

Axion model with the SU(6) unification

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We propose an SU(6) unified theory that solves the strong CP problem with a high-quality axion. This is achieved by an automatic global Peccei-Quinn $U(1)_{PQ}$ symmetry and the gauged discrete $Z_{4\mathcal{R}}$ symmetry. With the axion mass predictions of $m_a \sim \mathcal{O}(10^{-3}) - \mathcal{O}(0.1)$ eV, as well as a universal axion-photon coupling in the unification models, the QCD axion can be probed in the upcoming experiments of IAXO. An intermediate gauge symmetry breaking scale characterized by the axion decay constant is obtained by the Peccei-Quinn quality argument, which is likely to achieve a successful leptogenesis through the type-I seesaw mechanism. The Higgs sector at the electroweak scale is a type-II two-Higgs-doublet model. The gauge coupling unification is possible with the supersymmetric extension.

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I. INTRODUCTION

Grand unified theories (GUTs) [1,2] were proposed to unify all fundamental interactions. In the original Georgi-Glashow SU(5) GUT [1], it was assumed that there are no unobserved fermions beyond the Standard Model (SM). Aside from the aesthetic aspect of achieving the gauge coupling unification in its supersymmetric (SUSY) version, it is pragmatic to envision that a successful GUT could address as many physical issues beyond the SM as possible. One leading puzzle of the SM is the strong CP problem, which predicts the neutron electric dipole moment due to the nontrivial topological term of $\theta \frac{\alpha_s}{8\pi} G_{\mu\nu}^a \tilde{G}^{\mu\nu a}$ in the quantum chromodynamics (QCD) sector. However, the experimental upper limit of $|d_n| < 3.0 \times 10^{-26}$ e cm [3] leads to an extremely tiny upper bound of $|\bar{\theta}| \lesssim 10^{-10}$, where $\bar{\theta}$ is shifted from the θ parameter by the phase of the quark mass matrix M_q as $\bar{\theta} = \theta + \arg \det M_q$. Axion field, which transforms under a global $U(1)$ Peccei-Quinn (PQ) symmetry [4–6], is the most appealing candidate to solve the strong CP problem since its first proposal. Two widely studied benchmark scenarios of the invisible axion models are the Kim-Shifman-Vainshtein-Zakharov model [7,8] and the Dine-Fischler-Srednicki-Zhitnitsky model [9,10].

In the past four decades, GUTs were considered as frameworks for the axion models [11,12]. Most studies

based on the SU(5) [13–22], SO(10) [20,23–28], and E_6 [29] groups assumed a global $U(1)_{PQ}$ symmetry that commutes with the GUT groups. This leads to the PQ quality problem [13,30–34]. It is therefore desirable to have the global PQ symmetry arise automatically, and there have been discussions in models with extended gauge symmetries [35–49].

In this paper, we propose an SU(6) GUT with a high-quality $U(1)_{PQ}$. It was first noted by Dimopoulos, Raby, and Susskind (DRS) [50] that an anomaly-free $SU(N+4)$ gauge theory with N antifundamental chiral fermions and one rank-2 antisymmetric chiral fermion enjoys a global symmetry of

$$\mathcal{G}_{\text{global}} = SU(N)_F \otimes U(1)_{PQ}, \quad (1)$$

when $N \geq 2$. The SU(6) is thus the minimal GUT group with an automatic PQ symmetry. Historically, the SU(6) GUT was studied in various contexts [51–68], and found to alleviate the intrinsic doublet-triplet splitting problem [69–74] and the proton lifetime constraint [52,57,63] in the Georgi-Glashow SU(5) GUT [1].

The rest of the paper is organized as follows. In Sec. II, we setup our minimal SUSY SU(6) model with the possible symmetry breaking pattern to the SM gauge group. The physical SU(6) axion is derived in Sec. III. Our main observation is given in Sec. IV, where a high-quality QCD axion can arise with axion decay constant of $f_a \sim \mathcal{O}(10^8) - \mathcal{O}(10^{10})$ GeV in the SUSY-extended SU(6), and be probed in upcoming IAXO experiment. An intermediate symmetry of $\mathcal{G}_{331} = SU(3)_c \otimes SU(3)_L \otimes U(1)_N$ [66,75–80] is found to break at the scale of $v_{331} \sim \mathcal{O}(10^9) - \mathcal{O}(10^{11})$ GeV, which is obtained from the PQ-quality argument. The possible axion domain wall is briefly discussed in Sec. V. The gauge coupling unification is indicated in Sec. VI. We conclude and envision

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some future efforts in nonminimal GUTs in Sec. VII. In Appendix A, we prove that the non-SUSY SU(6) model cannot lead to a high-quality axion. The details of the SU(6) symmetry breaking and the spectrum can be found in Appendix B.

II. THE MINIMAL MODEL

The minimal setup of an anomaly-free SUSY SU(6) model is made up of three generational chiral superfields of $\bar{\mathbf{6}}_F^{\rho=1,II}$ and $\mathbf{15}_F$ that contain all necessary fermions. Two $\bar{\mathbf{6}}_F^\rho$ form an SU(2)_F doublet, which is free from the Witten anomaly [81]. The SU(6) undergoes two stages of symmetry breaking [65,66,82,83]

$$\text{SU}(6) \xrightarrow{\Lambda_{\text{GUT}}} \mathcal{G}_{331} \xrightarrow{v_{331}} \mathcal{G}_{\text{SM}},$$

$$\mathcal{G}_{\text{SM}} = \text{SU}(3)_c \otimes \text{SU}(2)_L \otimes \text{U}(1)_Y, \quad (2)$$

in our discussions.

Next, we determine the Higgs sector of the minimal SU(6) model by imposing the requirements that

- (1) A superfield of $\mathbf{35}_H$ [82] with its scalar vacuum expectation value (VEV) at Λ_{GUT} is necessary to achieve the first-stage symmetry breaking in Eq. (2).
- (2) All Yukawa couplings of the 125 GeV SM-like Higgs boson should be reproduced. This leads to at least one $\bar{\mathbf{6}}_{H I}$ and one $\mathbf{15}_H$.
- (3) Two $\bar{\mathbf{6}}_{H \rho}$ are necessary to keep the SU(2)_F-invariant couplings with two antifundamental superfields of $\bar{\mathbf{6}}_F^\rho$. Furthermore, only one of them (which we chose to be $\bar{\mathbf{6}}_{H II}$ without loss of generality) are allowed to develop VEV and give fermion masses at v_{331} . Otherwise the low-energy effective theory is not anomaly free.
- (iv) A superfield of $\mathbf{21}_H$ [66], together with the $\bar{\mathbf{6}}_{H II}$, achieve the second-stage symmetry breaking and the PQ-symmetry breaking through their scalar VEVs. Another conjugate superfield of $\bar{\mathbf{21}}_H$ is necessary for the anomaly cancellation.

Collectively, we tabulate the chiral superfields with their PQ and $Z_{4\mathcal{R}}$ charges in Table I. A discrete and gauged $Z_{4\mathcal{R}}$ symmetry with proper charge assignments is necessary to avoid the dangerous dimension-three PQ-breaking operators. The SU(2)_F is also gauged in order to avoid the constraint from the gravity. Indeed, the mixed gauge anomalies cancel as follows

TABLE I. The superfields with their SU(2)_F representations, and the charges under the U(1)_{PQ} and $Z_{4\mathcal{R}}$ symmetries, in the SUSY SU(6) model.

	$\bar{\mathbf{6}}_F^\rho$	$\mathbf{15}_F$	$\bar{\mathbf{6}}_{H \rho}$	$\mathbf{15}_H$	$\mathbf{21}_H$	$\bar{\mathbf{21}}_H$	$\mathbf{35}_H$
SU(2) _F	\square	1	$\bar{\square}$	1	1	1	1
U(1) _{PQ}	1	1	-2	-2	-2	0	0
$Z_{4\mathcal{R}}$	0	0	2	2	2	0	0

$$\begin{aligned} Z_{4\mathcal{R}}[\text{SU}(6)]^2 &= n_g \times [2 \times (-1) + 4(-1)] \\ &\quad + 2 \times 1 + 4 \times 1 + 8 \times 1 \\ &\quad + 12 = 0 \pmod{4}, \end{aligned} \quad (3a)$$

$$\begin{aligned} Z_{4\mathcal{R}}[\text{SU}(2)_F]^2 &= n_g \times 6 \times (-1) + 6 \times 1 + 4 \\ &= 0 \pmod{4}, \end{aligned} \quad (3b)$$

with $n_g = 3$. Accordingly, the superpotential includes the following terms

$$\begin{aligned} W_Y &= \mathbf{15}_F \bar{\mathbf{6}}_F^\rho \bar{\mathbf{6}}_{H \rho} + \mathbf{15}_F \mathbf{15}_F \mathbf{15}_H \\ &\quad + \epsilon_{\rho\delta} \bar{\mathbf{6}}_F^\rho \bar{\mathbf{6}}_F^\delta \mathbf{15}_H + \epsilon_{\rho\delta} \bar{\mathbf{6}}_F^\rho \bar{\mathbf{6}}_F^\delta \mathbf{21}_H. \end{aligned} \quad (4)$$

Schematically, we denote the Higgs VEVs and their hierarchies as follows

$$\begin{aligned} \langle \mathbf{35}_H \rangle &\sim \Lambda_{\text{GUT}}, \\ \langle \bar{\mathbf{6}}_{H II} \rangle &= v_3, \langle \mathbf{21}_H \rangle = v_6, \quad v_3 \sim v_6 \sim v_{331}, \\ \langle \bar{\mathbf{6}}_{H I} \rangle &= v_d = v_{\text{EW}} \sin \beta, \quad \langle \mathbf{15}_H \rangle = v_u = v_{\text{EW}} \cos \beta, \\ \Lambda_{\text{GUT}} &\gg v_{331} \gg v_{\text{EW}} = (\sqrt{2} G_F)^{-1/2} \simeq 246 \text{ GeV}, \end{aligned} \quad (5)$$

with $\tan \beta$ being the ratio between two EW Higgs VEVs.

After the first-stage symmetry breaking in Eq. (2), the $(\mathbf{1}, \bar{\mathbf{3}}, -\frac{1}{3})_{H II} \subset \bar{\mathbf{6}}_{H II}$ and the $(\mathbf{1}, \mathbf{6}, +\frac{2}{3})_H \subset \mathbf{21}_H$ will develop their VEVs to trigger the second-stage symmetry breaking. The Yukawa couplings of $\mathbf{15}_F \bar{\mathbf{6}}_F^\rho \bar{\mathbf{6}}_{H II} + \text{H.c.}$ give masses to the vectorlike D quarks of $m_D \simeq \mathcal{O}(v_{331})$. With the electric charge of $-1/3$, the D quarks can form $d = 4$ operators from the Yukawa couplings in Eq. (4), and the D -hadron lifetime [84–86] is found to be $\tau_D \sim m_D^{-1} \sim \mathcal{O}(10^{-36}) - \mathcal{O}(10^{-34})$ sec with $v_{331} \sim \mathcal{O}(10^9) - \mathcal{O}(10^{11})$ GeV through the PQ quality analysis below. This satisfies the cosmological constraint of $\tau_Q \lesssim 10^{-2}$ sec [87,88] from the big bang nucleosynthesis. In addition, the Yukawa couplings of $\bar{\mathbf{6}}_F^{[I} \bar{\mathbf{6}}_F^{II]} \mathbf{21}_H + \text{H.c.}$ gives heavy neutrino masses of $m_{N, N'} \simeq \mathcal{O}(v_{331})$, which satisfy the Davidson-Ibarra bound [89–91] of $M_N \gtrsim 10^9$ GeV for a successful leptogenesis. Two SU(2)_L EW Higgs doublets come from the $\bar{\mathbf{6}}_{H I}$ and the $\mathbf{15}_H$ and lead to the type-II 2HDM. Besides, a type-I seesaw mechanism can also be realized with the Yukawa couplings of $\bar{\mathbf{6}}_F^{[I} \bar{\mathbf{6}}_F^{II]} \mathbf{15}_H + \text{H.c.}$. All relevant Yukawa couplings in two stages of symmetry breaking will be explicitly given in Eqs. (B6) and (B8). Another remarkable feature is that the tree-level μ term of $\mu H_u H_d \subset \mu \bar{\mathbf{6}}_{H I} \mathbf{15}_H$ is impossible in the superpotential (4) due to the SU(6) gauge symmetry.

III. THE SU(6) AXION

The physical axion mainly comes from the $(\mathbf{1}, \bar{\mathbf{3}}, -\frac{1}{3})_{H II} \supset \frac{v_3}{\sqrt{2}} e^{ia_3/v_3}$ and the $(\mathbf{1}, \mathbf{6}, +\frac{2}{3})_H \supset \frac{v_6}{\sqrt{2}} e^{ia_6/v_6}$. It can be obtained from the orthogonality between the U(1)_{PQ} current and the U(1)_N current, and we arrive at

$$-\frac{1}{3}q_3(v_3)^2 + \frac{2}{3}q_6(v_6)^2 = 0. \quad (6)$$

The physical PQ charges of (q_3, q_6) for the (a_3, a_6) fields are linear combinations of the $U(1)_{\text{PQ}}$ charges and the $U(1)_N$ charges [28]

$$q \equiv c_1 \text{PQ} + c_2 N. \quad (7)$$

The coefficient of c_1 is determined by matching [92] the global anomaly factors of $U(1)_{\text{PQ}}[\text{SU}(6)]^2$ and $U(1)_{\text{PQ}}[\text{SU}(3)_c]^2$

$$N_{\text{SU}(6)} = -5, \quad (8a)$$

$$N_{\text{SU}(3)_c} = \sum_{\mathcal{R}_f \in \text{SU}(3)_c} \text{PQ}_f T(\mathcal{R}_f) = -5c_1, \quad (8b)$$

which leads to $c_1 = 1$. The explicit derivations are given in Eqs. (B5) and (B7). By denoting the overall size of the Higgs VEVs and their ratio as

$$v_{331}^2 = (q_3 v_3)^2 + (q_6 v_6)^2, \quad \tan \phi \equiv \frac{v_3}{2v_6}, \quad (9)$$

we find the physical PQ charges and axion decay constant of

$$q_3 = -3\cos^2\phi, \quad q_6 = -6\sin^2\phi, \quad (10a)$$

$$f_a = \frac{v_{331}}{2|N_{\text{SU}(3)_c}|} = \frac{v_{331}}{10} = \frac{3v_3 v_6}{5\sqrt{(v_3)^2 + 4(v_6)^2}}. \quad (10b)$$

The physical axion becomes $a_{\text{phys}} = \cos\phi a_3 + \sin\phi a_6$. In addition, the electromagnetic (EM) anomaly factor is

$$E = \sum_f \text{PQ}_f \dim(\mathcal{C}_f) \text{Tr} q_f^2 = -\frac{40}{3}, \quad (11)$$

with \mathcal{C}_f being the fermion representations under the $\text{SU}(3)_c$, and (PQ_f, q_f) being the PQ and EM charges of fermions. The explicit derivation is given in Eq. (B10).

IV. THE HIGH-QUALITY AXION

The leading PQ-breaking (with $\Delta\text{PQ} = -12$) operator that is invariant under the $\text{SU}(6)$, $\text{SU}(2)_F$, and the discrete $\mathbb{Z}_{4\mathcal{R}}$ symmetries in Table I reads

$$\begin{aligned} \mathcal{O}_{\text{PQ}}^{d=6} &= (\epsilon^{\rho\delta} \bar{\mathbf{6}}_{\mathbf{H}\rho} \bar{\mathbf{6}}_{\mathbf{H}\delta} \mathbf{15}_{\mathbf{H}})^2 \supset \left[\epsilon^{\rho\delta} \epsilon_{IJK} \left(\mathbf{1}, \bar{\mathbf{3}}, -\frac{1}{3} \right)_\rho^I \right. \\ &\quad \left. \times \left(\mathbf{1}, \bar{\mathbf{3}}, -\frac{1}{3} \right)_\delta^J \left(\mathbf{1}, \bar{\mathbf{3}}, +\frac{2}{3} \right)_\delta^K \right]^2, \end{aligned} \quad (12)$$

with (I, J, K) being the $\text{SU}(3)_L$ indices. In order not to reintroduce further PQ-breaking operators, it is reasonable

to expect the SUSY-breaking scale to be lower than v_{331} . The total axion effective potential and the induced effective $\bar{\theta}$ due to the Eq. (12) are

$$\begin{aligned} V &= V_{\text{QCD}} + V_{\text{PQ}}, \quad V_{\text{QCD}} = \Lambda_{\text{QCD}}^4 \left[1 - \cos\left(\frac{a_{\text{phys}}}{f_a}\right) \right], \\ V_{\text{PQ}} &= k \frac{\mathcal{O}_{\text{PQ}}^{d=6}}{M_{\text{pl}}^2} + \text{H.c.} \\ &\approx \frac{|k|(v_u v_d v_3)^2}{4M_{\text{pl}}^2} \cos\left(\frac{a_{\text{phys}}}{6f_a} + \delta\right), \end{aligned} \quad (13)$$

with $\delta = \text{Arg}(k)$ and $M_{\text{pl}} = 1.22 \times 10^{19}$ GeV. According to the PQ quality requirement [31–33], the contribution from the V_{PQ} to the energy density should be 10^{-10} times less than that of the QCD axion potential, and we find

$$f_a \lesssim \frac{3.7 \times 10^7 \cos\phi}{|k \sin(\delta)|^{1/2}} \left(\tan\beta + \frac{1}{\tan\beta} \right) \text{ GeV}. \quad (14)$$

In Fig. 1, we display upper limits to f_a . Within the reasonable parameter choices of $(\tan\beta, \tan\phi, |k \sin(\delta)|)$, we consider the high-quality axion window of

$$\begin{aligned} 10^8 \text{ GeV} &\lesssim f_a \lesssim 10^{10} \text{ GeV}, \\ \Rightarrow 10^9 \text{ GeV} &\lesssim v_{331} \lesssim 10^{11} \text{ GeV}, \end{aligned} \quad (15)$$

by using the relation in Eq. (10b). The corresponding axion masses [5,93] are

$$m_a = 5.70 \left(\frac{10^{12} \text{ GeV}}{f_a} \right) \mu\text{eV} \sim (10^{-4}, 10^{-2}) \text{ eV}. \quad (16)$$

This is distinguishable from the axion masses of $m_a \sim \mathcal{O}(10^{-9})$ eV in the $\text{SU}(5)$ GUT [17,19,21,22].

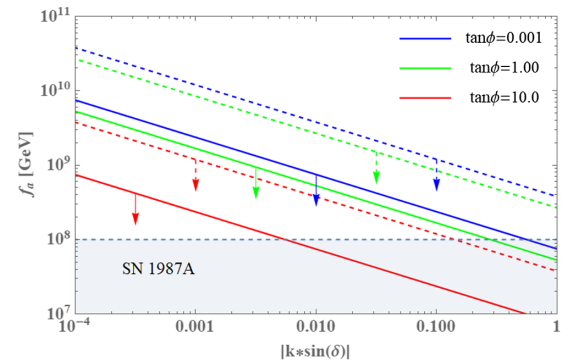


FIG. 1. The upper limit to the f_a from the dimension-six operator (12), with $\tan\beta = 1.0$ (solid lines) and $\tan\beta = 10.0$ (dashed lines). The shaded region represents the lower bound to the f_a from the supernovae 1987A neutrino burst [94], and arrows indicate that each line represents the upper bound to f_a .

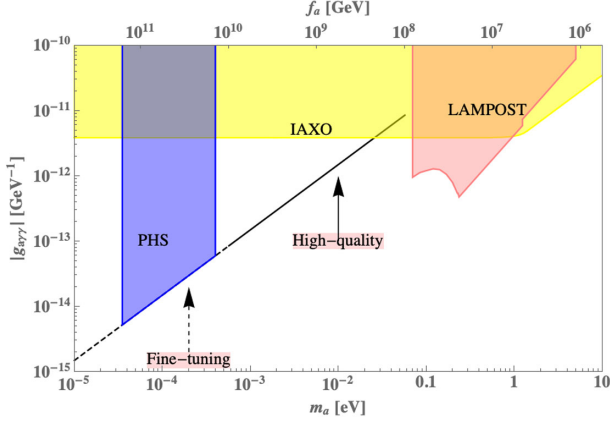


FIG. 2. The $(m_a, |g_{a\gamma\gamma}|)$ predictions for the high-quality SU(6) axion (solid line), together with the future projected search limits from the IAXO [96–98], LAMPOST [99], and Plasma halo scope (PHS) [100].

The effective axion-photon couplings are parameterized by

$$g_{a\gamma\gamma} = C_{a\gamma\gamma} \left(\frac{1.14 \times 10^{-3} \text{ GeV}}{f_a} \right) \text{ GeV}^{-1},$$

$$C_{a\gamma\gamma} = \frac{E}{N_{\text{SU}(3)_c}} - 1.92. \quad (17)$$

By using the QCD and EM anomaly factors in Eqs. (8b) and (11), we find that $C_{a\gamma\gamma} = 0.75$. Thus, the axion-photon couplings from the SU(6) GUT confirm the universal predictions as those in the SU(5) and SO(10) GUTs [21,22,28,95]. In Fig. 2, we present the benchmark models for the SU(6) axion in the $(m_a, |g_{a\gamma\gamma}|)$ plane. The high-quality axion (solid line) with mass range in Eq. (16) can be probed in the future IAXO [96–98] experiment.

V. THE AXION DOMAIN WALL

The V_{pQ} term in Eq. (13) plays a role as a biased term [87,101–105] to avoid the axion domain wall formation, which can be possible due to the periodicity of the effective potential term of V_{QCD} . One thus requires domain walls to decay before the domination epoch [106]

$$t_{\text{dec}} < t_{\text{form}}, \quad t_{\text{dec}} \sim 10^{-66} \text{ sec} \left(\frac{M_{\text{pl}} v_{331}}{v_u v_d} \right)^2 \left(\frac{10^{13} \text{ GeV}}{v_{331}} \right)^3,$$

$$t_{\text{form}} \sim 10^2 \text{ sec} \left(\frac{10^{13} \text{ GeV}}{v_{331}} \right), \quad (18)$$

where we use the axion domain wall tension of $\sigma_{\text{DW}} \approx 9m_a f_a^2$ [107,108] and Eq. (10b). With the high-quality range in Eq. (15), we find that $t_{\text{dec}} \sim \mathcal{O}(10^{-8}) - \mathcal{O}(10^{-6})$ sec and $t_{\text{form}} \sim \mathcal{O}(10^4) - \mathcal{O}(10^6)$ sec. Indeed,

the axion domain wall cannot be formed in the early Universe.

VI. THE GAUGE COUPLING UNIFICATION

We briefly present the gauge coupling evolutions in terms of the one-loop renormalization group equations (RGEs). The gauge couplings are $(\alpha_{3c}, \alpha_{3L}, \alpha_N)$ for the \mathcal{G}_{331} symmetry, and $(\alpha_{3c}, \alpha_{2L}, \alpha_Y)$ for the \mathcal{G}_{SM} symmetry. The $U(1)_N$ coupling should be normalized by $\alpha_1 = \frac{4}{3}\alpha_N$ for the unification.

The most general one-loop RGE solution for the gauge coupling α_i of the gauge symmetry \mathcal{G}_i is

$$\alpha_i^{-1}(\mu_2) = \alpha_i^{-1}(\mu_1) - \frac{b_i^{(1)}}{2\pi} \log \left(\frac{\mu_2}{\mu_1} \right), \quad (19)$$

with the one-loop β coefficients in the SUSY extension being

$$b_i^{(1)} = -3C_2(\mathcal{G}_i) + \sum_{\chi} T(\mathcal{R}_{\chi}^i). \quad (20)$$

Explicitly, the one-loop β coefficients for the SUSY SU(6) can be obtained by the spectrum in Table III and the branching rules in Eq. (B4) as follows

$$m_Z \leq \mu \leq v_{331}: (b_{\text{SU}(3)_c}^{(1)}, b_{\text{SU}(2)_L}^{(1)}, b_{\text{U}(1)_Y}^{(1)}) = (-3, 1, 11),$$

$$v_{331} \leq \mu \leq \Lambda_{\text{GUT}}: (b_{\text{SU}(3)_c}^{(1)}, b_{\text{SU}(3)_L}^{(1)}, b_{\text{U}(1)_1}^{(1)}) = \left(0, \frac{13}{2}, \frac{29}{2} \right). \quad (21)$$

To evaluate the RGEs, the tree-level matching conditions at the v_{331} scale are

$$\alpha_{3L}^{-1}(v_{331}) = \alpha_{2L}^{-1}(v_{331}),$$

$$\alpha_1^{-1}(v_{331}) = -\frac{1}{4}\alpha_{2L}^{-1}(v_{331}) + \frac{3}{4}\alpha_Y^{-1}(v_{331}). \quad (22)$$

The one-loop results are displayed in Fig. 3, with an intermediate scale of $v_{331} = 10^{10}$ GeV and the latest

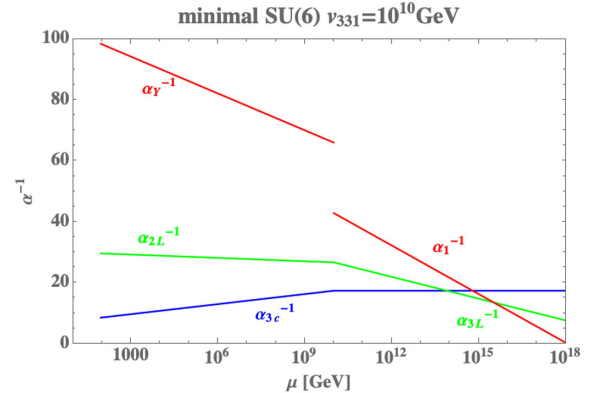


FIG. 3. The one-loop gauge coupling runnings for the SUSY SU(6) model, with $v_{331} = 10^{10}$ GeV.

electroweak precision measurements of $(\alpha_{3c}, \alpha_{\text{em}}, \sin^2 \theta_W)$ at the Z pole [109] as the inputs. A unification is indicated with $\Lambda_{\text{GUT}} \sim 10^{16}$ GeV for the SUSY SU(6). Notice that the intermediate \mathcal{G}_{331} -breaking scale usually requires the two-loop RGE analysis as well as the one-loop matching conditions with mass threshold effects [110,111]. Recent studies on the SO(10) and E_6 reveal the strong correlation to the proton lifetime predictions with these effects [112–116]. Thus, we defer the study of the proton lifetime predictions with the current constraint of $\tau_p \gtrsim 2.4 \times 10^{34}$ yrs from the Super-Kamionkande [117,118] to future work.

VII. CONCLUSIONS

We have put forth a SUSY SU(6) model for the strong CP problem, by utilizing the emergent global DRS symmetry in Eq. (1). Historically, the emergent global symmetry. A high-quality axion with its decay constant is found to be constrained from the PQ-quality requirement to the dimension-six PQ-breaking operator, and is most likely to be directly probed in the future IAXO searches. The type-I seesaw mechanism and a successful leptogenesis is also achievable at the corresponding PQ symmetry-breaking scale. Our work manifests that three seemingly unrelated issues of the strong CP problem, the neutrino mass origin, and the EW Higgs sector are coherently unified in the SU(6) framework. Though the current study of the SU(6) requires additional ingredients of discrete symmetries for suitable PQ-breaking operators, it is natural to extend SU(6) to higher unified groups, which enjoy the emergent global DRS symmetry in general. Historically, the nonminimal GUTs with gauge symmetries of $SU(N \geq 7)$ were considered to unify three generations of SM fermions [119–122]. It is therefore appealing to look for realistic GUTs that unify both the PQ quality problem and the flavor puzzle.

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APPENDIX A: NO HIGH-QUALITY AXION IN THE NON-SUSY SU(6) MODEL

We prove that the non-SUSY SU(6) model cannot lead to a high-quality axion with any discrete gauge symmetry of

TABLE II. The fermions and Higgs fields with their $SU(2)_F$ representations, and the charges under the $U(1)_{\text{PQ}}$ and \mathbb{Z}_k symmetries, in the non-SUSY SU(6) model.

	$\bar{\mathbf{6}}_F^\rho$	$\mathbf{15}_F$	$\bar{\mathbf{6}}_{H\rho}$	$\mathbf{15}_H$	$\mathbf{21}_H$	$\mathbf{35}_H$
$SU(2)_F$	\square	1	$\bar{\square}$	1	1	1
$U(1)_{\text{PQ}}$	1	1	-2	-2	-2	0
\mathbb{Z}_k	$q_{\bar{6}}$	q_{15}	q_{6_H}	q_{15_H}	q_{21_H}	q_{35_H}

\mathbb{Z}_k . To see this, we assign the most general \mathbb{Z}_k charges for fermions and Higgs fields in Table II, which are subject to the gauge anomaly cancellation of

$$\begin{aligned} \mathbb{Z}_k[SU(6)]^2 &= \sum_i 2T(\mathcal{R}_i)q_i = 2q_{\bar{6}} + 4q_{15} = 0 \pmod{k} \\ &\Rightarrow 2q_{\bar{6}} + 4q_{15} = k \cdot l, \quad l \in \mathbb{Z}. \end{aligned} \quad (\text{A1})$$

In addition, at least two of the following terms are necessary to reproduce the SM-like Higgs boson Yukawa couplings that are consistent with the current LHC results

$$\mathcal{L}_Y \supset \mathbf{15}_F \bar{\mathbf{6}}_F^\rho \bar{\mathbf{6}}_{H\rho} + \mathbf{15}_F \mathbf{15}_F \mathbf{15}_H + \text{H.c.} \quad (\text{A2})$$

The neutrality of the \mathbb{Z}_k charges in Eq. (A2) leads to

$$q_{\bar{6}} + q_{15} - q_{6_H} = k \cdot m, \quad (\text{A3a})$$

$$2q_{15} + q_{15_H} = k \cdot n. \quad (\text{A3b})$$

Apparently, the leading PQ-breaking and gauge-invariant operator is $\mathcal{O}_{\text{PQ}}^{d=3} = \epsilon^{\rho\delta} \bar{\mathbf{6}}_{H\rho} \bar{\mathbf{6}}_{H\delta} \mathbf{15}_H$, and it is neutral in \mathbb{Z}_k charge since

$$-2q_{6_H} + q_{15_H} = k \cdot (2m + n - l) = 0 \pmod{k} \quad (\text{A4})$$

according to Eqs. (A1), (A3a), and (A3b). The PQ-quality condition from the $\mathcal{O}_{\text{PQ}}^{d=3}$ clearly leads to an unrealistic constraint of

$$v_{\text{EW}}^2 v_{331} \lesssim 10^{-10} \frac{\Lambda_{\text{QCD}}^4}{M_{\text{pl}}} \Rightarrow v_{331} \lesssim 10^{-37} \text{ GeV}. \quad (\text{A5})$$

Thus, a high-quality axion is impossible in the non-SUSY SU(6) model.

APPENDIX B: THE SU(6) GAUGE SYMMETRY BREAKING AND SPECTRUM

In this section, we list the SUSY SU(6) spectrum and the Yukawa couplings following the breaking pattern of $SU(6) \rightarrow \mathcal{G}_{331} \rightarrow \mathcal{G}_{\text{SM}}$. The $U(1)_N$ charge for the SU(6) fundamental representation at the first-stage symmetry breaking is defined as follows

$$\begin{aligned}
 N &\equiv \frac{1}{3} \text{diag}(-1, -1, -1, +1, +1, +1), \\
 &= -\frac{\sqrt{6}}{3} T_{\text{SU}(6)}^{15} - \frac{\sqrt{10}}{5} T_{\text{SU}(6)}^{24} - \frac{2}{\sqrt{15}} T_{\text{SU}(6)}^{35}. \quad (\text{B1})
 \end{aligned}$$

Afterwards, the $U(1)_Y$ charge for the $SU(3)_L$ fundamental representation is given by

$$\begin{aligned}
 Y &\equiv \text{diag}\left(\frac{1}{3} + 2N, \frac{1}{3} + 2N, -\frac{2}{3} + 2N\right), \\
 &= \frac{2}{\sqrt{3}} T_{\text{SU}(3)}^8 + 2N \cdot \mathbb{I}_3, \quad (\text{B2})
 \end{aligned}$$

and the electric charges are quantized by

$$Q \equiv T_{\text{SU}(2)}^3 + \frac{Y}{2} \mathbb{I}_2. \quad (\text{B3})$$

$T_{\text{SU}(6)}^{15,24,35}$, $T_{\text{SU}(3)}^8$, and $T_{\text{SU}(2)}^3$ are Cartan generators of $SU(6)$, $SU(3)_L$, and $SU(2)_L$, respectively. The fermion spectrum from the superfields of $\bar{\mathbf{6}}_F \oplus \mathbf{15}_F$ are tabulated in Table III, with their representations under the \mathcal{G}_{331} and \mathcal{G}_{SM} . The first-stage branching rules of superfields containing Higgs are

$$\bar{\mathbf{6}}_{H\rho} = \left(\bar{\mathbf{3}}, \mathbf{1}, +\frac{1}{3}\right)_{H\rho} \oplus \left(\mathbf{1}, \bar{\mathbf{3}}, -\frac{1}{3}\right)_{H\rho}, \quad (\text{B4a})$$

$$\begin{aligned}
 \mathbf{15}_H &= \left(\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3}\right)_H \oplus \left(\mathbf{1}, \bar{\mathbf{3}}, +\frac{2}{3}\right)_H \\
 &\oplus (\mathbf{3}, \mathbf{3}, 0)_H, \quad (\text{B4b})
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{21}_H &= \left(\mathbf{6}, \mathbf{1}, -\frac{2}{3}\right)_H \oplus \left(\mathbf{1}, \mathbf{6}, +\frac{2}{3}\right)_H \\
 &\oplus (\mathbf{3}, \mathbf{3}, 0)_H, \quad (\text{B4c})
 \end{aligned}$$

$$\begin{aligned}
 \bar{\mathbf{21}}_H &= \left(\bar{\mathbf{6}}, \mathbf{1}, +\frac{2}{3}\right)_H \oplus \left(\mathbf{1}, \bar{\mathbf{6}}, -\frac{2}{3}\right)_H \\
 &\oplus (\bar{\mathbf{3}}, \bar{\mathbf{3}}, 0)_H, \quad (\text{B4d})
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{35}_H &= (\mathbf{1}, \mathbf{1}, 0)_H \oplus (\mathbf{8}, \mathbf{1}, 0)_H \oplus (\mathbf{1}, \mathbf{8}, 0)_H \\
 &\oplus \left(\mathbf{3}, \bar{\mathbf{3}}, -\frac{2}{3}\right)_H \oplus \left(\bar{\mathbf{3}}, \mathbf{3}, +\frac{2}{3}\right)_H. \quad (\text{B4e})
 \end{aligned}$$

TABLE III. The $SU(6)$ fermion representations under the \mathcal{G}_{331} and the \mathcal{G}_{SM} . The SM fermions are marked by solid underlines, the Kim-Shifman-Vainshtein-Zakharov vectorlike quarks are marked by dashed underlines, and other heavy leptonic states are marked by dotted underlines.

$SU(6)$	\mathcal{G}_{331}	\mathcal{G}_{SM}
$\bar{\mathbf{6}}_F^I$	$(\bar{\mathbf{3}}, \mathbf{1}, +\frac{1}{3})_F^I$	$(\bar{\mathbf{3}}, \mathbf{1}, +\frac{2}{3})_F^I: \underline{d_R^c}$
	$(\mathbf{1}, \bar{\mathbf{3}}, -\frac{1}{3})_F^I$	$(\mathbf{1}, \mathbf{2}, -1)_F^I: (\underline{e_L}, -\underline{\nu_L})$ $(\mathbf{1}, \mathbf{1}, 0)_F^I: \underline{N}$
$\bar{\mathbf{6}}_F^{II}$	$(\bar{\mathbf{3}}, \mathbf{1}, +\frac{1}{3})_F^{II}$	$(\bar{\mathbf{3}}, \mathbf{1}, +\frac{2}{3})_F^{II}: \underline{D_R^c}$
	$(\mathbf{1}, \bar{\mathbf{3}}, -\frac{1}{3})_F^{II}$	$(\mathbf{1}, \mathbf{2}, -1)_F^{II}: (\underline{e'_L}, -\underline{\nu'_L})$ $(\mathbf{1}, \mathbf{1}, 0)_F^{II}: \underline{N'}$
$\mathbf{15}_F$	$(\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})_F$	$(\bar{\mathbf{3}}, \mathbf{1}, -\frac{4}{3})_F: \underline{u_R^c}$
	$(\mathbf{1}, \bar{\mathbf{3}}, +\frac{2}{3})_F$	$(\mathbf{1}, \mathbf{2}, +1)_F: (\underline{\nu'_R}, \underline{e'_R})$ $(\mathbf{1}, \mathbf{1}, +2)_F: \underline{e_R^c}$
	$(\mathbf{3}, \mathbf{3}, 0)_F$	$(\mathbf{3}, \mathbf{2}, +\frac{1}{3})_F: (\underline{u_L}, \underline{d_L})$ $(\mathbf{3}, \mathbf{1}, -\frac{2}{3})_F: \underline{D_L}$

The scalar components of $(\mathbf{1}, \bar{\mathbf{3}}, -\frac{1}{3})_{HII} \subset \bar{\mathbf{6}}_{HII}$ and the $(\mathbf{1}, \mathbf{6}, +\frac{2}{3})_H \subset \mathbf{21}_H$ will develop VEVs of $\sim v_{331}$ for the symmetry breaking of $\mathcal{G}_{331} \rightarrow \mathcal{G}_{\text{SM}}$. We can determine the global $U(1)_{\text{PQ}}[SU(6)]^2$ anomaly factor from the fermions in Table III and Eq. (B4) as follows

$$\begin{aligned}
 N_{\text{SU}(6)} &= \left[\sum_{\rho} T(\bar{\mathbf{6}}_F^{\rho}) \text{PQ}(\bar{\mathbf{6}}_F^{\rho}) + T(\mathbf{15}_F) \text{PQ}(\mathbf{15}_F) \right] \times n_g \\
 &+ \sum_{\rho} T(\bar{\mathbf{6}}_{H\rho}) \text{PQ}(\bar{\mathbf{6}}_{H\rho}) + T(\mathbf{15}_H) \text{PQ}(\mathbf{15}_H) \\
 &+ T(\mathbf{21}_H) \text{PQ}(\mathbf{21}_H) = -5, \quad (\text{B5})
 \end{aligned}$$

with $n_g = 3$. The related trace invariants are $T(\mathbf{6}) = T(\bar{\mathbf{6}}) = \frac{1}{2}$, $T(\mathbf{15}) = 2$, and $T(\mathbf{21}) = 4$.

After the second-stage symmetry breaking, we find the following mass terms from the Yukawa couplings

$$\begin{aligned}
 Y_D \mathbf{15}_F \bar{\mathbf{6}}_F^{II} \bar{\mathbf{6}}_{HII} + \text{H.c.} &\supset Y_D \left[(\mathbf{3}, \mathbf{3}, 0)_F \otimes \left(\bar{\mathbf{3}}, \mathbf{1}, +\frac{1}{3}\right)_F^{II} \oplus \left(\mathbf{1}, \bar{\mathbf{3}}, +\frac{2}{3}\right)_F \otimes \left(\mathbf{1}, \bar{\mathbf{3}}, -\frac{1}{3}\right)_F^{II} \right] \otimes \left(\mathbf{1}, \bar{\mathbf{3}}, -\frac{1}{3}\right)_{HII} + \text{H.c.} \\
 &\Rightarrow m_D = m_{e'} = m_{\nu'} = Y_D v_3, \quad (\text{B6a})
 \end{aligned}$$

$$\begin{aligned}
 Y_N \bar{\mathbf{6}}_F^{I} \bar{\mathbf{6}}_F^{II} \mathbf{21}_H + \text{H.c.} &\supset Y_N \left(\mathbf{1}, \bar{\mathbf{3}}, -\frac{1}{3}\right)_F^I \otimes \left(\mathbf{1}, \bar{\mathbf{3}}, -\frac{1}{3}\right)_F^{II} \otimes \left(\mathbf{1}, \mathbf{6}, +\frac{2}{3}\right)_H + \text{H.c.} \\
 &\Rightarrow m_{N, N'} = Y_N v_6. \quad (\text{B6b})
 \end{aligned}$$

The global $U(1)_{\text{PQ}}[SU(3)_c]^2$ anomaly factor is determined as follows

$$\begin{aligned}
 N_{\text{SU}(3)_c} = & \left[\underbrace{\sum_{\rho} T(\bar{\mathbf{3}}_{\text{F}}^{\rho}) \left(c_1 + \frac{1}{3} c_2 \right)}_{\bar{\mathbf{6}}_{\text{F}}} + \underbrace{T(\bar{\mathbf{3}}_{\text{F}}) \left(c_1 - \frac{2}{3} c_2 \right) + T(\mathbf{3}_{\text{F}}) 3c_1}_{\mathbf{15}_{\text{F}}} \right] \times n_g + \underbrace{\sum_{\rho} T(\bar{\mathbf{3}}_{\text{H}\rho}) \left(-2c_1 + \frac{1}{3} c_2 \right)}_{\bar{\mathbf{6}}_{\text{H}\rho}} \\
 & + \underbrace{T(\bar{\mathbf{3}}_{\text{H}}) \left(-2c_1 - \frac{2}{3} c_2 \right)}_{\mathbf{15}_{\text{H}}} + \underbrace{T(\mathbf{3}_{\text{H}}) (-6c_1) + T(\mathbf{6}_{\text{H}}) \left(-2c_1 + \frac{2}{3} c_2 \right)}_{\mathbf{21}_{\text{H}}} + \underbrace{T(\mathbf{3}_{\text{H}}) (-6c_1) + T(\bar{\mathbf{6}}_{\text{H}}) \left(-\frac{2}{3} c_2 \right)}_{\bar{\mathbf{21}}_{\text{H}}} = -5c_1,
 \end{aligned} \tag{B7}$$

according to the physical PQ charges of $q \equiv c_1 \text{PQ} + c_2 N$.

The scalar components of $(\mathbf{1}, \mathbf{2}, -1)_{\text{H}\text{I}} \subset (\mathbf{1}, \bar{\mathbf{3}}, -\frac{1}{3})_{\text{H}\text{I}} \subset \bar{\mathbf{6}}_{\text{H}\text{I}}$ and the $(\mathbf{1}, \mathbf{2}, +1)_{\text{H}} \subset (\mathbf{1}, \bar{\mathbf{3}}, +\frac{2}{3})_{\text{H}} \subset \mathbf{15}_{\text{H}}$ will develop VEVs of $\sim v_{\text{EW}}$ for the EW symmetry breaking. The corresponding Yukawa couplings and mass terms are

$$\begin{aligned}
 Y_d \mathbf{15}_{\text{F}} \bar{\mathbf{6}}_{\text{F}}^{\text{I}} \bar{\mathbf{6}}_{\text{H}\text{I}} + \text{H.c.} \supset Y_d \left[\left(\mathbf{3}, \mathbf{2}, +\frac{1}{3} \right)_{\text{F}} \otimes \left(\bar{\mathbf{3}}, \mathbf{1}, +\frac{2}{3} \right)_{\text{F}}^{\text{I}} \oplus (\mathbf{1}, \mathbf{2}, -1)_{\text{F}} \otimes (\mathbf{1}, \mathbf{1}, +2)_{\text{F}}^{\text{I}} \right] \otimes (\mathbf{1}, \mathbf{2}, -1)_{\text{H}\text{I}} + \text{H.c.} \\
 \Rightarrow m_{d,\ell} = Y_d v_{\text{EW}},
 \end{aligned} \tag{B8a}$$

$$Y_u \mathbf{15}_{\text{F}} \mathbf{15}_{\text{F}} \mathbf{15}_{\text{H}} + \text{H.c.} \supset Y_u \left(\mathbf{3}, \mathbf{2}, +\frac{1}{3} \right)_{\text{F}} \otimes \left(\bar{\mathbf{3}}, \mathbf{1}, -\frac{4}{3} \right)_{\text{F}} \otimes (\mathbf{1}, \mathbf{2}, +1)_{\text{H}} + \text{H.c.} \Rightarrow m_u = (Y_u + Y_u^T) v_{\text{EW}}, \tag{B8b}$$

$$Y_{\nu} \bar{\mathbf{6}}_{\text{F}}^{\text{I}} \bar{\mathbf{6}}_{\text{F}}^{\text{II}} \mathbf{15}_{\text{H}} + \text{H.c.} \supset Y_{\nu} [(\mathbf{1}, \mathbf{2}, -1)_{\text{F}}^{\text{I}} \otimes (\mathbf{1}, \mathbf{1}, 0)_{\text{F}}^{\text{II}} \otimes (\mathbf{1}, \mathbf{2}, +1)_{\text{H}}] + \text{H.c.} \Rightarrow m_{\nu N'} = m_{\nu N} = (Y_{\nu} + Y_{\nu}^T) v_{\text{EW}}. \tag{B8c}$$

The $(\mathbf{1}, \mathbf{2}, -1)_{\text{H}\text{I}} \subset \bar{\mathbf{6}}_{\text{H}\text{I}}$ gives masses to down-type quarks and charged leptons, and $(\mathbf{1}, \mathbf{2}, +1)_{\text{H}} \subset \mathbf{15}_{\text{H}}$ gives masses to up-type quarks. This justifies the low-energy effective theory at the EW scale is the type-II 2HDM. The neutrino masses are realized through the type-I seesaw mechanism

$$(\nu, N') \cdot \begin{pmatrix} 0 & m_{\nu N'} \\ m_{\nu N'} & m_{N'} \end{pmatrix} \cdot \begin{pmatrix} \nu \\ N' \end{pmatrix} \Rightarrow m_{\nu} = (Y_{\nu} + Y_{\nu}^T)^2 Y_{N'}^{-1} \frac{v_{\text{EW}}^2}{v_6}. \tag{B9}$$

By combining the fermions in Table III and in Eqs. (B4), we can determine the electromagnetic anomaly factor as

$$\begin{aligned}
 E = \sum_f \text{PQ}_f \dim(\mathcal{C}_f) \text{Tr} q_f^2 = & \left[\underbrace{\left(3_c \left(\frac{1}{3} \right)^2 + (-1)^2 \right) \times 2}_{\bar{\mathbf{6}}_{\text{F}}} + \underbrace{3_c \left(-\frac{2}{3} \right)^2 + 1 + 1 + 3_c \times \left(\left(\frac{2}{3} \right)^2 + \left(-\frac{1}{3} \right)^2 + \left(-\frac{1}{3} \right)^2 \right)}_{\mathbf{15}_{\text{F}}} \right] \times n_g \\
 & + \underbrace{(-2) \times \left[3_c \left(\frac{1}{3} \right)^2 + (-1)^2 \right] \times 2}_{\bar{\mathbf{6}}_{\text{H}\rho}} + \underbrace{(-2) \times \left[3_c \left(-\frac{2}{3} \right)^2 + 1 \times 2 + 3_c \left(\frac{2}{3} \right)^2 + 3_c \left(-\frac{1}{3} \right)^2 \times 2 \right]}_{\mathbf{15}_{\text{H}}} \\
 & + \underbrace{(-2) \times \left[6_c \left(-\frac{2}{3} \right)^2 + 2^2 + 1 \times 2 + 3_c \left(\frac{2}{3} \right)^2 + 3_c \left(-\frac{1}{3} \right)^2 \times 2 \right]}_{\mathbf{21}_{\text{H}}} = -\frac{40}{3}.
 \end{aligned} \tag{B10}$$

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