770)^0$ with $\rho(770)^0$ decaying into K^+K^- or $K^0\bar{K}^0$ listed in Table III to hold the very small proportions of the total branching fractions for their corresponding three-body decay processes, and they are unlikely to be observed in the Dalitz plot analyses in the near future.

With the ω components in Eqs. (15) and (16), the branching fractions for quasi-two-body decays with the intermediate state $\omega(782)$ for $K\bar{K}$ could be easily obtained from Eq. (19) with the help of the existing experimental data. We predict the branching fractions for $D^0 \rightarrow K_S^0[\omega(782) \rightarrow]K^+K^-$ and $D_s^+ \rightarrow K^+[\omega(782) \rightarrow]K^+K^-$ to be $(4.59 \pm 0.25 \pm 0.40) \times 10^{-6}$ and $(5.72 \pm 1.64 \pm 0.50) \times 10^{-7}$, respectively, with the data $\mathcal{B}(D^0 \rightarrow K_S^0\omega(782)) = (1.11 \pm 0.06)\%$ and $\mathcal{B}(D_s^+ \rightarrow K^+\omega(782)) = (8.7 \pm 2.5) \times 10^{-4}$ [105]. The first error comes from the uncertainty the corresponding data, and the second one is induced by $c_{\omega(782)}^K = 1.113 \pm 0.019$ [52] and $f_\omega = 0.197 \pm 0.008$ GeV [119].

For the quasi-two-body decay $D^0 \rightarrow K^-\rho(1450)^+ \rightarrow K^-\pi^+\pi^0$, the color-favored tree amplitude T_P with $D^0 \rightarrow K^-$ transition and W -exchange nonfactorizable amplitude E_V are involved. Since the parameters $\chi_q^E = 0.25$ and $\phi_q^E = 1.73$ were fitted for the meson $\rho(770)$ in [88], one should not trust them in the decays with the subprocess $\rho(1450)^+ \rightarrow \pi^+\pi^0$. Fortunately, one has T_P to be the dominated amplitude for this quasi-two-body processes. By omitting the annihilation-type amplitude E_V , we estimate the branching fraction,

$$\mathcal{B} = (2.25 \pm 0.56) \times 10^{-3}, \quad (23)$$

for the $D^0 \rightarrow K^-\rho(1450)^+ \rightarrow K^-\pi^+\pi^0$ decay with the GS model measured in [109] for the pion form factor, where only the error coming from the uncertainty of $|c_{\rho(1450)}^\pi| = 0.158 \pm 0.018$ [109] has been taken into account. If we switch on the amplitude E_V and still adopt the parameters $\chi_q^E = 0.25$ and $\phi_q^E = 1.73$, we have $\mathcal{B} \approx 2.20 \times 10^{-3}$ for this quasi-two-body process, which is very close to the value in Eq (23). In

Ref. [83], a smaller coefficient $c_{\rho(1450)}^\pi = -0.119 \pm 0.011$ was fitted for the pion form factor $F_\pi(s)$, with which a small branching fraction $\mathcal{B} \approx 0.85 \times 10^{-3}$ could be obtained for the same quasi-two-body decay. In [105,116], the branching fraction $(8.2 \pm 1.8) \times 10^{-3}$ was claimed for the quasi-two-body decay $D^0 \rightarrow K^-\rho(1700)^+ \rightarrow K^-\pi^+\pi^0$. With the $F_\pi(s)$ measured in [109], we estimate its branching fraction to be about 0.74×10^{-3} . It seems the result for this decay process in [116] was overestimated.

In Fig. 3, the differential branching fraction is shown for the decay $D^0 \rightarrow K^-\rho(1450)^+ \rightarrow K^-\pi^+\pi^0$, but one cannot tell the feature of the resonance $\rho(1450)$ from the curve, it is more of a diagram for the background. The reason for this is that the subprocess $\rho(1450)^+ \rightarrow \pi^+\pi^0$ in this decay is, in essence, a virtual resonance decay. Although the pole mass of $\rho(1450)$ is larger than the threshold of pion pair, but the initial decaying state D^0 does not have the energy to make the resonance in $D^0 \rightarrow K^-\rho(1450)^+ \rightarrow K^-\pi^+\pi^0$ demonstrate its intact properties, it has already been terminated before the s for the pion pair arrive the position of $m_{\rho(1450)}^2$. Contrary to the virtual contribution of $\rho(770) \rightarrow K\bar{K}$ discussed above, the contribution from $\rho(1450)^+ \rightarrow \pi^+\pi^0$ in the quasi-two-body decay $D^0 \rightarrow K^-\rho(1450)^+ \rightarrow K^-\pi^+\pi^0$ arises only from the forepart of BW expression for the involved resonance. Similar virtual contributions from the forepart of BW for the resonances will also take place in other decays and shall be left for future studies.

For the resonance $\rho(1450)$ decaying into kaon pair, the s -dependent width $\Gamma_R(s)$ in Eq. (8) containing partial widths of various final states was adopted in Refs. [81,125,126]. In view of the decays $\rho(1450) \rightarrow \omega\pi$ and $\rho(1450) \rightarrow 4\pi$ are the two dominated modes for $\rho(1450)$, we adopt the $\Gamma_{\rho(1450)}(s)$, which is discussed in the Appendix with four decay channels. With Eq. (A3), the coefficient $c_{\rho(1450)}^K = -0.156 \pm 0.015$ [52], and the amplitude T_P , it's easy to estimate the branching fraction,

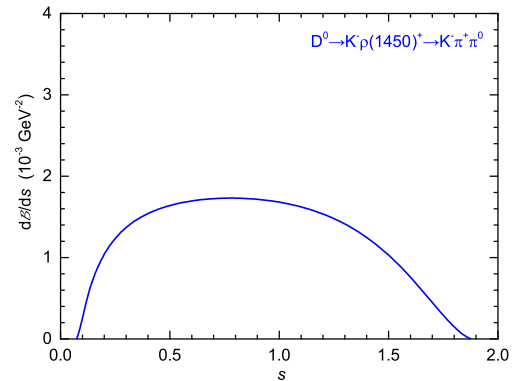


FIG. 3. The differential branching fractions for $D^0 \rightarrow K^-\rho(1450)^+ \rightarrow K^-\pi^+\pi^0$ decay.

$$\begin{aligned} \mathcal{B}(D^0 \rightarrow K^- \rho(1450)^+ \rightarrow K^- K^+ \bar{K}^0) \\ = (0.91 \pm 0.17) \times 10^{-4}, \end{aligned} \quad (24)$$

with the error coming from the uncertainty of $c_{\rho(1450)}^K = -0.156 \pm 0.015$ [52]. This result, just like the resonance contribution in the decay $D^0 \rightarrow K^- \rho(1450)^+ \rightarrow K^- \pi^+ \pi^0$, is the virtual contribution from the forepart of BW formula of $\rho(1450)$, and we need to stress here that this virtual contribution from $\rho(1450)$ doesn't depend on the $\Gamma_{\rho(1450)}(s)$ in Eq. (A3). Taking $\bar{K}^0 \rightarrow K_S^0$ into account, the resonance $\rho(1450)^+$ will contribute about $(1.03 \pm 0.19 \pm 0.07)\%$ for the total branching fraction of the tree-body decay $D^0 \rightarrow K_S^0 K^+ K^-$, where two errors come from the uncertainties of the estimated branching fraction and the data $\mathcal{B} = (4.42 \pm 0.32) \times 10^{-3}$ for the three-body decay, respectively.

When we put the decay amplitudes of two virtual contributions $\rho(770)^+ \rightarrow K^+ K_S^0$ and $\rho(1450)^+ \rightarrow K^+ K_S^0$ together for the tree-body decay $D^0 \rightarrow K_S^0 K^+ K^-$, we have the branching fraction,

$$\begin{aligned} \mathcal{B}(D^0 \rightarrow K^- \rho(770, 1450)^+ \rightarrow K^- K^+ K_S^0) \\ = (2.23 \pm 0.19) \times 10^{-4}. \end{aligned} \quad (25)$$

If we turn on three resonances $\rho(770)^+$, $\rho(1450)^+$, and $\rho(1700)^+$, the branching fraction will be enhanced to be about $\mathcal{B} = 3.54 \times 10^{-4}$, which means about 8.0%, comparable to the result $22_{-13}^{+8}\%$ in [50], of the total branching fraction for $D^0 \rightarrow K_S^0 K^+ K^-$. Nevertheless, we need stress that there is no precise measurement for kaon form factors like $F_\pi(s)$ in [109], and the coefficients of c_ρ^K 's are not necessary to be real values, the phase difference between BW expressions of $\rho(770)^+$, $\rho(1450)^+$, and $\rho(1700)^+$ for $F_K(s)$ could change the weight of the interferences between them.

For the quasi-two-body decay $D^0 \rightarrow \pi^- \rho(1450)^+ \rightarrow \pi^- K^+ \bar{K}^0$, the decay of the subprocess $\rho(1450)^+ \rightarrow K^+ \bar{K}^0$ is an ordinary process because of $(m_{\pi^-} + m_{\rho(1450)}) < m_{D^0}$. With Eq. (A3), one has its branching fraction,

$$\begin{aligned} \mathcal{B}(D^0 \rightarrow \pi^- \rho(1450)^+ \rightarrow \pi^- K^+ \bar{K}^0) \\ = (3.72 \pm 0.71) \times 10^{-5}, \end{aligned} \quad (26)$$

where the error comes from the uncertainty of $c_{\rho(1450)}^K = -0.156 \pm 0.015$. This result is close the virtual contribution from resonance $\rho(770)^+$ for the corresponding quasi-two-body decay process in the Table III. The other contributions of the kaon pair from resonance $\rho(1450)$ for the concerned D meson decays such as $D^0 \rightarrow \pi^- [\rho(1450)^+ \rightarrow] K^+ K_S^0$, the parameters like $D \rightarrow \rho(1450)$ transition form factors are absent in literature, and we leave them for the future studies.

IV. SUMMARY

In this work, we studied the contributions of the subprocesses $\rho(770, 1450) \rightarrow K\bar{K}$ for the three-body hadronic D meson decays. We constructed the theoretical framework for the quasi-two-body D decays with the help of the pion and kaon electromagnetic form factors and derived the relation connecting the branching fractions for the two-body with that for the corresponding quasi-two-body D meson decays and with which we obtained the numerical results for the concerned quasi-two-body D meson decay processes within the flavor $SU(3)$ symmetry.

We predicted the branching fraction $\mathcal{B} = (0.82 \pm 0.04) \times 10^{-4}$ for the decay $D^0 \rightarrow K^- \rho(770)^+ \rightarrow K^- K^+ K_S^0$, which is $(1.86 \pm 0.16)\%$ of the total branching ratio $(4.42 \pm 0.32) \times 10^{-3}$ in *Review of Particle Physics* for the three-body decay $D^0 \rightarrow K_S^0 K^+ K^-$. While the subprocess $\rho(1450)^+ \rightarrow K^+ \bar{K}^0$ was found to contribute a branching fraction $\mathcal{B} = (0.91 \pm 0.17) \times 10^{-4}$ for the quasi-two-body decay $D^0 \rightarrow K^- \rho(1450)^+ \rightarrow K^- K^+ \bar{K}^0$ in this work. The subprocess $\rho(770)^+ \rightarrow K^+ K_S^0$, in addition, could contribute about $(1.84_{-0.16}^{+0.21})\%$ of the total branching fraction for the $D^+ \rightarrow K^+ K_S^0 K_S^0$ decay, and the quasi-two-body decays $D^0 \rightarrow \pi^- [\rho(770)^+ \rightarrow] K^+ K_S^0$ and $D_s^+ \rightarrow K^+ [\rho(770)^0 \rightarrow] K^+ K^-$ were predicted to provide $(0.85 \pm 0.14)\%$ and $(0.73 \pm 0.14)\%$, respectively, for the total branching fractions of their corresponding three-body decay processes in this paper, which have the potential to be observed in the Dalitz plot analyses for related three-body D decays.

Different from $\rho(770) \rightarrow \pi\pi$, the subprocess $\rho(770) \rightarrow K\bar{K}$ will provide only the virtual contribution for the concerned three-body D meson decays in this work, and the virtual bound state $\rho(770)$ decaying into kaon pair can not completely present its properties in relevant decay processes because of the phase space limitation. The situation for $\rho(1450)^+$, which is decaying into $\pi^+ \pi^0$ or $K^+ \bar{K}^0$ in $D^0 \rightarrow K^- \rho(1450)^+$, is similar with that of $\rho(770)$ decaying to $K\bar{K}$. The pole mass of $\rho(1450)$ is larger than the threshold of pion and kaon pairs, but the initial decaying state D^0 does not have enough energy to make this resonance demonstrate its intact properties in $D^0 \rightarrow K^- \rho(1450)^+$. The virtual contributions from various resonant states widely exist in multibody D , J/ψ , B , etc., decays. We need to stress that the virtual contributions from specific known intermediate states is different from the nonresonant contributions demarcated in the experimental studies.

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APPENDIX: $\Gamma_R(s)$ FOR $\rho(1450)$

Different from the BW formula for $\rho(770, 1450)$ in [49,83,85,109], the s -dependent width $\Gamma_R(s)$ in Eq. (8) containing partial widths of various final states for resonances $\rho(770, 1450)$ was adopted in Refs. [81,125,126]. Since $\mathcal{B}(\rho(770) \rightarrow \pi\pi) \sim 100\%$ [105], $\Gamma_{\rho(770)}(s) = \Gamma_{\rho(770) \rightarrow \pi\pi}(s)$ is a very good approximation for the BW formula for its $\pi\pi$ and $K\bar{K}$ final states; besides, the virtual contributions of the subprocesses $\rho(770) \rightarrow K\bar{K}$ are unaffected by the full width of $\rho(770)$ for the concerned D meson decays, as discussed in Sec. III.

For the $\Gamma_R(s)$ in BW formula of resonance $\rho(1450)$, the s -dependent width with two channels,

$$\Gamma_{\rho(1450)}(s) = \Gamma_{\rho(1450)} \frac{m_R}{\sqrt{s}} \times \left[\frac{|\vec{q}|^3}{|q_0|^3} \theta(s - 4m_\pi^2) + \frac{1}{2} \frac{|\vec{q}_K|^3}{|q_{K0}|^3} \theta(s - 4m_K^2) \right], \quad (\text{A1})$$

and

$$\Gamma_{\rho(1450)}(s) = \Gamma_{\rho(1450)} \frac{m_R}{\sqrt{s}} \frac{2|\vec{q}|^3 + |\vec{q}_K|^3}{2|q_0|^3}, \quad (\text{A2})$$

were adopted in [81,125], respectively, where $\vec{q}_{K(0)}$ is the $\vec{q}_{(0)}$ with the replacement $m_\pi \rightarrow m_K$. In view of that the

decays $\rho(1450) \rightarrow \omega\pi$ and $\rho(1450) \rightarrow 4\pi$ are the two dominated modes for the resonance $\rho(1450)$, we adopt the $\Gamma_{\rho(1450)}(s)$ with four channels in this work as [126,127]

$$\Gamma_{\rho(1450)}(s) = \Gamma_{\rho(1450)} \frac{m_R}{\sqrt{s}} \times \left[\mathcal{B}(\rho(1450) \rightarrow \pi\pi) \frac{|\vec{q}|^3}{|q_0|^3} X^2(|\vec{q}|r_{\text{BW}}^R) + \mathcal{B}(\rho(1450) \rightarrow K\bar{K}) \frac{|\vec{q}_K|^3}{|q_{K0}|^3} X^2(|\vec{q}_K|r_{\text{BW}}^R) + \mathcal{B}(\rho(1450) \rightarrow \omega\pi) \frac{|\vec{q}_\omega|^3}{|q_{\omega 0}|^3} X^2(|\vec{q}_\omega|r_{\text{BW}}^R) + \mathcal{B}(\rho(1450) \rightarrow 4\pi) \frac{|q_{4\pi}^{\vec{}}|^3}{|q_{4\pi 0}^{\vec{}}|^3} X^2(|q_{4\pi}^{\vec{}}|r_{\text{BW}}^R) \right], \quad (\text{A3})$$

where [126]

$$|\vec{q}_\omega| = \frac{1}{2\sqrt{s}} \sqrt{[s - (m_\omega + m_\pi)^2][s - (m_\omega - m_\pi)^2]}, \quad (\text{A4})$$

and $|q_{4\pi}^{\vec{}}| = \frac{1}{2} \sqrt{s - 16m_\pi^2}$ [127], and $|q_{\omega 0}|$ and $|q_{4\pi 0}^{\vec{}}|$ are the $|q_{\omega 0}|$ and $|q_{4\pi}^{\vec{}}|$ at $s = m_{\rho(1450)}^2$, respectively. We have the branching fractions $\mathcal{B}(\rho(1450) \rightarrow \omega\pi) = 45\%$ and $\mathcal{B}(\rho(1450) \rightarrow 4\pi) = 40\%$ [126], and $\mathcal{B}(\rho(1450)^0 \rightarrow K^+K^-)/\mathcal{B}(\rho(1450)^0 \rightarrow \pi^+\pi^-) \approx 1/10$ [51].

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